UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL ATOMIC ENERGY AGENCY
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR.959 - 2

MINIWORKSHOP ON STRONG ELECTRON CORRELATIONS
"Disorder and Interaction in Quantum Systems and Their Classical Analogs"

(1 - 19 July 1996)

"Quantum Phase Transitions in Disordered Magnets"

A. Peter Young
University of California at Santa Cruz
Department of Physics
Santa Cruz, CA 95064
U.S.A.

These are preliminary lecture notes, intended only for distribution to participants.

Quantum Phase Transitions in Disordered Magnets

ICTP
July 1, '96

Why interesting?

- New Universality classes
- Unusual behavior in paramagnetic phase.

  Strong effects from rare regions (Griffiths, Mc Coy).

Will focus on one-dimensional Ising model in a transverse field.

Can work things out in great ~~de~~tail, both numerically and analytically.

Many features go over to higher dimensions.

Second Order Phase Transitions

Correlation length.

$$\xi \sim \delta^{-\nu}$$

δ is deviation from critical point.
e.g. $T - T_c$

ν is a critical exponent

- Relaxation time.

$$\tau \sim \xi^{z}$$

z is the dynamical exponent

$$\omega_c \sim 1/\tau \rightarrow 0 \quad \text{at criticality.}$$

$\Rightarrow$ Critical slowing down.

Quantum mechanics is unimportant when

$$\hbar \omega_c \ll k_B T$$

Always satisfied sufficiently close to the critical point provided $\boxed{T_c \text{ finite.}}$

- <u>Quantum Phase transition</u>, $T_c = 0$

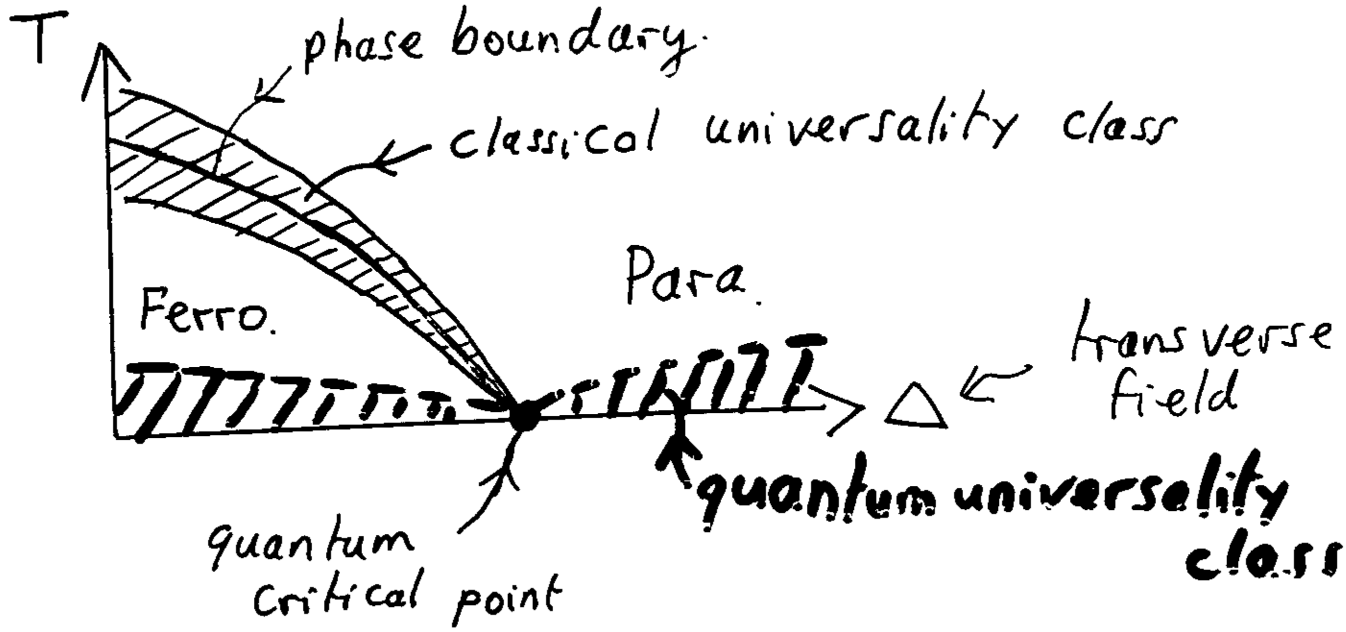
Vary some other parameter,
e.g. disorder, transverse field.

In this case, quantum fluctuations dominate.

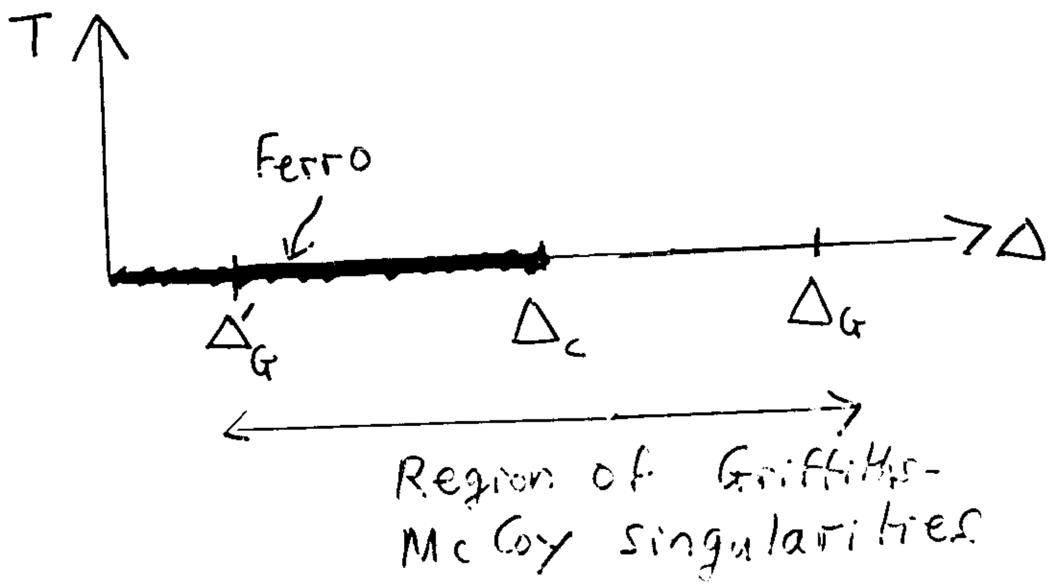
- <u>Statics and dynamics coupled.</u>

Need z even to get static critical behavior

Generic Phase Diagram



Phase Diagram in 1-d.

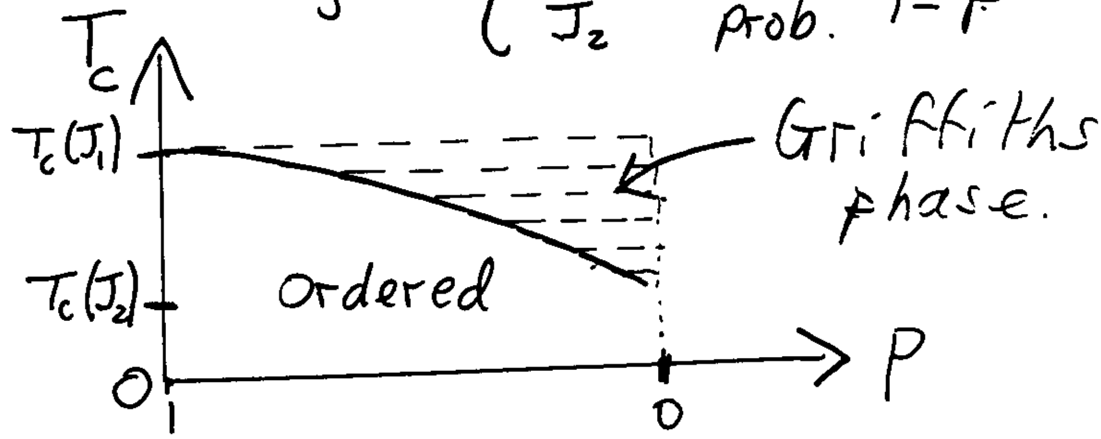


Griffiths Phase

Unique feature of disordered systems

- e.g. <u>classical</u> Ising model

$$J_{ij} = \begin{cases} J_1 & \text{prob. } p \\ J_2 & \text{prob. } 1-p \end{cases} \qquad (J_1 > J_2)$$



Prob. that a region of size L^d is locally in ordered region is $\sim e^{-\mu L^d}$

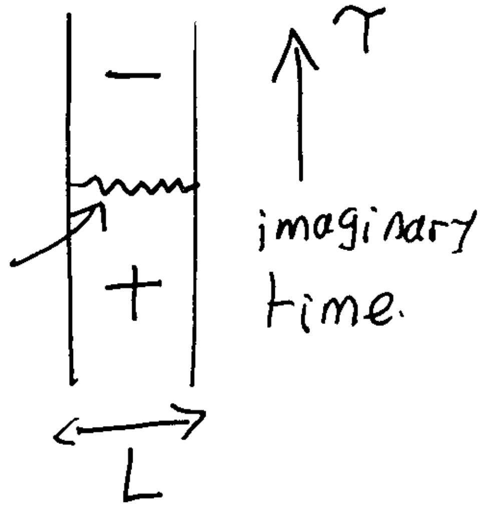
This gives a big response to a field, e.g. $\chi \sim \frac{1}{T} L^d$

But weak effect classically
$\Rightarrow$ essential singularity.

• <u>quantum</u> Ising model

Prob. of locally ordered region $\sim e^{-\mu L^d}$

domain wall energy σL^d



$\Rightarrow$ local relaxation time (local susceptibility) $\sim e^{\sigma L^d}$

$$\Rightarrow \quad P(\xi_\tau) \sim \frac{1}{\xi_\tau^{\,1+\mu/\sigma}}$$ $\leftarrow$ **distribution of local relaxation times.**

Average diverges if $\frac{\mu}{\sigma} < 1$

happens in 1-d.

Griffiths singularities are strong because the disorder is <u>perfectly correlated</u> in (imaginary) time.

For Quantum Ising model:

Griffiths phase is like a critical line with a continuously varying dynamical exponent, $z(\delta)$

$(\delta = \Delta - \Delta_c)$

$$z(\delta) \to 0 \quad \text{as} \quad \delta \to \delta_G \quad (\Delta \to \Delta_G)$$

- For quantum vector spin models, Effects of Griffiths singularities are less pronounced.

(1+1)-d Random Transverse Ising Chain

McCoy
McCoy + Wu
Shankar + Murphy
D. S. Fisher

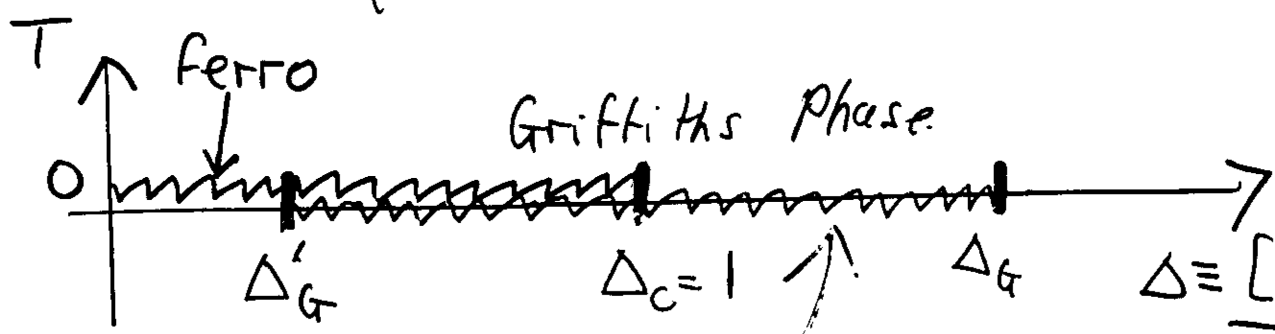
No frustration.
i.e. random ferromagnet.

$$\mathcal{H} = -\sum_i J_i \sigma_i^z \sigma_{i+1}^z - \sum_i h_i \sigma_i^x$$

Distributions $\rho(h)$, $\Pi(J)$

Self dual, i.e. at critical point, when

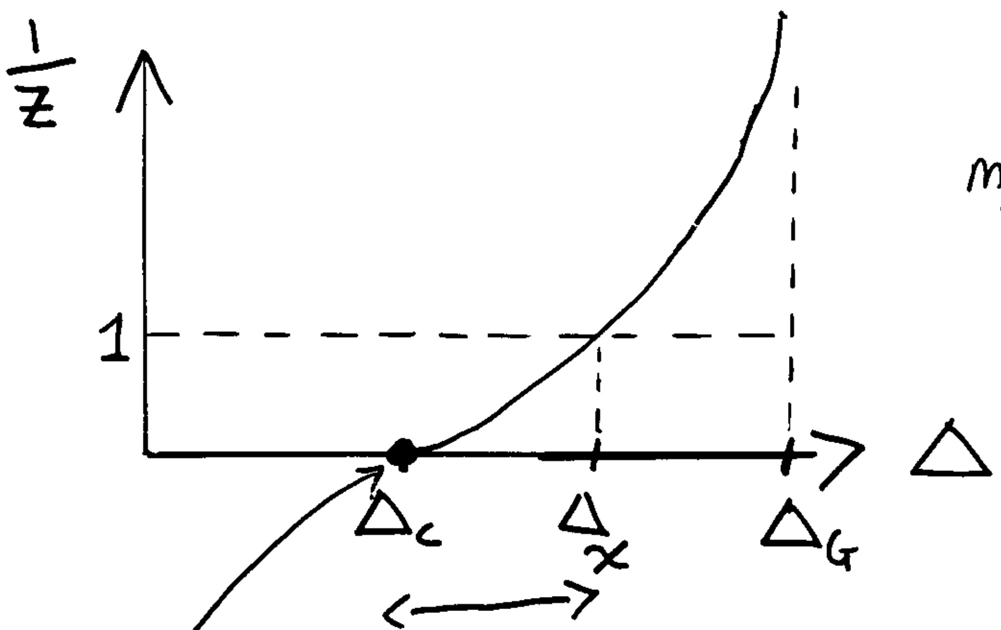
$$\rho(x) = \Pi(x)$$



$$\Delta \equiv \frac{[\ln h]_{av}}{[\ln J]_{av}}$$

$$m_{sing} \sim H_z^{1/z(\delta)}$$

$$\delta = \Delta - \Delta_c$$



$m_{sing} \sim |H_z|^{1/z}$

$z = \infty$ at criticality.

$[\chi]_{av}$ diverges here (McCoy)

Critical Behavior

mainly due to D. S. Fisher

In 1-d rare regions control not only the disordered phase but also critical behavior

At the critical point

$$C(r) = \langle \sigma_i^z \, \sigma_{i+r}^z \rangle$$

- $C_{av}(r) \equiv [C(r)]_{av} \sim \dfrac{1}{r^{2-\phi}}$, $\phi = \dfrac{1+\sqrt{5}}{2}$ Golden mean

but

- $C_{typ}(r) \equiv \exp\left[\ln C(r)\right]_{av} \sim e^{-c r^{1/2}}$

Typical being very different from average implies a broad distribution for $C(r)$ but what does it look like?

- In the disordered phase
  - $C_{av}(r) \sim e^{-r/\xi}$

    $\xi \sim \delta^{-\nu}$, average correlation length $\delta = \Delta - \Delta_c$

    $\underline{\nu = 2}$
  - $C_{typ}(r) \sim e^{-r/\tilde{\xi}}$

    $\tilde{\xi} \sim \delta^{-\tilde{\nu}}$ typical correlation length

    $\underline{\tilde{\nu} = 1}$

Motivation for my (numerical) calculations

- Confirm these surprising predictions
- Investigate the form of these broad distributions.

Free Fermion Representation

Liebs Schultz Mattis

$$\sigma_i^x = 2c_i^\dagger c_i - 1$$

$$\sigma_i^z = \exp\left(i\pi\sum_{j=1}^{i-1} c_j^\dagger c_j\right)\left(c_i^\dagger + c_i\right)$$

$$\sigma_i^y = \exp\left(i\pi\sum_{j=1}^{i-1} c_j^\dagger c_j\right)\left(\frac{c_i - c_i^\dagger}{i}\right)$$

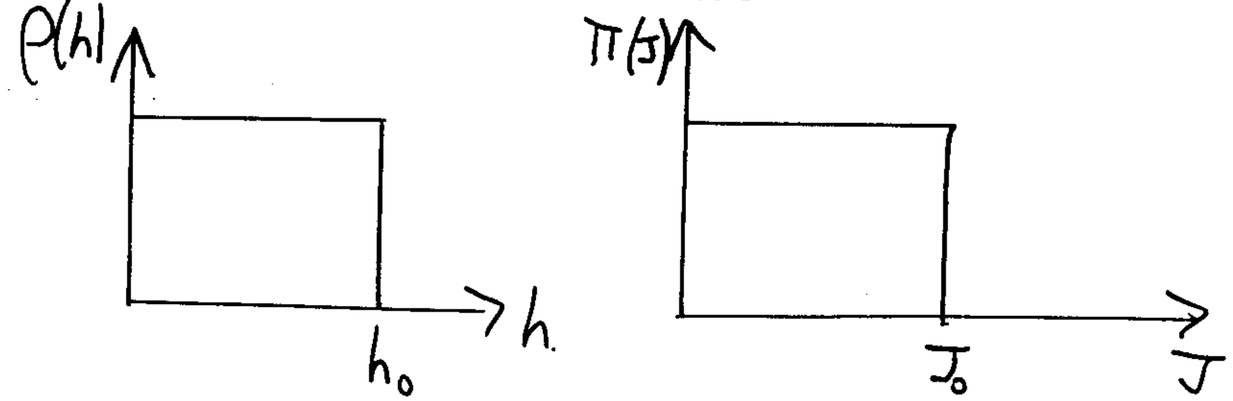
- $\Rightarrow \mathcal{H} = -\sum_i h_i\left(2c_i^\dagger c_i - 1\right) - \sum_i J_i\left(c_i^\dagger - c_i\right)\left(c_{i+1}^\dagger + c_{i+1}\right)$

(+boundary condition)

Find single particle spectrum numerically

$\Rightarrow$ • Energy (ground state)

• Gorrelation functions



Critical point, $\dfrac{h_0}{J_0} = 1$

• $\Delta_G = \infty$, $\Delta_G' = -\infty$, entire $T=0$ is the Griffiths phase

$$\Rightarrow \mathcal{H} = \sum_i \Big\{ h_i \left(c_i^\dagger c_i - c_i c_i^\dagger \right) - \frac{J_i}{2} \left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i \right) + \frac{J_i}{2} \left(c_i^\dagger c_{i+1}^\dagger - c_i c_{i+1} \right) \Big\}$$

Pure system: Fourier Transform + Bogoliubov-Valatin transform

Random system: Do BOTH numerically (diagonalize 1-body problem)

$\Rightarrow$ Ground State energy

energy gap

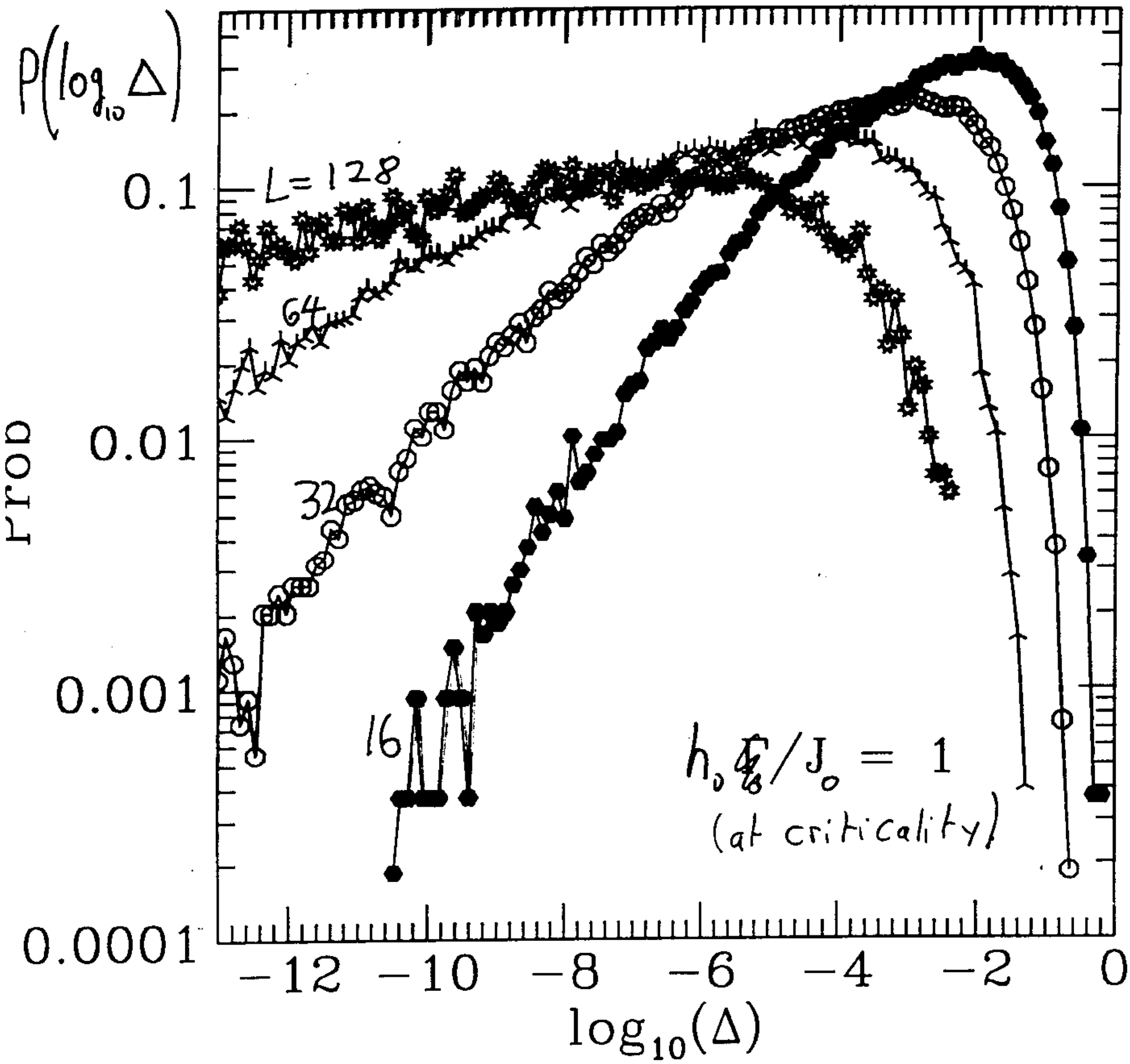
equal time correlation functions in ground state

Now lets look ~~for~~ at some of the results.

Free fermion representation

Lieb, Schultz + Mattis

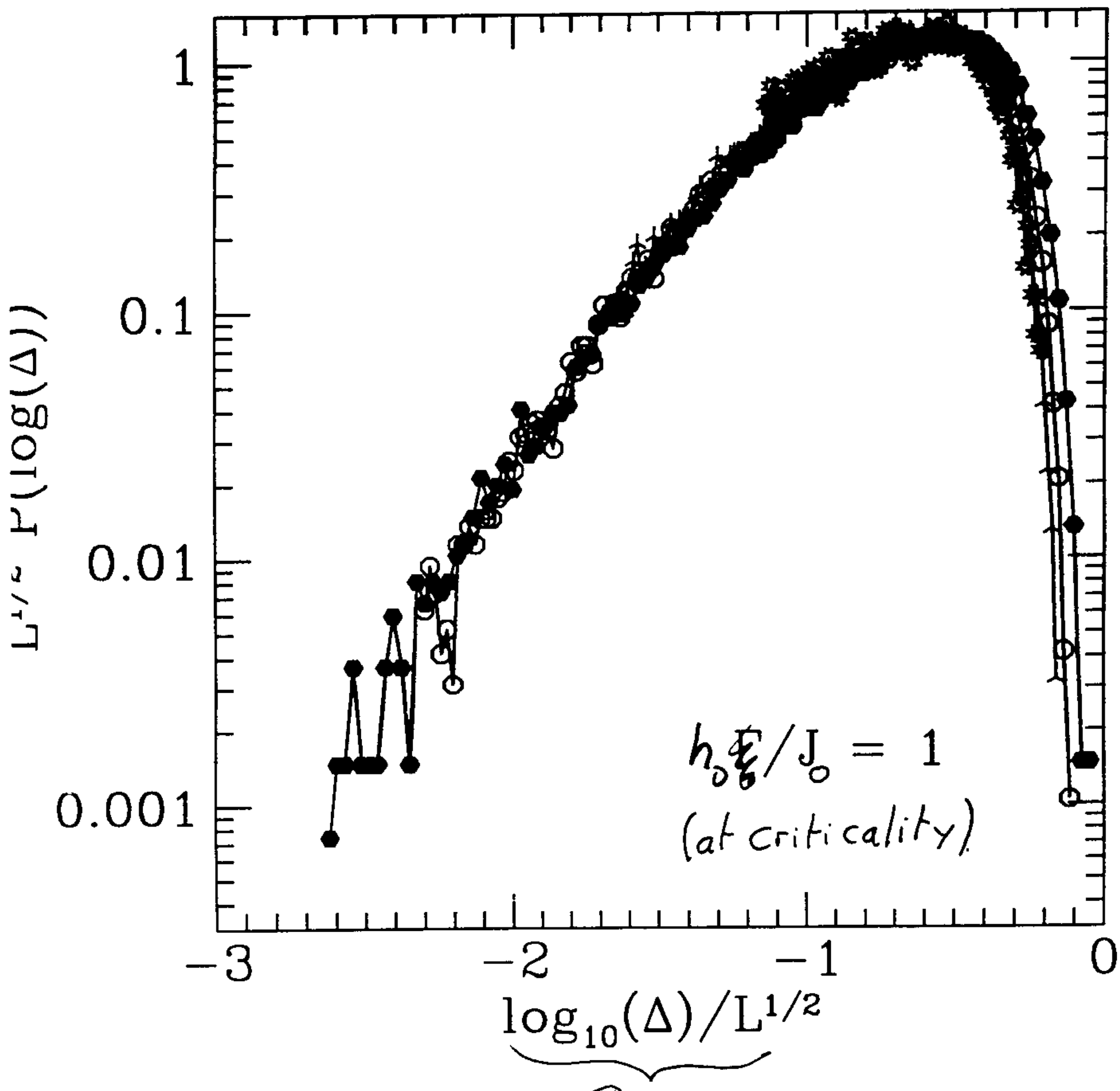
d=1 random transverse Ising model



Δ is the gap

about 50,000 samples

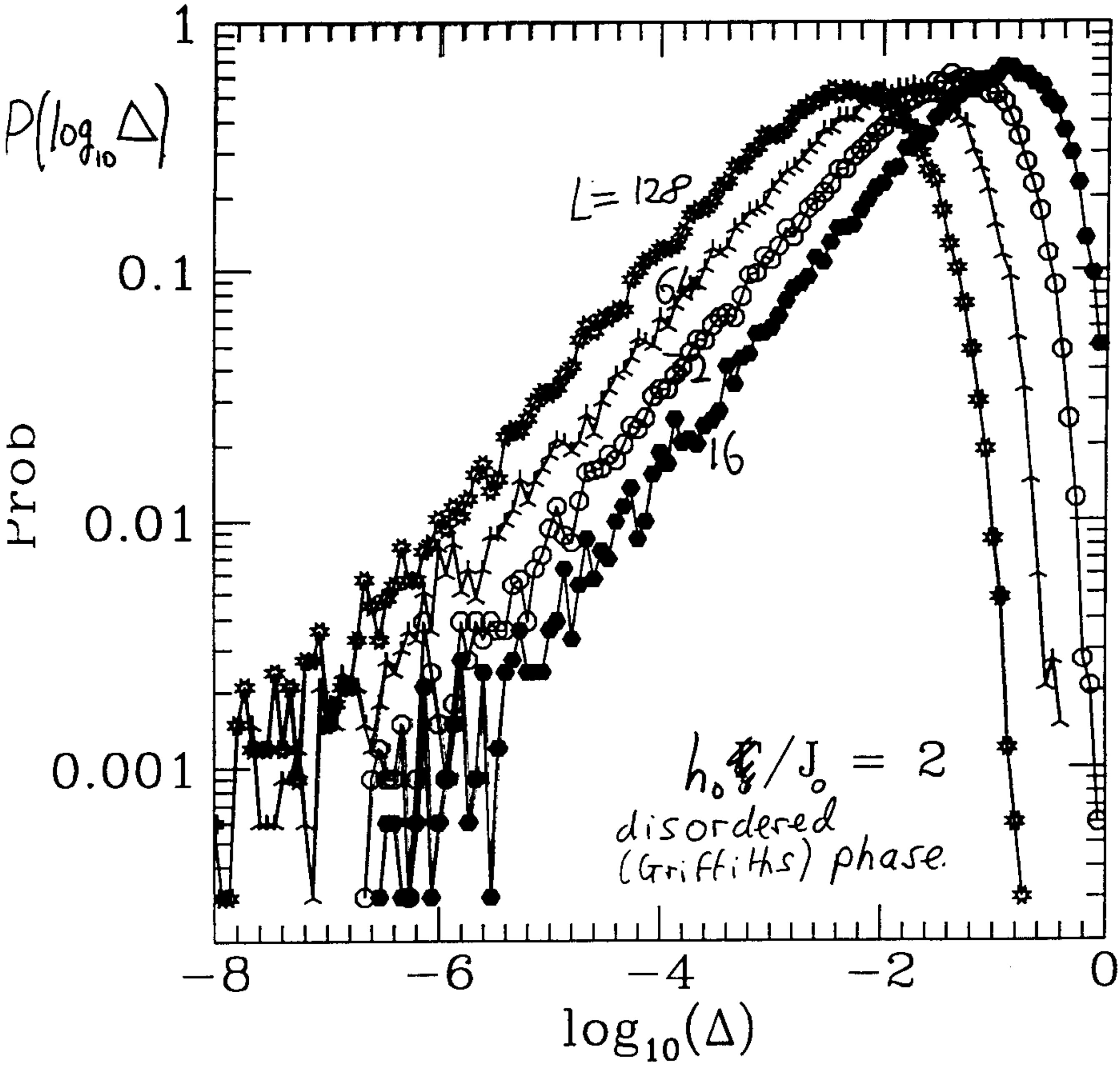
d=1 random transverse Ising model



n.b. $z = \infty$

50,000 samples

d=1 random transverse Ising model

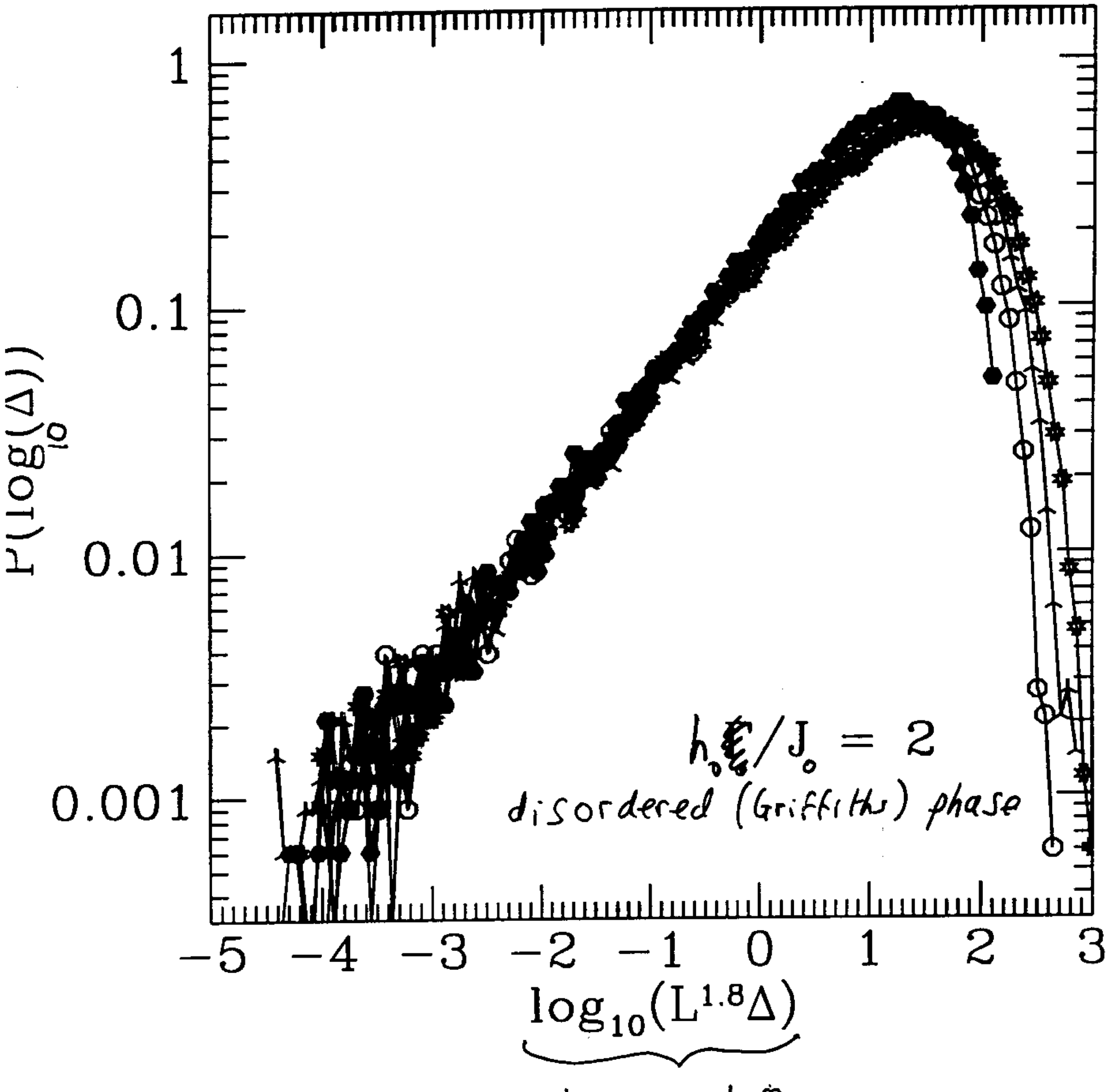


about 50,000 samples

Disordered Phase

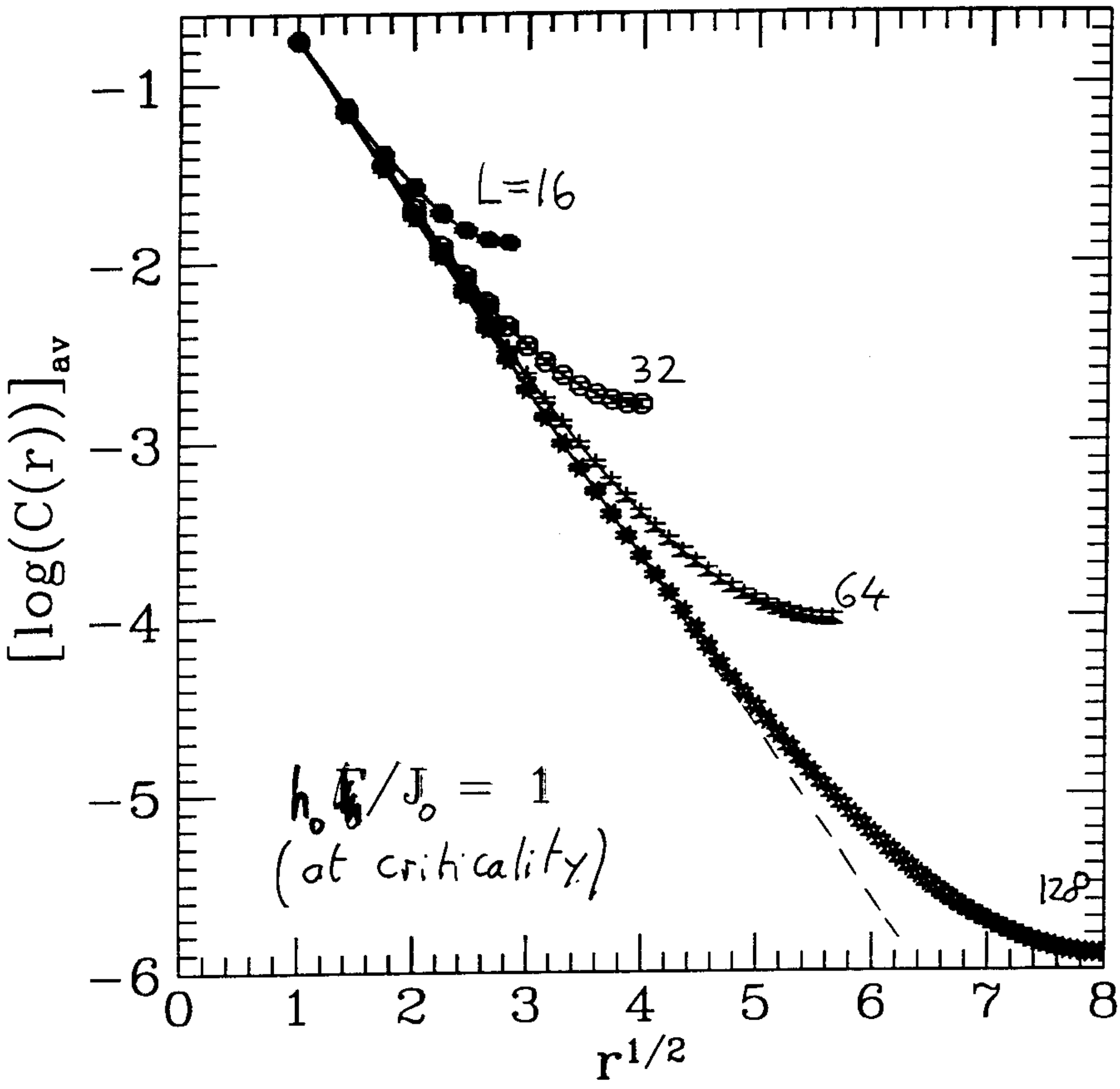
$[\chi_{loc}]_{av} \sim \left[\frac{1}{\Delta}\right]_{av}$ which diverges here

d=1 random transverse Ising model



Average of $\log(C(r)) \sim \log(C_{typ}(r))$

d=1 random transverse Ising model



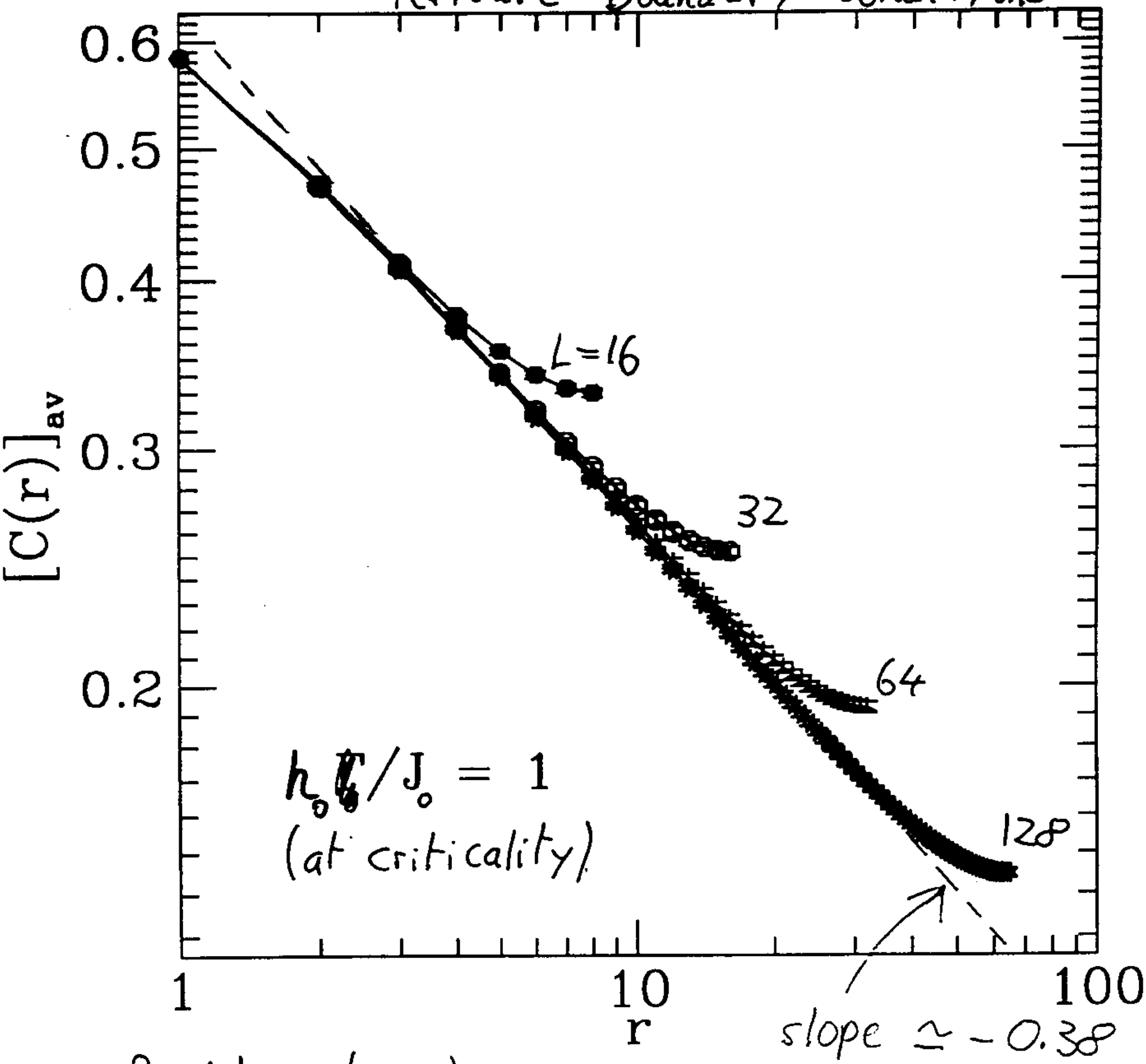
$[\log C(r)]_{av} \sim -r^{1/2}$ about 10,000 samples

so $C_{typ}(r) \sim e^{-c r^{1/2}}$

Average of C(r) (log-log plot)

d=1 random transverse Ising model

periodic Boundary conditions



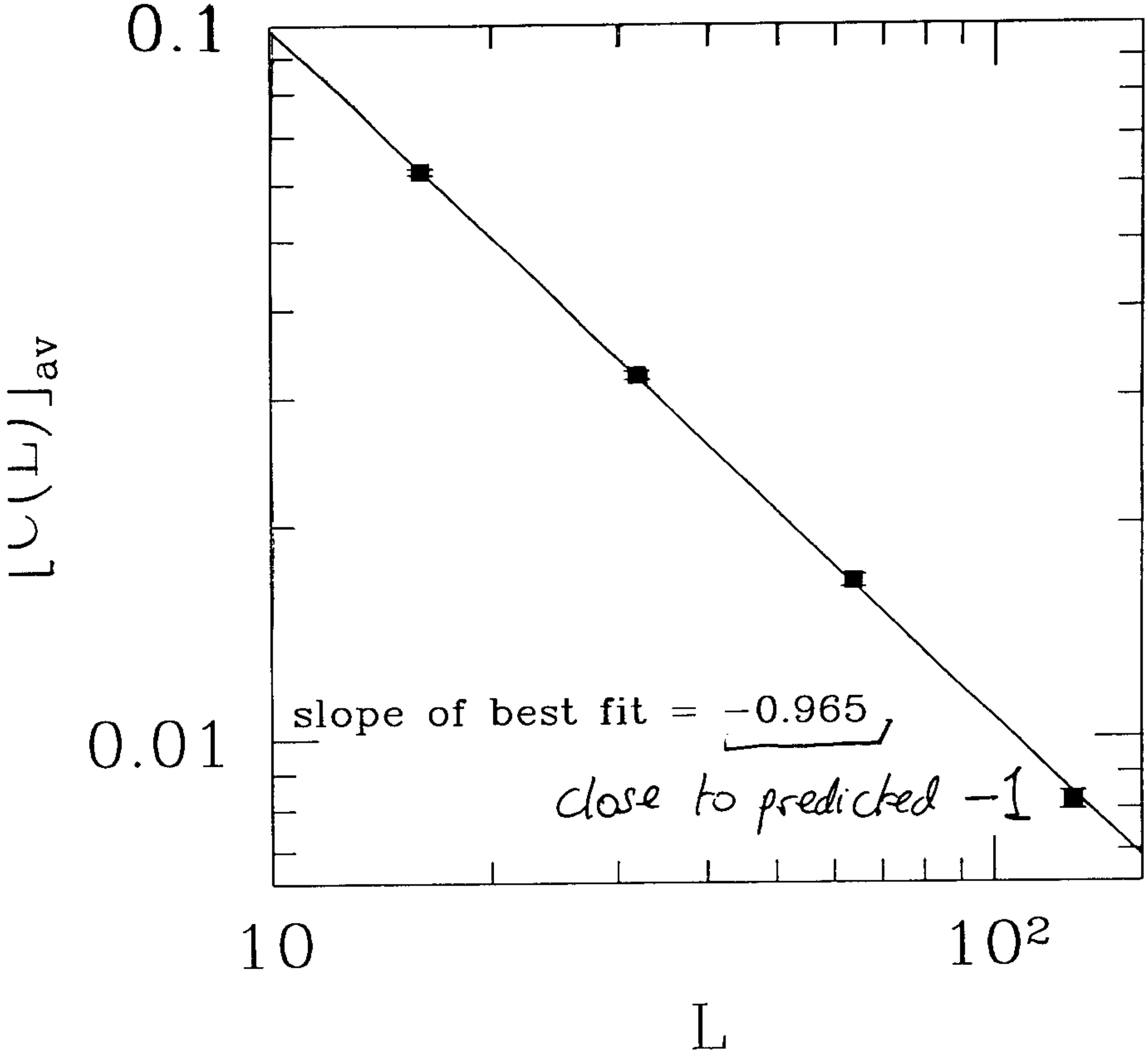
Predition (DSF)

$2-\phi$ where $\phi = \frac{1+\sqrt{5}}{2} \simeq 1.62$ is the golden mean

about 10,000 samples

Average (50,000 samples)

End to end correlation

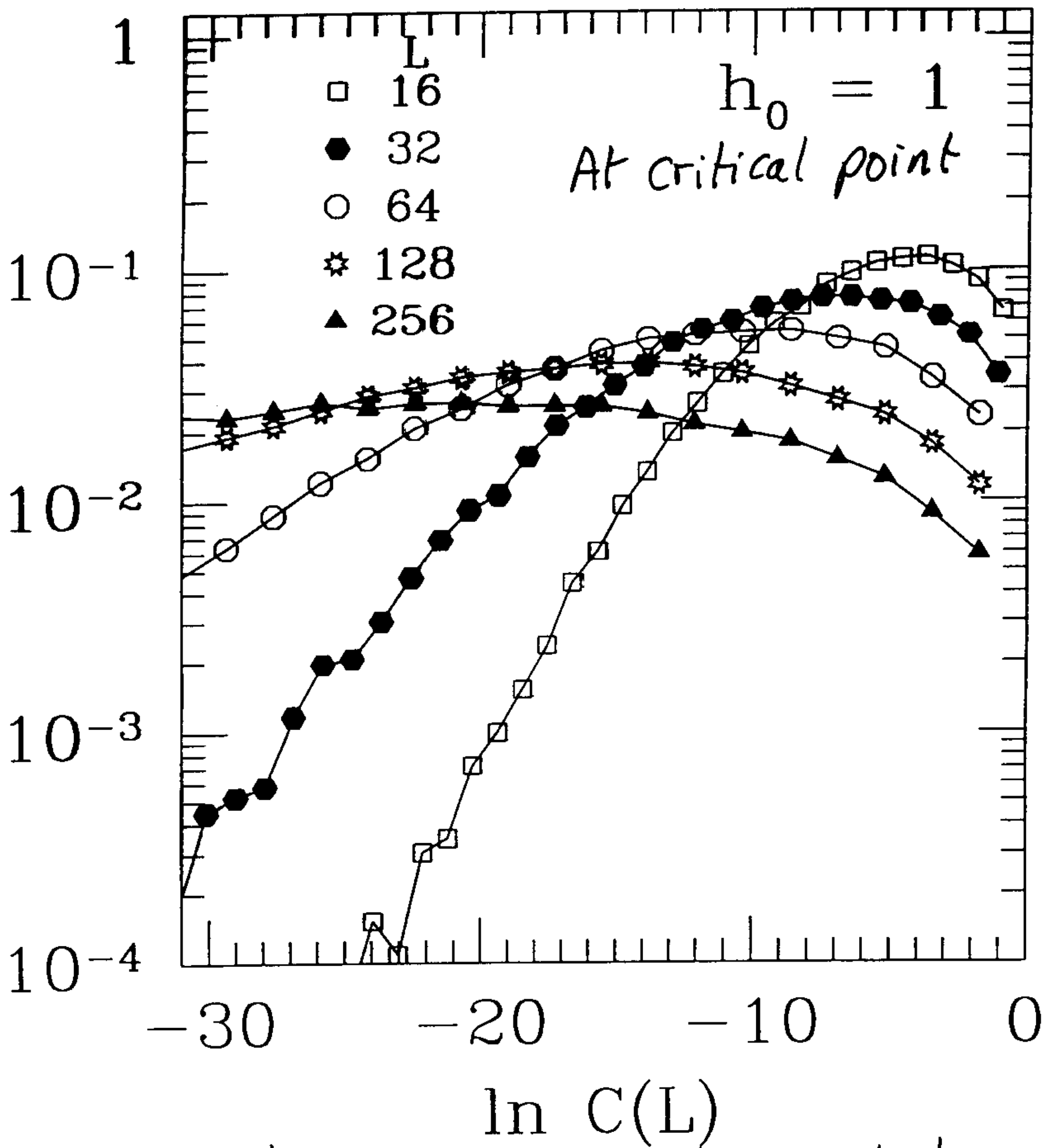


1-d random transverse Ising chain at criticality

$[C(L)]_{av} \sim \frac{1}{L}$ but $[C(L)]_{typ} \sim e^{-cL^{1/2}}$

50,000 samples

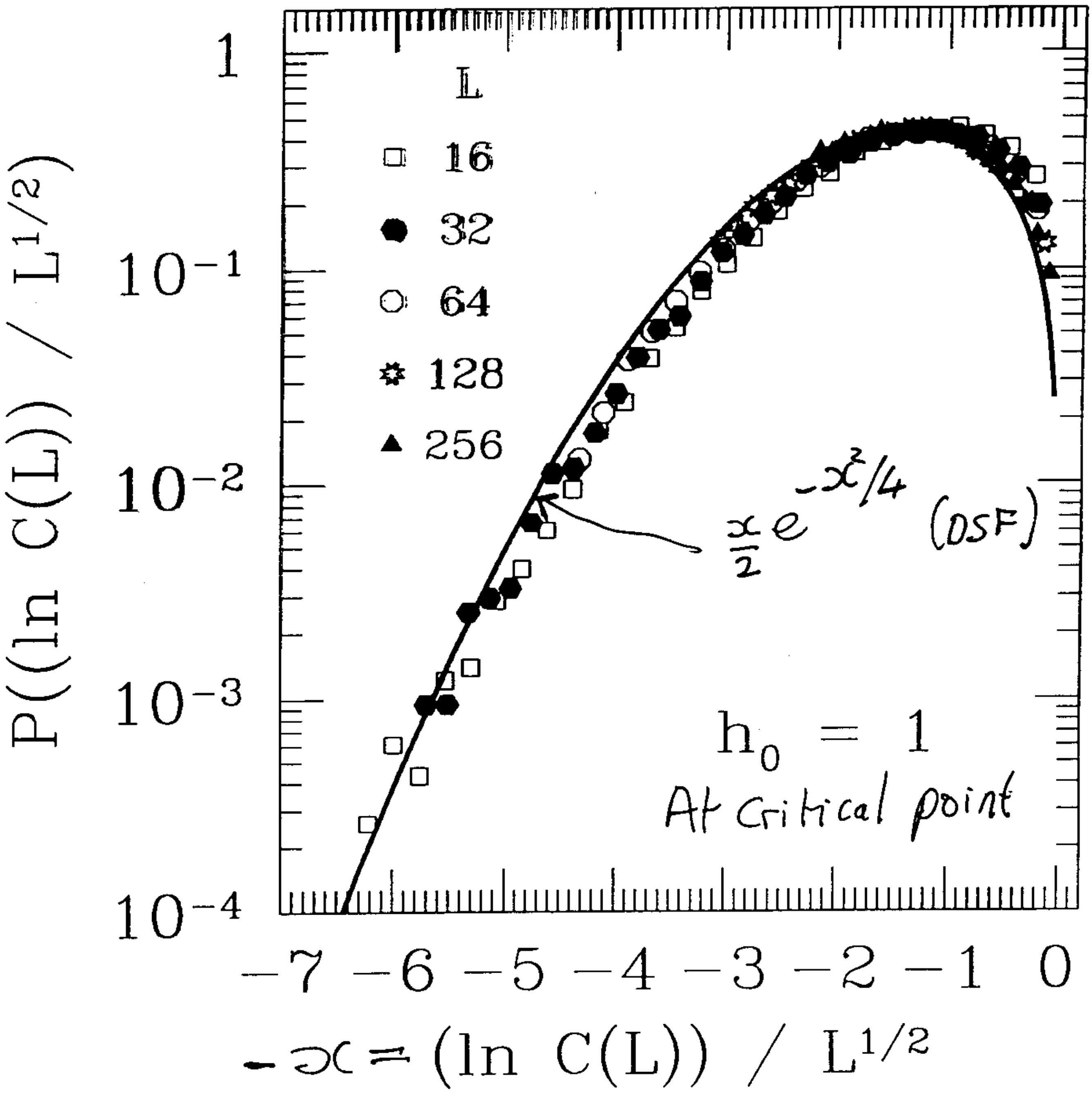
Free boundary conditions

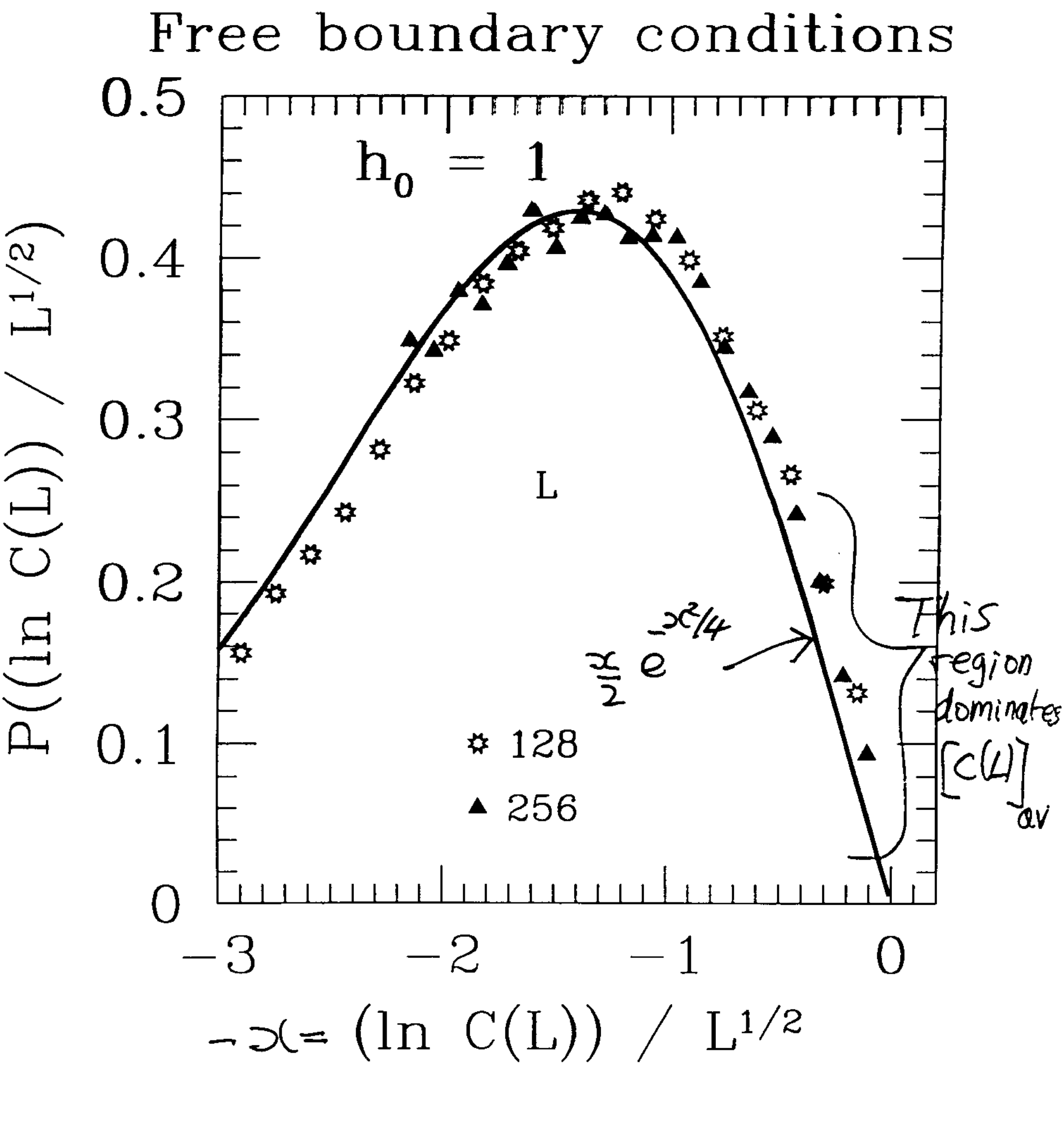


Notice how broad are the distributions

50,000 Samples

Free boundary conditions




Free boundary conditions
$h_0 = 1$
L
$\frac{x}{2} e^{-x^2/4}$
This region dominates $[C(L)]_{av}$
128
256
$P((\ln C(L)) / L^{1/2})$
$-x = (\ln C(L)) / L^{1/2}$
0.5
0.4
0.3
0.2
0.1
0
−3
−2
−1
0

Summary (1-d)

- Distributions very broad $\Rightarrow$ lose a lot of information by averaging.
- There is a Griffiths phase where distribution of energy gaps or local susceptibility is a power law.
- average susceptibility ~~may~~ diverges over part of the Griffiths phase.
- Griffiths phase is like a line of critical points with a continuously varying dynamical exponent, $z(\delta)$.
- $z \to 0$ at end of Griffiths phase.

  In 1-d, $z \to \infty$ at criticality.

What about higher-d ?

Quantum Spin Glass

Quantum Monte Carlo

(2+1)-d   H. Rieger + APY

(3+1)-d   Guo, Bhatt + Huse

- $z \neq 1$ (at criticality)
  $\Rightarrow$ anisotropic scaling.
- Griffiths phase $\rightarrow$ power laws
  continuously varying $z(\delta)$

But effects less spectacular than in 1-d

- $z \neq \infty$ at critical point
- Only small range of δ where $[\chi_{nl}]_{av}$ diverges
- No evidence for 2 correlation length exponents
- Unclear if rare regions dominate the critical behavior.