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SMR.959 - 29

MINIWORKSHOP ON STRONG ELECTRON CORRELATIONS
"Disorder and Interaction in Quantum Systems and Their Classical Analogs"

(1 - 19 July 1996)

"Quantum Phase Transitions in Boson and Spin Systems"

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These are preliminary lecture notes, intended only for distribution to participants.

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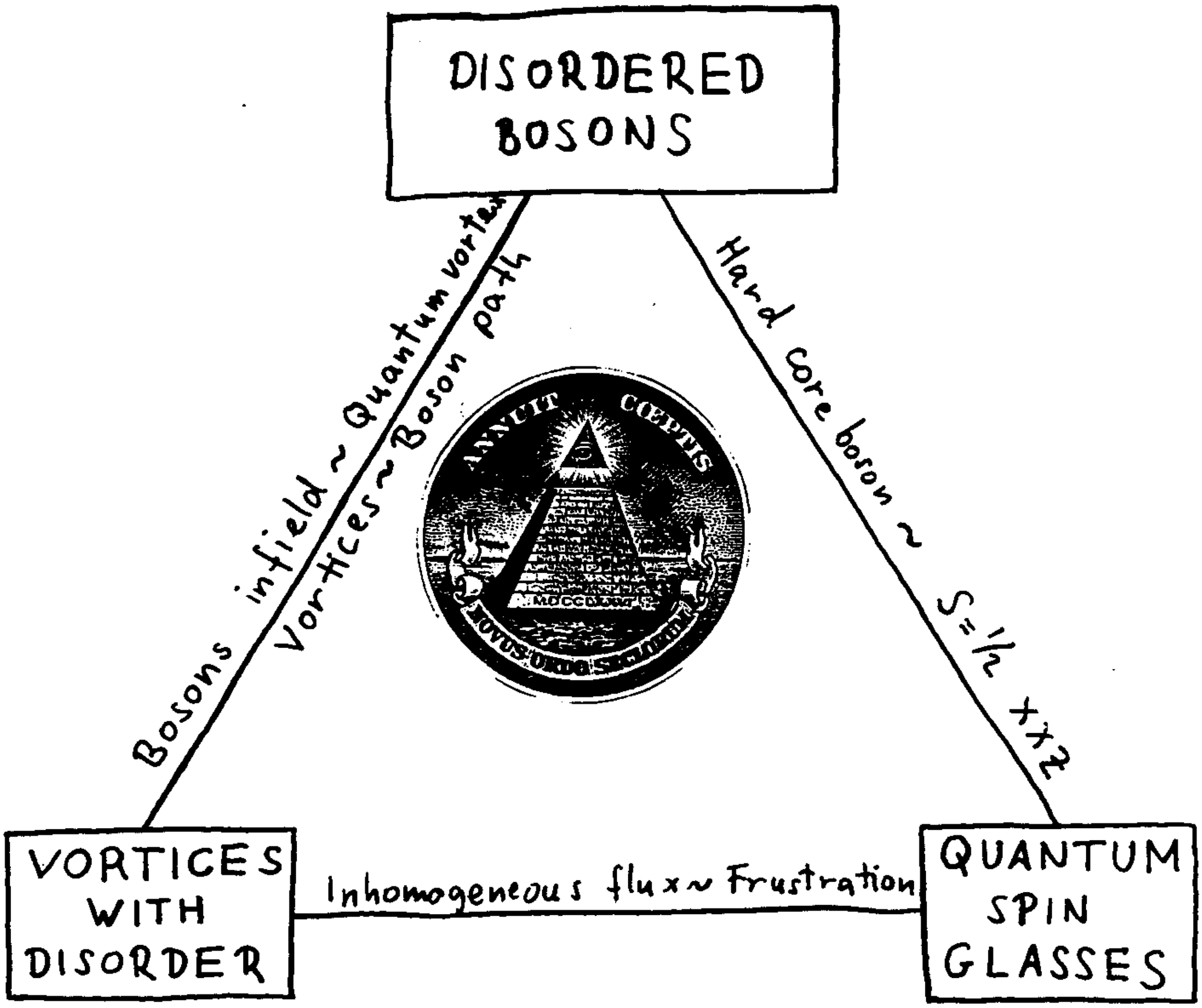
QUANTUM PHASE TRANSITIONS IN BOSON AND SPIN SYSTEMS

Ferenc PAZMANDI, Richard SCALETTAR

1.) The localization transition for hard core bosons

2.) Quantum spin glasses


DISORDERED BOSONS
Bosons in field ~ Quantum vortex
Vortices ~ Boson path
Hard core boson ~ S=1/2 XXZ
VORTICES WITH DISORDER
Inhomogeneous flux ~ Frustration
QUANTUM SPIN GLASSES
ANNUIT COEPTIS
MDCCLXXVI
NOVUS ORDO SECLORUM


THE UNITED STATES OF AMERICA
IN GOD WE TRUST
ONE
ONE DOLLAR

Disordered bosons are relevant for $\begin{cases} {}^4He \text{ in Vycor} \\ \text{disordered superconductor.} \end{cases}$

Fisher, Weichman, Grinstein, Fisher:

Strong enough disorder localizes bosons.

At $T=0$, instead of superfluid phase, Bose Glass.

Scaling in 1d : Giamarchi & Shulz

Numerical studies : Wallin, Sorensen, Girvin & Young
Krauth, Makivic & Trivedi
Scalettar, Batrouni & GTZ

Approximate analytic + numerical approaches: D. Lee & Gunn
K. Singh & Rokhsar
L. Zhang & M. Ma

Recent work (large N + ϵ expansion) : X. Wen ; P. Weichman

Hard to describe critical behavior in higher dimension.

Lack of field theory, grasping the localization / Bose Glass transition on mean field level, and allowing for a systematic expansion around this solution.

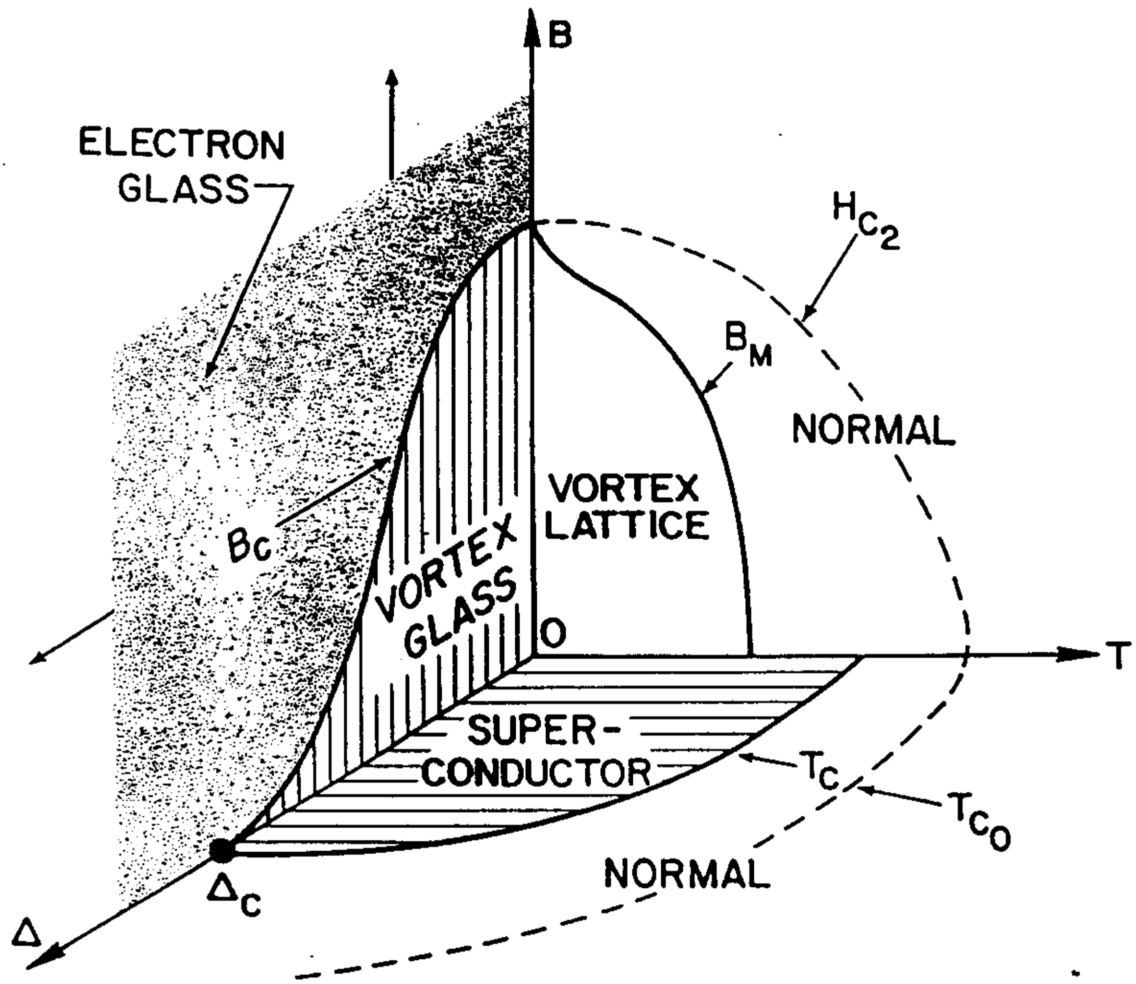


FIG. 1. Schematic phase diagram for disordered superconducting films. *Distinct* $T=0$ superconductor-insulator transitions occur at both critical disorder Δ_c *and* critical magnetic field B_c.

THE LOCALIZATION TRANSITION FOR DIRTY HARD CORE BOSONS

PRL, 75, 1356, (1995)

$$\mathcal{H} = -\frac{J_0}{N}\sum_{i\neq j} a_i^+ a_j - \sum_i (\mu+\epsilon_i)\, a_i^+ a_i$$

* infinite range hopping
* hard core interaction
* $\langle \epsilon_i \rangle = 0$, $\langle \epsilon_i^2 \rangle = \Delta$

magnetic equivalent:

$$\mathcal{H} = -\frac{J_0}{N}\sum_{i\neq j}\left(S_i^+ S_j^- + S_j^+ S_i^-\right) - \sum_i (H + h_i)\, S_i^z$$

* infinite range XY model
* random transverse field

What phases to expect?

BOSONS | MAGNET

MOTT

1 particle per site

large μ, H

QUANTUM PARAMAGNET

$\langle S^z \rangle \neq 0$, $\langle S^x \rangle = 0$

SUPERFLUID

small disorder

extended wave fc.

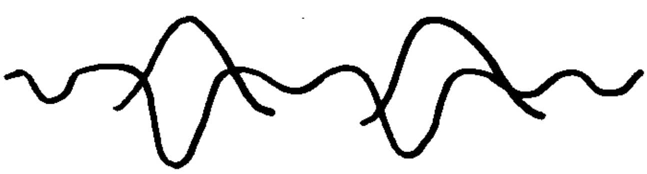
small μ, H

XY FERROMAGNET

$\langle S^x \rangle \neq 0$

BOSE GLASS

large disorder



localized particles

QUANTUM GRIFFITH

$\langle S^x \rangle = 0$,

localized spin excitations

large TEMPORAL correlations

Hard core: well defined ground state: 1 particle/site

Lower μ: low concentration of holes: one-particle approach

$$\sum_j \left(\frac{J_0}{N} + (\mu + \epsilon_i)\,\delta_{ij} \right) x_j = \omega\, x_i$$

The eigenvalue ω determined by self-consistency

$$1 = \frac{1}{N} \sum_i \frac{J_0}{\omega - (\mu + \epsilon_i)}$$

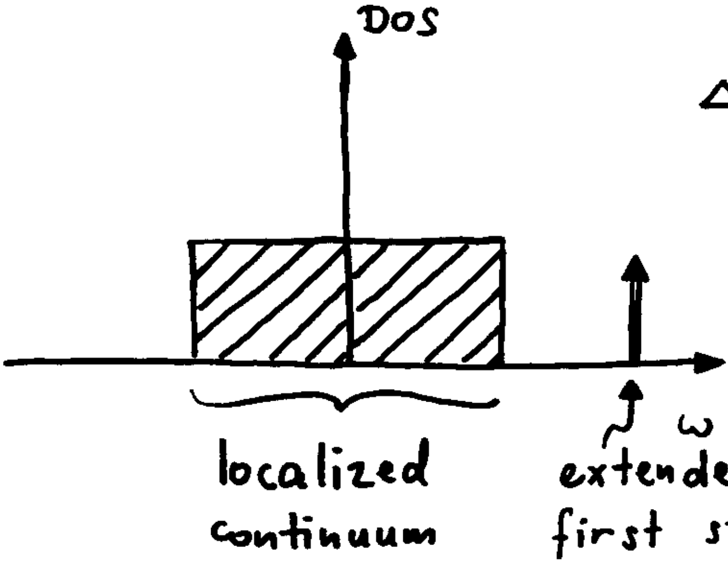
* Solutions in the range $\omega \in [-\Delta, \Delta]$ always exist, $\omega - \epsilon_i \sim 1/N$

* additional solution from

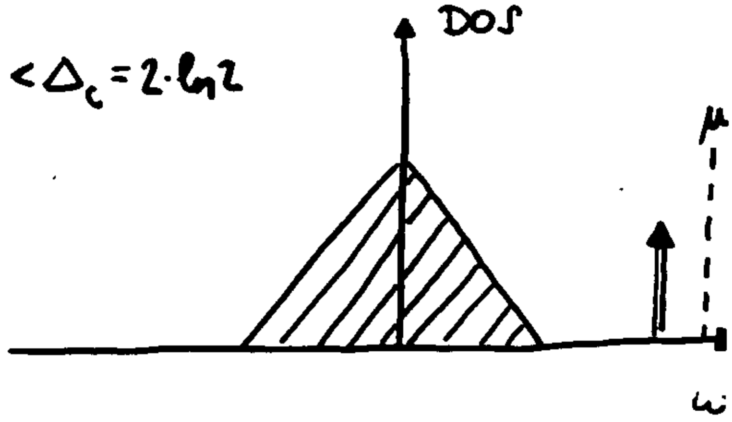
$$1 = J_0 \int_{-\Delta}^{\Delta} d\epsilon \, \frac{P(\epsilon)}{(\omega - \mu) - \epsilon}$$

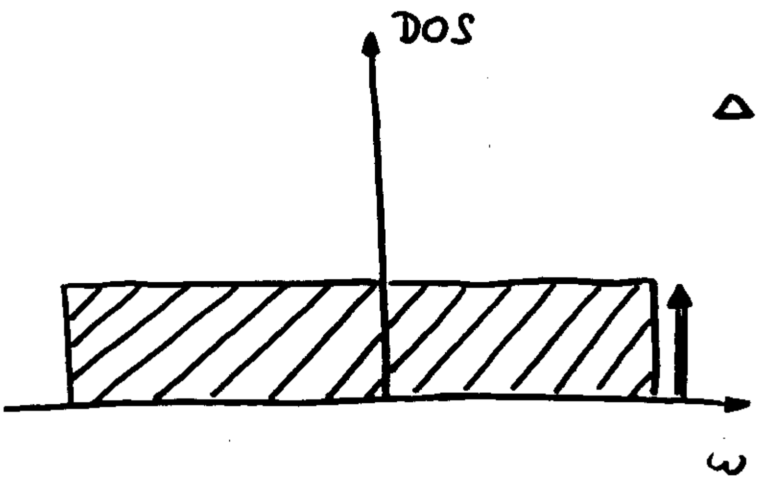
DEPENDS ON ASYMPTOTICS OF $P(\epsilon)$.

DENSITY OF STATES

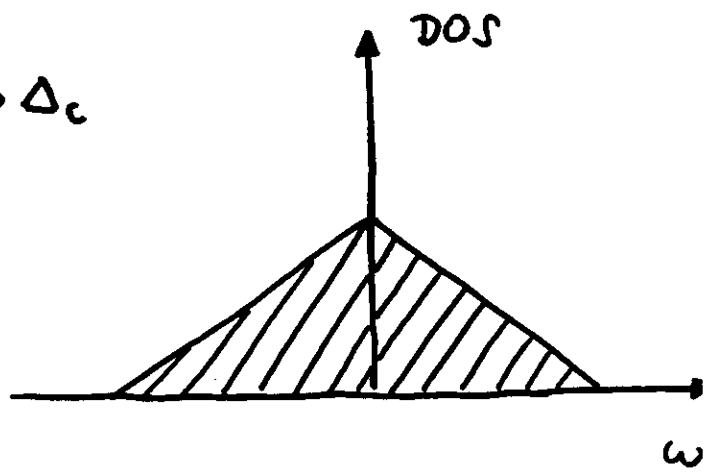


$\Delta < \Delta_c = 2 \cdot \ln 2$





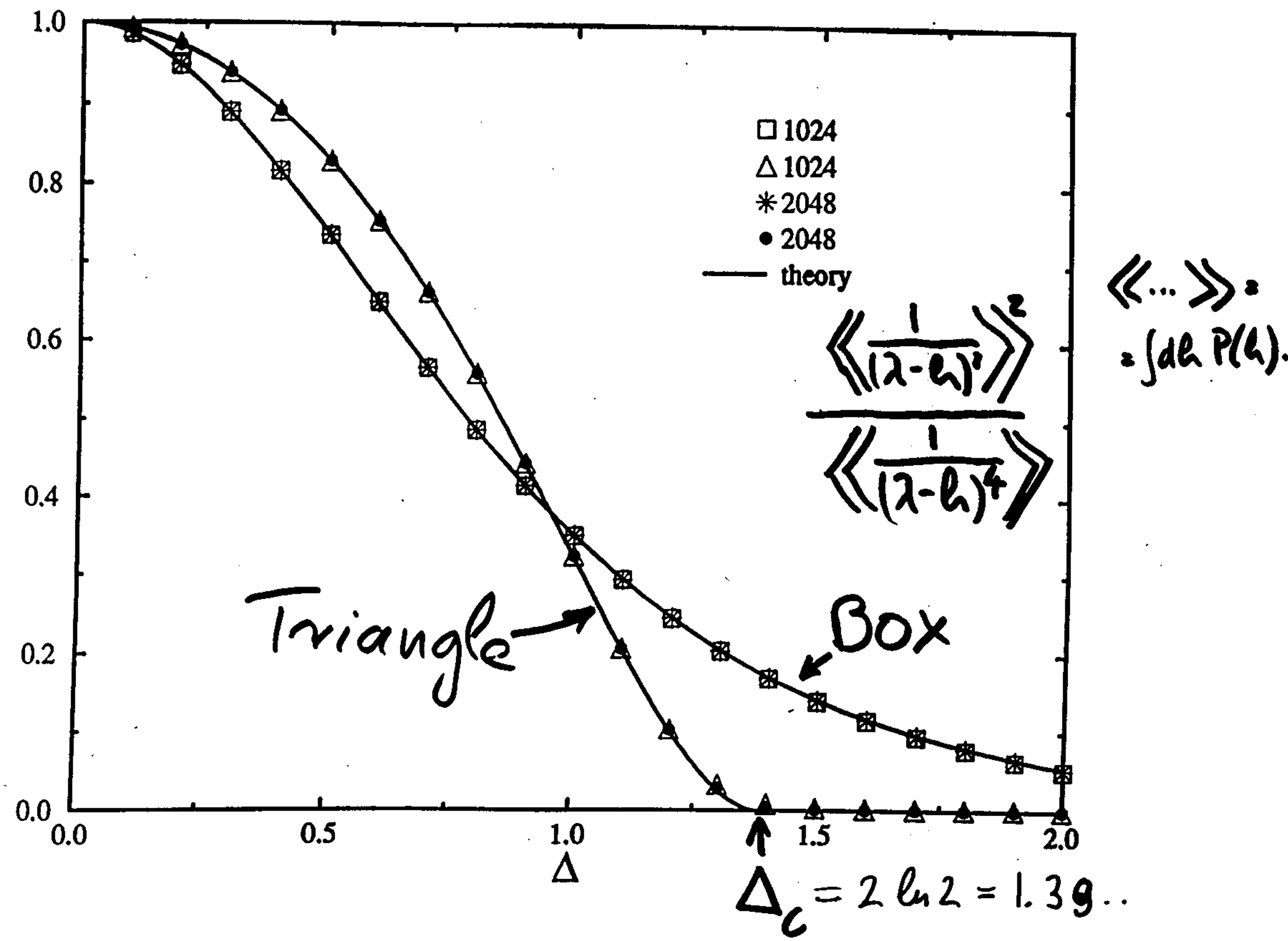
$\Delta > \Delta_c$



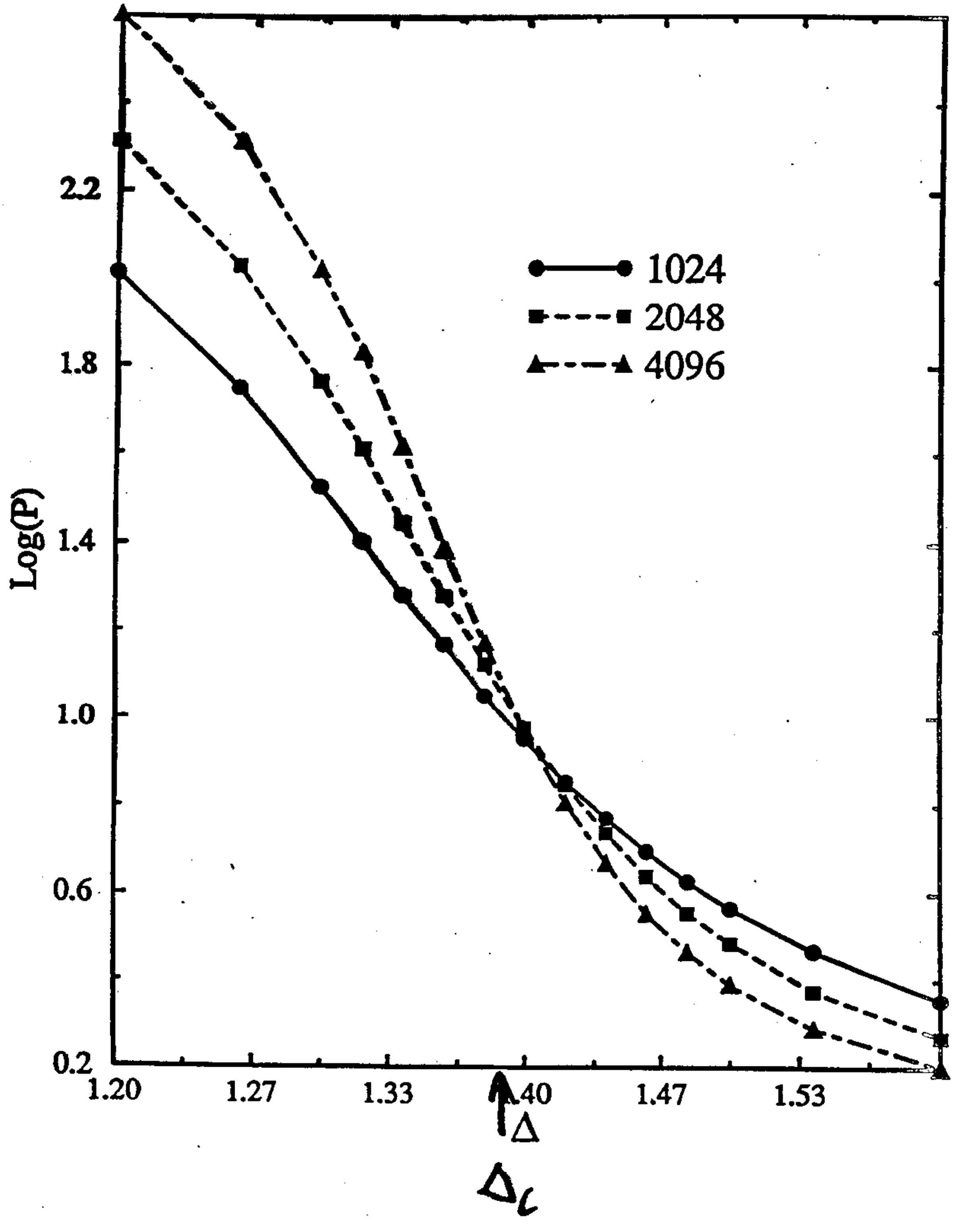
$\boxed{\Delta < \Delta_c}$ holes extended $\Longrightarrow$ form a superfluid

$\boxed{\Delta > \Delta_c}$ holes localized $\Longrightarrow$ form a glass

Participation ratio / N

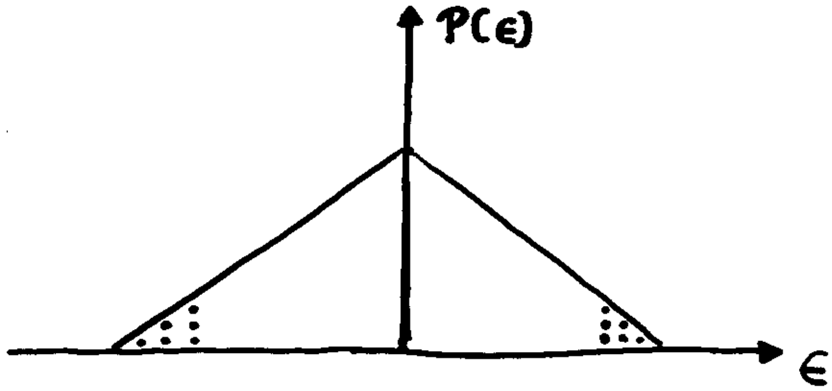


Triangulair



Physics ?

$$P(\epsilon) \sim (\Delta - |\epsilon|)^{\alpha}$$



Density of suitable low energy sites suppressed.

gain in kinetic energy $\sim \frac{1}{N}$

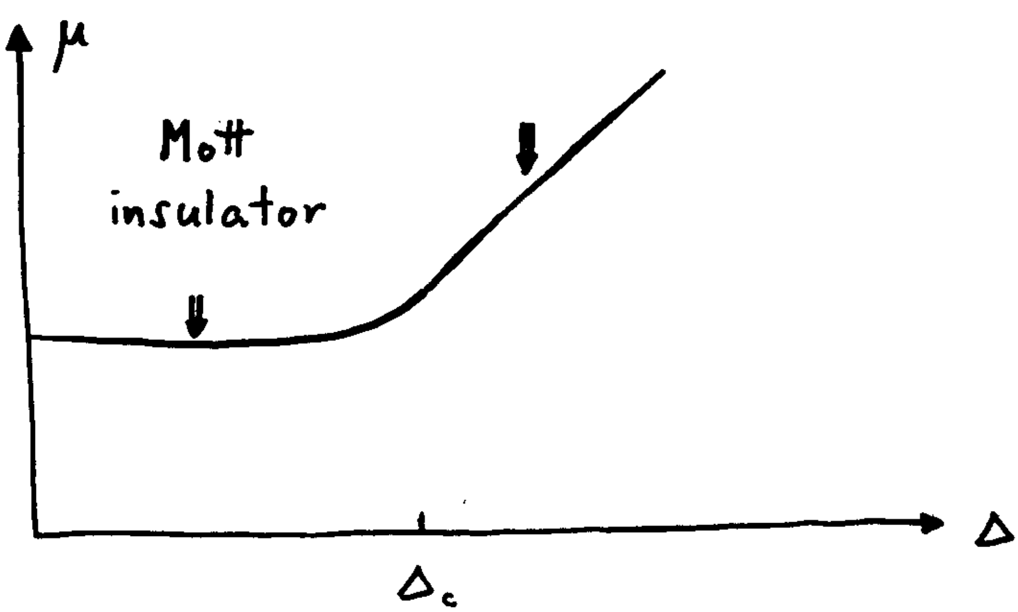
spacing to next localized level $\sim \frac{1}{N^{1/(\alpha+1)}}$

FOR $\alpha > 0$, STATES GAIN BY STAYING LOCALIZED.

TO CAPTURE THE LOCALIZATION TRANSITION, THE DISTRIBUTION OF THE DISORDER ALSO HAS TO BE PROPERLY SCALED WITH N.

CRITICAL BEHAVIOR

	WEAK DISORDER	STRONG DISORDER
XY SUSCEPTIBILITY	$\sim(\mu-\mu_c)^{-1}$	$\sim$ const.
GREEN FC. AT CRITICALITY	$\sim$ const.	$\sim \tau^{-(\alpha+1)}$



Role of α

* Participation ratio AT CRITICALITY

$$P_c \sim \mathcal{O}(1) \qquad 0 < \alpha < 1$$

$$P_c \sim \ln^2 N \qquad \alpha = 1$$

$$P \sim N^{2\frac{\alpha-1}{\alpha+1}} \qquad \alpha > 1$$

* Scaling of gap, between superfluid and continuum

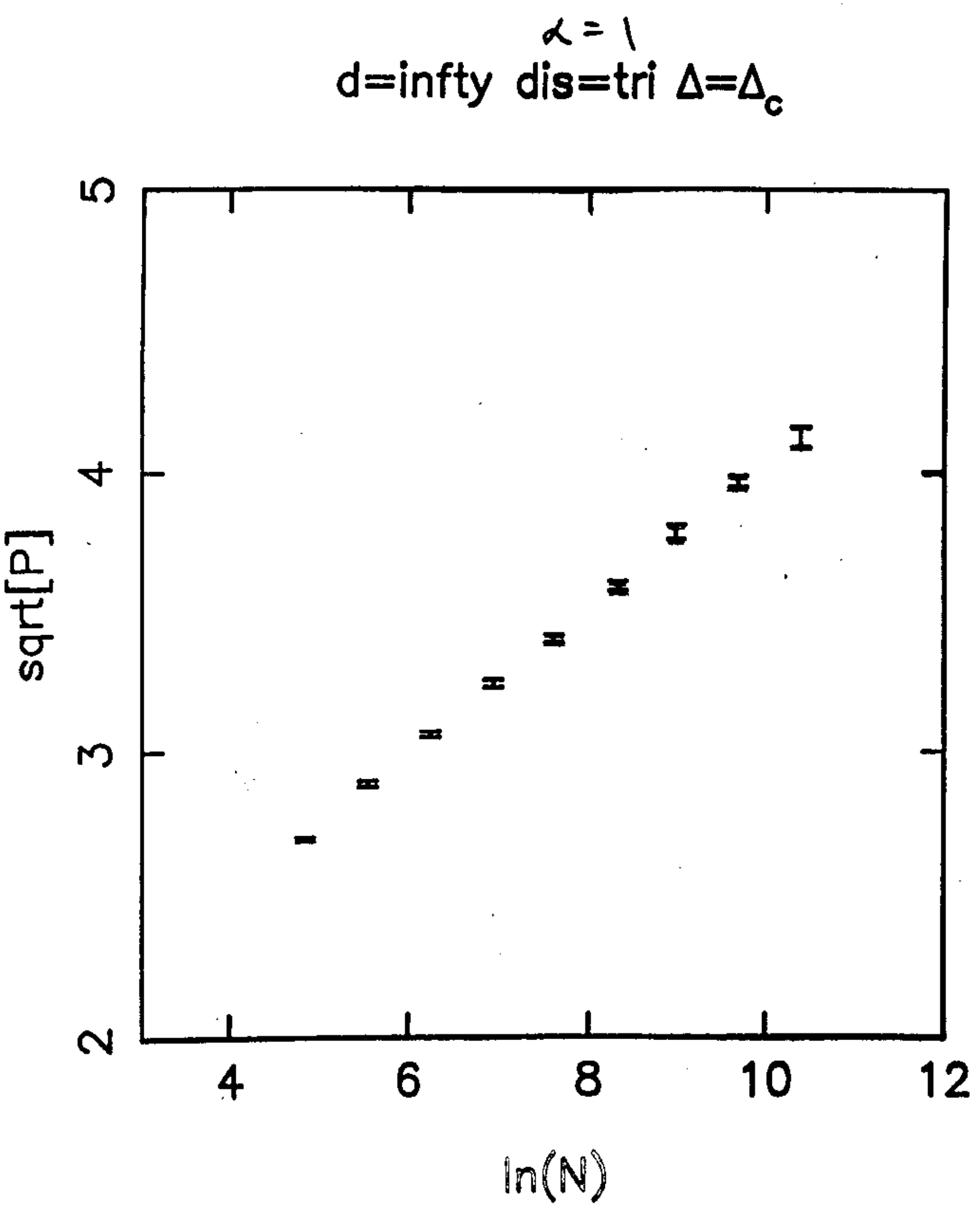
$$\omega \sim (\Delta_c - \Delta)^{\Theta} \qquad \Theta = \frac{1}{\alpha} \qquad 0 < \alpha < 1$$

$$\Theta = 1 \qquad \alpha > 1$$

$\alpha = 0$ "lower critical dimension"

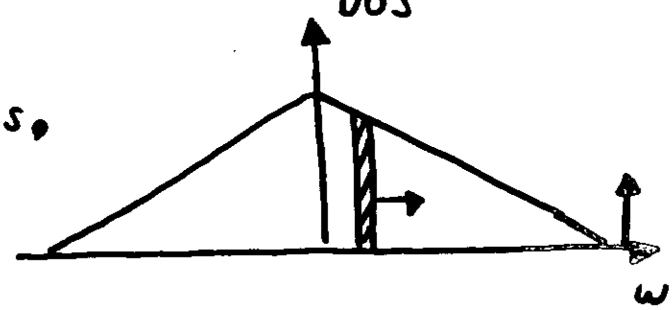
$\alpha = 1$ "upper critical dimension"

logarithmic corrections

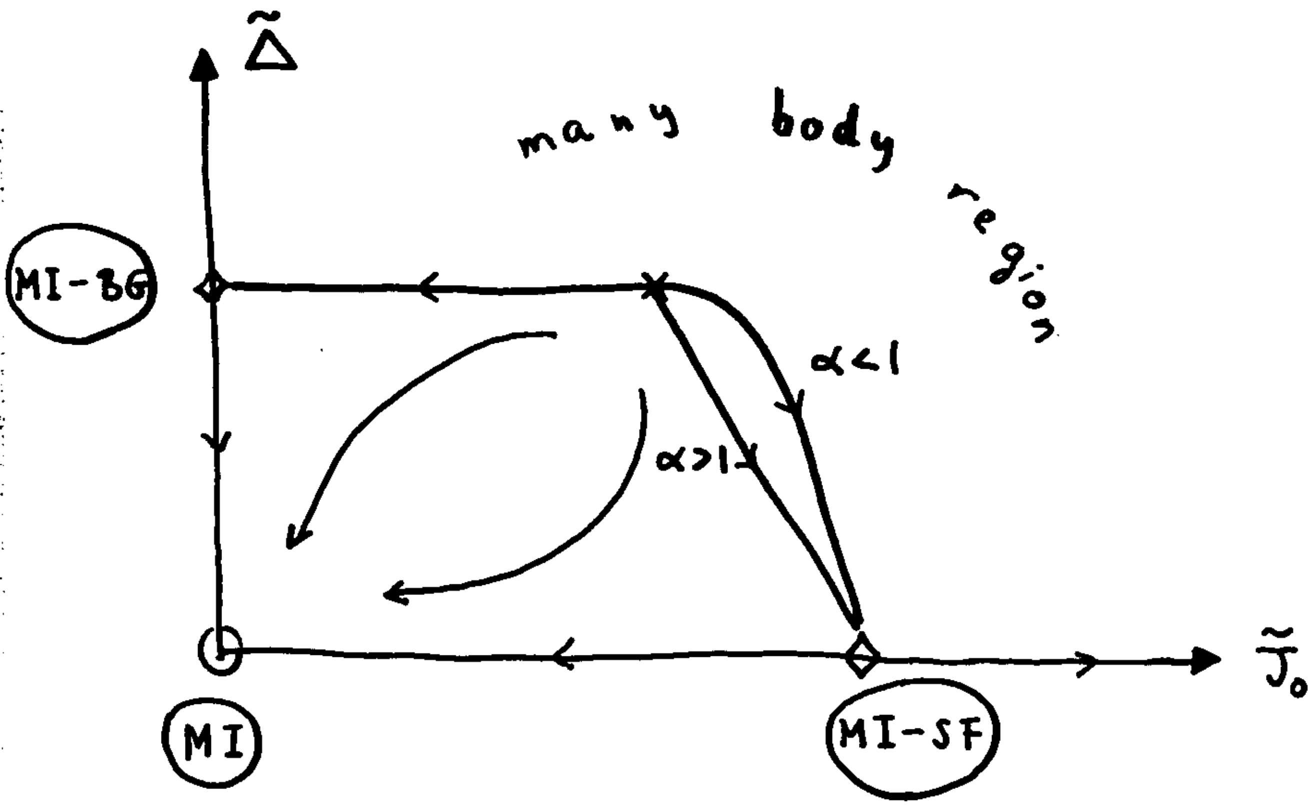

α = 1
d=infty dis=tri Δ=Δc
sqrt[P]
5
4
3
2
4
6
8
10
12
ln(N)

RENORMALIZATION GROUP

$$1 = \frac{1}{N} \sum_i \frac{J_0}{\omega - \mu - \epsilon_i}$$



Eliminate high energy states,
Adjust parameters.



dynamical crit. exp. $z = \tilde{\Delta}$

MANY BODY APPROACH

sites decoupled by Hubbard - Stratonovich transformation.

$$S(\psi) = \int_0^\beta d\tau\, |\psi(\tau)|^2 - \sum_i \ln \mathrm{Tr}\, \exp[-\beta(\mu+\epsilon_i) a_i^+ a_i] \cdot$$

$$\cdot\, T_\tau \exp\left[\sqrt{\frac{J}{N}} \int_0^\beta d\tau \left(\psi^*(\tau) a_i(\tau) + \psi(\tau) a_i^+(\tau)\right)\right]$$

AT the saddle point, for strong disorder

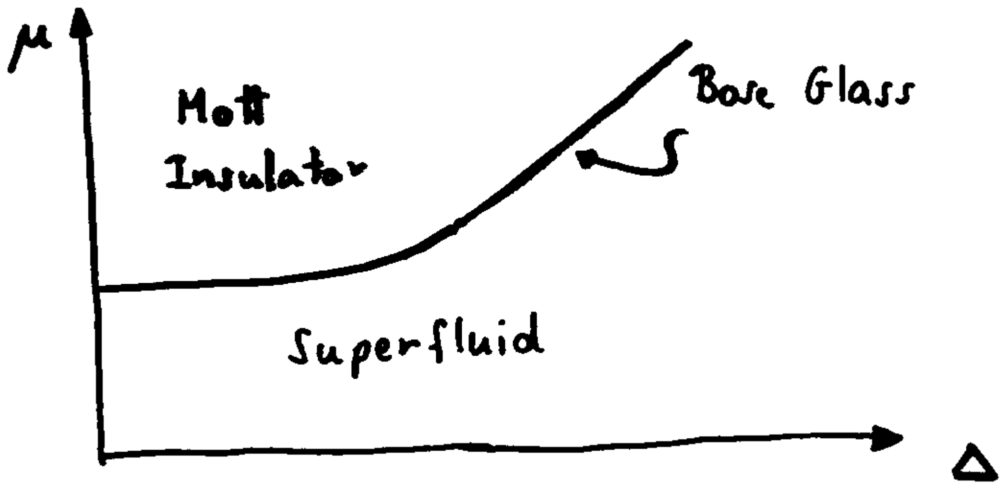
$$S(\psi) = \left[c + \delta \cdot \ln\left(\frac{\psi^2}{\delta}\right)\right] \psi^2 + u\,\psi^4 \qquad \delta \equiv (\mu_c - \mu)^\alpha$$

AT SMALL ψ THE GINZBURG-LANDAU ACTION is NONANALYTIC

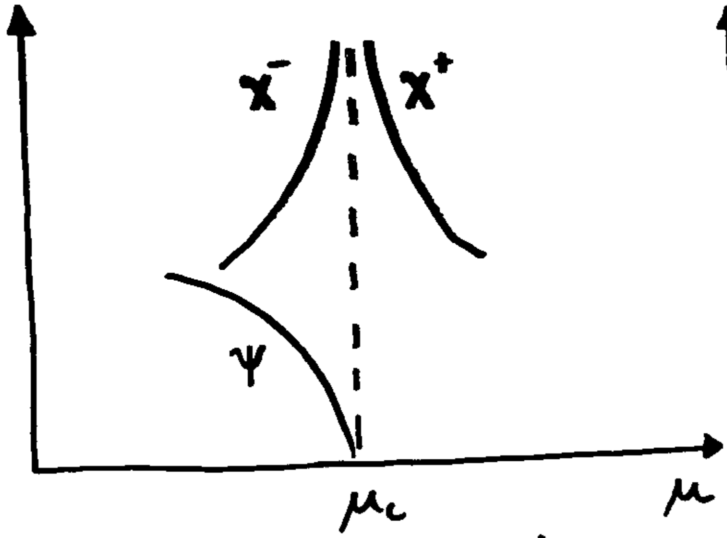
$$\psi \sim \sqrt{\delta}\, \exp\left[-\frac{a}{2\delta}\right]$$

$$\chi \sim \delta^{-1}$$

PHASE DIAGRAM



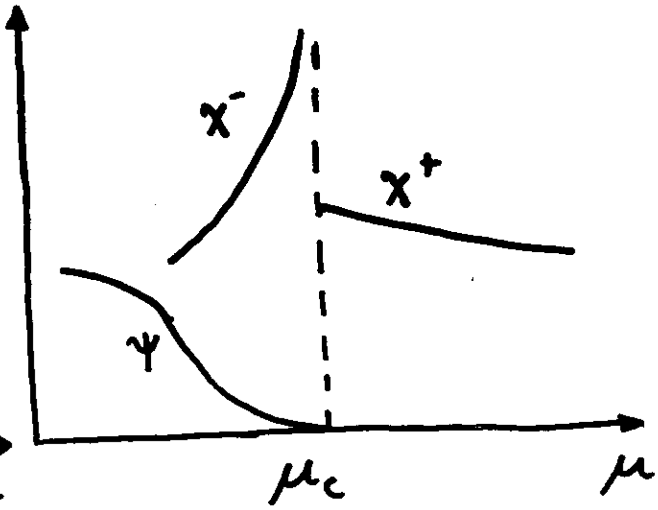
WEAK DIS.



$\chi^{\pm} \sim |\mu-\mu_c|^{-1}$

$\psi \sim (\mu_c-\mu)^{1/2}$

STRONG DIS.



$\chi^{+} \sim \text{const.}$

$\chi^{-} \sim |\mu-\mu_c|^{-1}$

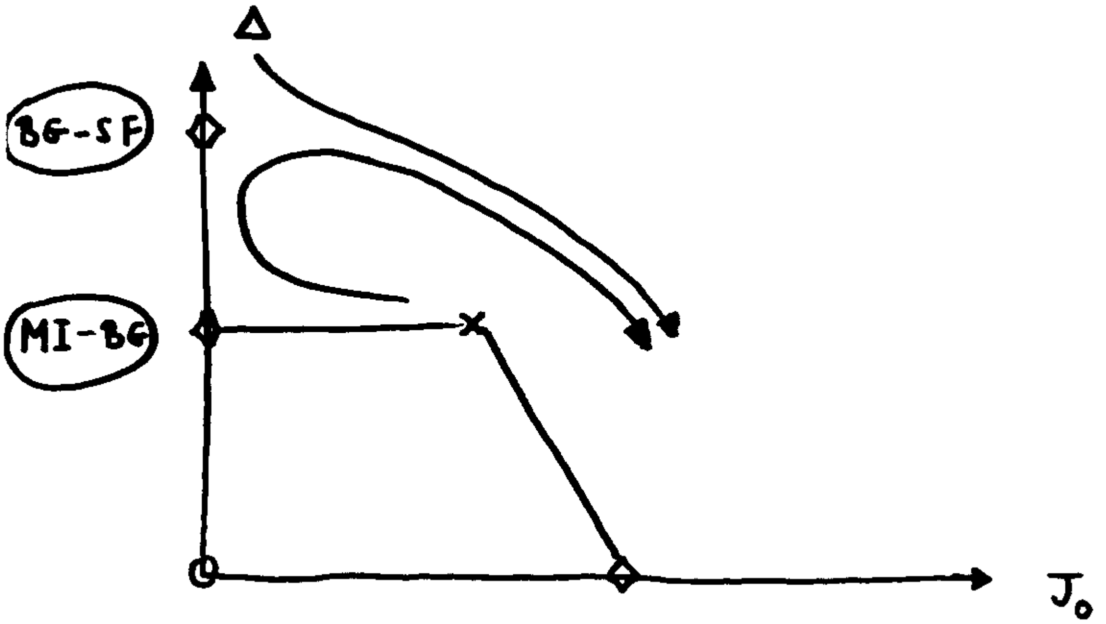
$\psi_a \sim \exp\left[-\frac{a}{\mu_c-\mu}\right]$

$\psi_{\text{crossover}} \sim (\mu_c-\mu)^{1/2}$

Very asymmetric critical behavior. Reason ?

CROSSOVER ENERGY SCALE FOR LARGE Δ.

Guess



Using the Ginzburg-Landau action, one can extend the model for finite range interaction/hopping.

2.) QUANTUM SPIN GLASSES

Classical spin glasses:

* Extensive frustration in the system gives rise to a rugged energy landscape in configuration space.
* Upon cooling system freezes into a LOCAL minimum.

Question:

* If quantum dynamics is introduced for the spins, will the quantum fluctuations / tunneling through barriers "melt" the glass?
* What is the nature, AT T=0, the quantum spin-glass — quantum paramagnet transition?

Simplest system: Ising model in transverse field

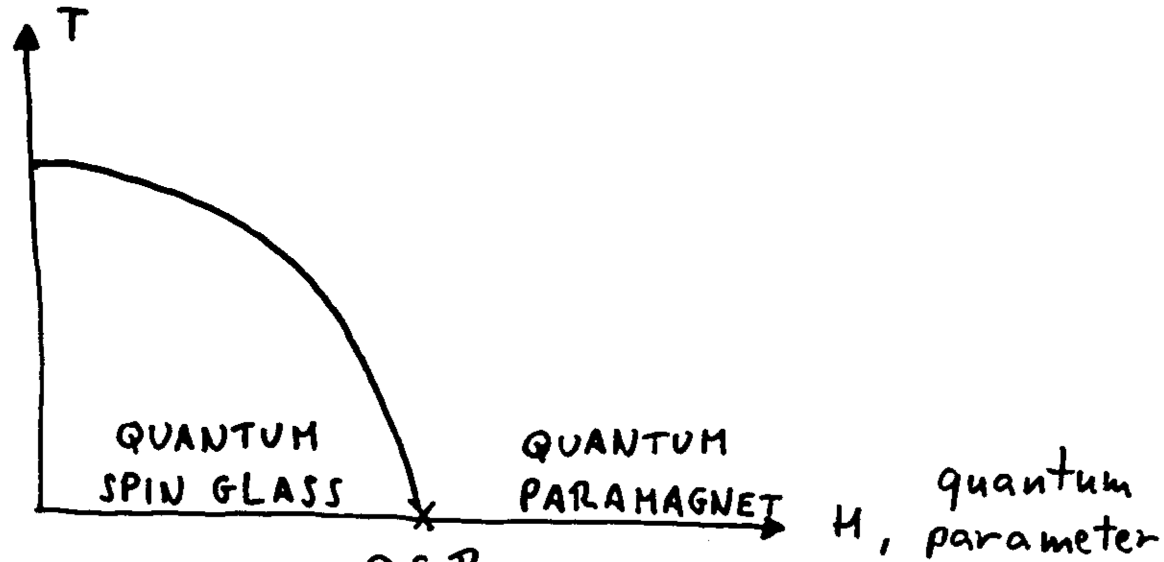
$$\mathcal{H} = \sum_{ij} J_{ij} \, S_i^z S_j^z - H \sum_i S_i^x$$

Real space renormalization in 1d : D. Fisher

Numerical studies in 1d : P. Young

Infinite range models, τ independent kernel approx. : Bray & Moore, Goldschmidt & Lai

Infinite range models, proper treatment of kernel : Ye, Sachdev & Read, Miller & Huse



Recent excitement:

In $LiHo_{0.167}Y_{0.833}F_4$, which is a diluted dipolar ferromagnet, indeed a similar phase diagram has been observed on the (T, H) plane.

Wu, Bitko, Rosenbaum & Aeppli

They measure $\left\{ \begin{array}{l} \text{nonlinear susceptibility} \\ \text{noise spectrum} \end{array} \right\}$.

* PHASE DIAGRAM: AS EXPECTED
* PROBLEM: AT LOW T,

AT THE TRANSITION

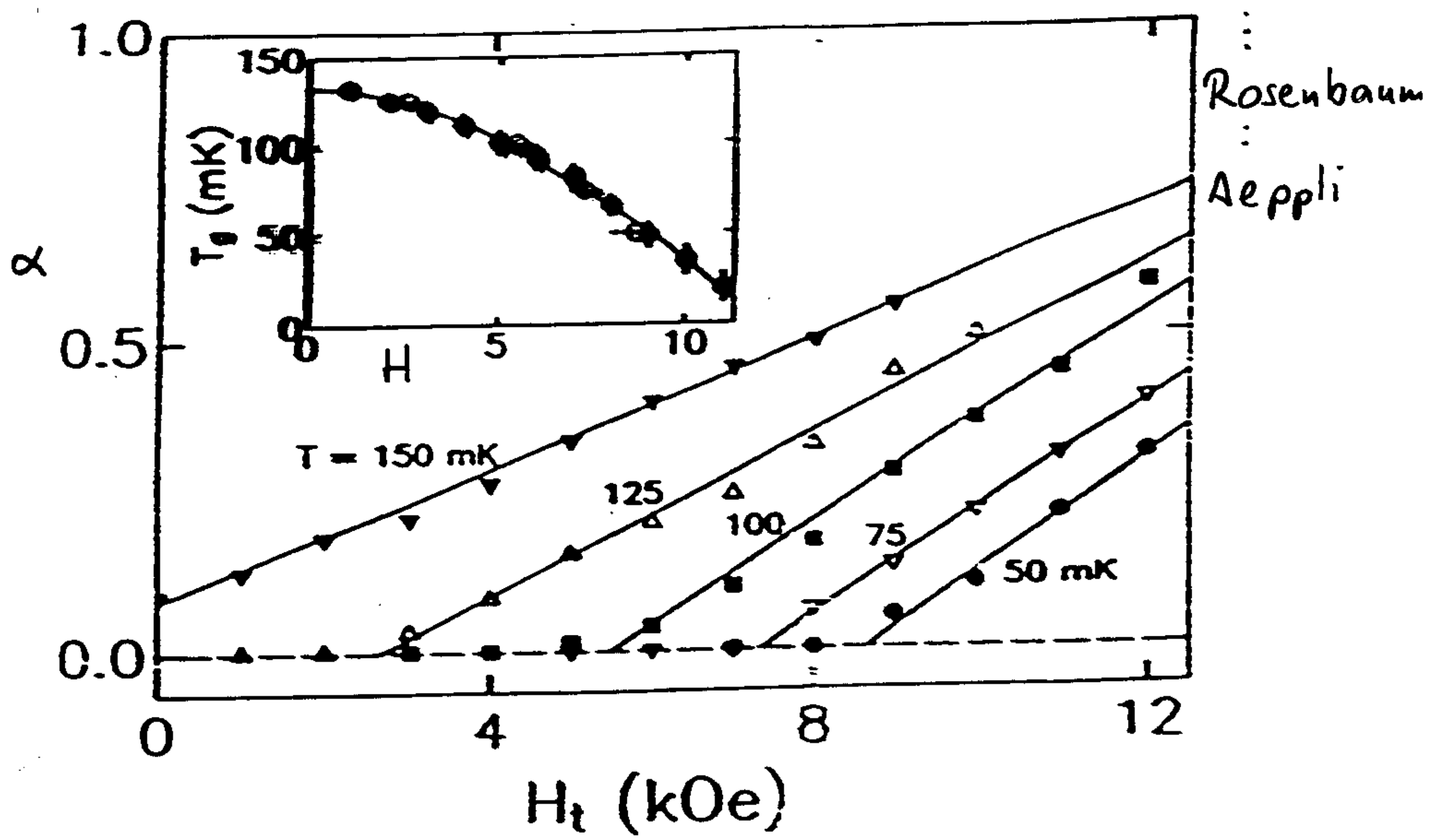
$\chi_{\text{non-linear}}$ DOES NOT DIVERGE

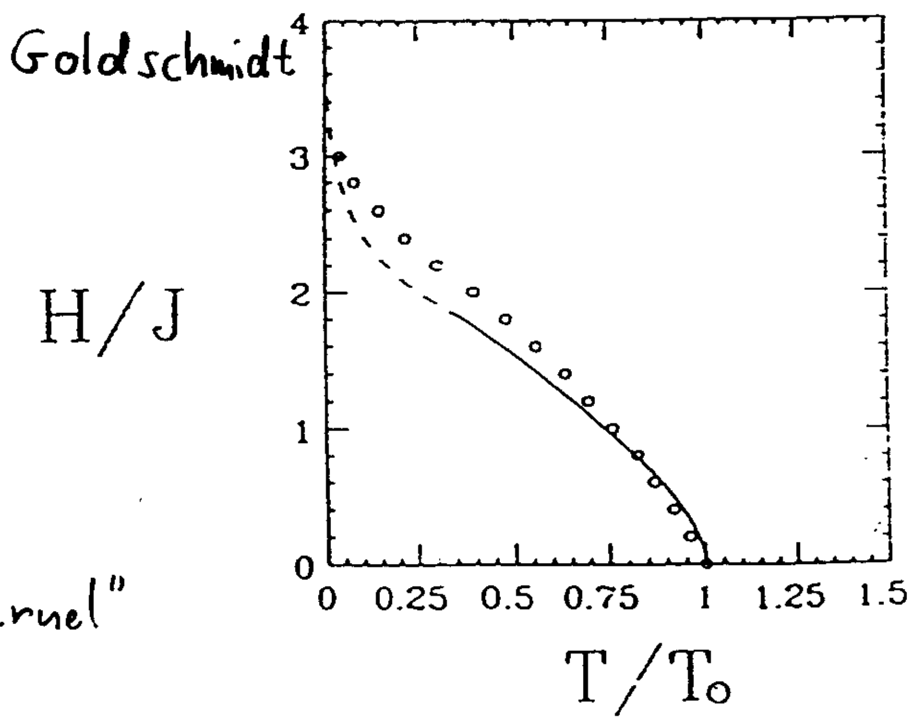
Numerical studies: Guo, Bhatt & Huse; Rieger & Young — CAN NOT ACCOUNT

Field theory, renormalization group for rotor models: Ye, Sachdev & Read

An undetermined exponent has to assume a quite large value.

$\mathrm{Im}\ \chi(\omega) \sim \omega^{-\alpha}$





- S.K. type model
- time independent kernel" approximation

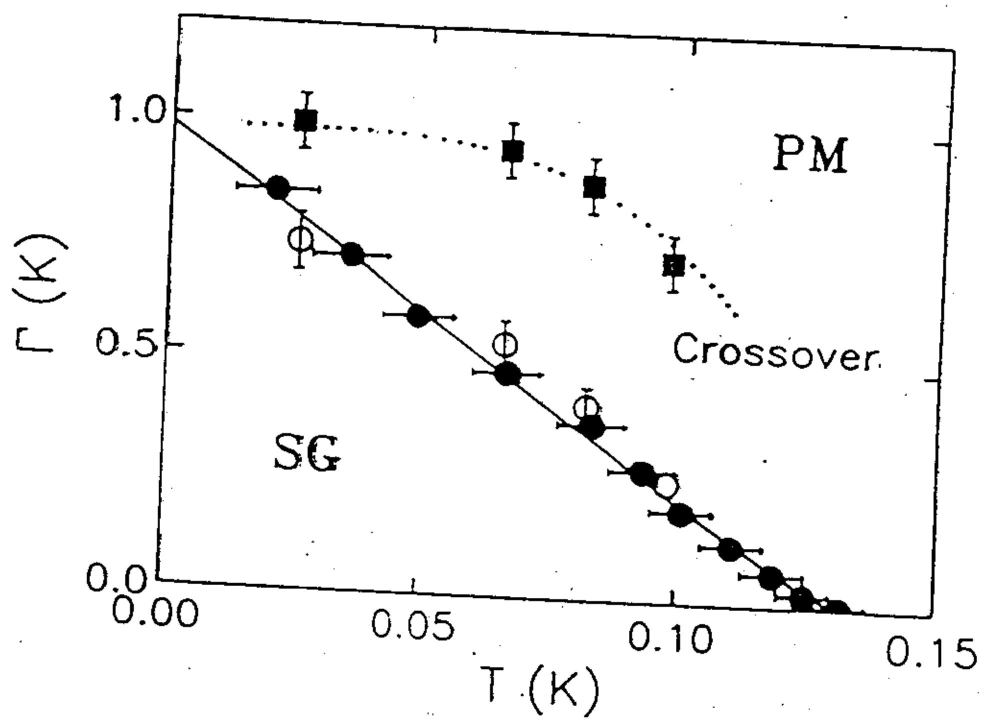


FIG. 1. Phase diagram of the diluted dipolar-coupled Ising spin glass $LiHo_{0.167}Y_{0.833}F_4$ in the transverse field (Γ)—temperature (T) plane. SG: spin glass; PM: paramagnet. Filled circles follow from dynamical measurements, open circles from the nonlinear susceptibility, and squares indicate where $\chi_1''(f=1.5\ \mathrm{Hz})$ begins to rise (see Fig. 3). The dotted line is the mean-field phase boundary associated with an ordered magnet whose critical temperature and field are the same as $T_g(\Gamma=0)$ and $\Gamma_g(T=0)$, respectively.

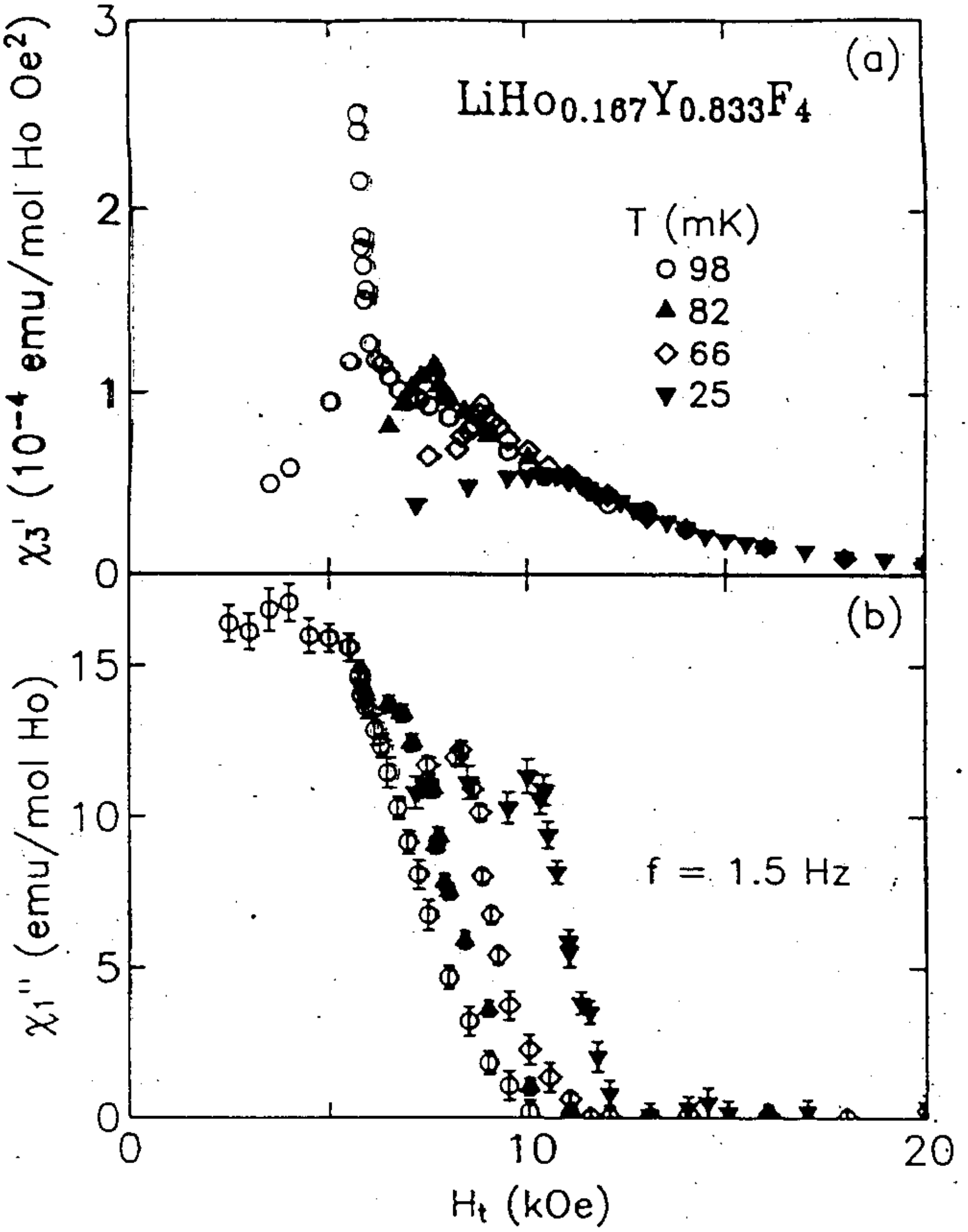


FIG. 3. (a) The nonlinear susceptibility as a function of transverse magnetic field at four temperatures. The clear divergence of χ_3 at the spin glass transition in the high T, small H_t (small Γ) classical limit becomes merely a small flat maximum in the low T, large H_t (large Γ) quantum limit. (b) Corresponding behavior of the imaginary part of the linear susceptibility. Here, there is a clear signature of the spin glass transition in both the classical and quantum regimes.

Motivation of model

1.) LiHoYF has dipolar, i.e. long range interactions.

⇒ A Sherrington-Kirkpatrick model might be a reasonable approximation.

Indeed, the clean system exhibits a ferromagnetic transition with the mean-field value 1 for γ.

2.) The dipolar interaction has two terms:

$$\frac{(\vec{S}_i \cdot \vec{S}_j)}{|r_i - r_j|^3} - 3\,\frac{(\vec{S}_i \cdot \vec{r}_i)(\vec{S}_j \cdot \vec{r}_j)}{|r_i - r_j|^5}$$

In the experiment $\vec{H} \parallel \hat{x}$, so $\langle S^x \rangle \neq 0$.

The second term then introduces a RANDOM TRANSVERSE FIELD, through $S_i^y \cdot \langle S^x \rangle$.

We consider

$$\mathcal{H} = -\sum_{ij} \left(J_{ij} \; S_i^+ S_j^- + h.c. \right) - \sum_i (H + h_i) \, S_i^z$$

$$< J_{ij} > = \frac{J_0}{N}$$

$$< |J_{ij}|^2 > = \frac{J}{N}$$ random (complex) number, with Gaussian distribution

h_i: randomly distributed over $[-\Delta, \Delta]$. $P(h) \sim (\Delta - |h|)^a$

Also describes bosons propagating in random, quenched flux: QUANTUM GAUGE GLASS.

* At large H we have a Quantum Paramagnet
(~ Mott insulator)

* Lowering H can give rise to an

→ XY ferromagnet

or a

→ glass

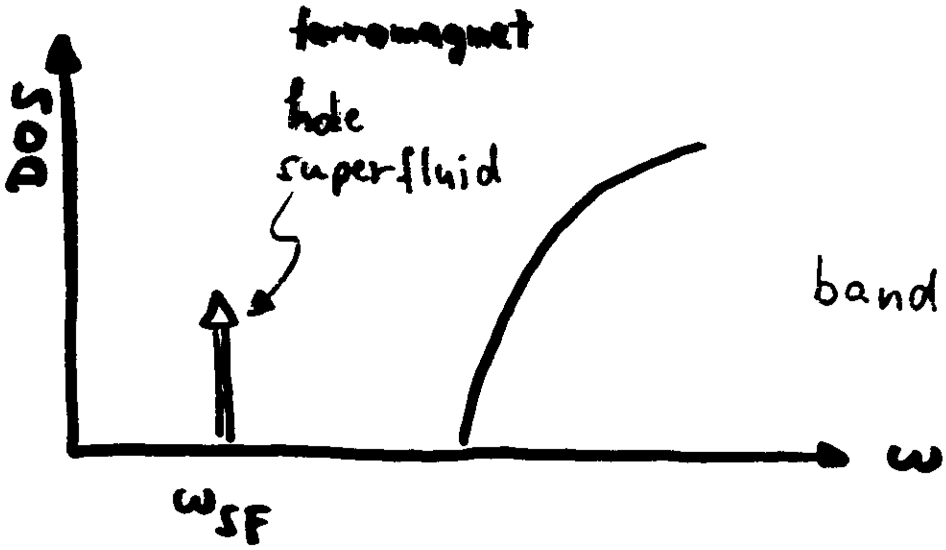
* For $\alpha > 1$ we find TWO GLASSES, $\begin{Bmatrix} \text{weak} \\ \text{strong} \end{Bmatrix}$

Again, we use one particle methods first.
We treat the site disorder exactly,
for large N exact for bonds, too.

Edwards method
Cavity method } gives same

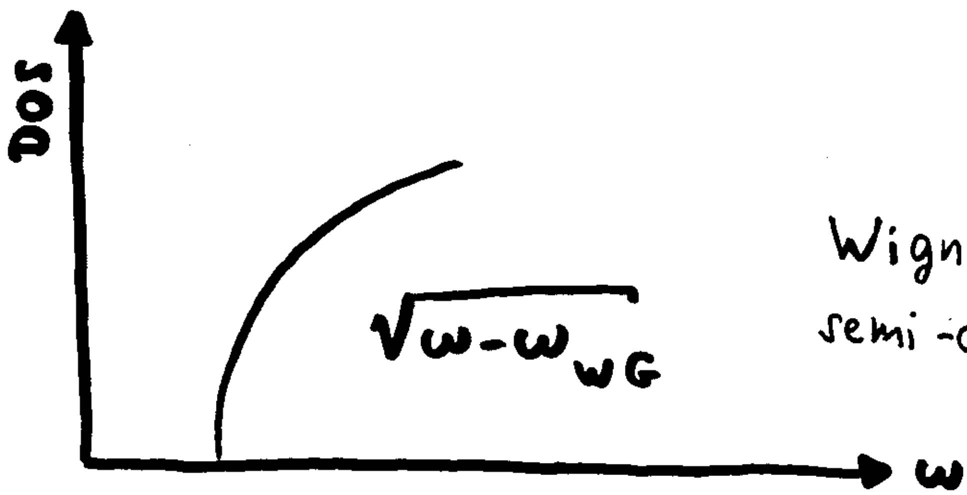
ONE PARTICLE SPECTRUM

XY FERRO MAGNET, SUPER FLUID



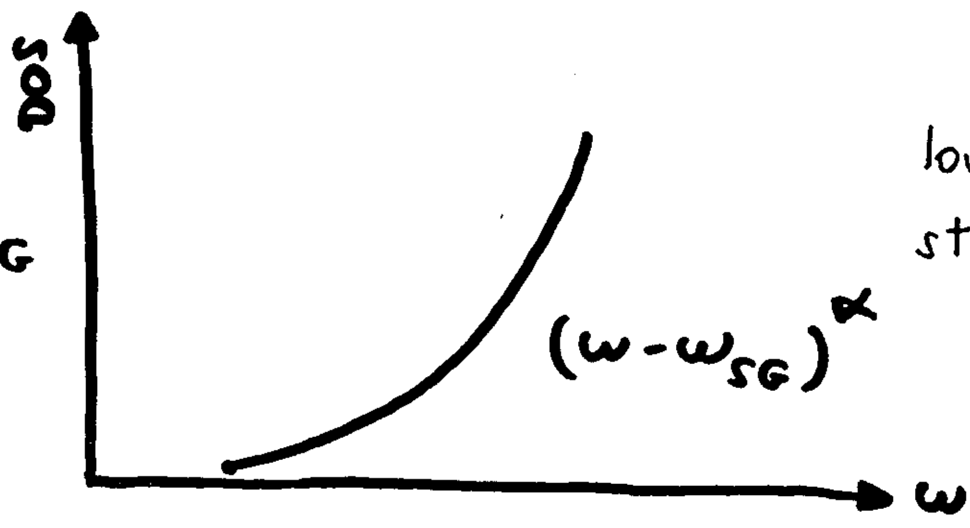
band

WEAK GLASS



Wigner's semi-circular band

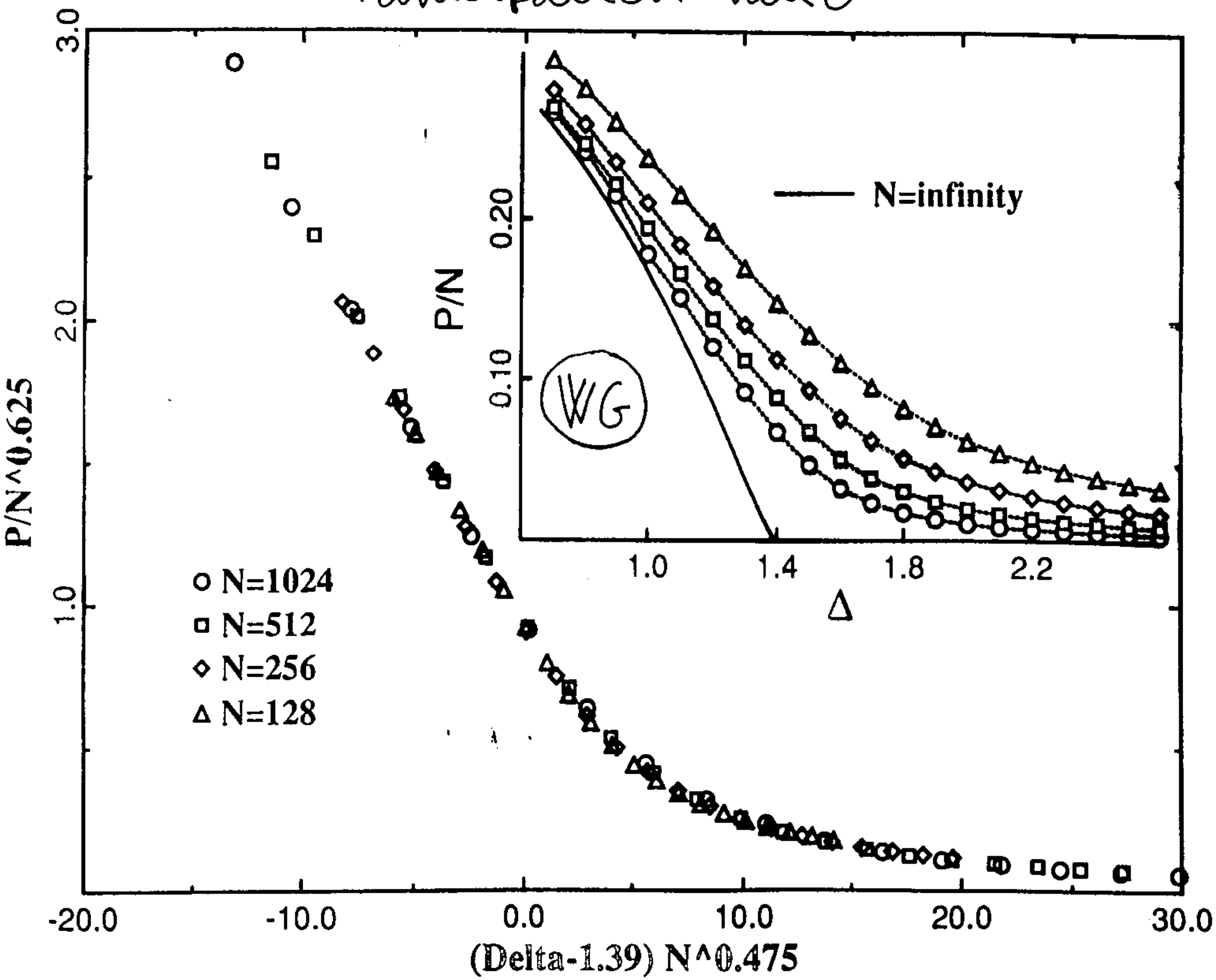
STRONG GLASS



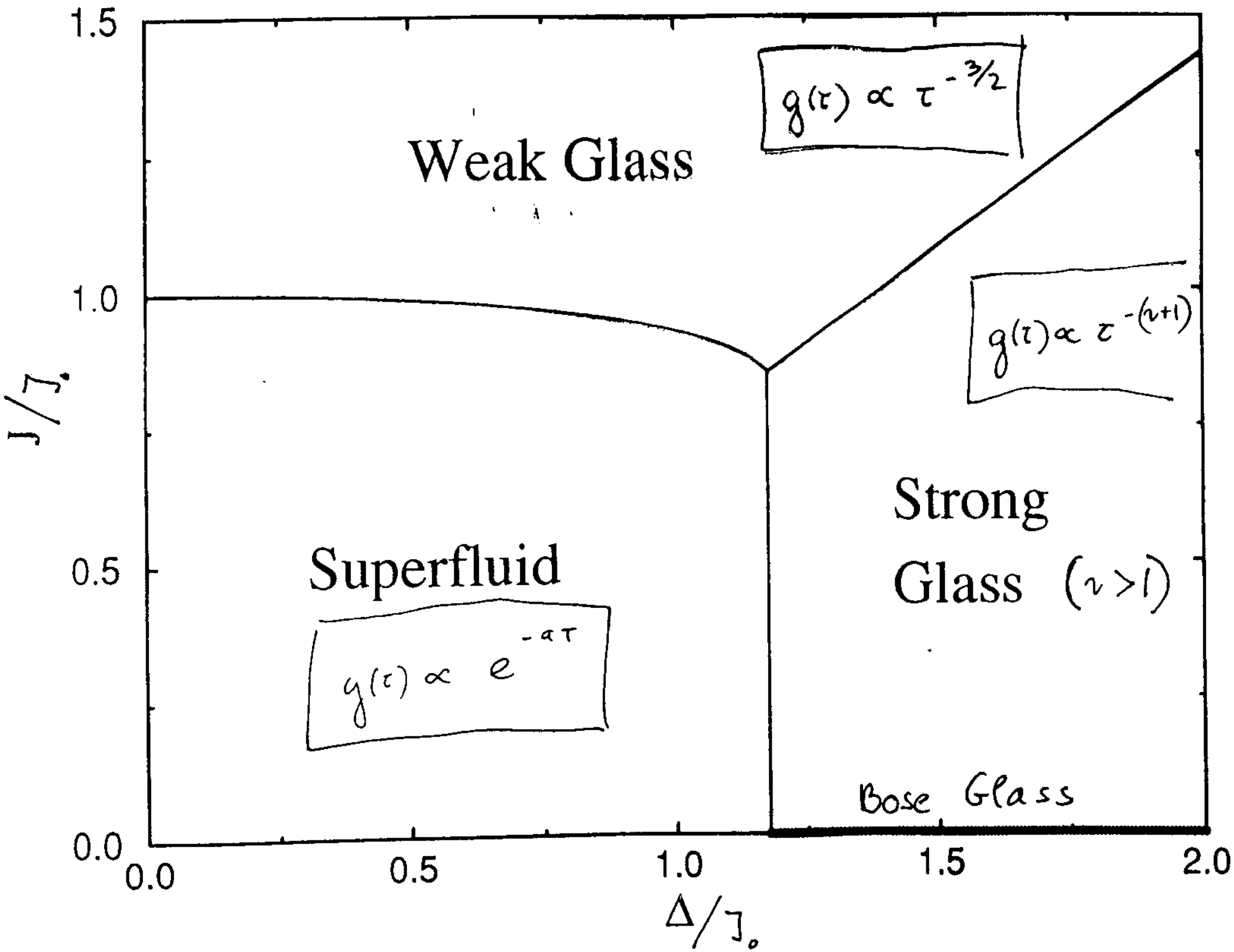
low energy states localized

Figure 2.

Participation ratio



$g(\tau) = \langle T_\tau a^+(\tau) a(0) \rangle$ Figure 1 $g(\tau) \propto \int d\omega \, e^{-\omega\tau} \rho(\omega)$



MANY BODY ANALYSIS

* Replica trick

* H-S decoupling: $Q_{\alpha\beta}(\tau,\tau') = \frac{1}{N}\sum_i \langle T_\tau \, a^+_\alpha(\tau)\, a_\beta(\tau') \rangle$

  frustration

* ADDITIONAL H-S decoupling : $\varphi_\alpha(\tau) = \frac{1}{N}\sum_i \langle a_i(\tau) \rangle$

  local field, from other spins

* Saddle point for Q

* G-L theory for φ

* work in self-consistent Hartree-Fock for φ.

$$Q_{\alpha\beta}(i\omega_n) = \delta_{\alpha\beta}\, G(i\omega_n) + \delta(i\omega_n)\,\beta q$$

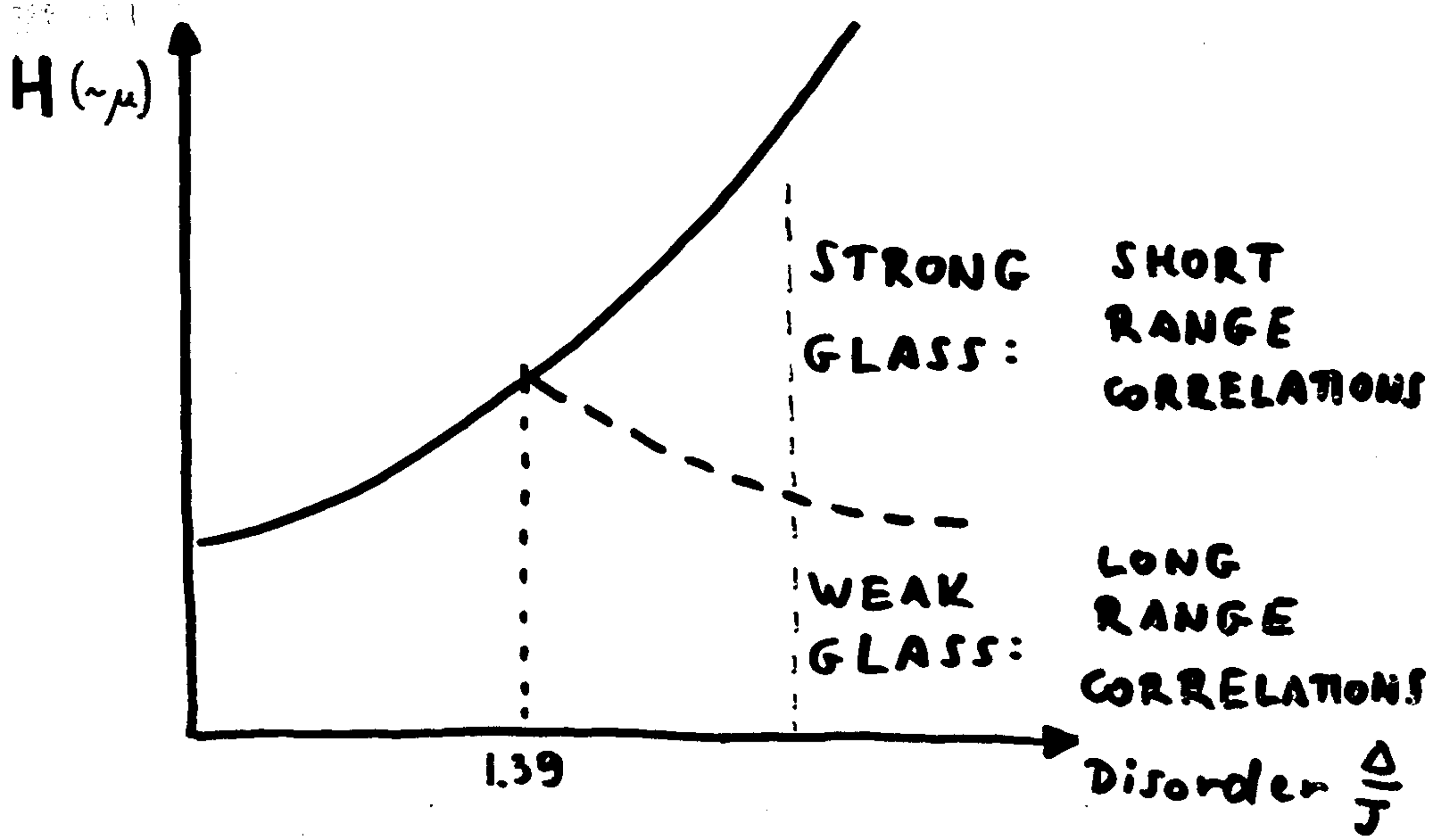
q : Edwards-Anderson order parameter

$\mathrm{Im}\, G(\omega)$: Density of states

RESULTS

1.) At Strong Glass transition the non-linear susceptibility does NOT diverge. ⟹ spin glass experiments

2.) Phase diagram



3.) critical exponents are different

	weak glass	strong glass
DOS: $\omega^{(\eta)}$	1/2	2
$q : (\mu-\mu_c)^{(\varepsilon)}$	1	4
$g(\tau) : \tau^{-(\delta)}$	2	3

4.) finite T : different transitions at low T and high T

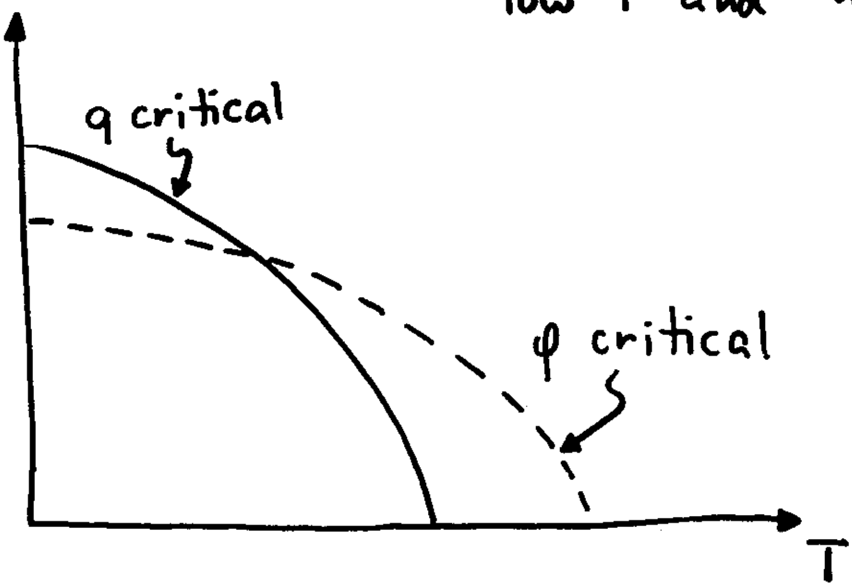
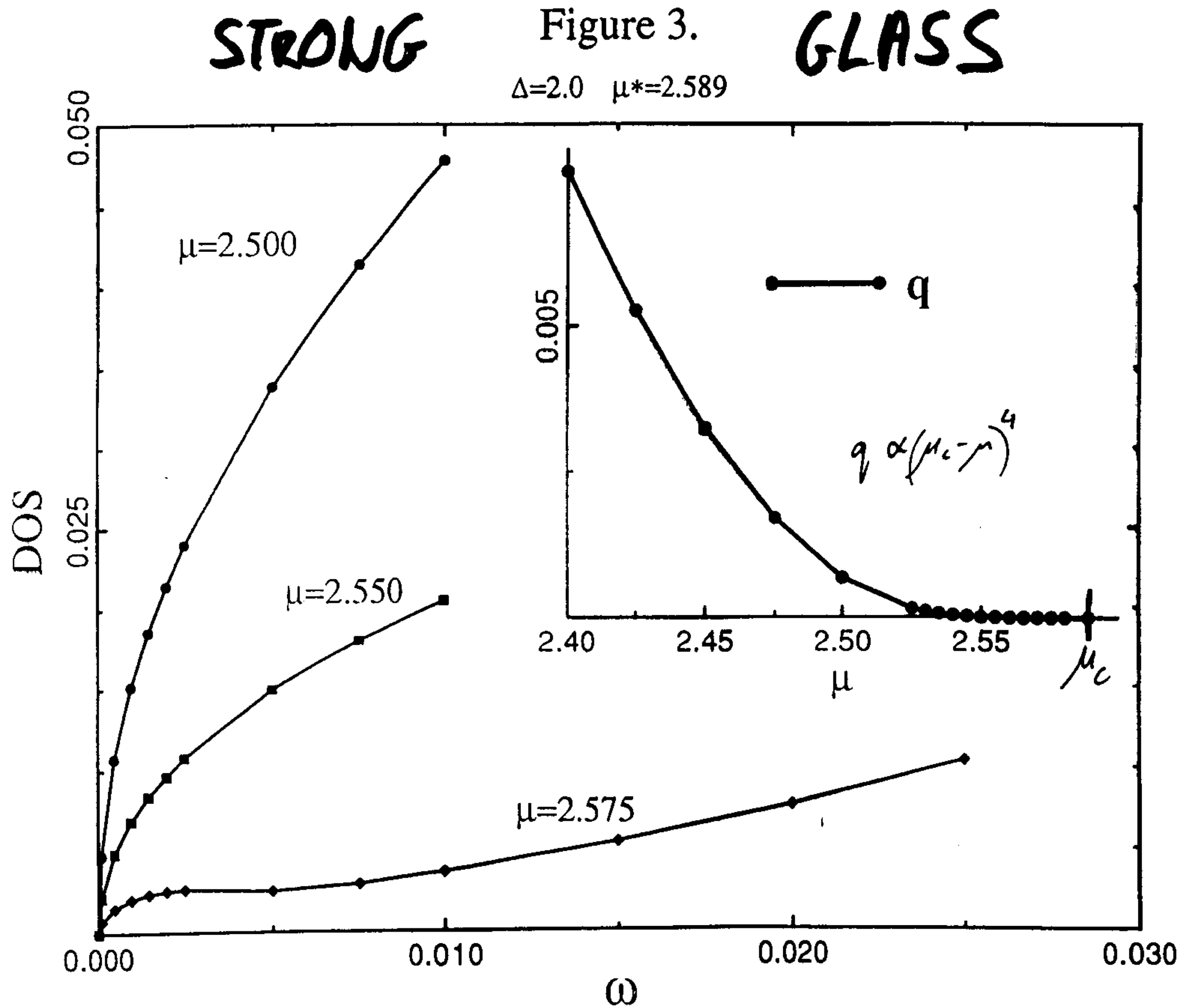
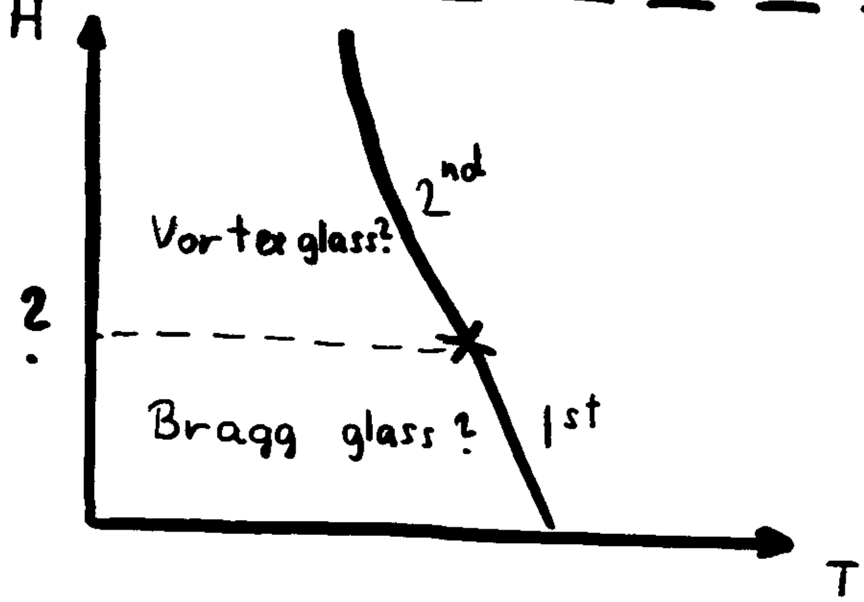


Figure 3.



STRONG
GLASS
Δ=2.0 μ*=2.589
DOS
0.050
0.025
0.000
0.010
0.020
0.030
ω
μ=2.500
μ=2.550
μ=2.575
q
0.005
2.40
2.45
2.50
2.55
μ
$q \propto (\mu_c - \mu)^4$
μ_c

I. QUANTUM GAUGE GLASS



classical gauge glass:

$$H = -J \sum_{\langle ij \rangle} \cos(\varphi_i - \varphi_j - A_{ij})$$

quantum gauge glass:

$$H = -\sum_{\langle ij \rangle} (J_{ij}\, e^{iA_{ij}} a_i^{+} a_j + h.c.) - \underbrace{\sum_i \mu_i a_i^{+} a_i}$$

* proper symmetry

* frustration

additional physics: site disorder

SUMMARY

1.) The localization/Bose Glass transition

For the infinite range model, using

* one particle approach
* numerical methods
* renormalization group
* non-analytic field theory

determined

* PHASE DIAGRAM
* CRITICAL BEHAVIOR $(\psi, \chi_\psi, G(\tau), \mathcal{H})$

2.) Quantum spin glasses in transverse random field

* same methods

determined

* PHASE DIAGRAM : TWO GLASS PHASES

CRITICAL BEHAVIOR: χ_{nl} is NON-DIVERGENT FOR QUANTUM-PARAMAGNET-TO-STRONG GLASS TRANSITION