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SMR.959 - 28

**MINIWORKSHOP ON STRONG ELECTRON CORRELATIONS**
***"Disorder and Interaction in Quantum Systems and Their Classical Analogs"***

**(1 - 19 July 1996)**

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# "Chiral Surface "Sheath" in the Bulk Quantum Hall Effect"

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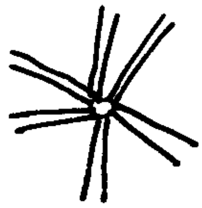
***These are preliminary lecture notes, intended only for distribution to participants.***

# Chiral Surface "Sheath" in the Bulk Quantum Hall Effect

with Matthew Fisher, ITP Santa Barbara
Martin Zirnbauer, ITP + Univ. Köln
Harsh Mathur, ITP + Case Western

# All roads lead to the QHE



Phenomena:

- Dissipationless current
- Accurate quantization $\sigma_{xy} = \frac{n}{m}\frac{e^2}{h}(1 \pm 10^{-5})$ 1981

$10^{-9}$ now

Theoretical Connections:

- Localization and disorder
- Topological invariants
- Gauge symmetry
- Random critical points
- Fractional charge / statistics
- Chern-Simons theory - statistical transmutation
- Superfluidity
- Charge-vortex duality
- Spin textures - skyrmions, merons
- Composite Fermions (non-Fermi liquid?)
- 1d chiral Fermions (c.f. doubling problem)
- 1d chiral Luttinger liquids - bosonization, CFT
- Point contacts, mesoscopics

* All this in two dimensions.

• This talk: Extension to three dimensions!

# Outline

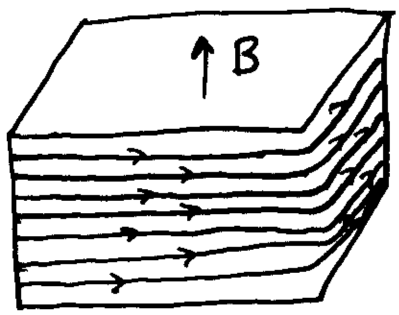
1. 2d QHE – Integral & Fractional

2. 3d QHE - types of systems

3. Edge states → surface "sheath"

   - 2d chiral electron liquid

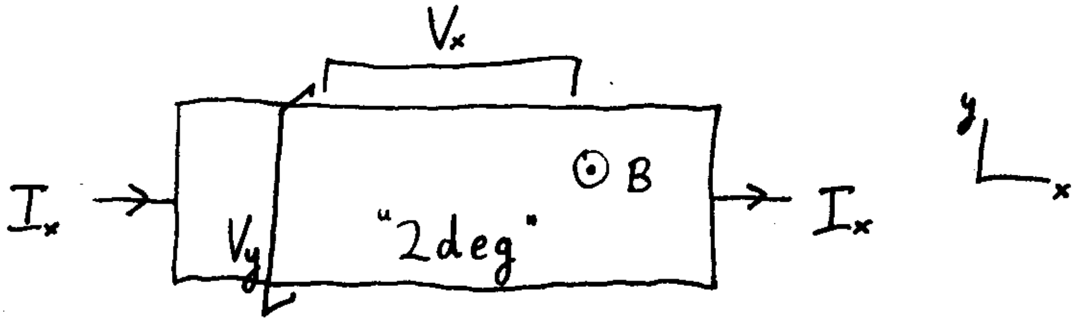
4. Experiments



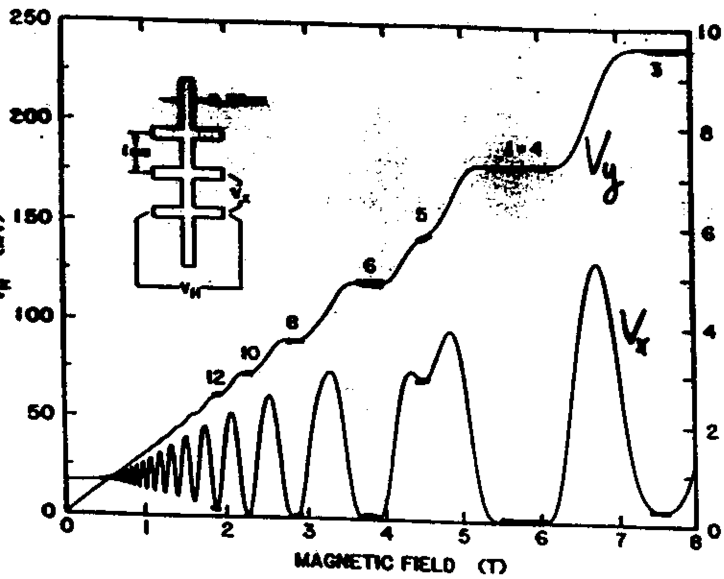
# Integral QHE (IQHE)

von Klitzing, Dorda & Pepper 1980

Nobel 1985 von Klitzing!




GaAs - AlGaAs
T = 1.2 K
$I_x$ = 25.5 μA




Cage et.al. 1985

$$\vec{E} = \underline{\rho}\,\vec{J} \quad \Leftrightarrow \quad \vec{J} = \underline{\sigma}\,\vec{E}$$

• On plateaus:

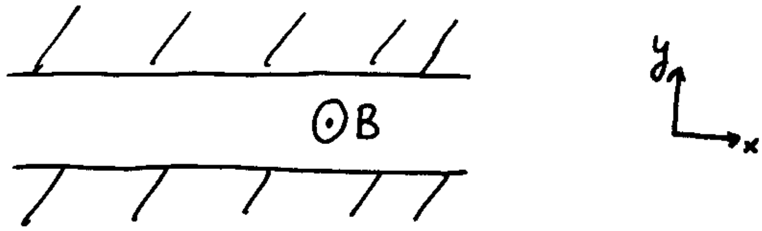
$$\rho_{xx} = 0 \qquad \longleftrightarrow \qquad \sigma_{xx} = 0$$

$$\rho_{xy} = \frac{h}{ne^2} \qquad \qquad \sigma_{xy} = \frac{ne^2}{h}$$

# Edge States

Halperin, 1982

Hall Bar

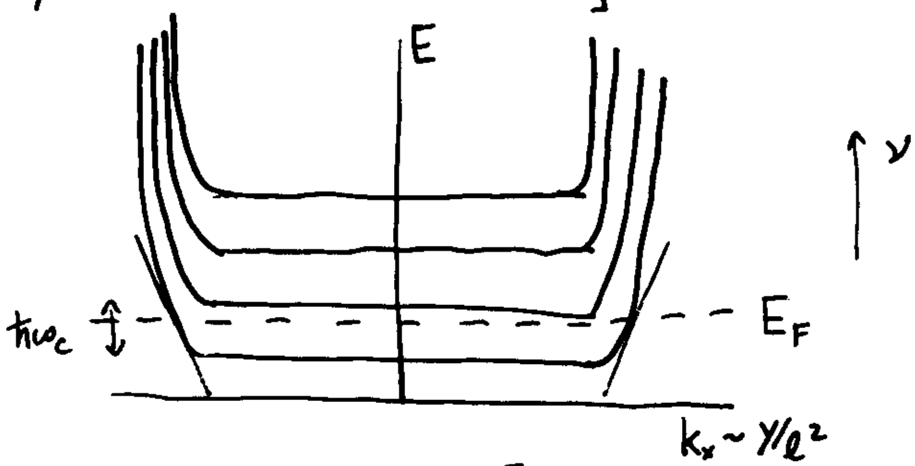



Non-interacting electrons - Landau gauge $A_x = By$

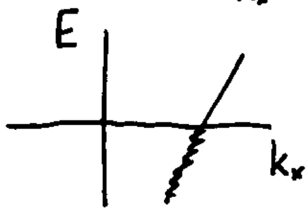
- $\psi = e^{ik_x x} u_{k_x}(y)$

$$\left[ \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial y^2} + \frac{1}{2} m \omega_c^2 (y - k_x \ell^2)^2 + V(y) \right] u_{k_x}(y) = E\, u_{k_x}(y)$$

$$\boxed{\begin{array}{l} \omega_c = \dfrac{eB}{mc} \\ \ell^2 = \varphi_0 = \dfrac{hc}{e} \end{array}}$$



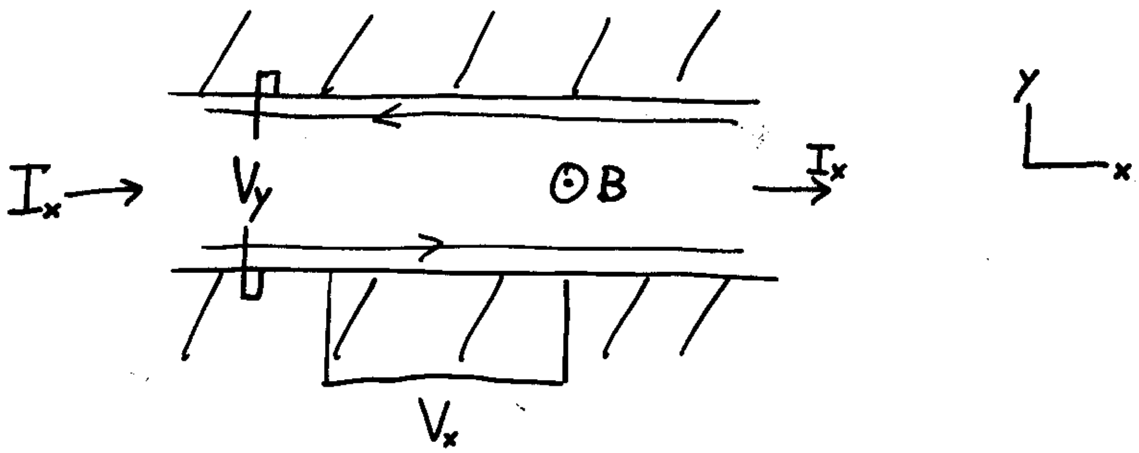


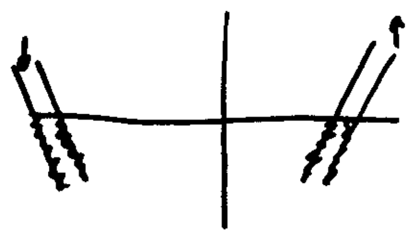



$$H_R = \int \frac{dp}{2\pi}\, v p\, \psi_R^\dagger \psi_R = \int dx\; \psi_R^\dagger\, i v \partial_x \psi_R$$

1-d Chiral Fermions

# Hall Effect?




- 

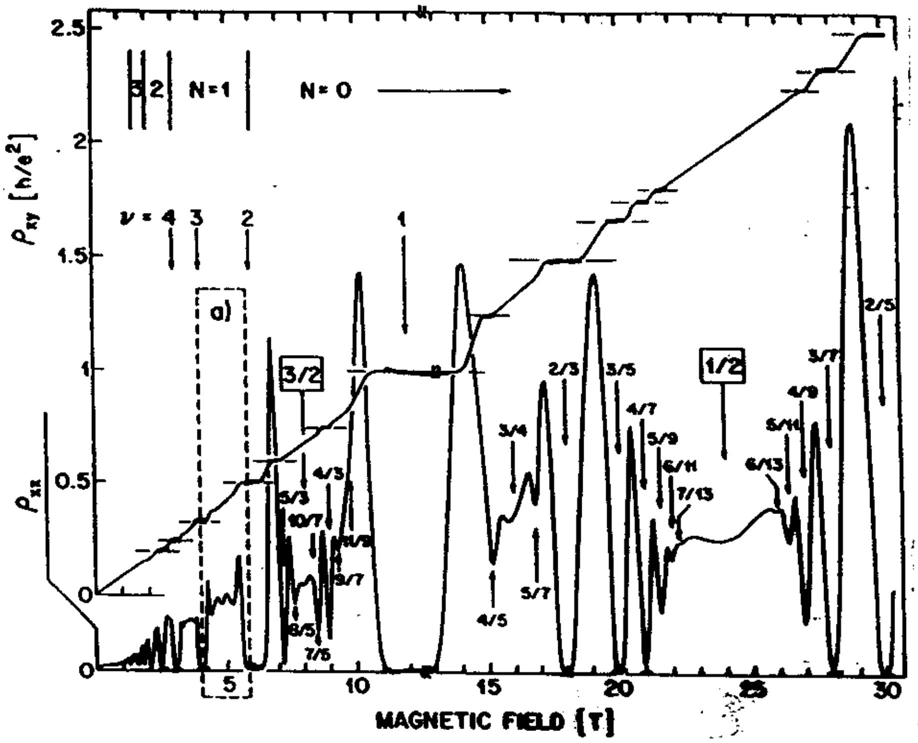
  $V_y \Rightarrow$ Chemical Potential Shift

$$I_x = I_R - I_L = e\,\Delta\left(\frac{vp}{h}\right) = \frac{e}{h}(eV_y) = \frac{e^2}{h}V_y$$

- $\sigma_{xy} = e^2/h \qquad \nu = 1$
- $\sigma_{xy} = ne^2/h \qquad \nu = n \qquad$ (parallel conductance

- $V_x = 0$ Edge Equilibration $\sigma_{xx} = 0$

# Fractional QHE (FQHE)  Tsui, Störmer, + Gossard, 1982




Willet et al., 1987

* Similar to IQHE, but $\sigma_{xy} = \frac{n}{m}\frac{e^2}{h}$ $\left(\rho_{xy} = \frac{m}{n}\frac{h}{e^2}\right)$

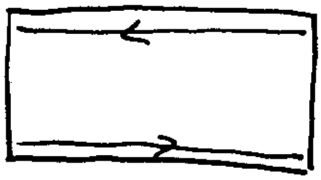
* Occur at very high magnetic fields
  for very clean samples

# FQHE

Laughlin, 1983

- Interactions → Incompressible Quantum Fluid (Bulk gap $E_g < \hbar\omega_c$)
- Fractionally charged quasiparticles $q = \nu e$ ($\nu = \frac{1}{3}, \frac{1}{5}, \ldots$)
  w/ fractional statistics!

Wen, 1990: FQHE Edges



- Edges carry quasiparticles, not electrons!

- Bosonized edge Lagrangian

$$L = \frac{1}{4\pi\nu} \int dx \; \partial_x \varphi \left( \partial_t + v \partial_x \right) \varphi \qquad \text{chiral Luttinger liquid}$$

$\mathcal{O} \sim e^{i\varphi}$ creates charge $\nu e$ quasiparticle

- Hall

$$I_x = \frac{\nu e}{h} \Delta(\nu\mu) = \frac{\nu e^2}{h} V_y$$

Aside: tested by tunneling conductance (point contact)

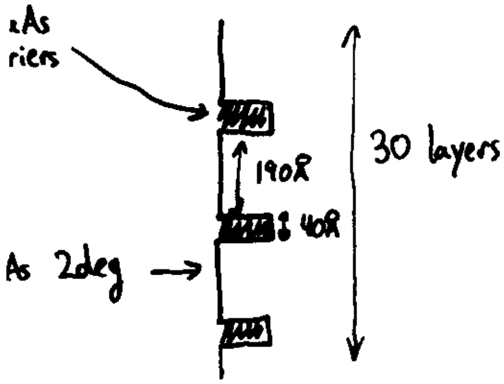
$$G(T) \sim T^{2(1/\nu - 1)}$$

theory: Kane + Fisher, Wen
expt: Milliken, Webb, Umbach

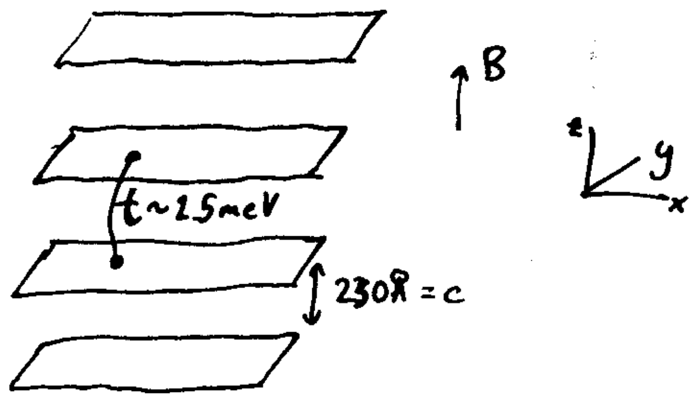
- other tests in progress

# 3d QHE

GaAs Multilayers    Expt: Störmer et.al., 1985




Experiment

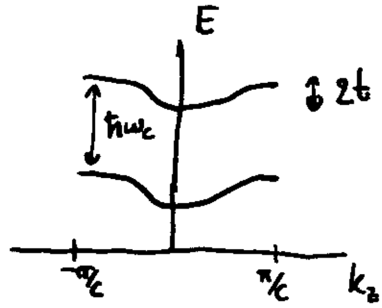



Model

..nd structure: $E = \hbar\omega_c(n+\tfrac{1}{2}) - t\cos q_z c + V(y)$

$\Rightarrow$ Gap for $t < \hbar\omega_c/2$

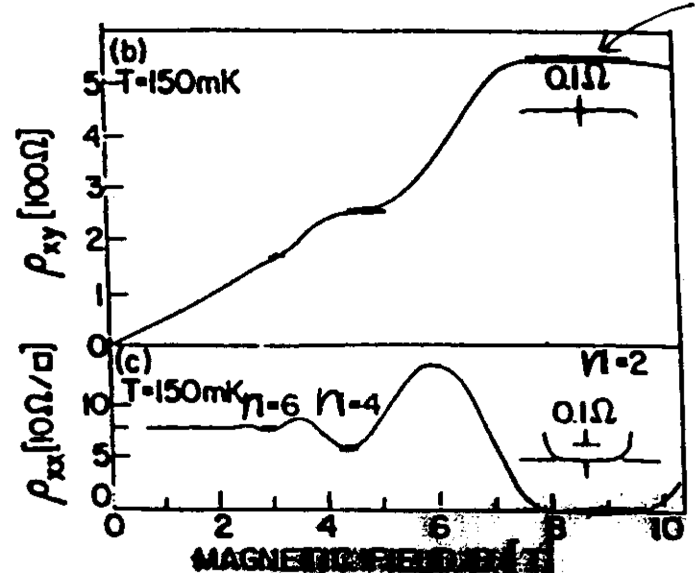
$\Rightarrow$ IQHE ✓




..ractional? $t \lesssim E_g/2$ more fragile

$\rho_{xy}^{\text{layer}} = \frac{h}{ne^2}(1 \pm 10^{-4})$

T = 150 mK

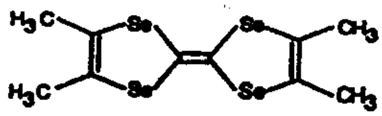



$\frac{\sigma_{xy}}{\text{layer}} = ne^2/h$

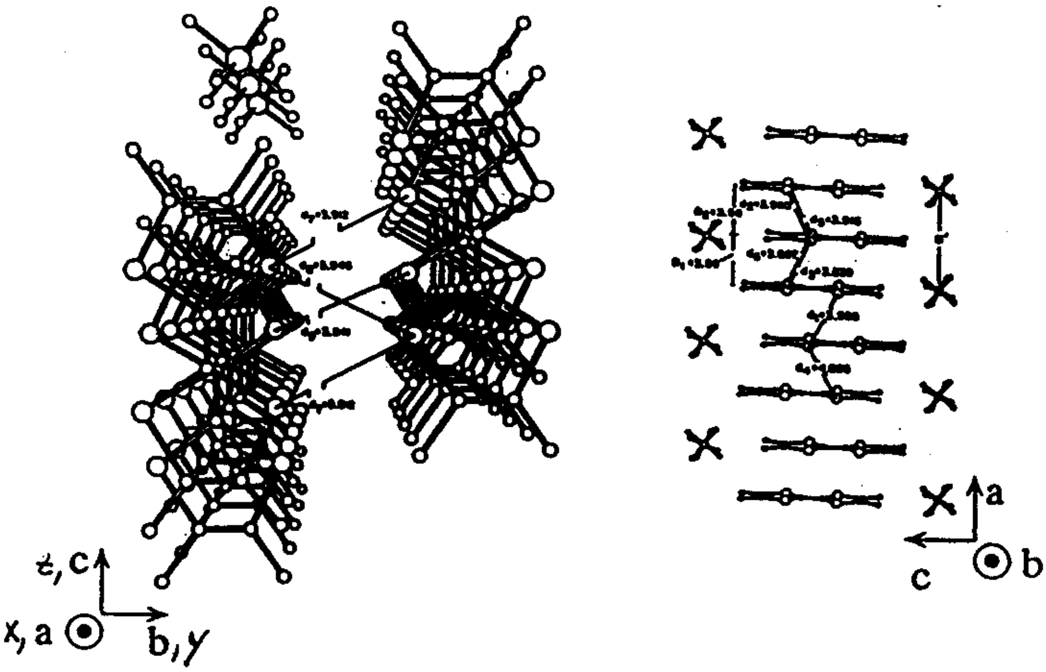
# Bechgaard Salts

Cooper et. al., 1989
Hannahs et. al., 1989

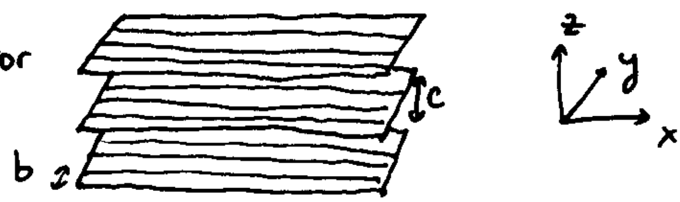
- Organics: $(TMTSF)_2 X$ $\quad X = PF_6, ClO_4, ReO_4, \ldots$




TMTSF = tetramethyltetraselenafulvalene




- Toy Model: chain conductor



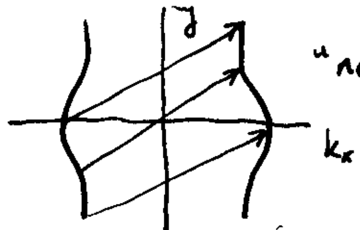

Very anisotropic

(room temperature) Conductivity $\quad \sigma_{xx} : \sigma_{yy} : \sigma_{zz} \sim 1 : 10^{-2} : 10^{-5} \quad \times 10^{3} (\Omega cm)^{-1}$

Bandwidths $\quad t_x : t_y : t_z \sim 1eV : .1eV : .01eV$

# Mechanism?

Lebed + Gorkov, 1984
Maki, 1986




"nesting"

- Quasi-1d $\rightarrow$ Field Induced Spin Density Wave (SDW)

$$\vec{S} \sim \vec{S}_0 \cos \vec{Q}\cdot\vec{r} \qquad \vec{Q} \approx (Q_x, \pi/b, \pi/c)$$

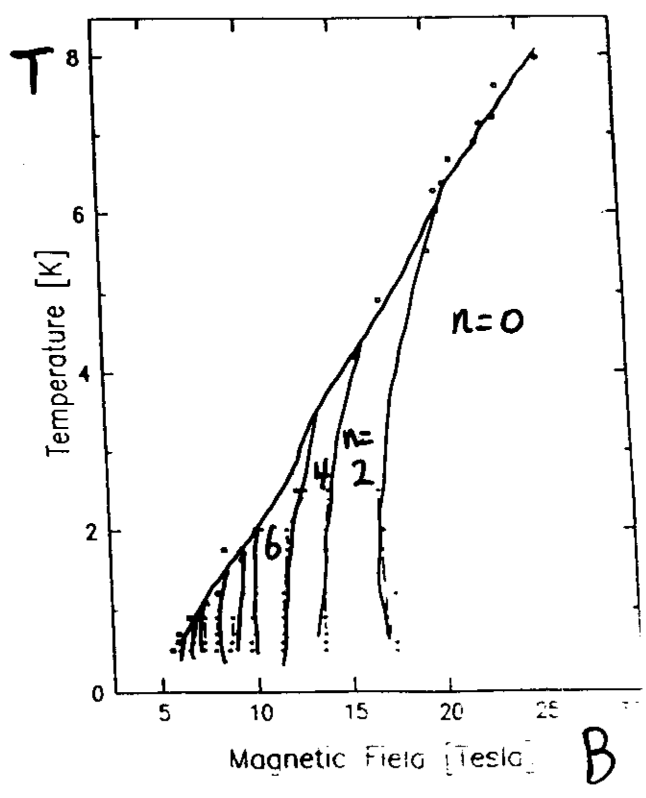
- Wavevector

  - $Q_x = 2k_{Fx} + 2\pi n/\ell_x$  magnetic length
  - Commensurate $\rightarrow$ Gap  "Hofstädter"

- Phase Diagram

$X = PF_6$  $P = 10.5$ kbar

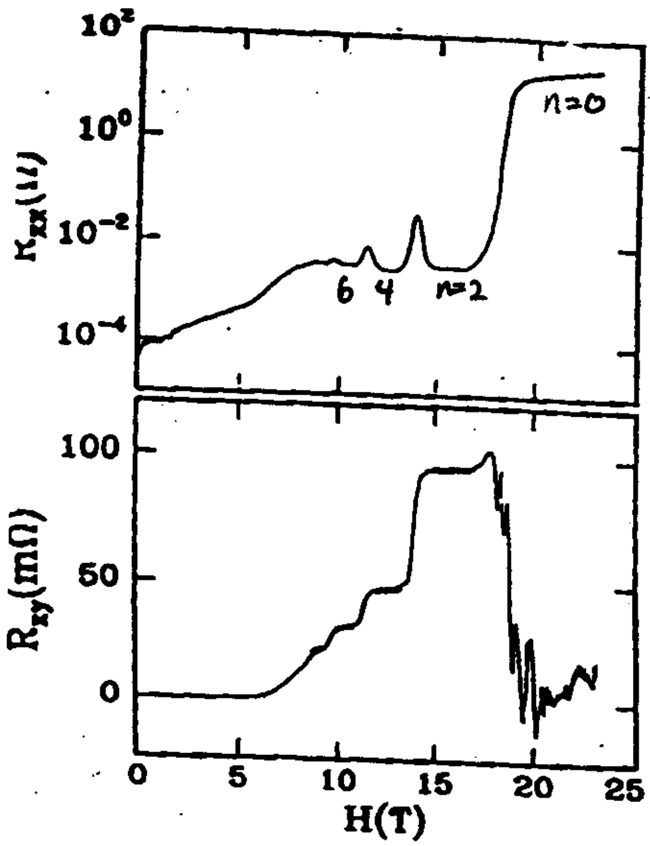
...nnahs et.al., 1989

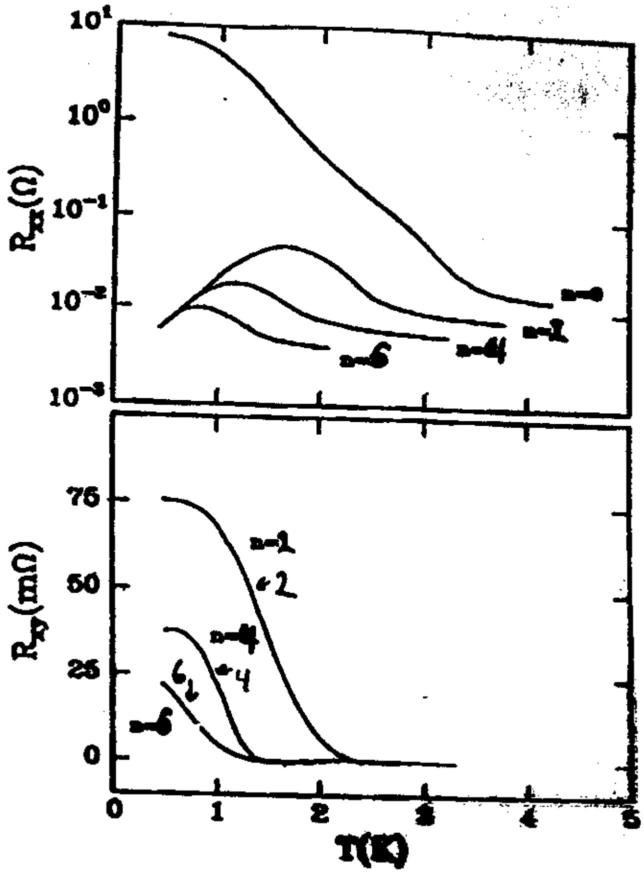


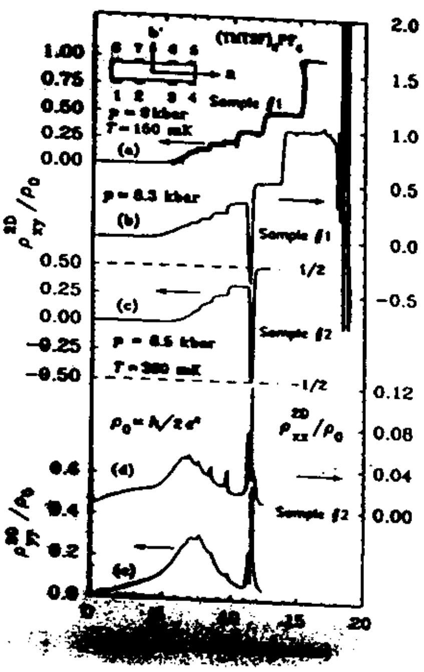

$$\frac{\sigma_{xy}}{\text{layer}} = \frac{ne^2}{h}$$

W. Kang et. al, 1992

$(TMTSF)_2\ PF_6$ $\quad P \sim 9\,kbar$



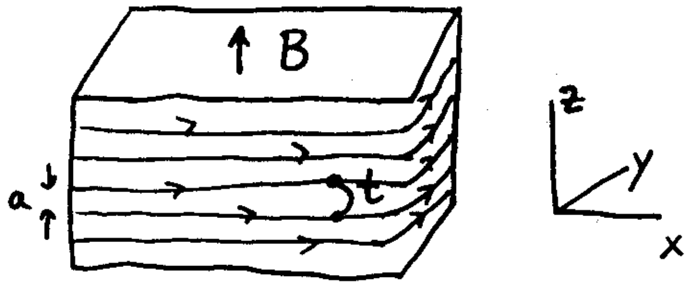




$$\frac{\sigma_{xy}}{\text{layer}} \simeq \frac{ne^2}{h}$$

Balicas et. al., 1995

# On Hall plateaus → Surface States

- $\nu = n$ edge states per layer

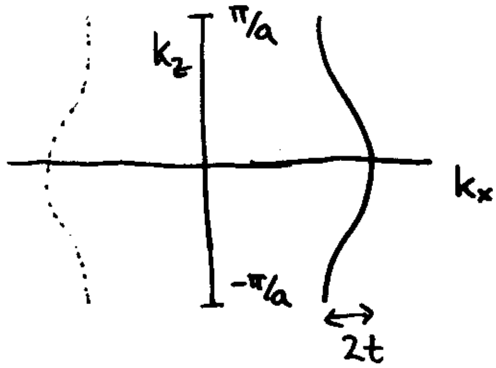



Model: 2d Chiral Fermions

$$H = \sum_n \int dx \left\{ \psi_n^\dagger i v \partial_x \psi_n - t\left(\psi_n^\dagger \psi_{n+1} + \psi_{n+1}^\dagger \psi_n\right) \right\}$$

$$= \sum_k \left(v k_x - t \cos k_z a\right) \psi_k^\dagger \psi_k$$

Fermi surface



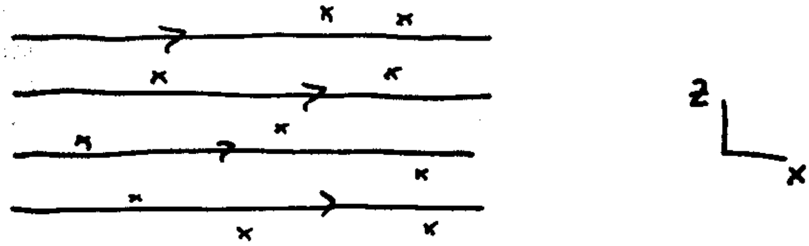

"right-moving Fermi surface"

Interactions: Chiral Fermi liquid theory

- Specific heat $C \sim T/av$ linear
- Conductivity $\sigma_{zz} \sim 1/T^2$ → need <u>disorder</u>

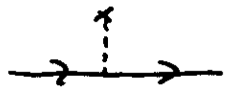
# Disorder

- Model: Random Potential




- Hamiltonian

  - $H_{imp} = \sum_n \int dx \; V(x, na)\, \psi_n^{+} \psi_n$



  - Gaussian $\quad \overline{V} = 0$

$$\overline{V(x,z)\,V(x',z')} = \Delta\, \delta(x-x')\, \delta_{zz'}$$

Guess: Diffusion?

– $\sigma_{zz} = e^2 \underbrace{\rho}_{\text{density of states}} D_z$ $\quad$ Einstein Relation

$D_z$: Diffusion Constant

Need $D_z$!

# Diffusion Kernel

$$K(\vec{r}) = \overline{|G(\vec{0},\vec{r};E+i\eta)|^2} = \text{Prob}(\vec{0} \longrightarrow \vec{r})$$

- SUSY Generating Functional

$$Z = \int \underbrace{d\Psi d\bar{\Psi}}_{\text{Fermions}} \; \underbrace{d\phi\, d\phi^*}_{\text{Bosons}} \; e^{-S_0} = 1 \; !$$

$$S_0 = \sum_{\vec{r}} \left\{ -i\phi_\uparrow^*(H - \tfrac{\omega}{2} + i\eta)\phi_\uparrow + i\phi_\downarrow^*(H + \tfrac{\omega}{2} - i\eta)\phi_\downarrow + \phi \leftrightarrow \Psi \right\}$$

$$= \sum_n \int dx \left\{ \phi_n^* \sigma^z \partial_x \phi_n - iV_n(x)\, \phi_n^* \sigma^z \phi_n + it\left[\phi_n^* \sigma^z \phi_{n+1} + \text{c.c.}\right] + \eta\, \phi_n^* \phi_n + \Psi \leftrightarrow \phi \right\}$$

"Spin"

Compare $\bar{\phi}\,\hbar\,\partial_\tau \phi$

Imaginary time QFT

$$Z = \mathrm{STr}\; e^{-L_x H}$$

# "Superspin" Chain

- Disorder Average $\longrightarrow$ SUSY QFT
  - Fermions $\{F_\alpha, F_\beta^\dagger\} = \delta_{\alpha\beta}$ $\quad \alpha,\beta = 1,2$
  - Bosons $[B_\alpha, \bar{B}_\beta] = \delta_{\alpha\beta}$

    $\boxed{\bar{B} = B^\dagger \sigma^z \quad \text{"Non-unitary Bosons"}}$

  - Superspin $J^\mu = \begin{pmatrix} F^\dagger \gamma^\mu F & F^\dagger \gamma^\mu B \\ \bar{B} \gamma^\mu F & \bar{B} \gamma^\mu B \end{pmatrix}$ $\quad \gamma^\mu = (1, \vec{\sigma})/2$
  - Hamiltonian

    "Supertrace"

$$H = \underbrace{-2D}_{\text{Ferromagnetic}} \sum_n \mathrm{STr}\, J_n^\mu J_{n+1}^\mu + 2\eta \sum_n \mathrm{Tr}\, J_n^2$$

- Exact Ground State $|0\rangle \sim |\downarrow\downarrow\downarrow\cdots\rangle$

$$J^z_{TOT}|0\rangle = \sum_n J^z_n |0\rangle = -N|0\rangle$$

$$J^+_{TOT}|0\rangle = 0$$

$$H|0\rangle = 0 \quad \text{SUSY}$$

- Exact Single Magnon States $\quad \vec{S}_n = F_n^\dagger \frac{\vec{\sigma}}{2} F_n$

$$S_k^+ \equiv \sum_n S_n^+ e^{ikn}$$

$$|k\rangle = S_k^+ |0\rangle$$

$$H|k\rangle = E_k |k\rangle$$

dispersion $\quad E_k = 2D(1-\cos k) + 2\eta \approx 2\eta + Dk^2$

$z = 2$ FM

- Diffuson

$$K_n(x) = \langle T_x S_n^-(x) S_0^+(0) \rangle$$

$$\Downarrow$$

$$K(k_x, k_z) = \frac{1}{ik_x + E_k} \approx \frac{1}{2\eta + ik_x + Dk_z^2}$$

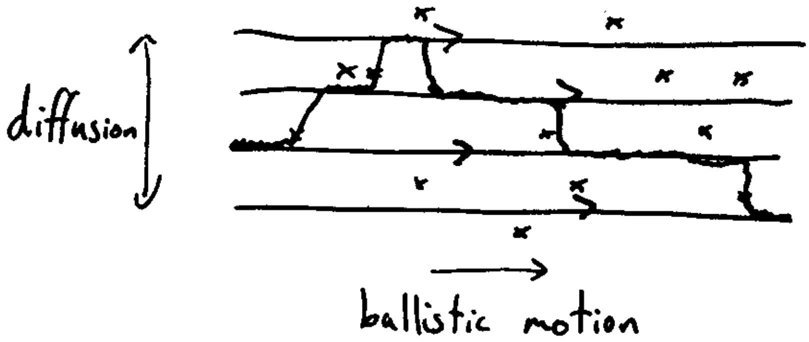
ballistic along x axis

diffusive along z axis

$\Rightarrow$ Finite z-axis conductivity at $T=0$

$$\sigma_{zz} = \frac{\partial n}{\partial \mu} D \neq 0$$

# Contrast IQHE Sheath w/ Normal 2d Electrons





* Anisotropic

  - $\rho_{xx} = 0$ ballistic along x
  - $\rho_{zz} = \frac{h}{e^2}\left(\frac{\hbar^2 v}{t^2 \tau a}\right) \neq 0$ diffusive along z

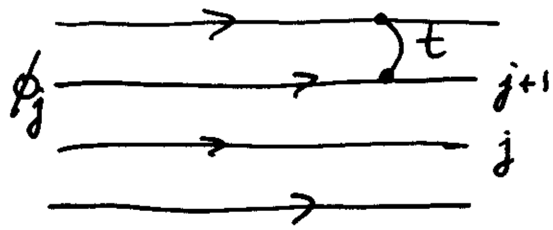
* No minimum metallic conductivity!

  - $\rho_{zz} \gg h/e^2$ $\not\Rightarrow$ Localization
  - Physics: No backscattering

"2d dirty metal"

# FQHE Surface States

$\nu = 1/3, 1/5, \ldots$





Tunneling between Luttinger liquids is non-trivial

Model

$$\mathcal{L} = \sum_j \int dx \left\{ \frac{1}{4\pi\nu} \partial_x\varphi \left(\partial_t\varphi + v\partial_x\varphi\right) + t \cos\left(\underbrace{\tfrac{1}{\nu}\left(\varphi_j - \varphi_{j+1}\right)}_{\text{electron tunneling}}\right) \right\}$$

$$q = \tfrac{1}{\nu}(\nu e) = e$$

Linear Renormalization Group

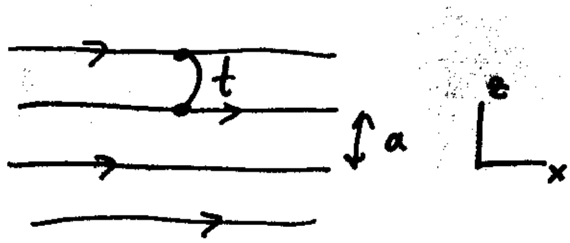
$$\frac{\partial t}{\partial(\ln L)} = \underbrace{\left(2 - 1/\nu\right)}_{<0\,!} t \rightarrow$$ z-axis tunneling is irrelevant

$\Rightarrow$ Electrons are "confined"

- Trivial 2d non-Fermi liquid

# FQHE Surface Transport

- Incoherent tunneling at $T \neq 0$




- Scaling

$$G_{zz} = \frac{L_x}{L_z} \sigma_{zz}$$

$$\Rightarrow \quad \sigma_{zz} = \frac{a}{\xi_x} f\left( \xi_x^{2-1/\nu} t, \ \xi_x T \right)$$

ballistic $z=1$ scaling
$\omega \sim \xi^{-1}$

- Rescale until $T_R \sim \xi_x T = 1$

$$\sigma_{zz} \sim t^2 T^{2/\nu - 3} \qquad \sim T^3 \qquad \nu = 1/3 \quad \text{no disorder!}$$

$$\sim t^2 T^{2/\nu - 4} \qquad \sim T^4 \qquad \nu = 1/3 \quad \text{with disorder}$$

Maximal anisotropy $\left.\begin{array}{l} \rho_{xx} \to 0 \\ \rho_{zz} \to \infty \end{array}\right\} T \to 0$

Incoherent since

tunneling rate $\to \Gamma_t \ll \Gamma_\varphi \sim T$ ← dephasing rate

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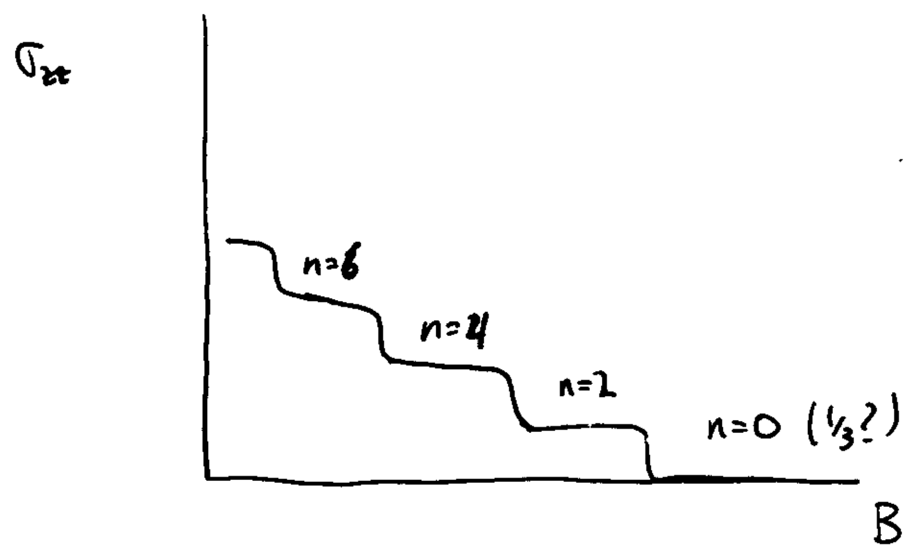
# Predictions for Transport

* Experiment: vary $B$ $\rightarrow$ need to understand bulk phase diagram

(1) Organics

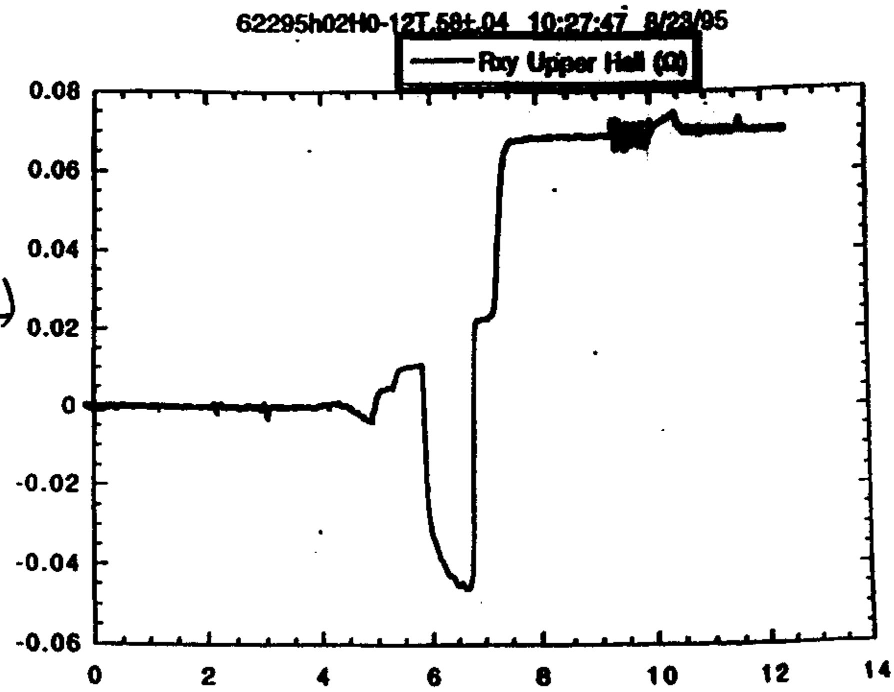
* Direct transitions between QH states
* Expect $\sigma_{zz} \propto n$ (number of sheaths)
  - temperature independent
* Critical behavior? 1st order -or- 3d?



Chaikin et. al, unpublished 1995

$(TMTSF)_2 ClO_4$

$R_{xy}(\Omega)$

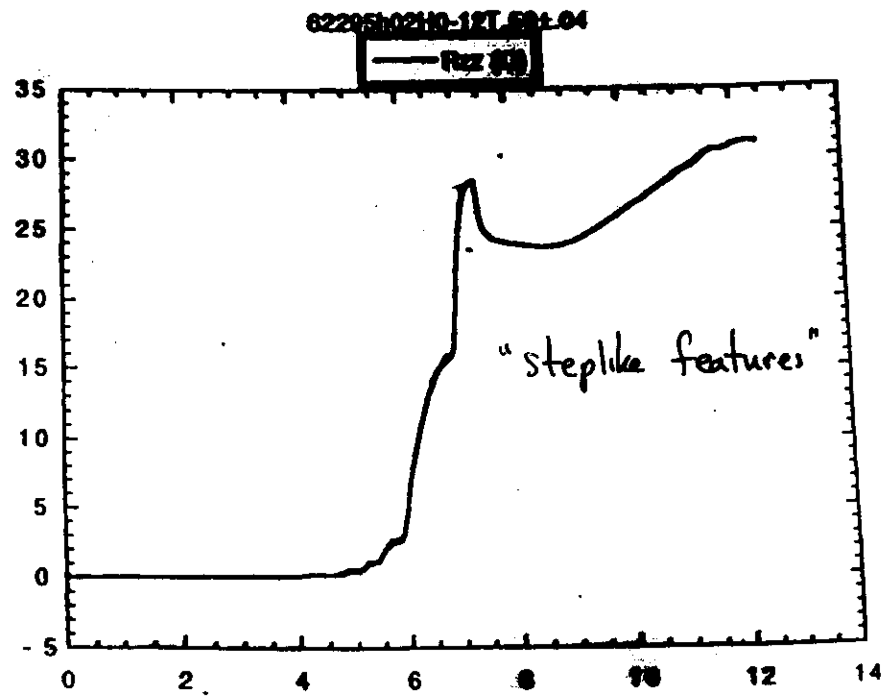



$T = 0.6$ K

B (Tesla)

$R_{zz}(\Omega)$

$\left(= \frac{1}{G_{zz}}\right)$

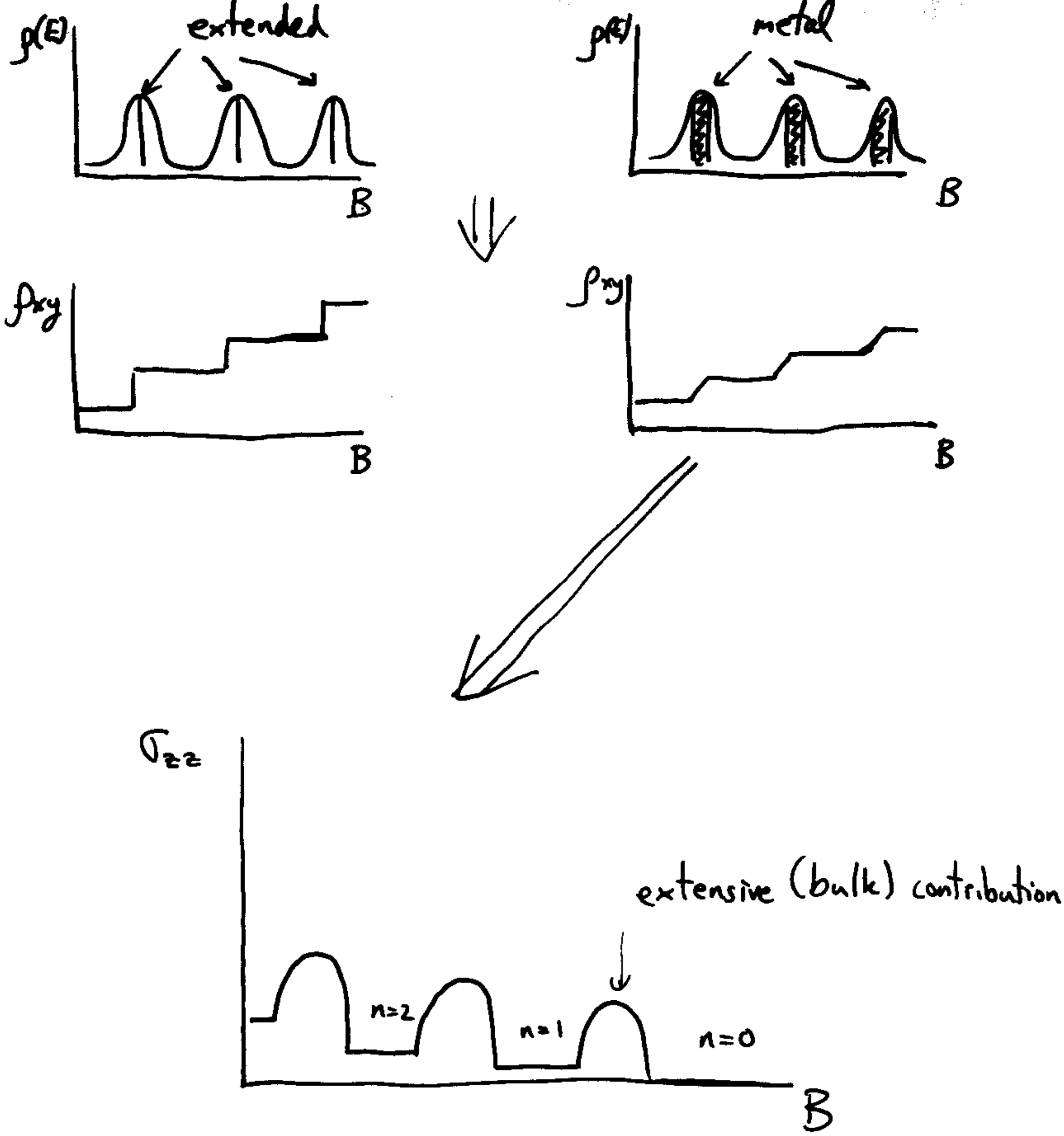



B (Tesla)

## ② Multilayers

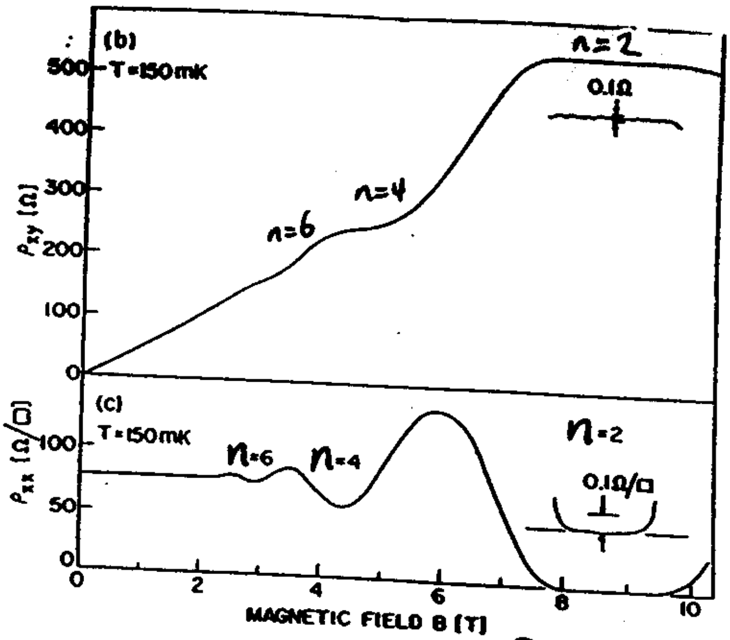
(c.f. Chalker & Dohmen, 1995)

- Expect band of extended states between plateaus



Störmer et.al., 1986

$\rho_{xy}$



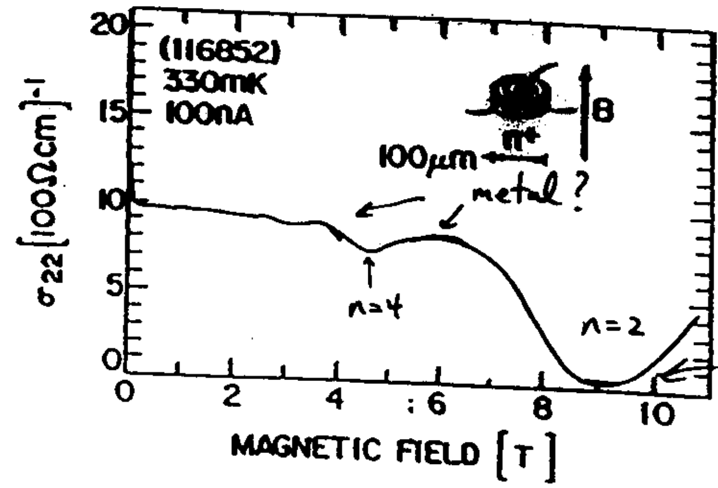

$\rho_{xx}$

$$\frac{\sigma_{xy}}{\text{layer}} = \frac{ne^2}{h}$$

T=150 mK

B

$\sigma_{zz}$




T=330 mK

n=2 sheath

B

# Specific Heat

* 2d QHE

$$C = \underbrace{C_{edge}}_{\frac{\pi}{6}\frac{k_B^2 T}{v}\cdot L} + \underbrace{C_{bulk}}_{\rho T L^2}$$

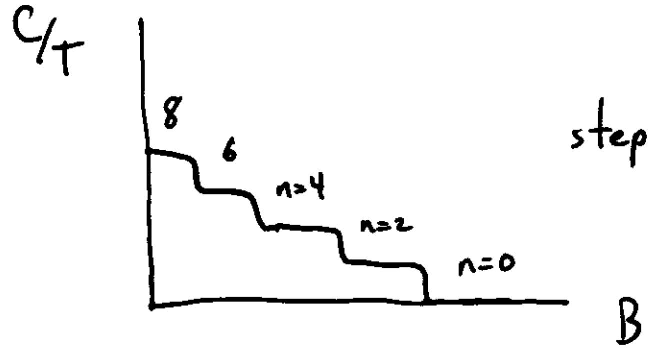
localized states swamp edge contribution

* 3d Multilayers $\quad C_{bulk} \gg C_{sheath} \quad$ still swamped

Organics

- Bulk Gap $\quad C_{Bulk} \sim e^{-E_g/T}$
- $T \ll E_g \quad$ surface sheath dominates!

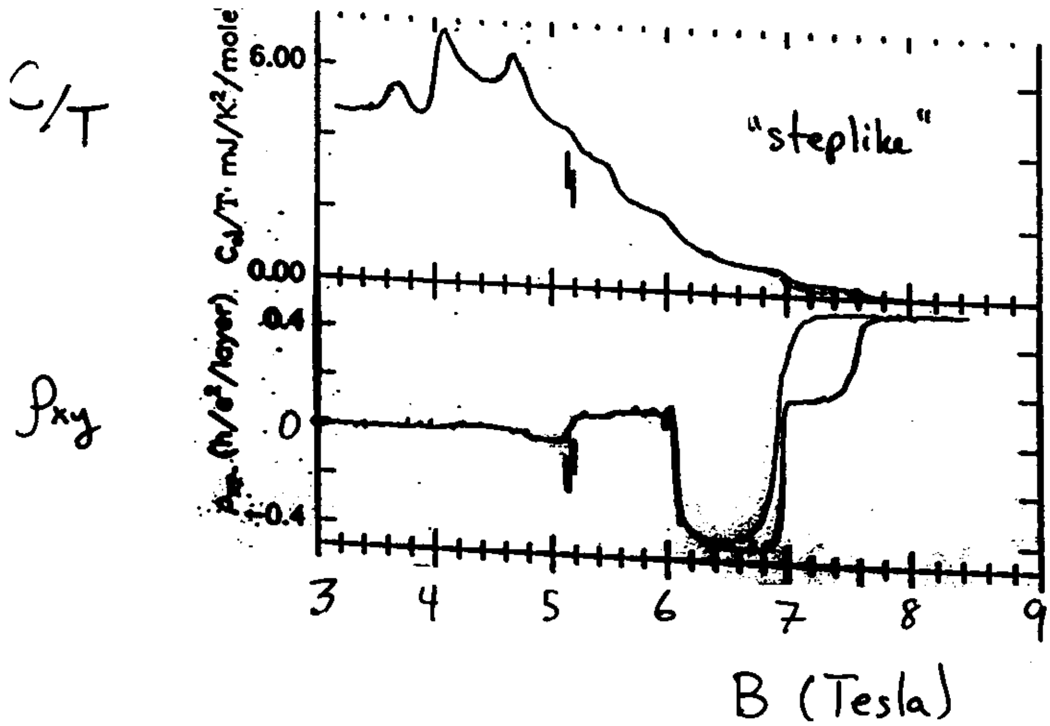
$$\frac{C}{L^2} \sim n \frac{\pi}{6} \frac{k_B^2 T}{a v}$$




steps!

Scheven et.al., 1995

$(TMTSF)_2 ClO_4$




$T \simeq .35\ K$

# Conclusion

* Bulk QHE $\Rightarrow$ Novel 2d Chiral Quantum Liquid!

* IQHE "surface sheath"
  - coherent z-axis transport
  - anisotropic "dirty metal"

    No minimum conductivity $0 < \sigma_{zz} \ll e^2/h$

* FQH sheath
  - confined non-Fermi liquid
  - Maximally anisotropy

$$\left.\begin{array}{l}\rho_{xx} \to 0 \\ \rho_{zz} \to \infty\end{array}\right\} T \to 0!$$

* Future Work:
  * Experiment: z-axis transport, spin?
  * Theory: 3d transition with disorder?

    $\nu = 2/3$ "composite" sheath, spin