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SEISMIC SOURCE STUDY BY A COMPARATIVE ANALYSIS OF SURFACE AND BODY WAVES

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Results of joint application of parametric techniques for surface waves inversion and of body waveform analysis for seismic source description are presented. Solving the problem by the first of these techniques we consider a realistic model of two-dimensional source describing its distribution in space and in time by few integral characteristics. By the second technique we solve the problem in much more particular approximation of a point source propagating along a line in the fault plane, but as a result we have a detailed description of such a model. The results of application of these two techniques supplement each other, and if they are in a good agreement we consider them as more reliable.

We used such an approach to study the Susamyr earthquake, 1992. This earthquake in Kirghizstan (M_s = 7.4, M_b = 6.8) is the one of strongest events occurred in this century in the region of Central Asia.

This research was conducted in collaboration with R.Madariaga and J.M.Gomez (IPGP, Paris).

Surface wave inversion

For surface waves inversion we used a technique based on the description of seismic source distribution in space and in time by integral moments (see Bukchin *et al.*, 1994; Bukchin, 1995; Gomez, 1996 a, b). We assume that the time derivative of stress glut tensor $\dot{\Gamma}$ can be represented by following product:

$$\dot{\Gamma}(\mathbf{x},t) = f(\mathbf{x},t)\mathbf{M}\,, \tag{1}$$

where $f(\mathbf{x},t)$ is non-negative source time function and $\mathbf{M}$ is a uniform normalised seismic moment tensor. Following Backus and Mulcahy, 1976 we will define the source region by the condition that function $f(\mathbf{x},t)$ is not identically zero and the source duration is the time during which nonelastic motion occurs at various points within the source region, i.e., $f(\mathbf{x},t)$ is different from zero.

Spatial and temporal integral characteristics of the source can be expressed by corresponding moments of the function $f(\mathbf{x},t)$ (Backus, 1977; Bukchin *et al.*, 1994). These moments can be estimated from the seismic records using the relation between them and the displacements in seismic waves, which we will consider later. In the general case stress glut moments of spatial degree 2 and higher are not uniquely determined by the displacement field. But in the case when equation (1) is valid such a uniqueness takes place (Bukchin,1995).

Following equations define the spatio-temporal moments of function $f(\mathbf{x},t)$ of total degree (both in space and time) 0, 1, and 2 with respect to point $\mathbf{q}$ and instant of time τ.

$$f^{(0,0)} = \int_V dV \int_0^\infty f(\mathbf{x},t)dt, \quad f_i^{(1,0)}(\mathbf{q}) = \int_V dV \int_0^\infty f(\mathbf{x},t)(x_i - q_i)dt,$$

$$f^{(0,1)}(\tau) = \int_V dV \int_0^\infty f(\mathbf{x},t)(t-\tau)dt, \quad f^{(0,2)}(\tau) = \int_V dV \int_0^\infty f(\mathbf{x},t)(t-\tau)^2 dt,$$

$$f_i^{(1,1)}(\mathbf{q},\tau) = \int_V dV \int_0^\infty f(\mathbf{x},t)(x_i - q_i)(t-\tau)dt, \tag{2}$$

$$f_{ij}^{(2,0)}(\mathbf{q}) = \int_V dV \int_0^\infty f(\mathbf{x},t)(x_i - q_i)(x_j - q_j)dt$$

Using these moments we will define integral characteristics of the source. Source location is estimated by the spatial centroid $\mathbf{q}_c$ of the field $f(\mathbf{x},t)$ defined as

$$\mathbf{q}_c = \mathbf{f}^{(1,0)}(\mathbf{0}) / M_0 , \tag{3}$$

where $M_0 = f^{(0,0)}$ is the scalar seismic moment.
Similarly, the temporal centroid τ_c is estimated by the formula

$$\tau_c = f^{(0,1)}(0) / M_0 . \tag{4}$$

The source duration is estimated by, where

$$(\Delta\tau)^2 = f^{(0,2)}(\tau_c) / M_0 . \tag{5}$$

The spatial extent of the source is described by matrix $\mathbf{W}$, where

$$\mathbf{W} = \mathbf{f}^{(2,0)}(\mathbf{q}_c) / M_0 . \tag{6}$$

The mean source size in the direction of unit vector $\mathbf{r}$ is estimated by value $2l_r$, defined by formula

$$l_r^2 = \mathbf{r}^T \mathbf{W} \mathbf{r}, \tag{7}$$

where $\mathbf{r}^T$ - transposed vector. From (6) and (7) we can estimate the principal axes of the source. There directions are given by the eigenvectors of the matrix $\mathbf{W}$, and the lengths are defined by correspondent eigenvalues: the length of the minor semi-axis is equal to the least eigenvalue, and the length of the major semi-axis is equal to the greatest eigenvalue.

In the same way, from the coupled space time moment of order (1,1) the mean velocity $\mathbf{v}$ of the instant spatial centroid (Bukchin, 1989) is estimated as

$$\mathbf{v} = \mathbf{w} / (\Delta\tau)^2 , \tag{8}$$

where

$$\mathbf{w} = \mathbf{f}^{(1,1)}(\mathbf{q}_c, \tau_c) / M_0 .$$

Now we will consider the low frequency part of the spectra of the i^{th} component of displacements in Love or Rayleigh wave $u_i(\mathbf{x},\omega)$. It is assumed that the frequency ω is small, so that the duration of the source is small in comparison with the period of the wave, and the source size is small as compared with the wavelength. It is assumed that the origin of coordinate system is located in the point of spatial centroid $\mathbf{q}_c$ (i.e. $\mathbf{q}_c = \mathbf{0}$) and that time is measured from the instant of temporal centroid, so that $\tau_c = 0$. With this choice the first degree moments with respect to the spatial origin **x=0** and to the temporal origin t=0 are zero, i.e. $\mathbf{f}^{(1,0)}(\mathbf{0}) = \mathbf{0}$ and $f^{(0,1)}(0) = 0$.

Under this assumptions the relation between the spectrum of the displacements $u_i(\mathbf{x},\omega)$ and the spatio-temporal moments of the function $f(\mathbf{x},t)$ can be expressed by following formula (Bukchin,1995)

$$u_i(\mathbf{x},\omega) = \frac{1}{\mathrm{i}\omega} M_0 M_{jl} \frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega) + \frac{1}{2\mathrm{i}\omega} f_{mn}^{(2,0)}(\mathbf{0}) M_{jl} \frac{\partial}{\partial y_m}\frac{\partial}{\partial y_n}\frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega)$$

$$- f_m^{(1,1)}(\mathbf{0},0) M_{jl} \frac{\partial}{\partial y_m}\frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega) + \frac{\mathrm{i}\omega}{2} f^{(0,2)}(0) M_{jl} \frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega), \quad (9)$$

i,j,l,m,n = 1,2,3 and the summation convention for repeated subscripts is used. $G_{ij}(\mathbf{x},\mathbf{y},\omega)$ in equation (9) is the spectrum of Green function for the chosen model of medium and wave type (see Levshin, 1985; Bukchin, 1990). We assume that the paths from the earthquake source to seismic stations are relatively simple and are well approximated by weak laterally inhomogeneous model (Woodhouse, 1974; Babich *et al.*, 1976). The surface wave Green function in this approximation is determined by the near source and near receiver velocity structure, by the mean phase velocity of wave, and by geometrical spreading. The amplitude spectrum $|u_i(\mathbf{x},\omega)|$ defined by formula (9) does not depend on the average phase velocity of the wave. In such a model the errors in source location do not affect the amplitude spectrum (Bukchin, 1990).

If all characteristics of the medium, depth of the best point source and seismic moment tensor are known (determined, for example, using the spectral domain of longer periods) the representation (9) gives us a system of linear equations for moments of the function $f(\mathbf{x},t)$ of total degree 2. But the average phase velocities of surface waves are usually not well known. For this reason, we use only amplitude spectrum of surface waves for determining these moments, in spite of non linear relation between them.

Let us consider a plane source. All moments of the function $f(\mathbf{x},t)$ of total degree 2 can be expressed in this case by formulas (3)-(8) in terms of 6 parameters: Δt - estimate of source duration, $l_{\max}$ - estimate of maximal mean size of the source, φ_l - estimate of the angle between the direction of maximal size and strike axis, $l_{\min}$ - estimate of minimal mean size of the source, v - estimate of the absolute value of instant centroid mean velocity $\mathbf{v}$ and φ_v - the angle between $\mathbf{v}$ and strike axis.

Using the Bessel inequality for the moments under discussion we can obtain the following constrain for the parameters considered above (Bukchin, 1995):

$$v\Delta t^2\left(\frac{\cos^2\varphi}{l^2_{max}}+\frac{\sin^2\varphi}{l^2_{min}}\right)\le 1\,, \tag{10}$$

where φ is the angle between major axis of the source and direction of **v**. Assuming that the source is a plane fault and representation (1) is valid let us consider a rough grid in the space of 6 parameters defined above. These parameters have to follow inequality (11). Let models of the media be given and the moment tensor be fixed as well as the depth of the best point source. Let the fault plane (one of two nodal planes) be identified. Using formula (9) we can calculate the amplitude spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed amplitude spectra give us a residual $\varepsilon^{(i)}$ for every point of observation, every wave and every frequency ω. Let $u^{(i)}(\mathbf{r},\omega)$ be any observed value of the spectrum, i = 1,...,N; $\varepsilon^{(i)}$- corresponding residual of $|u^{(i)}(\mathbf{r},\omega)|$. We define the normalized amplitude residual by formula

$$\varepsilon(\Delta t, l_{max}, l_{min}, \varphi_l, v, \varphi_v)=\left[\left(\sum_{i=1}^{N}\varepsilon^{(i)^2}\right)\Big/\left(\sum_{i=1}^{N}|u^{(i)}(\mathbf{r},\omega|^2\right)\right]^{1/2} \tag{11}$$

The optimal values of the parameters that minimize ε we consider as estimates of these parameters. We search them by a systematic exploration of the six dimensional parameter space. To characterize the degree of resolution of every of these source characteristics we calculate partial residual functions. Fixing the value of one of varying parameters we put in correspondence to it a minimal value of the residual ε on the set of all possible values of the other parameters. In this way we define 6 functions of the residual corresponding to the 6 varying parameters: $\varepsilon_{\Delta t}(\Delta t)$, $\varepsilon_{l_{max}}(l_{max})$, $\varepsilon_{l_{min}}(l_{min})$, $\varepsilon_{\varphi_l}(\varphi_l)$, $\varepsilon_v(v)$ and $\varepsilon_{\varphi_v}(\varphi_v)$. The value of the parameter for which the corresponding function of the residual attains its minimum we define as estimate of this parameter. At the same time these functions characterize the degree of resolution of the corresponding parameters. From geometrical point of view these functions describe the boundaries of projections of the 6-D surface of functional ε on the coordinate planes. A sketch of such a 6-D surface is presented at the Fig.1. Here one of 6 parameters is picked out as 'parameter 1', and one of coordinate axis corresponds to this parameter. The another coordinate axis we consider formally as 5-D space of the rest 5 parameters. Plane Σ is orthogonal to the axis 'parameter 1' and cross it in a point p_0. Curve L is the intersection of the plane Σ and the surface of functional ε. As one can see from the figure the point $\varepsilon_1(p_0)$ belong to the boundary of projection of the surface of functional ε, and at the same time it corresponds to a minimal value of the residual ε on the set of all possible values of the other 5 parameters while 'parameter 1' is equal to the value p_0. So, as it is accepted in engineering we characterize our surface by its 6 projections on coordinate planes.

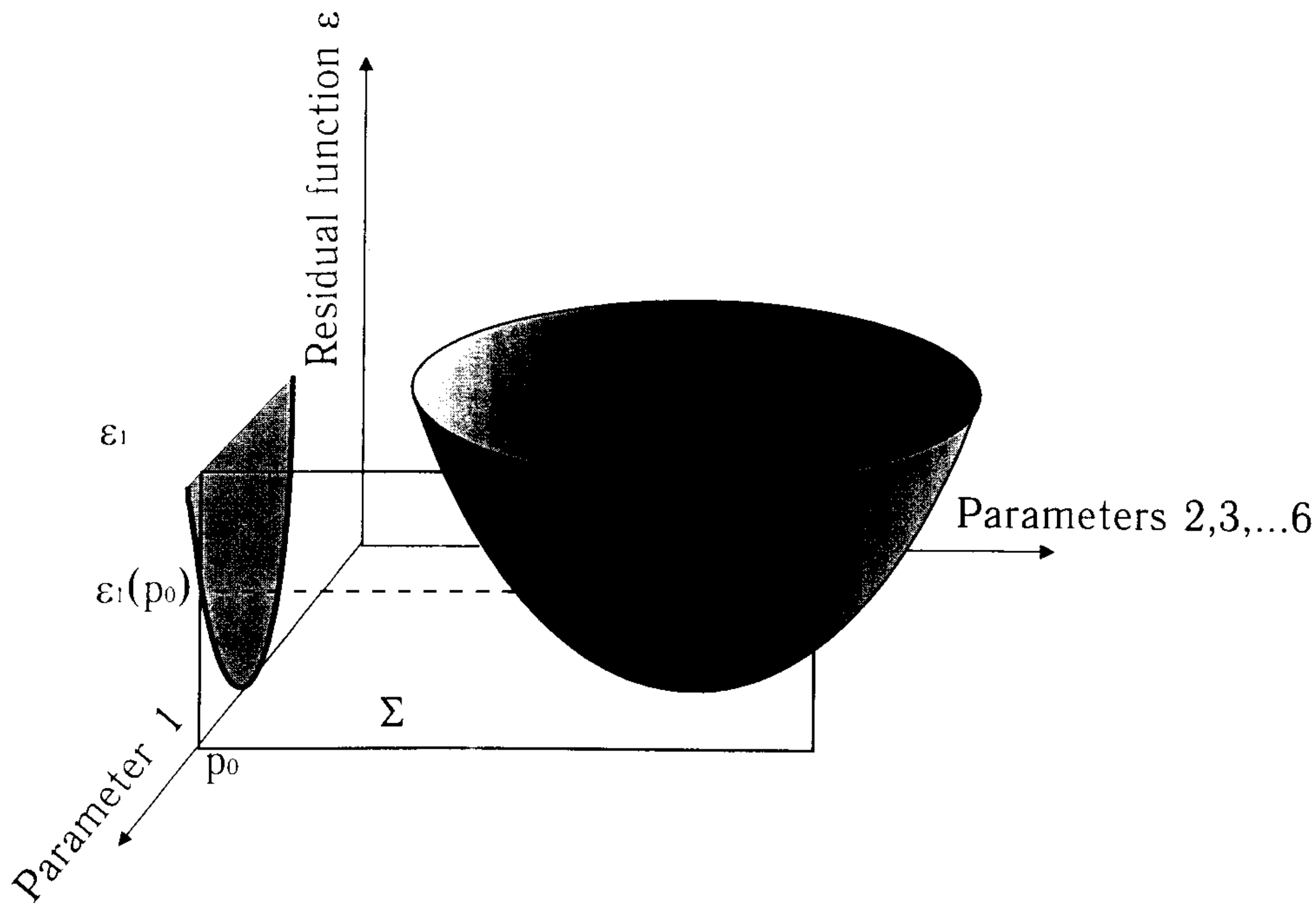

Fig.1 6-D Resolution function.

Body wave inversion

For body wave inversion we used the technique of source parameters estimation from a waveform modelling of P and SH waves (Nabelek, 1984; Campos *et al.*, 1994; Gomez *et al.*, 1996a; Gomez *et al.*, 1996b). This approach uses the maximum likelihood principle to obtain the source parameter solution that produce the best fit between observed and synthetic waveforms. A point or multiple sources can be considered. As the relation between the theoretical model and the source parameters is non linear, the fitting and minimization is achieved at each step by an iterative procedure, based on a linearized model. The source function is convolved with mantle response, geometrical spreading, instrument response and the receiver crust response in order to build the synthetic waveform (Nabelek, 1984). This is a standard technique, so we will not describe it in detail. We will describe the results of its application to study of the Susamyr earthquake, 1992. This earthquake in Kirghizstan (M_s = 7.4, M_b = 6.8) is the one of strongest events occurred in this century in the region of Central Asia. Then we will present results of surface wave analysis. And in conclusion we will compare the estimates obtained by these two techniques.

Results of body waves analysis

The body wave analysis for this earthquake was performed by J.M.Gomez and R.Madariaga (IPGP, Paris). P and SH waves recorded by GEOSCOPE and IRIS networks were analysed in the period range from 10 s up to 100 s. Following focal mechanism was obtained from these data in approximation of a point source: strike 92°, dip 56° and rake 107°. The estimate of seismic moment is $5.3 \cdot 10^{19}$ n·m and depth is about 14 km. The source time function has a roughly triangular shape with duration of about 18 s.

Then the effect of propagating rupture was studied by introducing the Doppler effect in the source time function (Nabelek, 1984). The azimuth of the direction of propagation was estimated by 315° and rupture velocity by 2.1 km/s.

And at last the inversion was performed in approximation of a point source propagating in the fault plane along a line with estimated azimuth. The length of such a source was estimated to be about 40 km, and duration of the source about 18-20 s.

Results of surface waves analysis

The estimation of source parameters from surface wave records was done by using spectra of Love and Rayleigh fundamental modes in the spectral domain from 40 to 250 s. The distribution of stations is shown in Figure 2.

For inversion we selected those records for which we considered surface wave polarization anomalies were weak, using the polarization technique proposed by Lander (1989b). In the retrieving of the Love and Rayleigh fundamental modes from observed surface wavetrains we used the frequency-time analysis (FTAN) and time-variable filters (Ewing *et al.*, 1959; Dziewonski *et al.*, 1969; Dziewonski *et al.*, 1972; Levshin *et al.*, 1972; Cara, 1973; Lander, 1989a). From the long period part of the spectra (100-250 s), we determined the following focal mechanism of the source: strike 90°, dip 60° and rake 105°. The estimate of seismic moment is $6.8 \cdot 10^{19}$ n·m. The best point source depth was found to be about 15 km. To model surface wave spectra in this long period domain we used only the first term in equation (9) (see Bukchin *et al.*, 1994).

To estimate duration and geometry of the source we have used amplitude spectra of fundamental modes of Love and Rayleigh waves in the spectral domain from 35 to 50 seconds. The plane dipping to the South was identified as a fault plane. Results of direct trial of possible values of the unknown parameters are shown in the Figure 3. Function $\varepsilon_{\Delta t}$ (see Fig. 3a) attains its minimum at the value of duration equal to 13 s. The estimate of v - the absolute value of instant centroid mean velocity is found to be 1.0-1.3 km/s (Fig. 3b). Residuals $\varepsilon_{l_{max}}$ and $\varepsilon_{l_{min}}$ attain there minima for a major axis estimate 30 km and minor axis 10-22 km (Fig. 3c and 3d). The functions ε_{φ_l} and (Fig. 3e and 3f) defining direction of maximal mean size of the source and direction of the instant centroid velocity - two last curves in Figure 3. These residuals were calculated for all possible values of angles φ_l (from 0° to 180°) and φ_v (from 0° to 360°) while other parameters were fixed equal to their estimates obtained before. Angles are measured in the foot wall of the fault plane clockwise round from the strike axis. These angles estimates are 115° and 240° for φ_l and φ_v respectively. These estimates of φ_l and φ_v correspond to following directions characterising the projection of the source on horizontal free surface: direction of maximal source extension with azimuth 43°, and the direction of rupture propagation with azimuth 310°.

As a result of surface wave spectra analysis we produced a model of the source which is given in Figure 4. The stereographic projections of nodal planes are presented at the same figure. The ellipse represents here the integral estimates of source geometry. $\Delta\mathbf{U}$ - slip vector, $\mathbf{V}$ - direction of the instant centroid velocity.

Comparison of results of body and surface waves inversion

As one can see the moment tensor and the source depth estimates obtained from two different kinds of data are in a very good agreement. To compare the other results of application of these two techniques, we calculated the second integral moments of the distribution of moment rate obtained from body wave inversion for the point source propagating along a line in the fault plane. We obtained the integral estimate of duration to be equal 9s, which is less then surface wave estimate 11-13s. The integral estimate of the source length was obtained equal to 18 km. As we mentioned above the azimuth of instant centroid velocity on the free surface is equal to 310°, which is close to 315° obtained for azimuth of rupture propagation from body waves inversion. Using formula (7) and estimates l_{max} , l_{min} and φ_l, we calculated the integral estimate for the source size in this direction be equal 20 km, which is only 2km more then the integral estimate of the source length obtained from body waves. It appears that rupture velocity estimated from body waves (2.1 km/s) differs much from its surface wave estimate (1.0-1.3 km/s). But as one can see from formula (8), the instant centroid velocity is defined by second temporal moment $(\Delta\tau)^2$ as well as by coupled space-time moment $\mathbf{w}$. If calculating this velocity we will use surface wave estimate for the coupled moment and value correspondent to 9 s (obtained from body waves) for the second temporal moment, we will obtain for velocity 2.2 km/s.

On the other hand fixing the duration estimate equal to 9 s and varying all other parameters in surface wave inversion we obtain for velocity estimate 2 km/s. So, the results of surface and body waves inversion are in a good agreement. The main difference is obtained for the integral estimate of duration, which calculated from body waves is from 2 to 4 s shorter then its surface wave estimate. This results obtained from seismological data are in a good agreement with the field study of the epicentral zone of this earthquake (see Gomez *et al.*, 1996b).

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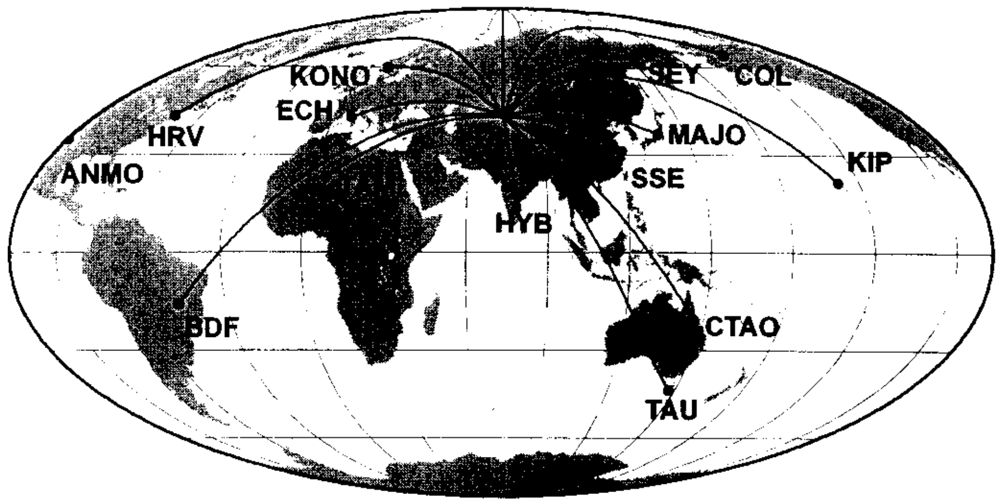

Fig.2 Distribution of stations

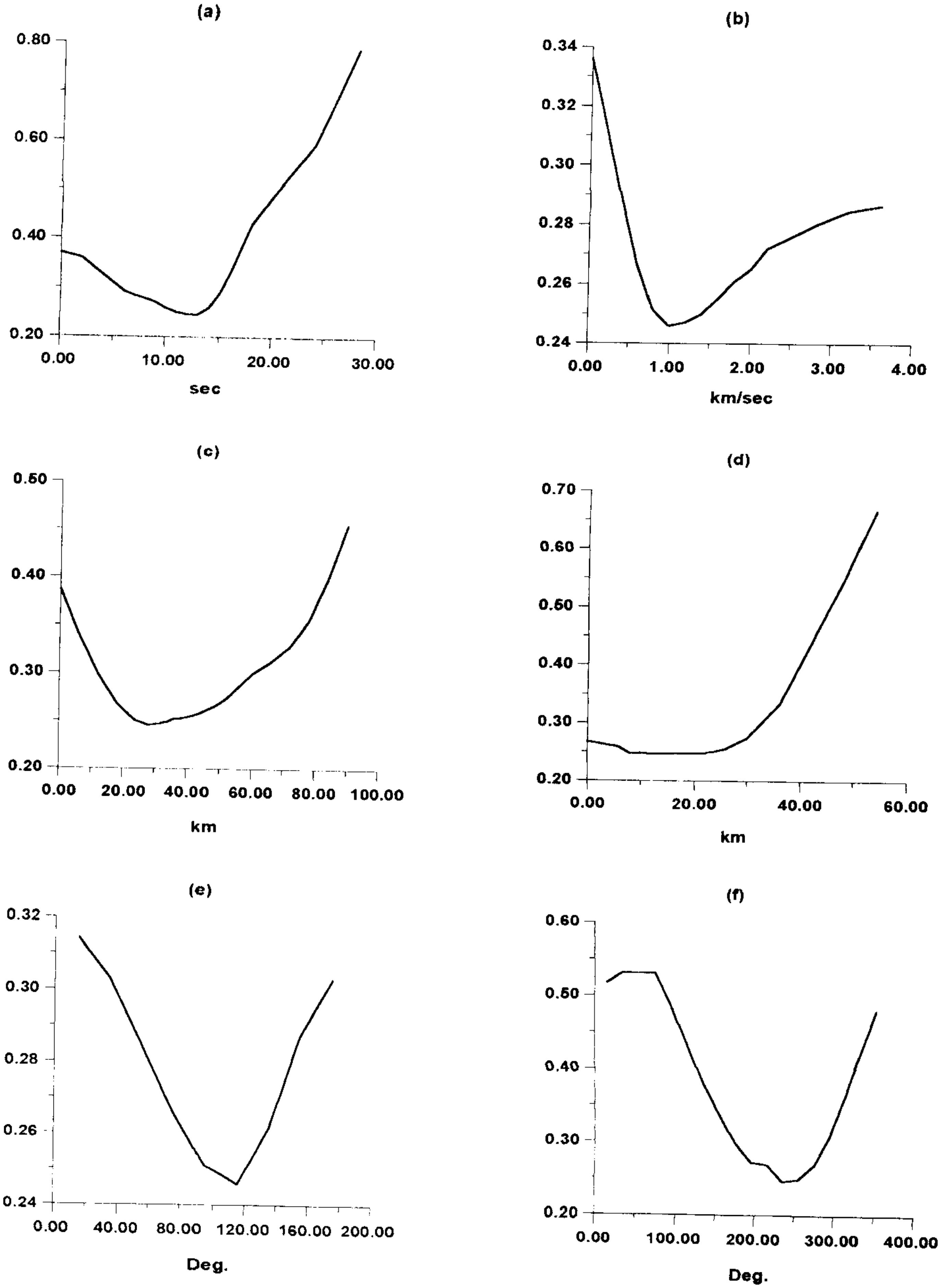

Fig.3 Residual functions

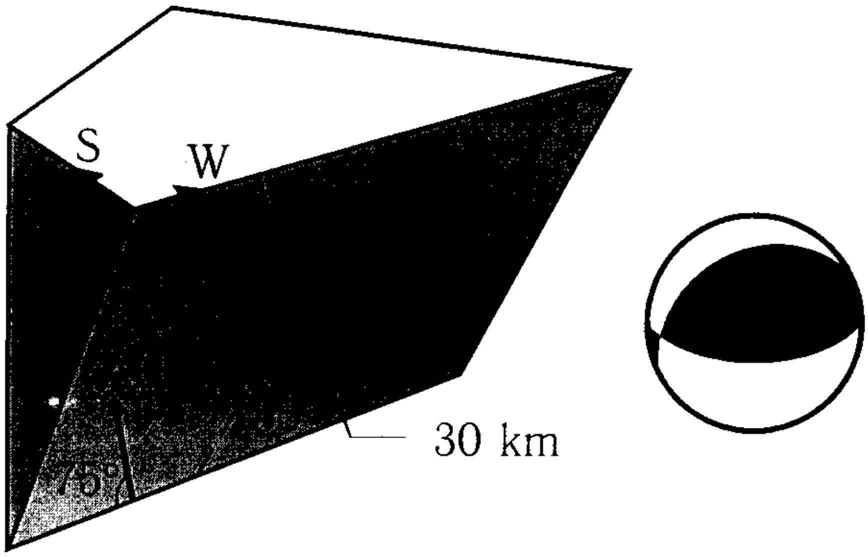

Fig.4 Focal mechanism and source model.