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Third Workshop on 3D Modelling of Seismic Waves Generation Propagation and their Inversion

4 - 15 November 1996

Fundamentals of Kinematics Sources

Point Sources - I
Seismic Moment Tensor - II
Source Dimensions - III

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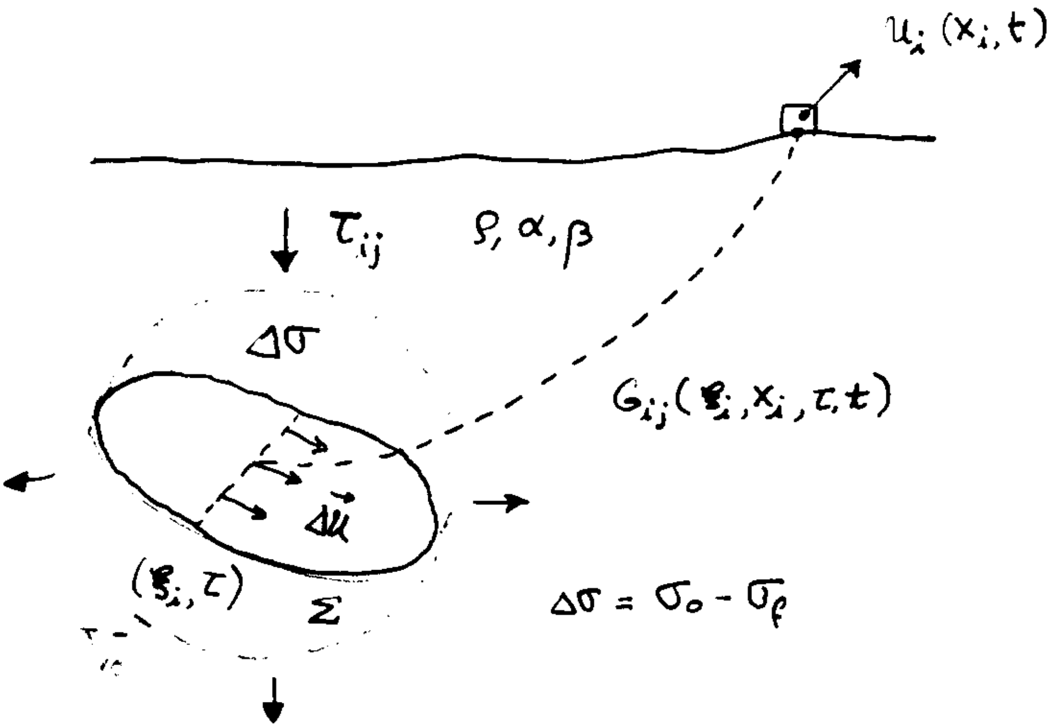
SOURCE MECHANISM OF EARTHQUAKES



Kinematics models $\rightarrow$ $\Delta\vec{u}$ $\longrightarrow$ $\vec{u}(x_i,t)\,[\Delta u(\xi_i,\tau)]$

dynamics models $\longrightarrow$ $\vec{\Delta\sigma}$ $\longrightarrow$ $u(x_i,t)\,[\vec{\Delta\sigma}(\xi_i,\tau)]$

Point source – orientation of Σ. (ϕ, δ, λ)
Scalar moment – $M_0 = \mu \Delta u\, S$
source time function – $s(\tau)$ (rise time τ_0)

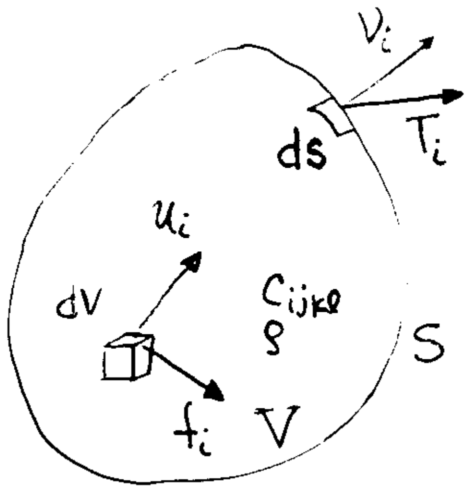
Extended source –
Dimension of Σ – L, r
fracture velocity – v
Distribution of $\Delta\vec{u}(\xi_i,\tau)$ on Σ

Distribution of v, τ_0 on Σ

Direct problem – given $\vec{\Delta u}$ calculate $\vec{u}$

Inverse problem – given $\vec{u}$ calculate $\vec{\Delta u}$

Equation of motion



$$\int_V f_i \, dV + \int_S T_i \, dS = \int_V \rho \ddot{u}_i \, dV$$

$$\int_V (\rho \ddot{u}_i - f_i) \, dV = \int_S \tau_{ij} \nu_j \, dS$$

$$\int_V (\rho \ddot{u}_i - f_i) \, dV = \int_S c_{ijkl} u_{k,l} \nu_j \, dS$$

$$T_i = \tau_{ij} \nu_j \;;\quad \tau_{ij} = c_{ijkl} e_{kl} \;;\quad e_{kl} = \tfrac{1}{2}(u_{k,l} + u_{l,k})$$

$$c_{ijkl} u_{k,l} = \lambda u_{k,k} \delta_{ij} + \mu (u_{i,j} + u_{j,i})$$

$$\int_V (\tau_{ij,j} + f_i) \, dV = \int_V \rho \ddot{u}_i \, dV$$

$$\int_V (c_{ijkl} u_{k,lj} - \rho \ddot{u}_i) \, dV = -\int_V f_i \, dV$$

$$\alpha^2 \nabla \Theta - \beta^2 \nabla \times \vec{\omega} + \frac{1}{\rho} \vec{f} = \ddot{\vec{u}} \qquad \begin{cases} \nabla \cdot \vec{u} = u_{k,k} = \Theta \\ \nabla \times \vec{u} = e_{ijk} u_{k,j} = \vec{\omega} \end{cases}$$

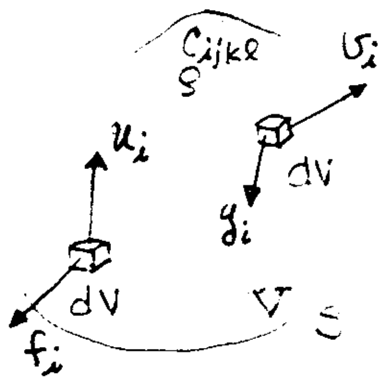
$$\nabla^2 \Theta = \frac{1}{\alpha^2} \ddot{\Theta} \;;\quad \nabla^2 \vec{\omega} = \frac{1}{\beta^2} \ddot{\vec{\omega}} \qquad \alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

for $\vec{f} = 0$. $\qquad \beta = \sqrt{\dfrac{\mu}{\rho}}$

$$(\lambda + \mu) u_{k,ki} + \mu u_{i,jj} + f_i = \rho \ddot{u}_i$$

$$(\lambda + \mu) \nabla (\nabla \cdot \vec{u}) + \mu \nabla^2 \vec{u} + \vec{f} = \rho \ddot{\vec{u}}$$

Representation theorem - Betti



$$\int_V (f_i - \rho \ddot{u}_i)\, dV = - \int_S \overset{u}{T}_i \, dS$$

$$\int_V (g_i - \rho \ddot{v}_i)\, dV = - \int_S \overset{v}{T}_i \, dS$$

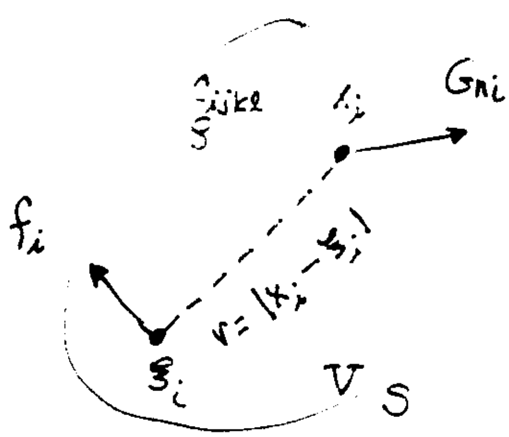
$$\int_{-\infty}^{\infty} dt \int_V \rho\, (\ddot{u}_i v_i - \ddot{v}_i u_i)\, dV = \int_{-\infty}^{\infty} dt \int_V (v_i f_i - u_i g_i)\, dV + \int_{-\infty}^{\infty} dt \int_S \left(v_i \overset{u}{T}_i - u_i \overset{v}{T}_i \right) d$$

If for $t=0$; $v_i = \dot{v}_i = u_i = \dot{u}_i = 0$

$$\int_{-\infty}^{\infty} dt \int_V (u_i g_i - v_i f_i)\, dV = \int_{-\infty}^{\infty} dt \int_V (v_i c_{ijkl} u_{k,lj} - u_i c_{ijkl} v_{k,lj}) d$$

$$= \int_{-\infty}^{\infty} dt \int_S (v_i \overset{u}{T}_i - u_i c_{ijkl} v_{k,l} \nu_j)$$

Green Function



$$f_i(\xi_i, \tau) = \delta(x_i - \xi_i)\, \delta(t - \tau)\, \delta_{in}$$

$$u_i(x_i, t) \rightarrow G_{ni}(x_i, t; \xi_i, \tau)$$

$$\int_V [\rho \ddot{G}_{ni} - \delta(x_s - \xi_s)\, \delta(t - \tau)\, \delta_{in} =$$

$$= \int_S c_{ijkl} G_{nk,l} \nu_j \, dS$$

For a given V, S, c_{ijkl} and ρ - $G_{ni}(x_i, t, \xi_i, \tau)$

Green function of the medium - elastic impulse response.

$$\int_{-\infty}^{\infty} dt \int_V [u_j\, \delta(x_i-\xi_i)\, \delta(t-\tau)\, \delta_{ni} - f_i\, G_{ni}]\, dV = \int_{-\infty}^{\infty} dt \int_S G_{ni}\, T_i\, ds$$

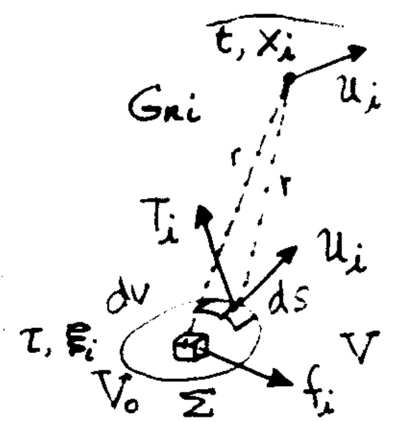
$$+ \int_{-\infty}^{\infty} dt \int_S u_i\, C_{ijkl}\, G_{nk,l}\, \nu_j\, ds .$$

$v_i = G_{ni}$; $g_i = \delta(x_i-\xi_i)\,\delta(t-\tau)\,\delta_{ni}$

$$u_i(x_i,t) = \int_{-\infty}^{\infty} d\tau \int_V f_i\, G_{in}\, dV + \int_{-\infty}^{\infty} d\tau \int_S T_i\, G_{ni}\, ds + \int_{-\infty}^{\infty} d\tau \int_S u_i\, C_{ijkl}\, G_{nk,l}\, \nu_j$$

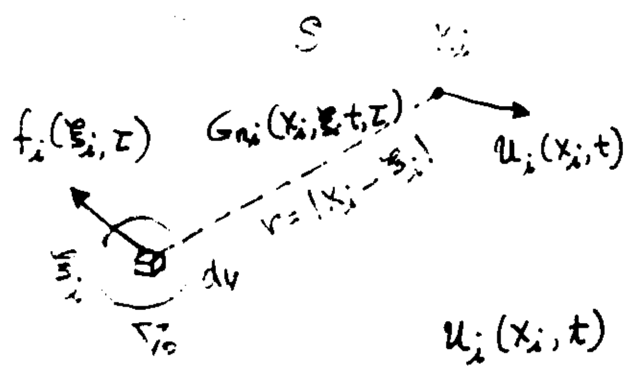
$V = V' + V_0$

$S + \Sigma$



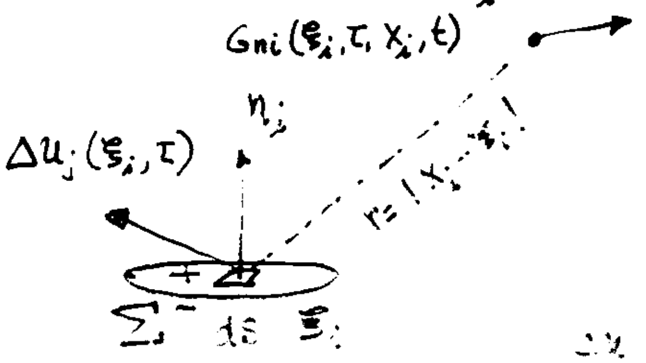
$$u(x_i,t) = \int_{-\infty}^{\infty} d\tau \int_{V_0} f_i(\xi_i)\, G_{in}\, dV$$

$$u(x_i,t) = \int_{-\infty}^{\infty} d\tau \int_{\Sigma} u_i\, C_{ijkl}\, G_{nk,l}\, n_j\, d$$



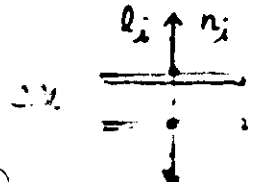
On Σ $\quad u_i^+ - u_i^- = \Delta u_i$

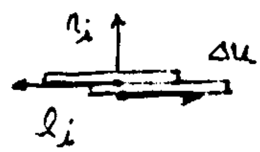
$$u(x_i,t) = \int_{-\infty}^{\infty} d\tau \int_{\Sigma} \Delta u_i\, C_{ijkl}\, G_{nk,l}\, n_j\, d$$



$\Delta u_i(\xi_i,\tau) = \Delta u\, l_i\, s(\tau)$

$s(\tau)$ = source time function





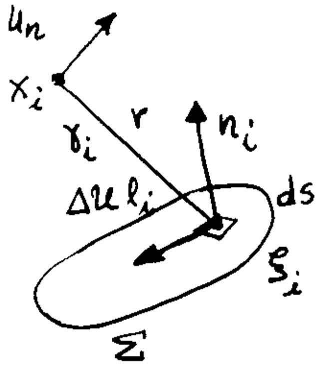
(6) dilatation shear

Shear fracture

$$n_i \, \Delta u_j = 0 \quad ; \quad \Delta u_j = \Delta u \, l_j$$

$$c_{ijkl} \, \Delta u_k \, n_l = \Delta u \, \mu \, (l_i n_j + l_j n_i)$$

$$u_n = \int_{-\infty}^{\infty} d\tau \int_{\Sigma} \mu \, \Delta u \, (l_i n_j + l_j n_i) \, G_{in,j} \, ds$$



$\Delta u(\xi_i, t)$ - dislocation or slip.

orientation: l_i, n_i

$$\int_{\Sigma} ds = S$$

Point source

$$u_n(x_i, t) = \mu \, (l_i n_j + l_j n_i) S \int_{-\infty}^{\infty} \Delta u(\tau) \, G_{in,j}(t-\tau) \, d\tau$$

Green function far-field

$$G_{ij}^{P} = \frac{1}{4\pi\rho\alpha^2 r} \gamma_i \gamma_j \, \delta\left(t - \frac{r}{\alpha}\right)$$

$$G_{ij}^{S} = \frac{-1}{4\pi\rho\beta^2 r} (\gamma_i \gamma_j - \delta_{ij}) \, \delta\left(t - \frac{r}{\beta}\right)$$

Derivatives

$$G_{ki,j}^{P} = \frac{1}{4\pi\rho\alpha^3 r} \gamma_i \gamma_k \gamma_j \, \dot{\delta}\left(t - \frac{r}{\alpha}\right)$$ - far-field

$$G_{ki,j} = \frac{\partial G_{ki}}{\partial \xi_j} \quad ; \quad \frac{\partial r}{\partial \xi_j} = -\gamma_j \quad ;$$

$$\int_{-\infty}^{\infty} \Delta u(\tau) \, \dot{\delta}\left(t - \frac{r}{\alpha} - \tau\right) d\tau = \Delta \dot{u}\left(t - \frac{r}{\alpha}\right)$$

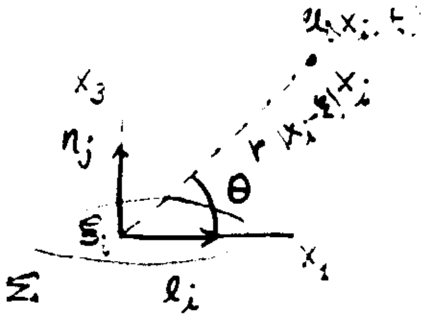
$$u_k^{P} = \frac{\mu \, \Delta \dot{u} \, S}{4\pi\rho\alpha^3 r} \gamma_k \gamma_i \gamma_j \, (n_i l_j + n_j l_i)$$ far field P wave displ.

Radiation Pattern for a shear point dislocation

$$u_k^P = \frac{\mu S \Delta \dot{u}}{4\pi\rho\alpha^3 r} \gamma_k \gamma_i \gamma_j (n_i l_j + n_j l_i)$$

$$u_k^S = -\frac{\mu S \Delta \dot{u}}{4\pi\rho\beta^3 r} (\gamma_i \gamma_k - \delta_{ik}) \gamma_j (n_i l_j + n_j l_i)$$

since - $C_{ijkl} \Delta u_l \, l_i n_j = \mu \Delta u \, (l_i n_j + n_i l_j)$

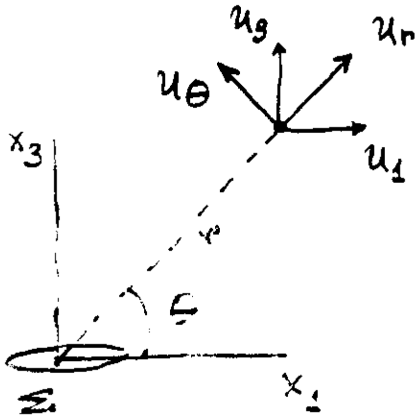


$n_i = (0,0,1)$; $l = (1,0,0)$

$\gamma_1 = \cos\theta$, $\gamma_3 = \operatorname{sen}\theta$

$$u_1^P = \frac{A}{r} 2\gamma_1^2\gamma_3 = \frac{A}{r} \operatorname{sen}2\theta \cos\theta$$

$$u_3^P = \frac{A}{r} 2\gamma_3^2\gamma_1 = \frac{A}{r} \operatorname{sen}2\theta \operatorname{sen}\theta$$



$$u_r^P = \frac{A}{r} \operatorname{sen} 2\theta$$

$$u_\theta^P = 0$$

$$u_1^S = \frac{B}{r} \gamma_3(1 - 2\gamma_1^2) = -\frac{B}{r} \cos 2\theta \operatorname{sen}\theta$$

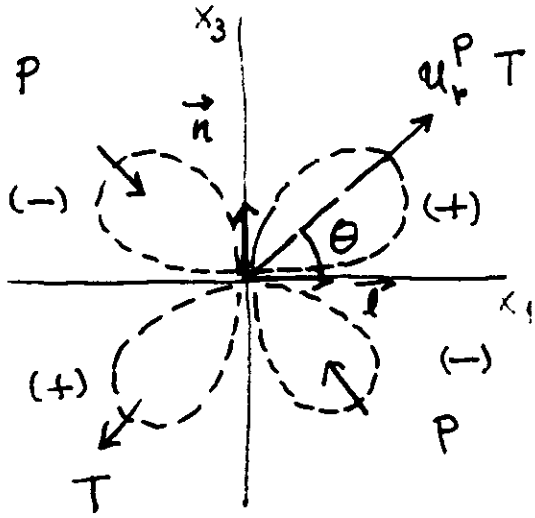
$$u_3^S = \frac{B}{r} \gamma_1(1 - 2\gamma_3^2) = \frac{B}{r} \cos 2\theta \cos$$

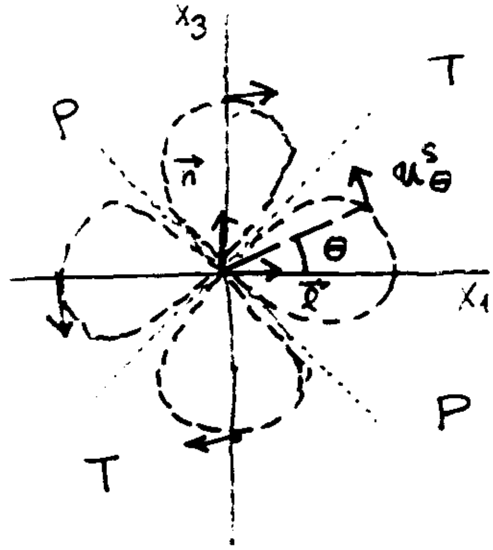
$$u_r^S = 0$$

$$u_\theta^S = \frac{B}{r} \cos 2\theta$$

$\Delta\dot{u}(\xi_i, \tau) = \Delta u(\xi_i) \, s(\tau)$ - $s(\tau)$ - source time function

if $\Delta u(t) = H(t) \rightarrow \Delta\dot{u}(t) = \delta(t)$.

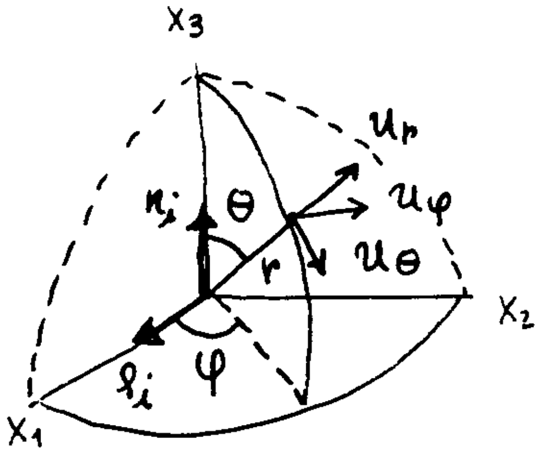




In Space

horizontal plane

<u>En el espacio</u> X_3 vertical; (X_1, X_2) plano horizontal



$\gamma_1 = \operatorname{sen}\Theta \cos\varphi$ $\qquad n_i = (0, 0, 1)$

$\gamma_2 = \operatorname{sen}\Theta \operatorname{sen}\varphi$ $\qquad l_i = (1, 0, 0)$

$\gamma_3 = \cos\Theta$

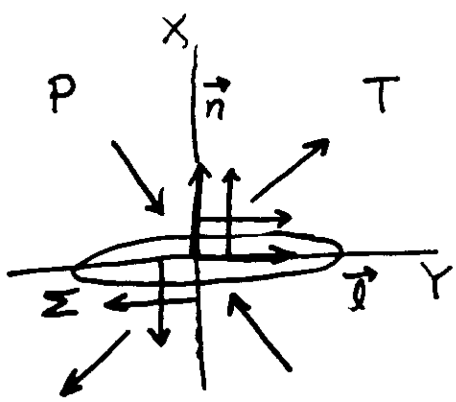
$$P - u_r^P = \frac{A}{r} \operatorname{sen} 2\Theta \cos\varphi$$

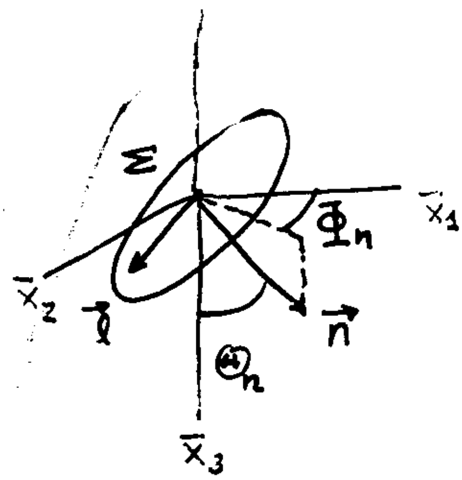
$$SV - u_\Theta^s = \frac{B}{r} \cos 2\Theta \cos\varphi$$

$$SH - u_\varphi^s = \frac{-B}{r} \cos\Theta \operatorname{sen}\varphi$$

Geometry of Shear source

<u>Geometría de la fuente de cizalla</u>

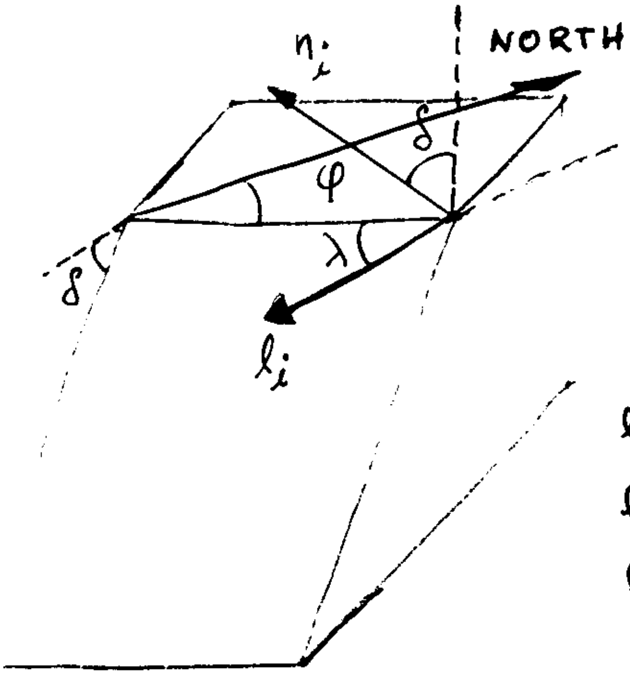




$n_1 = \operatorname{sen}\Theta_n \cos\Phi_n$ $\qquad l_1 = \operatorname{sen}\Theta_l \cos\Phi_l$

$n_2 = \operatorname{sen}\Theta_n \operatorname{sen}\Phi_n$ $\qquad l_2 = \operatorname{sen}\Theta_l \operatorname{sen}\Phi_l$

$n_3 = \cos\Theta$ $\qquad l_3 = \cos\Theta$



$n_1 = -\sin\delta \, \sin\varphi$

$n_2 = \text{sen}\,\delta \, \cos\varphi$

$n_3 = -\cos\delta$

$l_1 = \cos\lambda \cos\varphi + \cos\delta \, \text{sen}\,\lambda \, \sin\varphi$

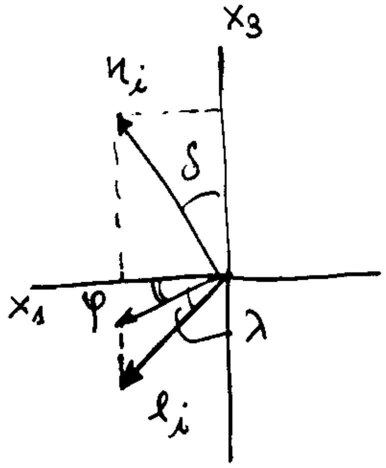
$l_2 = \cos\lambda \, \sin\varphi - \cos\delta \, \sin\lambda \, \cos\varphi$

$l_3 = -\sin\lambda \, \text{sen}\,\delta$

$\varphi = \Phi_n - \frac{\pi}{2}$

$\delta = \Theta_n$

$\lambda = \sin^{-1}\left(\frac{\cos\Theta_l}{\sin\Theta_n}\right)$



Orientation : (Θ_n, Φ_n) (Θ_l, Φ_l)

shear F. φ, δ, λ

P and T. (Θ_T, Φ_T) (Θ_P, Φ_P)

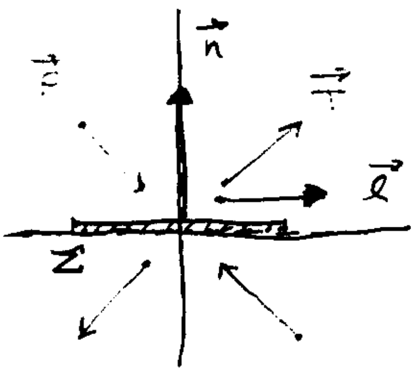
Double C. (Θ_X, Φ_X), (Θ_Y, Φ_Y)

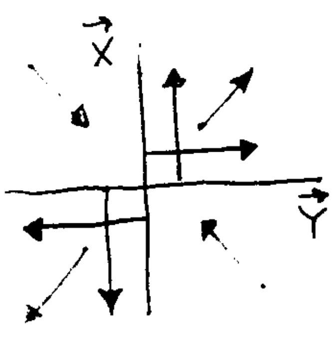
Orthogonality only 3 parameter

φ, δ, λ

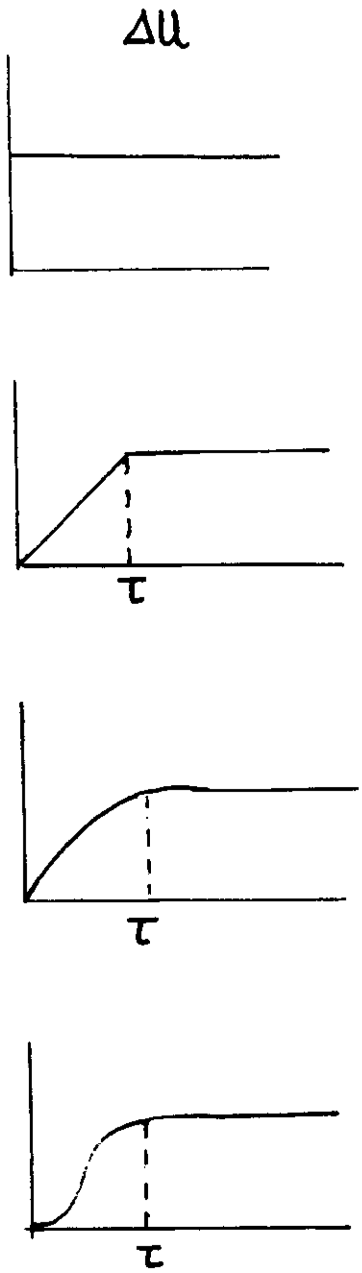
Θ_T, Φ_T, Φ_P

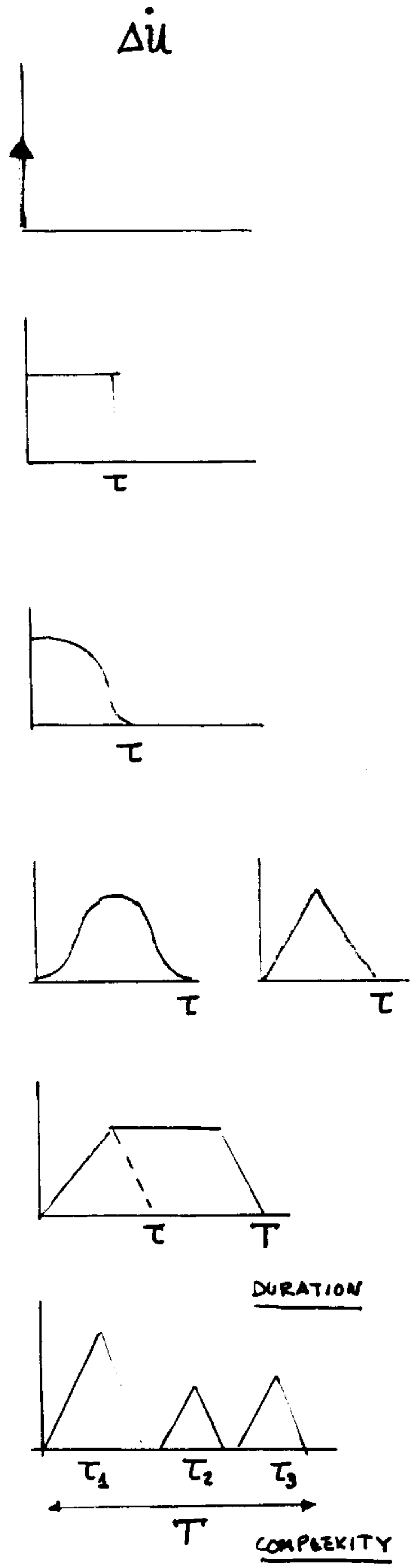
Θ_X, Φ_X, Φ_Y





SOURCE TIME FUNCTION



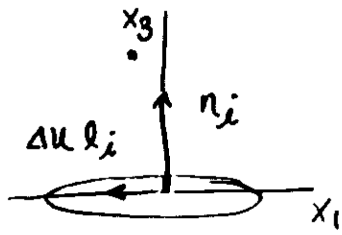


τ: Rise Time

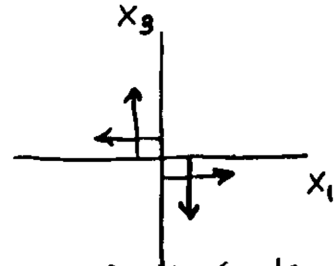
T: Duration

(II) fuerzas equivalentes, dislocaciones y tensor momento

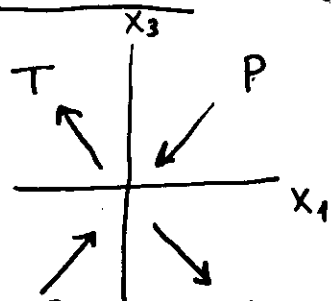
Equivalent Forces, dislocations and moment Tensor



Dislocation



Double Couple



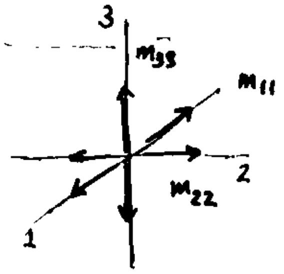
Pressure-Tension

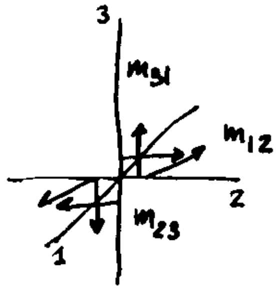
Tensor Momento Sismico
Seismic Moment tensor

$$m_{kl} = C_{ijkl}\,\Delta u_i\, n_j \quad \text{(dislocations)} \qquad F_i = -\frac{\partial m_{ij}}{\partial x_j} \quad \text{(Forces)}$$

$$u_n = \int_{-\infty}^{\infty} d\tau \int_\Sigma m_{kl}\, G_{nk,l}\, dS$$ – Dislocation on a surface Σ

$$M_{ij} = \int_\Sigma m_{ij}\, dS$$





$$C_{ijkl}\,\Delta u_k n_l = \Delta u\left[\mu\,(l_i n_j + l_j n_i) + \lambda n_k l_k\, \delta_{ij}\right]$$ general displacement

$$m_{ij} = \Delta u\,\mu\,(l_i n_j + l_j n_i)$$ – shear fracture. $n_k l_k = 0$

$$M_{ij} = \Delta\bar{u}\,\mu\, S\,(l_i n_j + l_j n_i) = M_0\,(l_i n_j + l_j n_i)$$

$M_0 = \Delta\bar{u}\,\mu\, S$ – scalar seismic moment

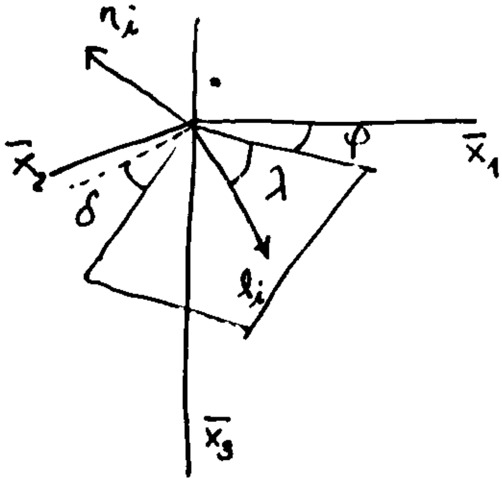
Forces on a volume V

$$G_{ni}(\xi_i) = G_{ni}(0) + \xi_j \frac{\partial}{\partial \xi_j} G_{ni} + \dots$$

$$u_n = \int_{-\infty}^{\infty} d\tau \int_V f_i\, G_{ni}(0)\, dV + \int_{-\infty}^{\infty} d\tau \int_V f_i\, \xi_j \frac{\partial G_{ni}}{\partial \xi_j}\, dV$$

$$u_n = \int_{-\infty}^{\infty} d\tau \int_V m_{ij}\, G_{ni,j}\, dV \;; \qquad m_{ij} = f_i \xi_j \;; \qquad f_i = -\frac{\partial m_{ij}}{\partial \xi_j} \;;$$

$$u_n = M_{ij} * G_{ni,j}$$: Foco puntual



$0 \leq \varphi \leq 360°$ – Azimut (strike)

$0 \leq \delta \leq 90°$ – Buzamiento (dip)

$-180 \leq \lambda \leq 180°$ – Deslizamiento (slip, rake)

$$\varphi = \Phi_n - \frac{\pi}{2}$$

$$\delta = \Theta_n$$

$$\lambda = \operatorname{sen}^{-1}\left\{\frac{\cos\Theta_l}{\operatorname{sen}\Theta_n}\right\}$$

Tensor Momento Sismico

$$m_{ij} = (n_i l_j + n_j l_i)$$

$$m_{11} = 2n_1 l_1 , \qquad m_{22} = 2n_2 l_2 \qquad m_{33} = 2n_3 l_3$$

$$m_{12} = n_1 l_2 + n_2 l_1 \qquad m_{13} = n_1 l_3 + n_3 l_1 \qquad m_{32} = n_3 l_2 + n_2 l_3$$

$$n_1 = -\operatorname{sen}\delta \operatorname{sen}\varphi \qquad l_1 = \cos\lambda\cos\varphi + \cos\delta \operatorname{sen}\lambda \operatorname{sen}\varphi$$

$$n_2 = \operatorname{sen}\delta\cos\varphi \qquad l_2 = \cos\lambda \operatorname{sen}\varphi - \cos\delta \operatorname{sen}\lambda \cos\varphi$$

$$n_3 = -\cos\delta \qquad l_3 = -\operatorname{sen}\lambda \operatorname{sen}\delta$$

$$m_{11} = -\operatorname{sen}\delta\cos\lambda \operatorname{sen}2\varphi - \operatorname{sen}2\delta \operatorname{sen}^2\varphi \operatorname{sen}\lambda$$

$$m_{22} = \operatorname{sen}\delta\cos\lambda \operatorname{sen}2\varphi + \operatorname{sen}2\delta\cos^2\varphi \operatorname{sen}\lambda$$

$$m_{33} = \operatorname{sen}2\delta \operatorname{sen}\lambda$$

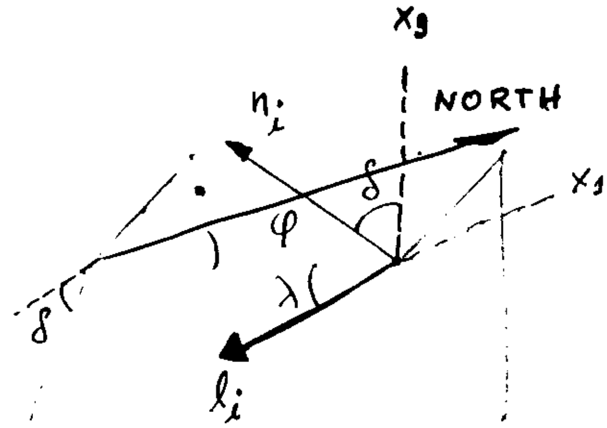
$$m_{12} = \operatorname{sen}\delta\cos\lambda\cos2\varphi + \frac{1}{2}\operatorname{sen}2\delta \operatorname{sen}2\varphi \operatorname{sen}\lambda$$

$$m_{13} = -\operatorname{sen}\lambda \operatorname{sen}\varphi \cos2\varphi - \cos\delta\cos\lambda\cos\varphi$$

$$m_{23} = \cos\varphi \operatorname{sen}\lambda\cos2\delta - \cos\delta\cos\lambda \operatorname{sen}\varphi$$

$$m_{11} = m_{\theta\theta} \qquad m_{22} = m_{\varphi\varphi} \qquad m_{33} = m_{rr}$$

$$m_{12} = -m_{\varphi\theta} \qquad m_{13} = m_{\theta r} \qquad m_{32} = -m_{r\varphi}$$



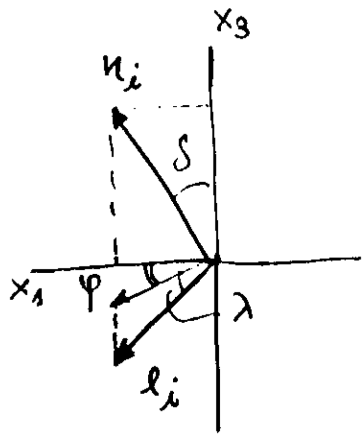
$$n_1 = -\sin\delta \sin\varphi$$
$$n_2 = \sin\delta \cos\varphi$$
$$n_3 = -\cos\delta$$

$$l_1 = \cos\lambda\cos\varphi + \cos\delta \sin\lambda \sin\varphi$$
$$l_2 = \cos\lambda \sin\varphi - \cos\delta \sin\lambda \cos\varphi$$
$$l_3 = -\sin\lambda \sin\delta$$

$$\varphi = \Phi_n - \frac{\pi}{2}$$

$$\delta = \Theta_n$$

$$\lambda = \sin^{-1}\left(\frac{\cos\Theta_l}{\sin\Theta_n}\right)$$



<u>Orientation</u> : (Θ_n, Φ_n) (Θ_l, Φ_l)

shear F. φ, δ, λ

P and T. (Θ_T, Φ_T) (Θ_P, Φ_P)

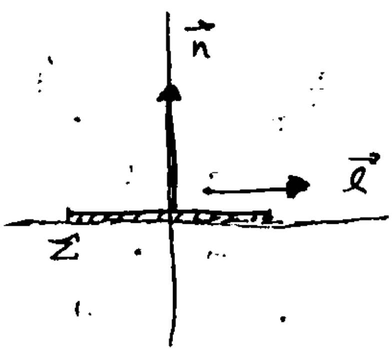
Double C. (Θ_X, Φ_X), (Θ_Y, Φ_Y)

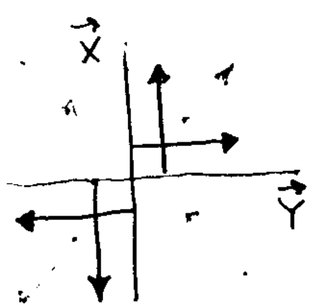
Orthogonality only 3 parameter

φ, δ, λ

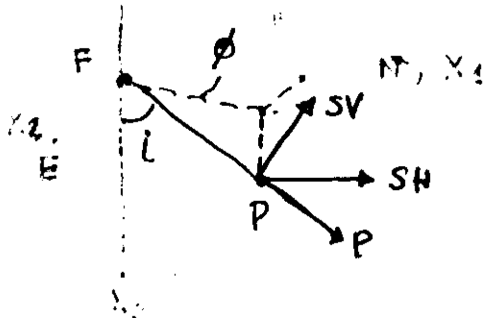
Θ_T, Φ_T, Φ_P

Θ_X, Φ_X, Φ_Y





Desplazamientos P, SV, SH



P:

$$u_k^P = \frac{M_0\,\delta(t-\frac{r}{\alpha})}{4\pi\alpha^3 r}\,\gamma_k\gamma_i\gamma_j\,m_{ij}$$

Componente radial normalizado

$$u_r^P = \gamma_i\gamma_j\,m_{ij} = \gamma_1^2 m_{11} + \gamma_2^2 m_{22} + \gamma_3^2 m_{33} + 2\gamma_1\gamma_2 m_{12} + 2\gamma_1\gamma_3 m_{13} + 2\gamma_2\gamma_3 m_{23}$$

$$u_r^P = \operatorname{sen}^2 i\,(\cos^2\varphi\, m_{11} + \operatorname{sen}^2\varphi\, m_{22} + \operatorname{sen}2\varphi\, m_{12}) + \cos^2 i\, m_{33} + \operatorname{sen}2i\,(\cos\varphi\, m_{13} + \operatorname{sen}\varphi\, m_{23})$$

$$\begin{cases}\gamma_1 = \operatorname{sen} i\cos\varphi\\ \gamma_2 = \operatorname{sen} i\operatorname{sen}\varphi\\ \gamma_3 = \cos i\end{cases}$$

S:
$$u_k^S = \frac{M_0\,\delta(t-\frac{r}{\beta})}{4\pi\beta^3 r}\,(\gamma_i\gamma_k - \delta_{ik})\,\gamma_j\,m_{ij}$$

Componente SV normalizado

$$SV_k\gamma_k = 0$$

$$u_{SV} = (\gamma_i\gamma_k - \delta_{ik})\,SV_k\,\gamma_j\,m_{ij} = -SV_i\gamma_j\,m_{ij}$$
$$= SV_1\gamma_1 m_{11} + SV_2\gamma_2 m_{22} + SV_3\gamma_3 m_{33} + m_{12}(SV_1\gamma_2 + SV_2\gamma_1) + m_{13}(SV_1\gamma_3 + SV_3\gamma_1) + m_{23}(SV_2\gamma_3 + SV_3\gamma_2)$$

$$u_{SV} = \tfrac{1}{2}\operatorname{sen}2i\,(\cos^2\varphi\, m_{11} + \operatorname{sen}^2\varphi\, m_{22} - m_{33} + \operatorname{sen}2\varphi\, m_{12}) + \cos 2i\,(\cos\varphi\, m_{13} + \operatorname{sen}\varphi\, m_{23})$$

Componente SH normalizado

$$\begin{cases}SH_3 = 0\\ SH_k\gamma_k = 0\end{cases}$$

$$u_{SH} = (\gamma_i\gamma_k - \delta_{ik})\,SH_k\,\gamma_j\,m_{ij} = SH_i\gamma_j\,m_{ij}$$

$$u_{SH} = SH_1\gamma_1 m_{11} + SH_2\gamma_2 m_{22} + m_{12}(SH_1\gamma_2 + SH_2\gamma_1) + m_{13}SH_1\gamma_3 + m_{23}SH_2\gamma_3$$

$$u_{SH} = \operatorname{sen} i\left[\tfrac{1}{2}\operatorname{sen}2\varphi\,(m_{22} - m_{11}) + \cos 2\varphi\, m_{12}\right] + \cos i\,(\cos\varphi\, m_{23} - \operatorname{sen}\varphi\, m_{13})$$

Valores y vectores propios — Eigen values and eigen vectors

$$\begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \rightarrow \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$$

$$(m_{ij} - \delta_{ij}\sigma_l) V_j^l = 0 \; ; \quad \mathrm{Det}\, | m_{ij} - \delta_{ij}\sigma | = 0$$

a) $\sigma_1 = \sigma_2 = \sigma_3$; $\sigma_1 + \sigma_2 + \sigma_3 = 3K\Delta u$ - Expansion

b) $\sigma_3 = -\sigma_1$; $\sigma_2 = 0$; $\sigma_1 + \sigma_2 + \sigma_3 = 0$ - cizalla

c) $\sigma_1 \neq \sigma_2 \neq \sigma_3$; $\sigma_1 + \sigma_2 + \sigma_3 = 0$ - desviatoria

d) $\sigma_1 \neq \sigma_2 \neq \sigma_3$; $\sigma_1 + \sigma_2 + \sigma_3 \neq 0$ - general

- $V_j^1 = P$, $V_j^2 = B$, $V_j^3 = T$; $\sigma_1 > \sigma_2 > \sigma_3$

Separacion del tensor momento sismico
separation of the seismic moment tensor

$$\bar{m}_{ij} = m_{ij} - \frac{1}{3} m_{kk}\delta_{ij}$$

$$M_{ij}^D = M_{ij} - M_{ij}^{ISO}$$

$$\begin{cases} \bar{m}_{ij} - \left\{ \begin{matrix} \text{deviatory} \\ \text{desviatorio} \end{matrix} \right\} - M_{ij}^D \\ \bar{m}_{11} + \bar{m}_{22} + \bar{m}_{33} = 0 \end{cases}$$

1)
$$M_{ij}^D \quad \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} = \underbrace{\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\sigma_1 \end{pmatrix}}_{\text{DC- Mayor}} + \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & -\sigma_2 \end{pmatrix}}_{\text{DC- Menor}}$$

$\sigma_1 + \sigma_2 + \sigma_3 = 0$
$\sigma_3 = -\sigma_2 - \sigma_1$
$|\sigma_1| > |\sigma_2|$

2)
$$M_{ij}^D \quad \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} = \underbrace{\begin{pmatrix} (\sigma_1 - \sigma_3)\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2}(\sigma_1 - \sigma_3) \end{pmatrix}}_{\text{DC}} + \underbrace{\begin{pmatrix} -\frac{\sigma_2}{2} & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & -\frac{\sigma_2}{2} \end{pmatrix}}_{\text{CLVD}}$$

$\sigma_1 + \sigma_2 + \sigma_3 = 0$

3) $\left\{ \begin{matrix} \text{best} \\ \text{Mejor} \end{matrix} \right.$ DC $\rightarrow$ $| \bar{m}_{ij} - m_{ij}(DC) | =$ $\overset{\text{least}}{\text{mínimo}}$ -

$$M_{ij}^D = M_{ij}^{DC1} + M_{ij}^{DC2}$$

CLVD: Compensated linear vector dipole

$$M_{ij}^D = M_{ij}^{DC} + M_{ij}^{CLVD}$$

SEISMIC MOMENT TENSOR

$$M_{ij} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$$

$$m_{ij} = \mu\ \Delta u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Separation of moment tensor

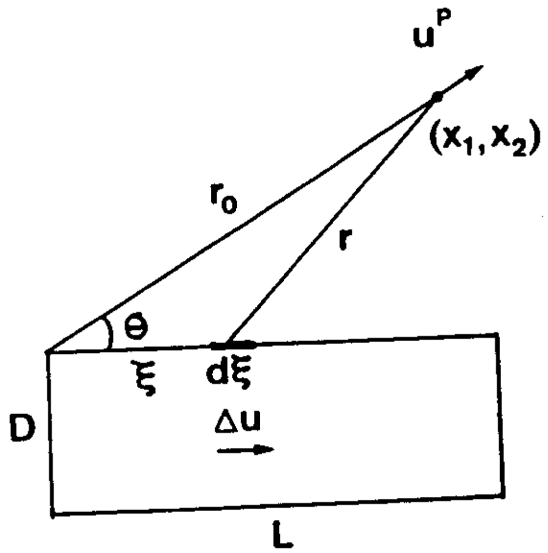
$$M_{ij} = M_{ij}^{ISO} + M_{ij}^{DC} + M_{ij}^{R}$$

$$\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & -\sigma_1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\sigma_3 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$$

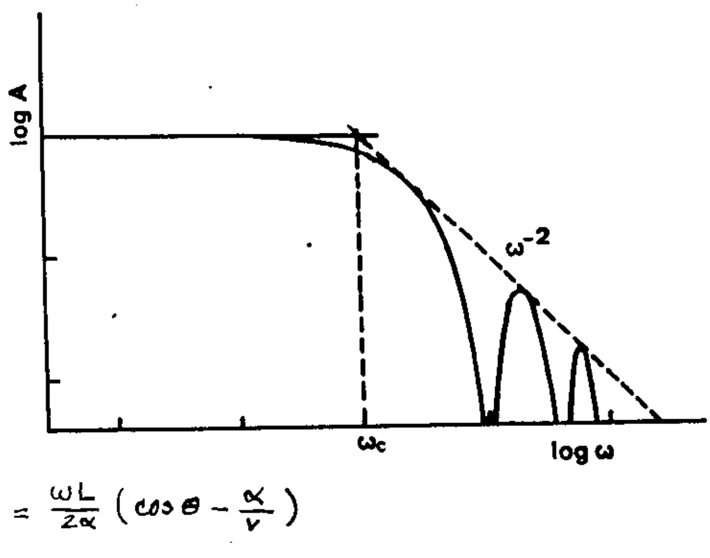
$$\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(\sigma_1-\sigma_3) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2}(\sigma_1-\sigma_3) \end{pmatrix} + \begin{pmatrix} -\sigma_2/2 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & -\sigma_2/2 \end{pmatrix}$$

Source dimensions

$$u_i^p = \frac{\mu}{4\pi\rho\alpha^3 r}\, \gamma_j\gamma_k\gamma_i\, (n_j l_k + n_k l_j) \int_\Sigma \Delta\dot{u}(\xi, t - \frac{r}{\alpha})\, dS$$



$$\int_\Sigma \Delta\dot{u}(\xi, t - \frac{r}{\alpha})\, dS = D \int_0^L \Delta\dot{u}(t - \frac{r}{\alpha} - \frac{\xi}{v})\, d\xi$$



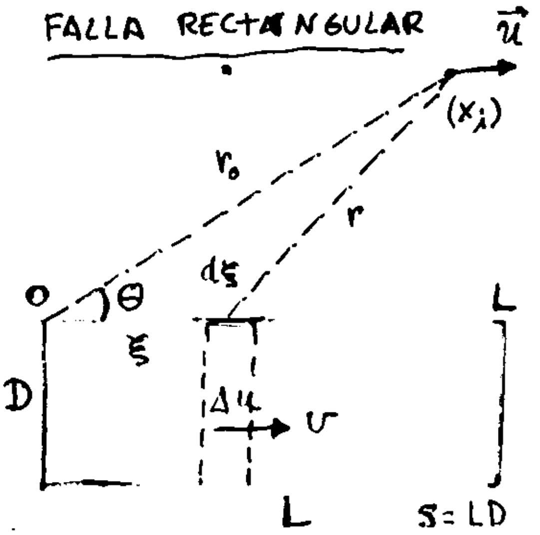
$$X = \frac{\omega L}{2\alpha}\left(\cos\theta - \frac{\alpha}{v}\right)$$

$$U_i^p = \frac{M_o}{4\pi\alpha^3 r\rho}\, \gamma_i\gamma_k\gamma_j (l_k n_j + l_j n_k)\, \frac{1}{1+i\omega\tau}\, \frac{\operatorname{sen} X}{X}\, e^{i(\frac{\omega r}{\alpha} + X - \frac{\pi}{2})}$$

$$\Delta u_1(\rho, t) = \Delta u_1\, H(t - \rho/v)\, [1 - H(\rho - a)]$$

DIMENSIONES DE LA FRACTURA (Haskell)

FALLA RECTANGULAR



$S = LD$

$$\int_\Sigma \Delta\dot{u}\left(\xi, t-\frac{r}{\alpha}\right) ds$$

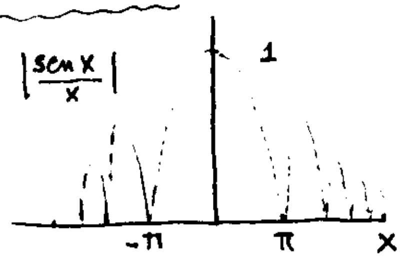
$$= D\int_0^L \Delta\dot{u}\left(t-\frac{r}{\alpha}-\frac{\xi}{v}\right) d\xi$$

$$= D\int_0^L \Delta\dot{u}\left[t-\frac{r_o}{\alpha}-\frac{\xi}{\alpha}\left(\cos\theta-\frac{\alpha}{v}\right)\right] ds$$

$$u_i^P = \frac{\mu}{4\pi\rho\alpha^3 r}\gamma_j\gamma_k\gamma_i(n_j l_k + n_k l_j)\int_\Sigma \Delta\dot{u}\left(\xi, t-\frac{r}{\alpha}\right) d\xi .$$

transformada de Fourier

$$\begin{cases} U^P(\omega, x) = \text{T.F. de } u^P(t,x) \\ \Delta U(\omega) = \text{TF de } \Delta u(t) \\ i\omega\,\Delta U(\omega) = \text{TF de } \Delta\dot{u}(t) \end{cases}$$



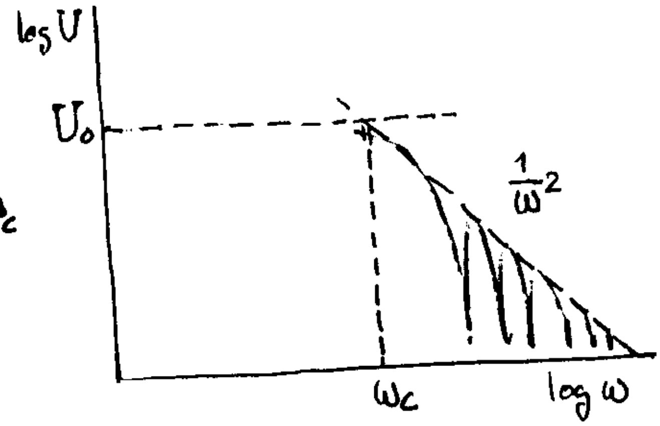
espectro de amplitud

$$|U^P(\omega, x)| \sim \underbrace{\mu\,\Delta U(\omega)\, S}_{M_o}\,\omega\frac{\operatorname{sen} X}{X}\,R(\gamma_i, n_i, l_i)\frac{1}{4\pi\rho\alpha^3 r}$$

$$X = \frac{\omega L}{2\alpha}\left(\cos\theta - \frac{\alpha}{v}\right)$$

forma del espectro

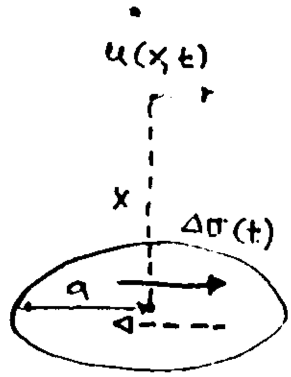
$$\begin{cases} \Delta U(\omega) \sim \dfrac{1}{\omega^2} ; \\ U^P(\omega) \sim M_o , \quad \omega \to 0 \\ \quad\quad \sim \dfrac{1}{\omega^2} , \quad \omega \gg \omega_c \end{cases}$$



frecuencia de esquina: ω_c $\quad \omega_c = \dfrac{C\beta}{L}$

$$U_o \sim M_o$$

FALLA CIRCULAR : Modelo de Brune



$$\Delta\sigma(x,t) = \Delta\sigma\, H\left(t - \frac{x}{\beta}\right)$$

$$\Delta u(0,t) = H(t)\,\frac{\Delta\sigma}{\mu}\,\beta t$$

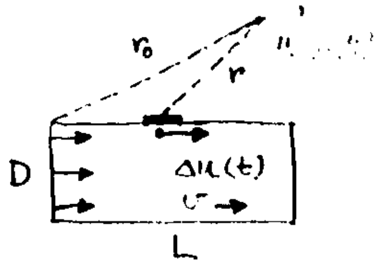
$$\Delta U(\omega) = -\frac{\Delta\sigma\,\beta}{\mu\,\omega^2}$$

Onda S campo lejano: $$u(t) = \frac{\Delta\sigma\beta}{\mu}\left(t - \frac{r}{\beta}\right) e^{-\alpha\left(t-\frac{r}{\beta}\right)}$$

$$U(\omega) = \frac{\Delta\sigma\beta}{\mu}\,\frac{1}{\omega^2 - \alpha^2}$$

$$\alpha = \frac{2.33\,\beta}{a} = \omega_c \;;\quad a = \frac{2.33\,\beta}{\omega_c}$$

FALLA RECTANGULAR : Modelo de Haskell



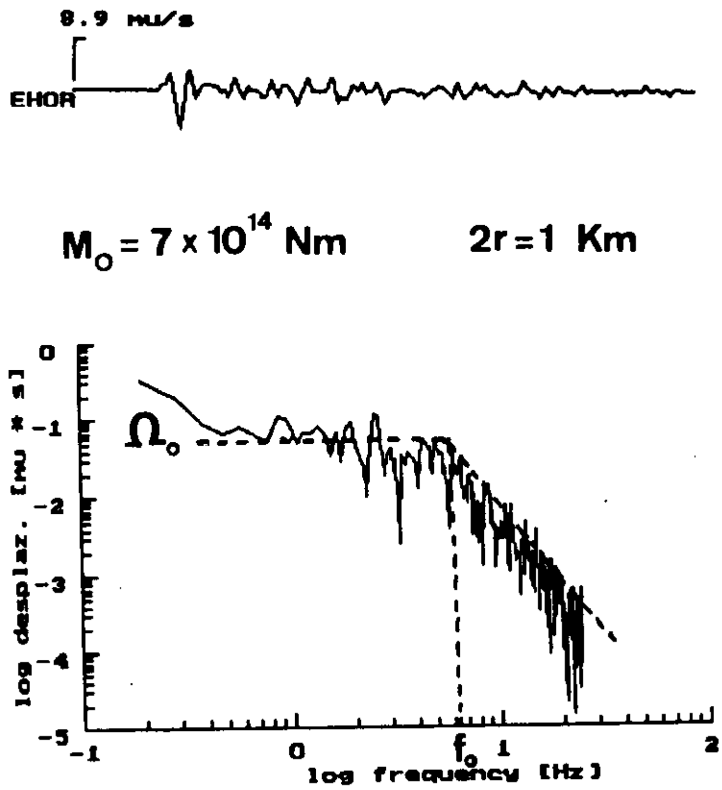
$$|U(\omega)| = \Delta U(\omega)\; LD\; \omega \left|\frac{\operatorname{sen} X}{X}\right|$$

$$X = \frac{\omega L}{2\alpha}\left(\frac{\alpha}{v} - \cos\theta\right)$$

$$\Delta u(t) = \begin{cases} \Delta u \dfrac{t}{\tau_0} & 0<t<\tau_0 \\ \Delta u & t \geq t_0 \end{cases}$$

$$\Delta U(\omega) = \frac{\Delta u\,(1 - e^{-i\omega\tau_0})}{\tau_0\,\omega^2}$$

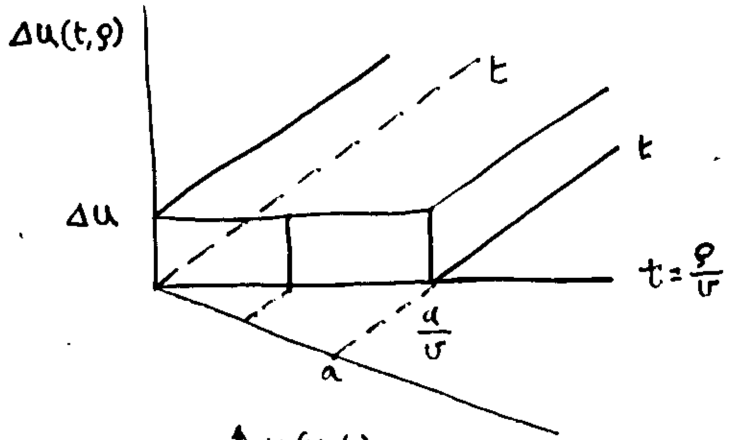
$$\sqrt{LD} = \frac{1.7\,\alpha}{\omega_c^P} = \frac{3.8\,\beta}{\omega_c^S}.$$


8.9 mu/s
EHOR
M_O = 7 x 10^14 Nm
2r = 1 Km
Ω_o
f_o
log desplaz. [mu * s]
log frequency [Hz]
0
-1
-2
-3
-4
-5
-1
0
1
2

NUCLEACION, PROPAGACION Y PARADA

Falla circular - modelo de Savage

(Parada del movimiento independiente de la parada en $\rho = a$)

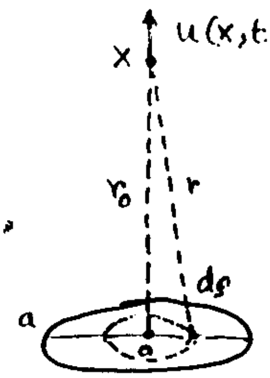


$$\Delta u(\rho,t) = \Delta u\, H\left(t-\frac{\rho}{v}\right)\left[1-H(\rho-a)\right]$$

$$\Delta u = 0\,,\quad t<\frac{\rho}{v}$$

$$\Delta u = \Delta u\,,\quad t \geq \frac{\rho}{v}$$

$$\Delta u = 0\,,\quad \rho \lessgtr a$$

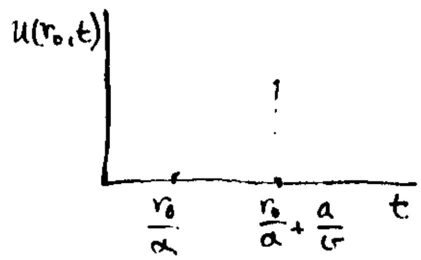


$$u^{\rho}(x,t) = 2\pi\int_0^a \Delta\dot{u}\left(t-\frac{r_0}{\alpha}-\frac{\rho}{v}\right)\rho\, d\rho$$

$$u^{\rho}(x,t) = 2\pi\Delta u\, v^2\left(t-\frac{r_0}{\alpha}\right)\left\{1-H\left[\left(t-\frac{r_0}{\alpha}\right)v-a\right]\right\}$$

$$= 2\pi\,\Delta u\, v^2\left(t-\frac{r_0}{\alpha}\right) \;:\; v\left(t-\frac{r_0}{\alpha}\right)<a$$

$$= 0 \;:\; \geq a$$



Modelo para $\Delta\dot{u}(\rho,t)$ (Molnar et al.)

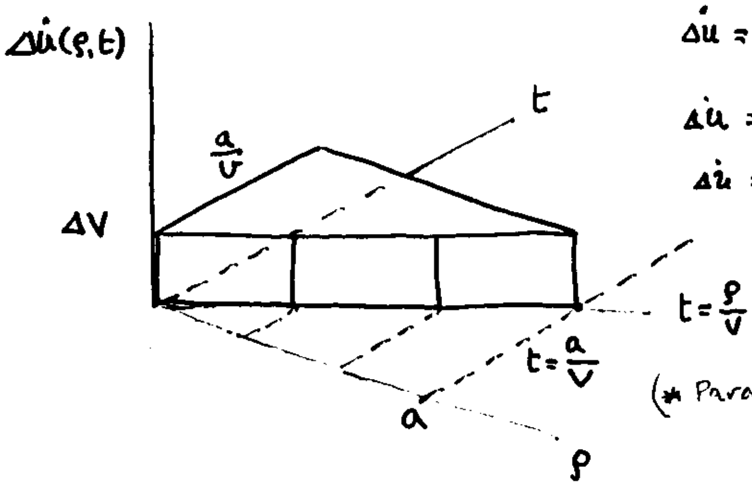
$$\Delta\dot{u} = \Delta V\left[H\left(t-\frac{\rho}{v}\right)-H\left(t-\frac{a}{v}+\frac{\rho}{v}\right)\right]H(a-\rho)$$

$$\Delta\dot{u} = 0 \;:\; t<\frac{\rho}{v}$$

$$\Delta\dot{u} = \Delta V \;:\; \frac{\rho}{v}\leq t\leq\frac{a}{v}$$

$$\Delta\dot{u} = 0 \;:\; t>\frac{a}{v}$$

$$\Delta\dot{u} = 0 \;:\; \rho>a$$

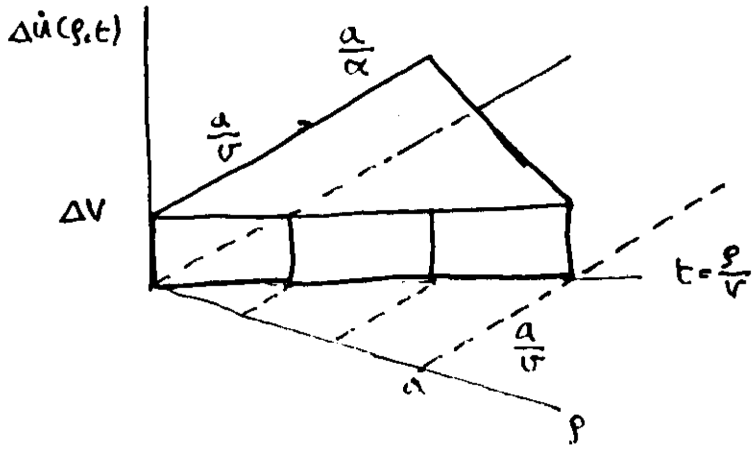


(* Parada del movimiento cuando $\rho = a$)

NUCLEACION, PROPAGACION Y PARADA

- Parada del movimiento cuando llega a cada punto ρ la información de la parada en $\rho = a$.

$$\Delta \dot{u}(\rho,t) = \Delta V \left[H\left(t - \frac{\rho}{v}\right) - H\left(t - \frac{a}{v} - \frac{a-\rho}{\alpha}\right)\right] H(a-\rho)$$



$\Delta\dot{u} = 0 \quad : \quad t < \frac{\rho}{v}$

$\Delta\dot{u} = \Delta V \quad : \quad \frac{\rho}{v} < t \leq \frac{a}{v} + \frac{a-\rho}{\alpha}$

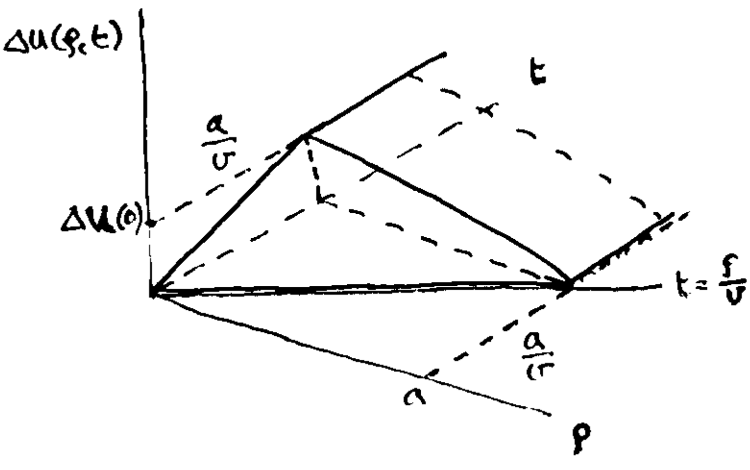
$\Delta\dot{u} = 0 \quad : \quad t > \frac{a}{v} + \frac{a-\rho}{\alpha}$

$\Delta\dot{u} = 0 \quad : \quad \rho > a$

Modelo de Sato y Hirasawa (1973)

$$\Delta u(\rho,t) = \frac{24}{7\pi}\frac{\Delta\sigma}{\mu} v \sqrt{t^2 - \frac{\rho^2}{v^2}}\, H\left(t - \frac{\rho}{v}\right)\left[1 - H(\rho - a)\right]; \quad t < \frac{a}{v}$$

$$= \frac{24}{7\pi}\frac{\Delta\sigma}{\mu}\sqrt{a^2-\rho^2}\left[1 - H(\rho - a)\right]; \quad t > \frac{a}{v}$$



$\Delta u = 0 \quad , \quad t < \frac{\rho}{v}$

$\Delta u = K\sqrt{t^2 - \frac{\rho^2}{v}} \quad ; \quad t > \frac{\rho}{v}$

$\Delta u = 0 \quad ; \quad \rho \geq a$

[$\Delta u = \frac{24}{7\pi}\frac{\Delta\sigma}{\mu}\sqrt{a^2-\rho^2}$ - solución del problema estático de una fractura circular

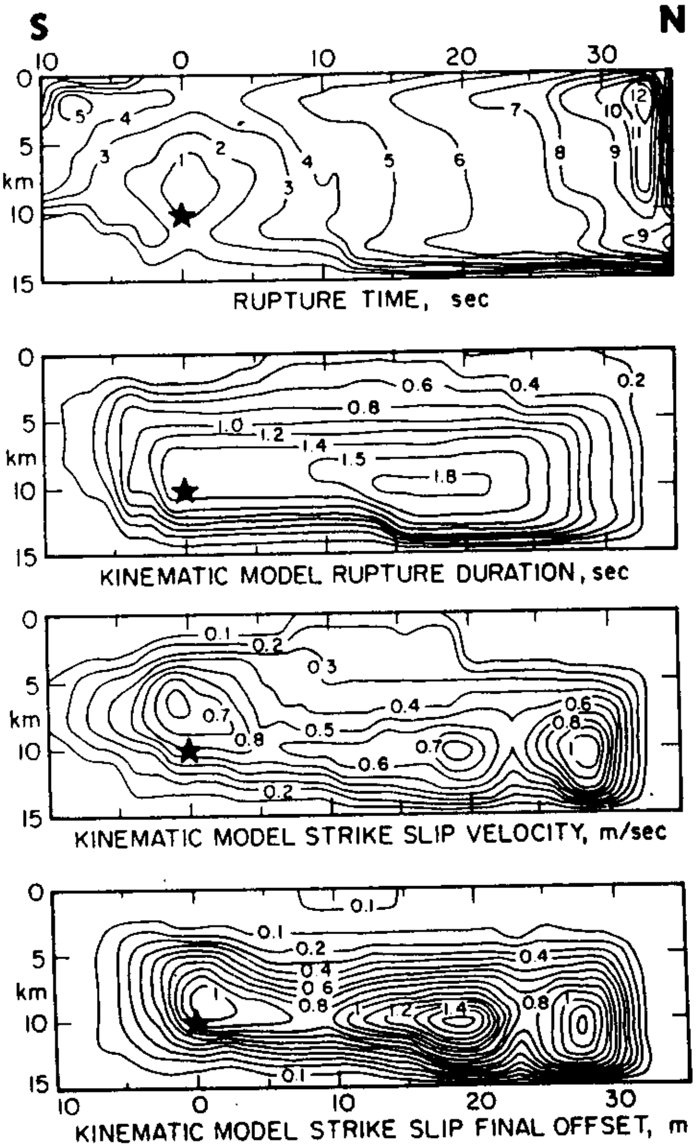
$L = 35$ km

$D = 15$ km

$S = 525$ km^2

$M_0 = 2.3 \times 10^{18}$ Nm

KINEMATIC FAULT PARAMETERS



$\Delta u(\xi_i, t)$

$t = \frac{r}{v}$

$\bar{v} \sim 3$ km/s

$\tau_0(\xi_i)$

$\bar{\tau}_0 \sim 1$ s

$\Delta \dot{u}(\xi_i)$

$\overline{\Delta \dot{u}} \sim 0.4$ m/s

$\Delta u_{max}(\xi_i)$

$\overline{\Delta u} \sim 0.6$ m

Fig. 4.20 Results of Archuleta's (1984) kinematic model of the Imperial Valley earthquake. The origin of the horizontal scale is at the epicenter with north to the right. From top to bottom are the positions of the rupture front at different times, the duration of slip, the particle velocity, and the net slip.