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MINIWORKSHOP ON STRONG ELECTRON CORRELATIONS
"Disorder and Interaction in Quantum Systems and Their Classical Analogs"

(1 - 19 July 1996)

"Spin Order in (Some) Quantum Magnets with Quenched Disorder"

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These are preliminary lecture notes, intended only for distribution to participants.

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SPIN ORDER IN (SOME) QUANTUM MAGNETS WITH QUENCHED DISORDER

—

Ravin Bhatt

I. INTRODUCTION : Quantum Antiferromagnets with large randomness.

- The a.f. nearest neighbor spin ½ chain
- Higher dimensional analogs.

*,† II. EFFECT OF BOND DISORDER IN GAPPED SPIN CHAINS

- The dimer spin ½ chain
- The spin-1 chain
- Majumdar-Ghosh nnn spin ½ chain

* III A HIGHLY DISORDERED SPIN SYSTEM IN 3 DIMENSIONS – The strange case of doped diluted magnetic semiconductors.

* coll: KUN YANG
ROSS HYMAN
STEVE GIRVIN

* coll: ADAM DURST
P. A WOLFF

† INSPIRED BY ICTP SPRING COLLEGE 1994.

THE ANTIFERROMAGNETIC n.n. SPIN-$\frac{1}{2}$ CHAIN (HEISENBERG)

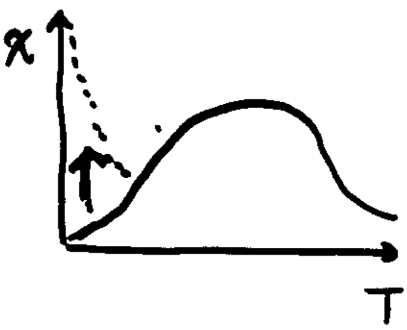
$$H = \sum_i J\,(\vec{S}_i \cdot \vec{S}_{i+1}) \qquad J>0$$

Bethe ansatz soln. Power law correlations. (Critical)

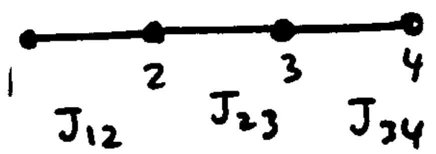
Random bonds $\quad J \rightarrow J_i\ (>0)\ P(J)$

Studied by Dasgupta & Ma (79), Jose & Hirsch (80)
Bondeson & Soos (80), Bhatt & Lee (81) ... etc

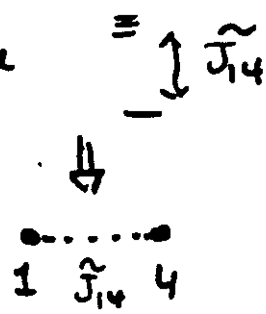
$\Rightarrow$ Divergent susceptibility at low T, for arbitrarily weak disorder



Physics:

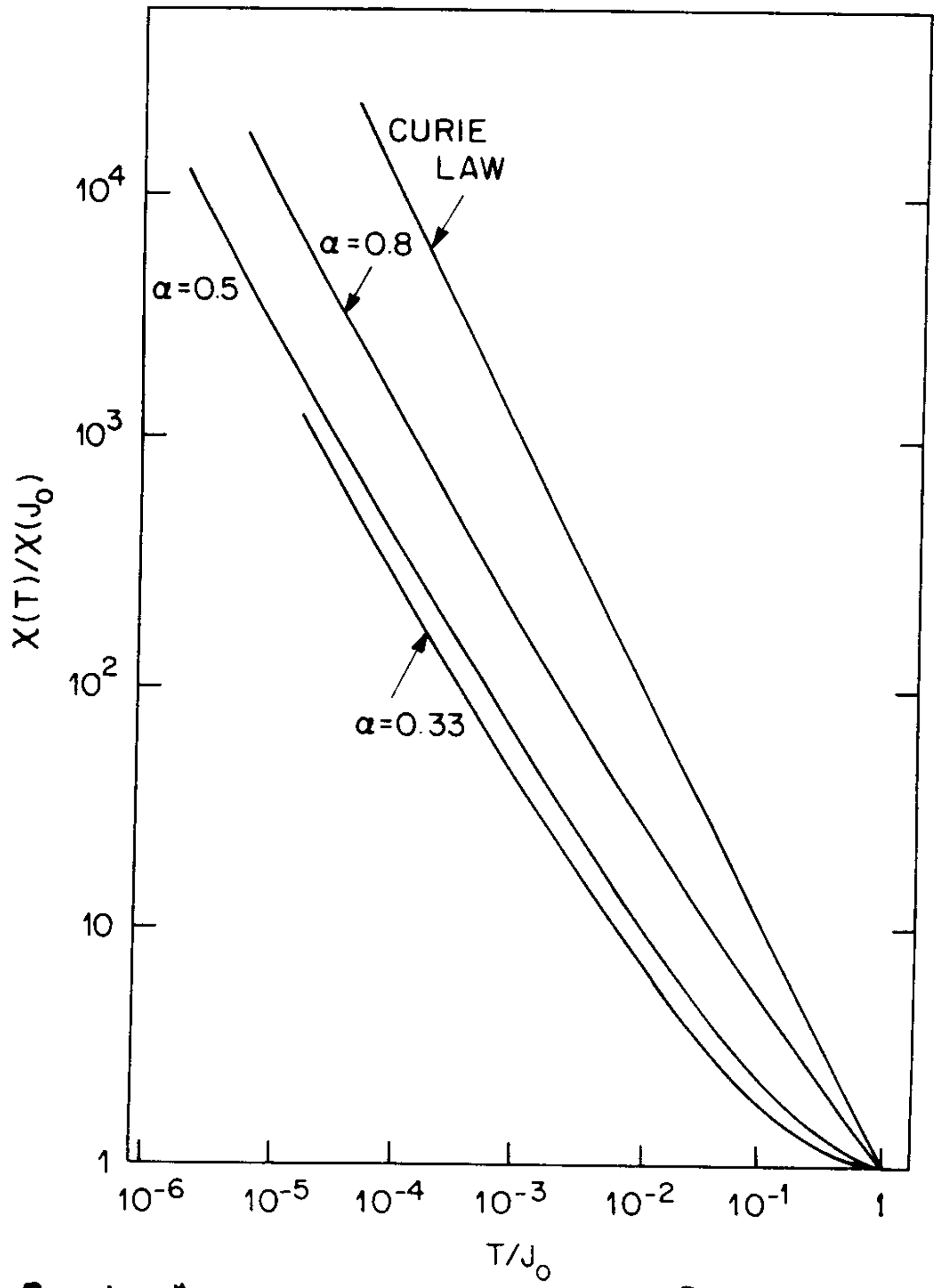


If $J_{23} \gg J_{12}, J_{34}$, low energy excitation structure



$$\tilde{J}_{14} = \frac{J_{12}J_{34}}{2J_{23}} \quad \text{(very small)}$$

Bhatt & Lee, J. Appl. Phys 52, 1703 (81)

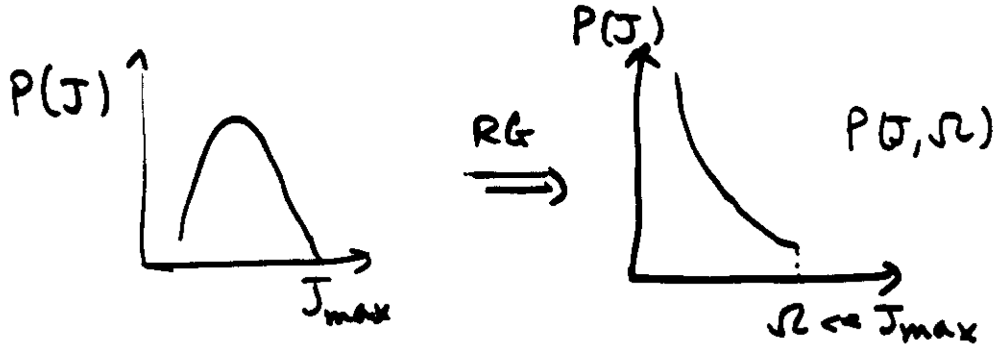


Similar ~~fo~~ results obtained by Dasgupta & Ma, and by Jose and Hirsch.

R.G. procedure generates weak bonds

$P(J)$ broad on logarithmic scale

$$\left(\ln \tilde{J}_{14} = \ln J_{12} + \ln J_{34} - \ln J_{23} - \ln 2 \right)$$



$$P(J,\Omega) \sim J^{-1+\lambda}$$

$$\lambda \simeq \frac{1}{|\ln \Omega|} \to 0$$

⇒ Random Singlet or Valence-Bond-Glass Phase

Thermo**dy**namics : Decimate till $\Omega \sim T$

$$N_{spins} \sim \frac{1}{|\ln^2 T|}$$

$$\chi \sim \frac{1}{T|\ln^2 T|} \quad ; \quad C \sim \frac{1}{|\ln^3 T|} \qquad \text{universal}$$

Detailed analysis by Fisher (94)

$$\langle \vec{S}_i \cdot \vec{S}_j \rangle \sim \frac{1}{|i-j|^2} \qquad \text{universal}$$

Experiments Qn$(TCNQ)_2$, NMP TCNQ, Acridinium-TCNQ etc

$\chi \sim T^{-\alpha}$ α dependent on sample (0.6-0.8)

ANALOG IN THREE DIMENSIONS

Doped semiconductors in dilute limit

Localized, hydrogenic bound states, with Bohr radius >> lattice constant.
~ 30 Å $\quad\quad \sim$ Å

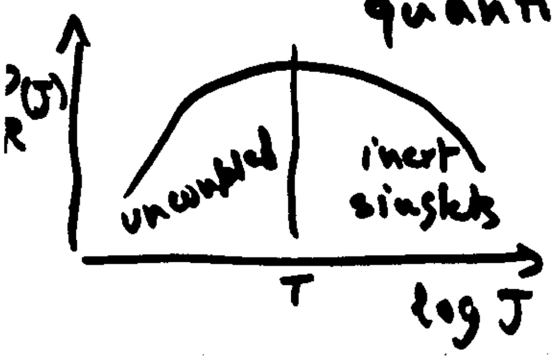
For $n \ll n_c \, (\sim a_B^{-3})$, effective Heisenberg Hamiltonian (for uncompensated case),

$$\mathcal{H} = \sum_{i<j} J(\vec{R}_{ij})\, \vec{S}_i \cdot \vec{S}_j$$

with $J(\vec{R}) \sim \exp(-2R/a_B)$, and ANTIFERRO.
↑
Short range

Physics (Bhatt + Lee) —
Decimate strongly coupled pairs, generate effective couplings for low T

Results: Broad distribution of J, remains broad, can calculate thermodynamic quantities perturbatively

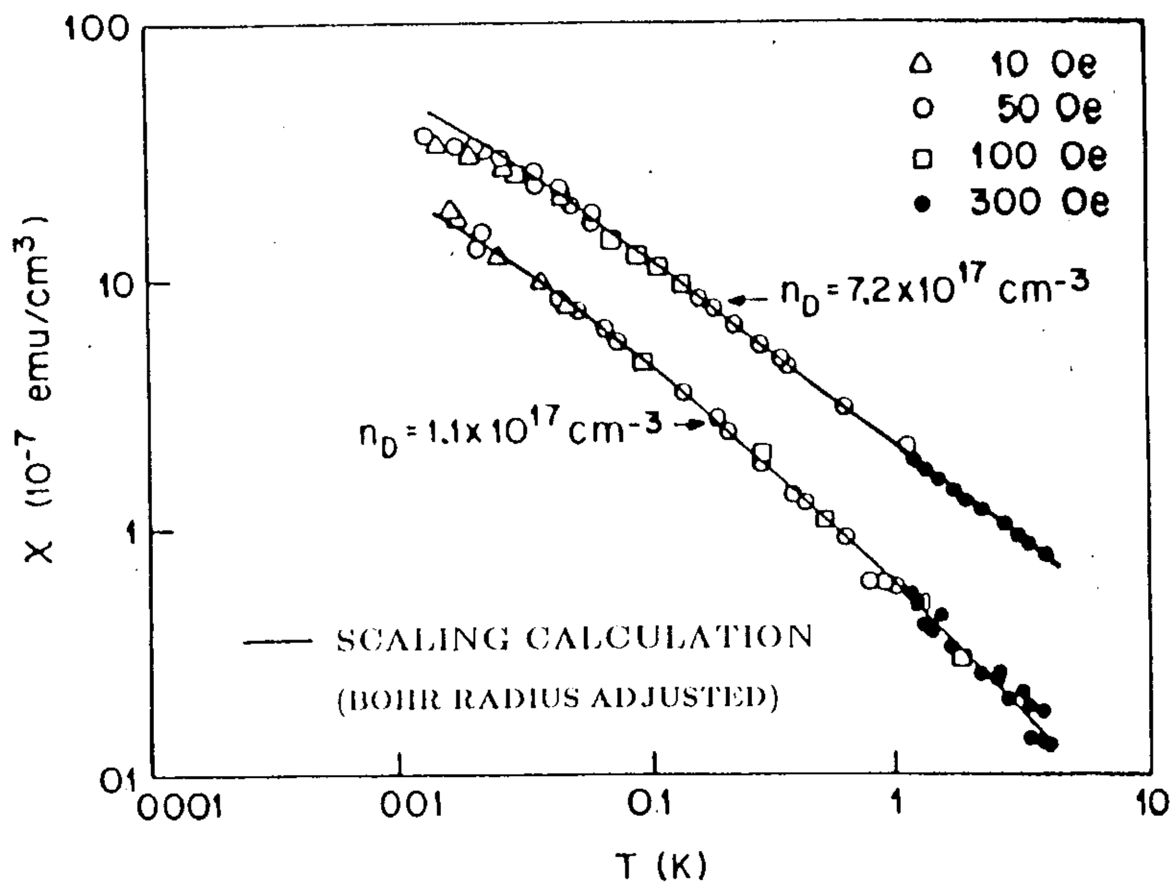


Renormalized distribution of pairs
(Random Singlet)

SEQUENCE NO.

TOP
DO NOT AFFIX OVERLAYS ALONG THIS SURFACE

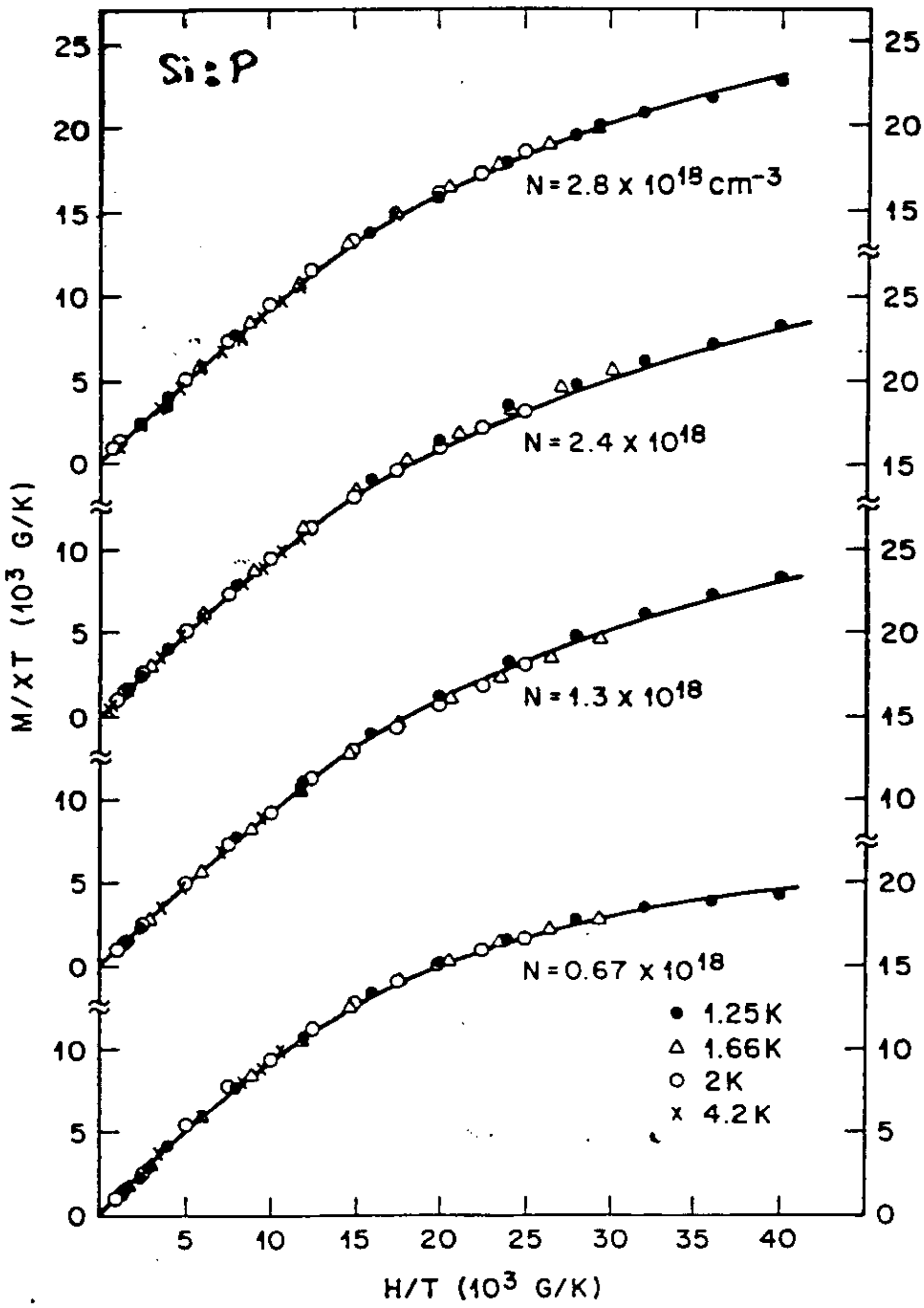
NOTES:



Data: K. Andres et al, Phys Rev B 24, 244 (81)
Theory: Bhatt & Lee, PRL 48, 344 (82)

AT&T Bell Laboratories

Sarachik et al (CCNY) + R.N.B. (Bell)

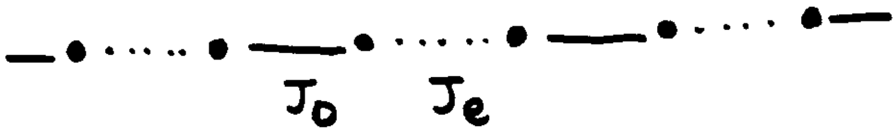


Sarachik et. al., Phys. Rev. B 34, 387 (1986)

$M/\chi T = f_\alpha (H/T)$

II. GAPPED SPIN CHAINS

- SPIN - 1 AF CHAIN (Haldane 83)
- DIMERIZED SPIN-½ CHAIN



- (nn + nnn) SPIN-½ CHAIN (Majumdar-Ghosh) 1969

$$\mathcal{H} = J \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{2} \vec{S}_i \cdot \vec{S}_{i+2} \right)$$

SPONTANEOUSLY DIMERIZED GROUND STATE

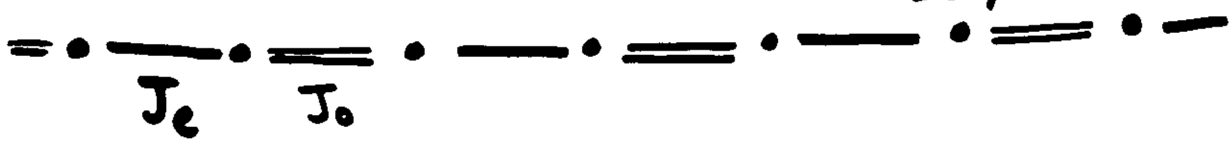
ADD RANDOMNESS:

Naive Expectation: Models have gaps, no effect at low T for small randomness, then Random Singlet Phase at high randomness.

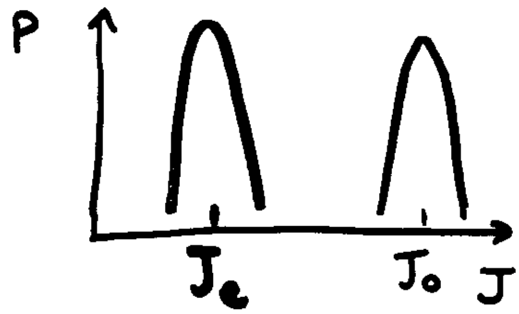
Caution (Haldane): Non-trivial Topological Order.

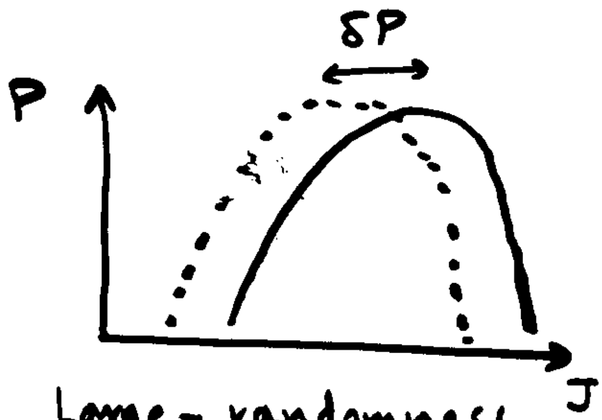
RANDOM DIMERIZED SPIN-½ CHAIN

[Hyman, Yang, Bhatt & Girvin PRL 76, 839(96)]



Small Randomness – Gap remains
(non-overlapping dist.)





Large-randomness
overlapping distributions
What happens?

Look at effect of
δP on RG procedure

$$\frac{d\,\delta P}{d|\ln\Omega|} = +\,\delta P \quad \Rightarrow \text{RELEVANT !}$$

Basically in RG procedure

the middle (large) bond more likely to be odd, leaving very weak even bond instead of two moderate even bonds.
P_{even} scales to zero coupling faster than P_{odd}, leaving uncoupled dimers.

In the end, a soluble RANDOM DIMER model
(dist. determined by initial dist.)

For small initial dimerization(δ) can show

$$P_{odd}(J) \sim J^{-1+\gamma} \qquad \gamma \propto \delta \ll 1$$

$\rightarrow$ distance from criticality

$$\chi(T) \sim T^{-1+\gamma}$$

$$C(T) \sim T^{\gamma}$$

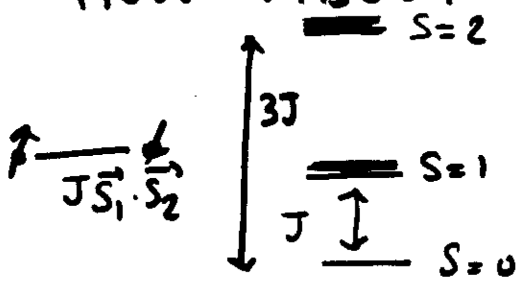
$$\langle \vec{S}_i \cdot \vec{S}_j \rangle \sim e^{-|i-j|/\xi} \qquad \xi \sim |\delta|^{-2}$$

$$\Rightarrow \nu = 2 \ \left(= \frac{2}{d}\right)$$

cf. $\nu = \frac{2}{3}$ without randomness

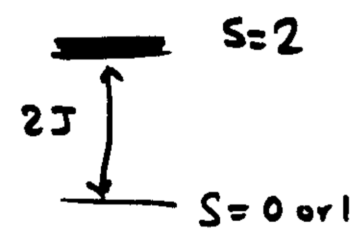
In random spin-$\frac{1}{2}$
$\langle \vec{S}_i \cdot \vec{S}_j \rangle \sim |i-j|^{-2}$
uniform spin-$\frac{1}{2}$
$\langle \vec{S}_i \cdot \vec{S}_j \rangle \sim |i-j|^{-1/2}$

HOW ABOUT SPIN-1 CHAIN?



AKLT model

$$J(\vec{S}_1 \cdot \vec{S}_2) + \frac{J}{3}(\vec{S}_1 \cdot \vec{S}_2)^2$$



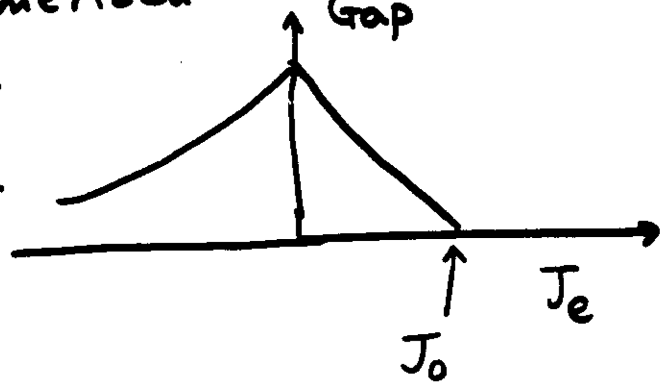
Can try to decimate S=2 state. Looking at various cases, appears to be similar to dimerized spin-$\frac{1}{2}$ case

Reason for similarity - Same Phase
(Hida 92)

$0 < J_e < J_o$ spin-$\frac{1}{2}$ dimerized

$J_e = 0$ trivial dimer

$J_e = -\infty$ spin-1 chain



Conjecture: Same results
(numerical work underway).

TOPOLOGICAL ORDER (Haldane)

SPIN-1 $\langle \vec{S}_i \cdot \vec{S}_j \rangle \sim (-1)^{i-j} \, e^{-|i-j|/\xi}$

Structure + followed by 0's & -, not another +

$+ - 0 0 + 0 0 - 0 0 0 +$, $+ 0 0 0 +$
okay, not okay

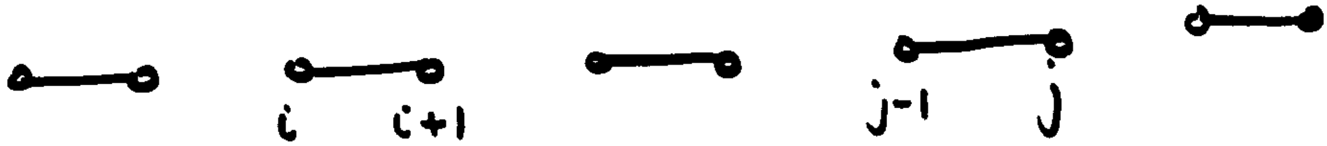
$$T_{ij} = \langle S_i^z \exp[i\pi \sum_{i<k<j} S_k^z] S_j^z \rangle \underset{|i-j| \to \infty}{=} \text{const} \quad \left(-\frac{4}{9}\right)$$

DIMERIZED SPIN-$\frac{1}{2}$

eventual order

$\langle \vec{S}_i \cdot \vec{S}_j \rangle \underset{|i-j| \to \infty}{=} 0$ T_{ij} = non zero for i odd, j even
zero otherwise. SURVIVES RANDOMNESS!

In Dimerized Spin-$\frac{1}{2}$ chain



$$\langle \vec{S_i} \cdot \vec{S_j} \rangle = 0 \quad \text{for } |i-j|>1 \qquad \xi = 0$$

$$T_{ij} = \langle S_i^z \, e^{i\pi \sum_{i<k<j} S_k^z} \, S_j^z \rangle$$

If i odd, j even

$$T_{ij} = \langle S_i^z \, e^{i\pi S_{i+1}^z} \, e^{i\pi S_{j-1}^z} \, S_j^z \rangle$$

$$= -4 \langle S_i^z S_{i+1}^z S_{j-1}^z S_j^z \rangle = -\tfrac{1}{4} \neq 0$$

$(T_{ij} = 0$ otherwise$)$

With randomness

$$T = \lim_{|j-i| \to \infty} \left(\overline{T}_{2i,2j-1} - \overline{T}_{2i+1,2j} \right)$$

$$\sim -|\delta|^{2\beta} \operatorname{sign}(\delta) \qquad \beta = 2$$

$\Rightarrow$ T order exists all throughout for dimerized chain

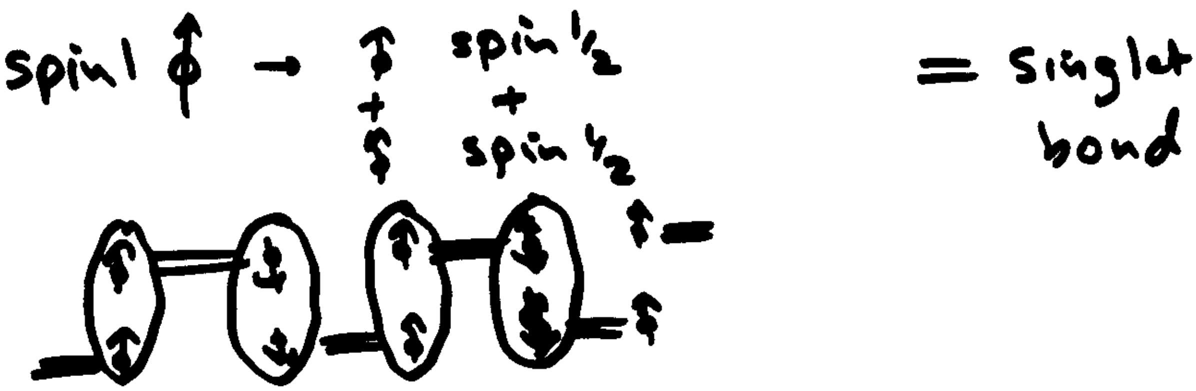
a

In SPIN-1 case, look at AKLT model:

$$H = \sum_i J_i \left[\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3} (\vec{S}_i \cdot \vec{S}_{i+1})^2 \right]$$

$J_i > 0$

Ground state identical for all J_i configurations



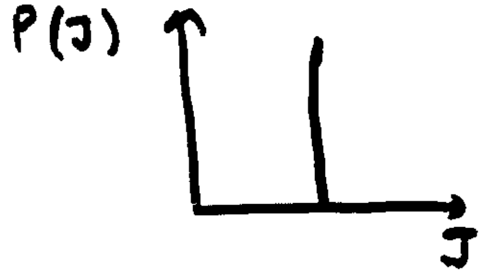
Thermodynamics (at low T also)

depends on $P(J_i)$

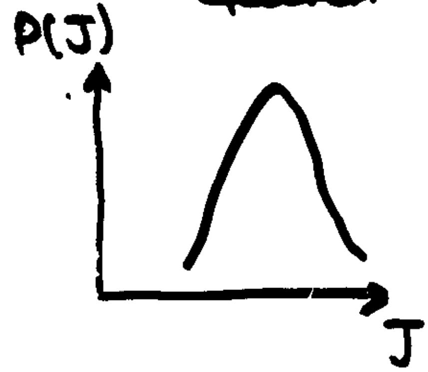
but ground state order persists.
topological

Is this generic ?

SPIN-1 CHAIN (Preliminary Results Hyman+Yang) [redacted]

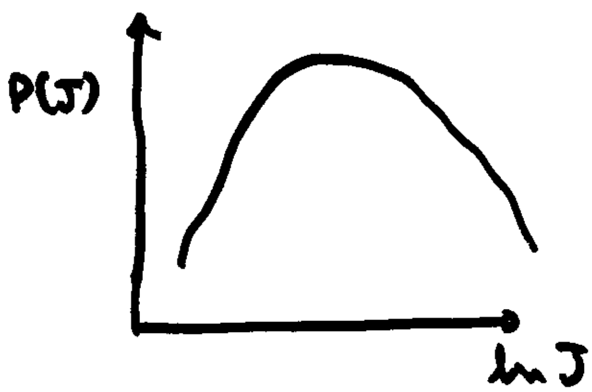


Haldane Gap
Topological Order
$\chi(T) \to 0$ as $T \to 0$
exponentially



at a Critical width
$\chi(T) \to \infty$ as $T \to 0$
__BUT__ Topological order persists
Non-universal low T behaviour $\chi(T) \sim T^{-\alpha}$
varying α

still broader P(J)

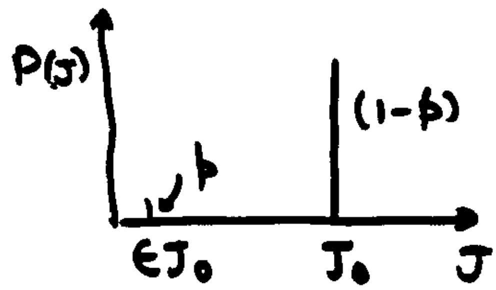


Random Singlet Phase
Topological order destroyed
$\chi(T) \to \infty$ as $T \to 0$
$\sim (T \ln^2 \frac{1}{T})^{-1}$
Universal low T behaviour as in spin-1/2

Spin-1 chain with "diluted" bonds

$$P(J) = p\,\delta(J - \epsilon J_0) + (1-p)\,\delta(J - J_0)$$

Goes to "Griffiths" phase right away?

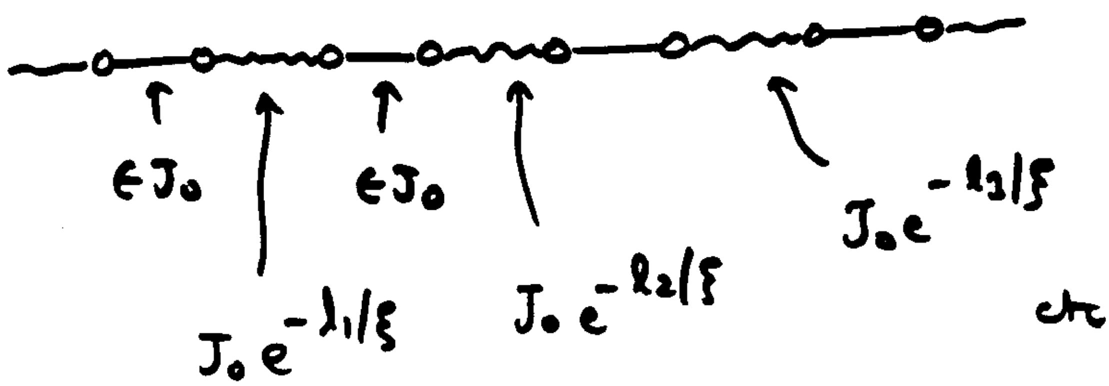


$\epsilon J_0 \ll J_0$

o – spin 1

o – spin-½

$\Rightarrow$



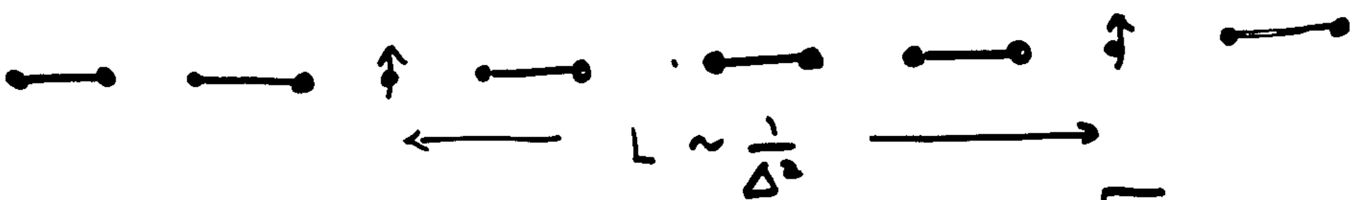
$\Rightarrow$ like dimerized spin-½ case

always in Griffiths phase $\chi(T) \sim T^{-\alpha}$

MAJUMDAR-GHOSH MODEL $H = \sum J (\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{2} \vec{S}_i \cdot \vec{S}_{i+2})$

Spontaneous Dimerization – two ground states

With randomness Δ, for arbitrarily weak Δ, create unpaired spins $\left(\begin{smallmatrix}\text{topological}\\ \text{solitons}\end{smallmatrix}\right)$ in regions between two phases



Volume energy gain $\propto \sqrt{L}$

$\Rightarrow$ UNSTABLE AGAINST WEAK RANDOMNESS

$$\chi \sim \frac{\Delta^2}{T |\ln^2 T|}$$

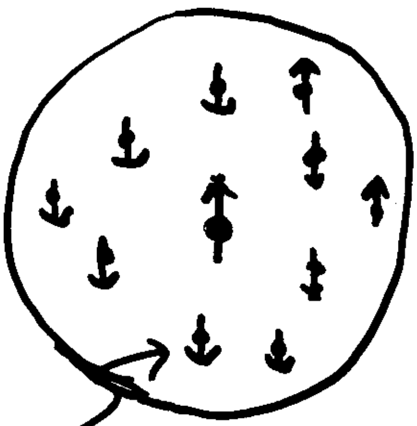
TOPOLOGICAL ORDER DESTROYED, IMMEDIATELY.

SINGLE MAGNETIC POLARON

Donor Electron – Bohr radius ~ 20 Å
Binding Energy ~ 20 meV

Scale of A.F. interaction with Mn
~ 20 K ~ 2 meV

Assume Small Mn conc. (neglect Mn-Mn int.)



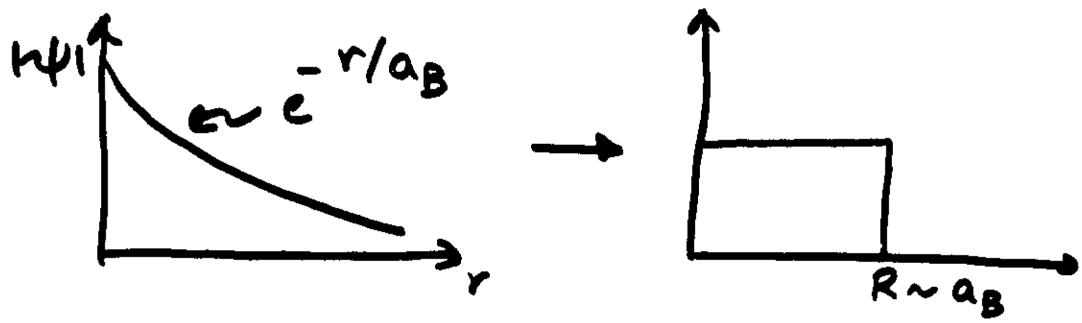
Typical # of Mn in wavefn ~ 20 - 50

$\updownarrow \; \vec{s} = 1/2 \qquad \downarrow \; \vec{S}_i = 5/2$

$$\mathcal{H} = \alpha \sum_i (\vec{s} \cdot \vec{S}_i) |\psi_i|^2$$

$$\downarrow$$

$$\mathcal{H}_0 = K \sum_{|\vec{R}_i| < R} \vec{s} \cdot \vec{S}_i$$



Justification of $\mathcal{H}_0$:

Feynman Variational Principle

$$F \le F_0 + \langle \mathcal{H} - \mathcal{H}_0 \rangle$$

Use K and R as variational parameters

minimize R.H.S. get "best" $\mathcal{H}_0(T)$.

Result: High T, Mn spins all free,

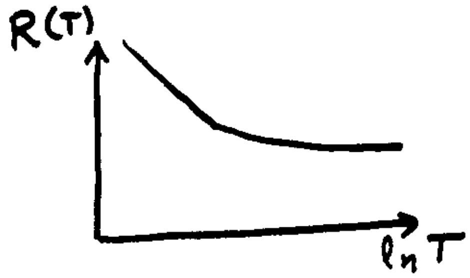
$\mathcal{H}_0$ tries to match wave fn. as best as it can. $R = 1.16\, a_B$.

$K \sim .06\, \alpha / a_B^3$

Low T: competition between entropy of free spins outside sphere, and energy gain due to exchange interaction within

expect $\alpha |\Psi(R)|^2 \sim k_B T$

or $R \sim a_B \ln\left(\frac{T_0}{T}\right)$



Reasonable.

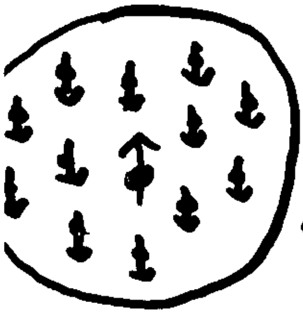
Soln of $\mathcal{H}_0 = K\,\vec{s}\cdot\vec{S}$ $\qquad \vec{S} = \sum_i (\vec{S}_i)$

QM: $\qquad H_0 = \frac{1}{2} K \left[(\vec{s}+\vec{S})^2 - \vec{s}^{\,2} - \vec{S}^{\,2} \right]$

Eigenvalues of $\vec{s}+\vec{S} = S \pm \frac{1}{2}$, Energies

$$E_0 = -\frac{K}{2}(S+1), \qquad +\frac{KS}{2}$$

$$(S-\tfrac{1}{2}) \qquad\qquad (S+\tfrac{1}{2})$$



← Ground state

Classically (S), with QM. (s) $\qquad E_0 = \pm \frac{KS}{2}$

Can solve at all T:

$$Z = 2 \iint d\Omega_1 \ldots d\Omega_N \cosh(\gamma S) \qquad S = \sqrt{\left(\sum_j \vec{S}_j\right)^2}$$

$$= 2\left[\mathcal{F}(\gamma)\right]^N \left[1 + \gamma \frac{N\,\mathcal{F}'(\gamma)}{\mathcal{F}(\gamma)} \right] \qquad \gamma = \beta\frac{K}{2}$$

$$\mathcal{F}(\gamma) = e^{5/2\,\gamma} + e^{3/2\,\gamma} + \ldots + e^{-5/2\,\gamma} \longleftarrow 2\pi \int_{-1}^{+1} e^{-\gamma S_z \mu}\, d\mu$$

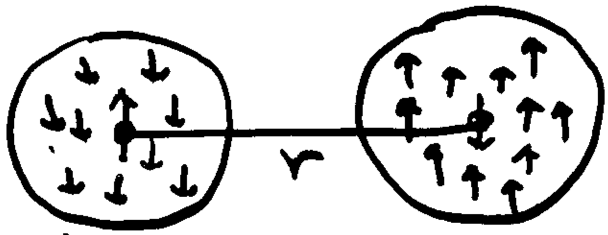
$\Rightarrow$ Can get all thermodynamic, magnetic quantities.

$$\longrightarrow \quad Z = 2 \int \mathcal{D}(S) \cosh(\gamma S)\, d^3S$$

TWO POLARONS

Include AF exchange between donor electron spins : $J\,\vec{s}_1\cdot\vec{s}_2$ $\quad J(\vec{r}) \sim e^{-2r/a_B}$

$$H = K(\vec{s}_1\cdot\vec{S}_1 + \vec{s}_2\cdot\vec{S}_2) + J\,\vec{s}_1\cdot\vec{s}_2$$



$(J \ll K)$

At low T, $\vec{S}_\mu$ fully polarized, i.e. $|\vec{S}_\mu| = S_{max} = S$

$$E = \underbrace{-2\cdot\frac{K(S+1)}{2}}_{\vec{S}_\mu \parallel \vec{s}_\mu} \pm \begin{cases} J/4 & \vec{s}_1, \vec{s}_2 \text{ parallel} \\ -3J/4 & \text{" " anti- "} \end{cases}$$

$\boxed{\mu = 1, 2}$

G. State $\Rightarrow S_{tot} = 0$ at $T = 0$

With same classical approx for $\vec{S}_\mu$, can solve exactly, within fully polarized approx.

$$Z = \iint e^{\frac{\beta K}{2}(S_1+S_2)}\,\delta(S_1)\,\delta(S_2)\,d^3S_1\,d^3S_2$$

(allow for diffrent angles bet. $\vec{S}_1$ & $\vec{S}_2$)

$$= \frac{1}{2} Z_1 Z_2 \,\frac{\sinh(\beta J/4)}{(\beta J/4)}$$

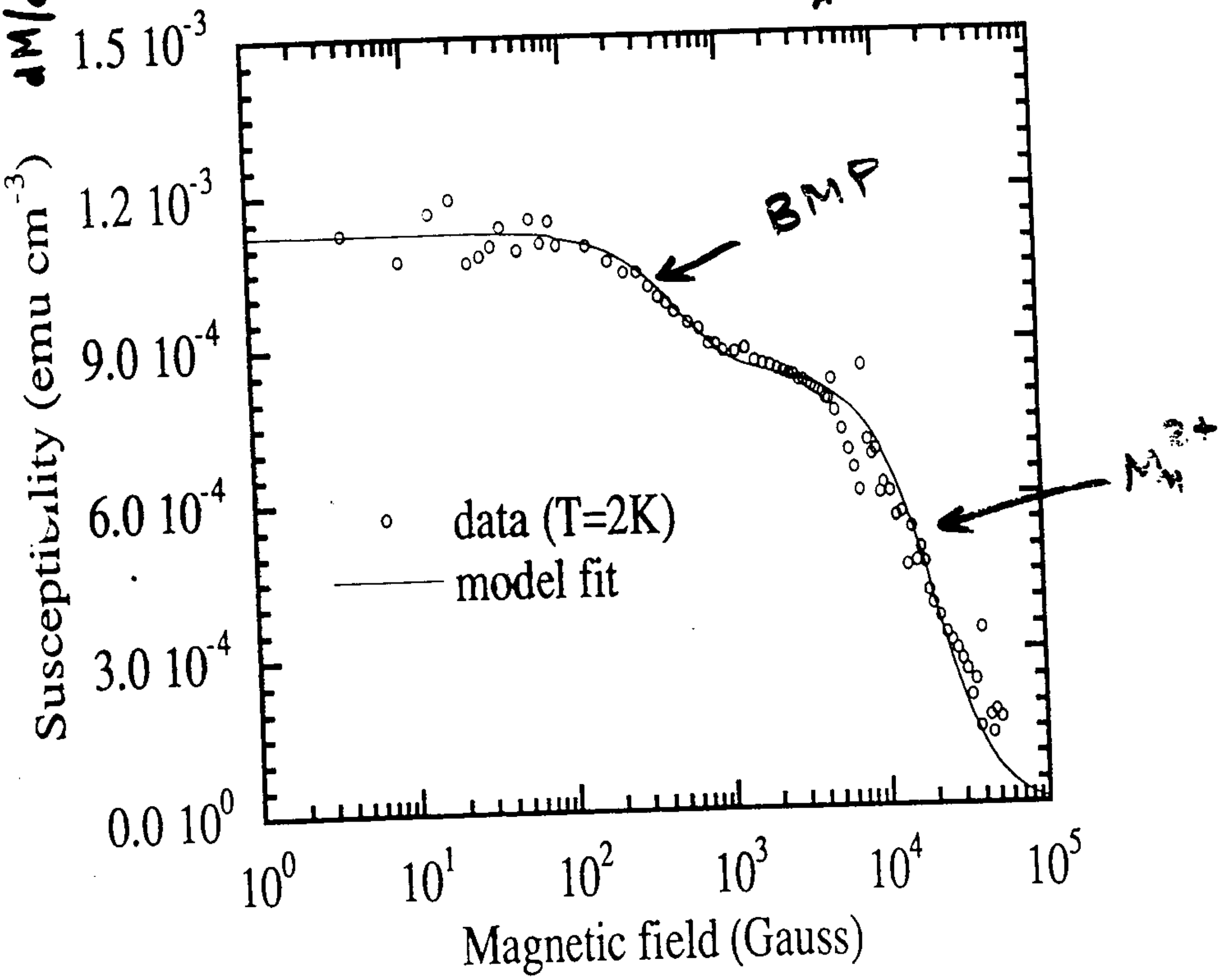
$$\Rightarrow \quad \langle \cos\theta_{12}\rangle = -\left[\coth\left(\frac{\beta J}{4}\right) - \frac{4}{\beta J}\right]$$

Exptl Data on p-type $Zn_{1-x}Mn_xTe : P$

J. Z. Liu, NEC

$x \sim 0.1$

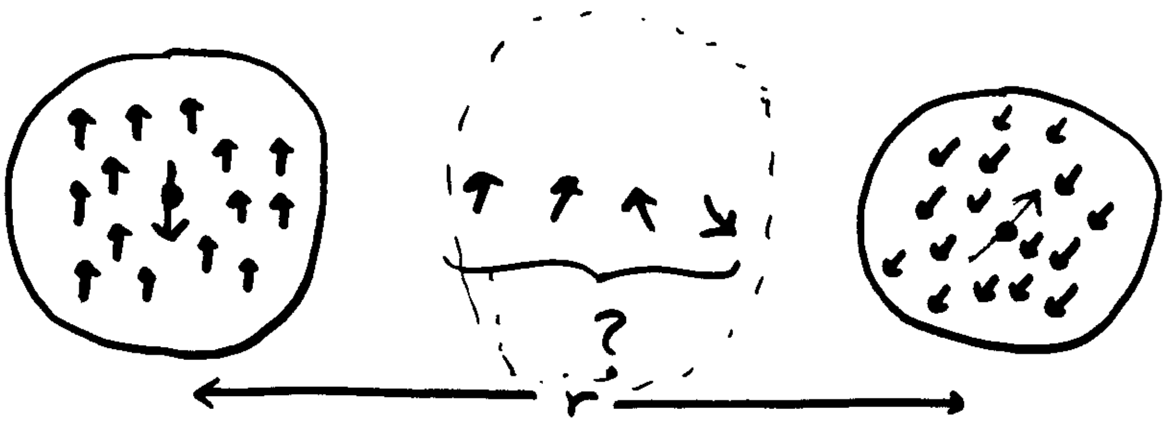
$n_A \sim 3\times10^{17}\ cm^{-3}$



Because of separation of field scales, can analyse BMP & Mn separately. (Langevin / Brillouin fn. with effective interaction as Curie temp.)

Experimental Data $\Rightarrow$ "Ferromagnetic interaction" between polarons, ie. $\chi \sim (T - T_0)^{-1}$ with $T_0 > 0$.

$\longrightarrow$ LEFT OUT INTERVENING Mn !



$$\mathcal{H} = \alpha \sum_i \left(\vec{s}_1 \cdot \vec{S}_i \, |\psi_{1i}|^2 + \vec{s}_2 \cdot \vec{S}_i \, |\psi_{2i}|^2 \right) + J \vec{s}_1 \cdot \vec{s}_2$$

$$\downarrow$$

$$\mathcal{H}_0 = K \sum_{R_{i1} < R} \vec{s}_1 \cdot \vec{S}_i + K \sum_{R_{j2} < R} \vec{s}_2 \cdot \vec{S}_j + J \vec{s}_1 \cdot \vec{s}_2$$

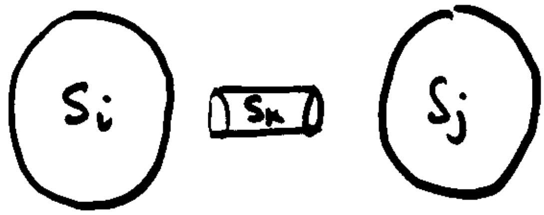
$$+ K' \sum_{\text{inter.}} \vec{S}_k \cdot (\vec{s}_1 + \vec{s}_2)$$

For large r, expect $K' \sim |\psi(\frac{r}{2})|^2 \sim e^{-r/a_B}$

$$\gg J \sim e^{-2r/a_B}$$

Q.M. can solve $\mathcal{H}_0$ with K' also. For ground state, $\sum_i S_i = S_{max} = \sum_j S_j$, $\sum_k S_k = S'_{max}$
$= S$ $\qquad = S'$

Two competing states



$\vec{s}_1 + \vec{s}_2 = 0$

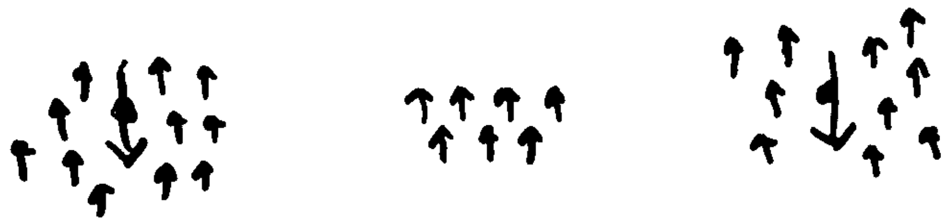
$$E = -K(S+1) - \frac{3J}{4}$$

$\vec{s}_1 + \vec{s}_2 = 1$, $\quad \vec{s}_1 + \vec{s}_2 + \sum_k \vec{S}_k = S' - 1$

$$E = -K(S+1) + \frac{J}{4} - K'(S'+1)$$

Latter wins when $\boxed{K'(S'+1) > J}$

Configuration:



$S_{tot} = 2S + S' - 1$

cf. $S_{ferro} = 2S + S' + 1$

Treating S, S' as classical, can solve with prior assumptions the 2 polaron problem:

$$Z = Z_1 Z_2 \int_0^{\sqrt{2}} e^{-\frac{\beta J}{4}(x^2-1)} \left(\mathcal{F}\left(\frac{\beta K' x}{\sqrt{2}}\right)\right)^{N_3} x\,dx$$

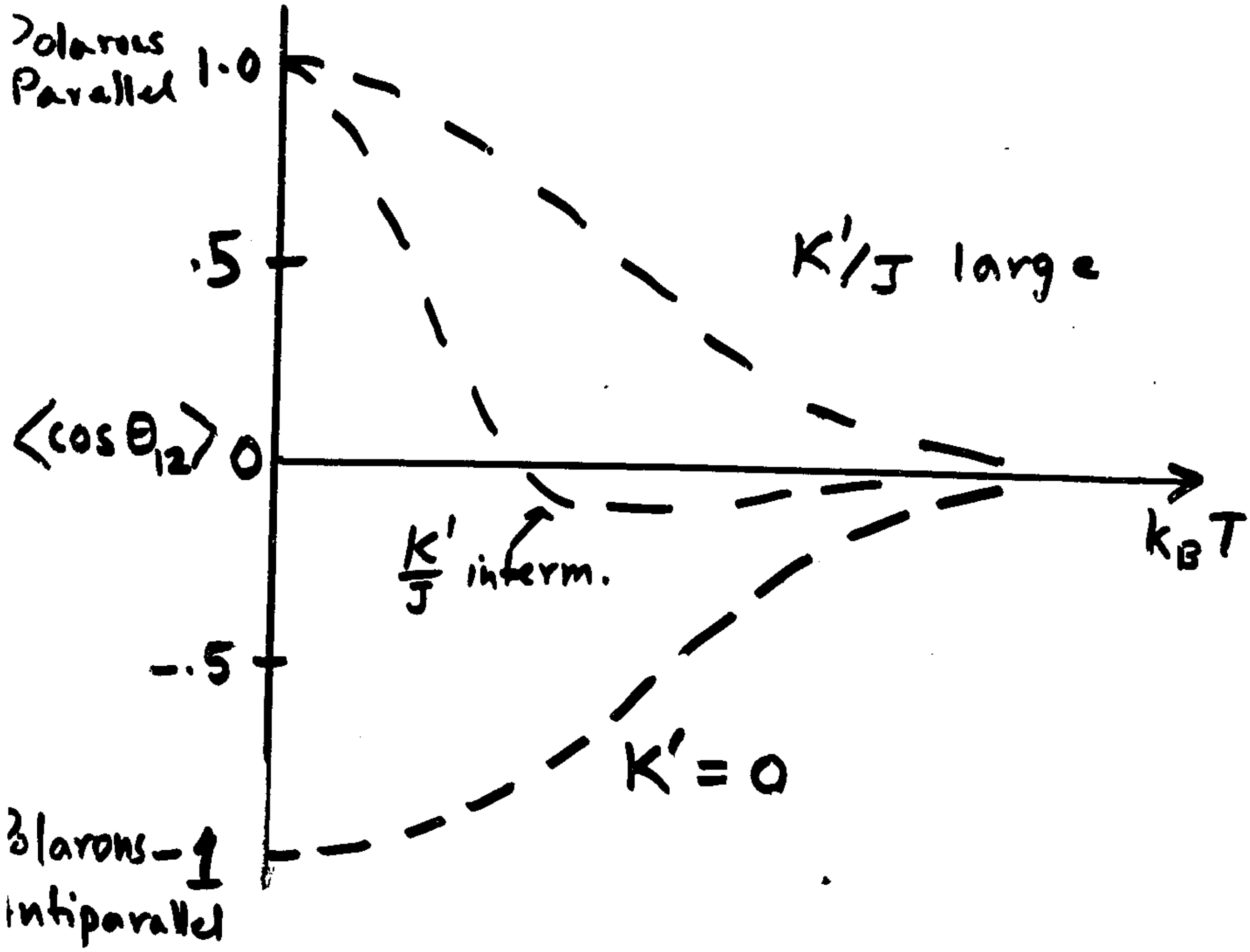
$$\Rightarrow \langle \cos\theta_{12} \rangle = \frac{\int_0^{\sqrt{2}} e^{-\beta J(x^2-1)/4} \left\{\mathcal{F}\left(\frac{\beta K' x}{\sqrt{2}}\right)\right\}^{N_3} (x^2-1)\, x\,dx}{\int_0^{\sqrt{2}} e^{-\beta J(x^2-1)/4} \left\{ \quad \right\}^{N_3} x\,dx}$$

etc.

MANY POLARONS

If "superexchange" wins, ferromagnetism likely outcome, even if some cases af. Decimate like Bhatt-Lee, to long distances $K' > J$, then ferromag. takes over.

Avg. angle between polaron moments



SUMMARY

$\boxed{\text{SPIN CHAINS}}$: Not all Gaps are the same.

Dimerized spin-½ and spin-1 chain in Haldane phase retain non-trivial topological structure even when gap→0 (finite randomness)

Non-universal thermodynamics, but similar to RS phase

Short range spin-spin corr., but persisting Topological order

Eventual transition to RS in spin-1 case, but weak signature in thermodynamics.

spontaneously dimerized (MG or biquadratic spin-1) unstable to arbitrarily weak disorder, divergent χ immediately.

DILUTED MAGNETIC SEMICONDUCTORS (3D):

Intervening free Mn give rise to effective Ferromagnetic coupling between polarons.

At T=0 (Ground State) ALMOST Ferromagnetic with only AF interactions in $\mathcal{H}$. $\left(\frac{\delta M}{M} \sim 10^{-4}\right)$

Ferromagnetic Insulator-Metal Transition.

QUANTUM (ANTIFERRO) MAGNETS WITH QUENCHED DISORDER HAVE MANY INTERESTING PHASES, WITH MORE SURPRISES TO COME.