UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL ATOMIC ENERGY AGENCY
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR.959 - 33

MINIWORKSHOP ON STRONG ELECTRON CORRELATIONS
"Disorder and Interaction in Quantum Systems and Their Classical Analogs"

(1 - 19 July 1996)

"Vortex Pinning and Elementary Quantum Mechanics"

Valerii Vinokur
Argonne National Laboratory
Materials Science Division
Argonne, IL 60439
U.S.A.

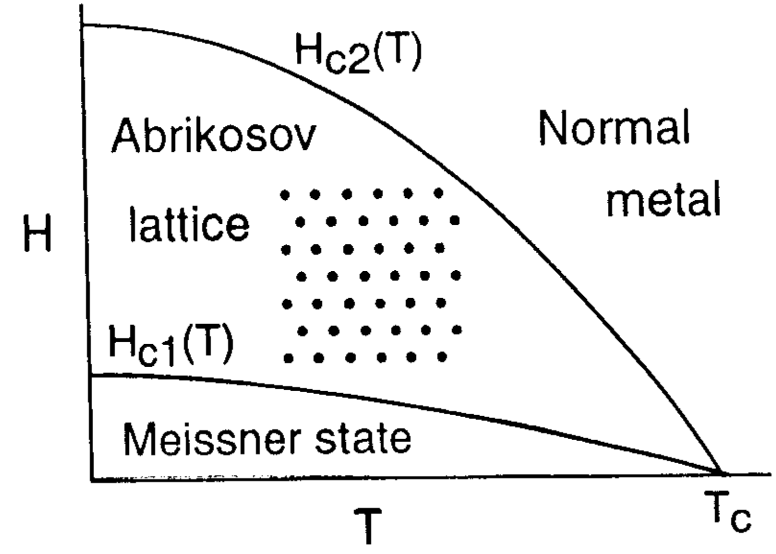
These are preliminary lecture notes, intended only for distribution to participants.

VORTEX PINNING AND ELEMENTARY QUANTUM MECHANICS

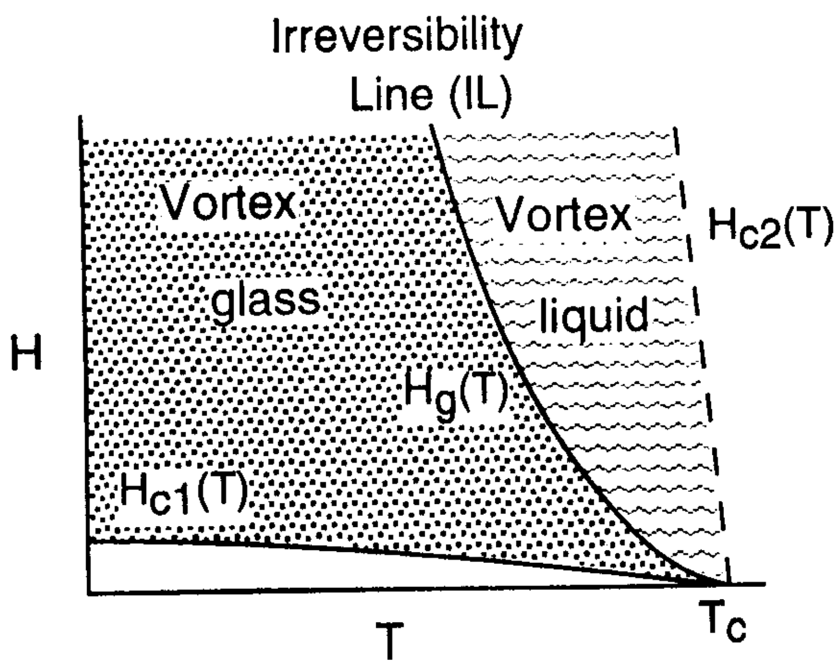
A. I. LARKIN
D. R. NELSON
V. V.

ARGONNE NATIONAL LABORATORY
ARGONNE, IL 60439, USA

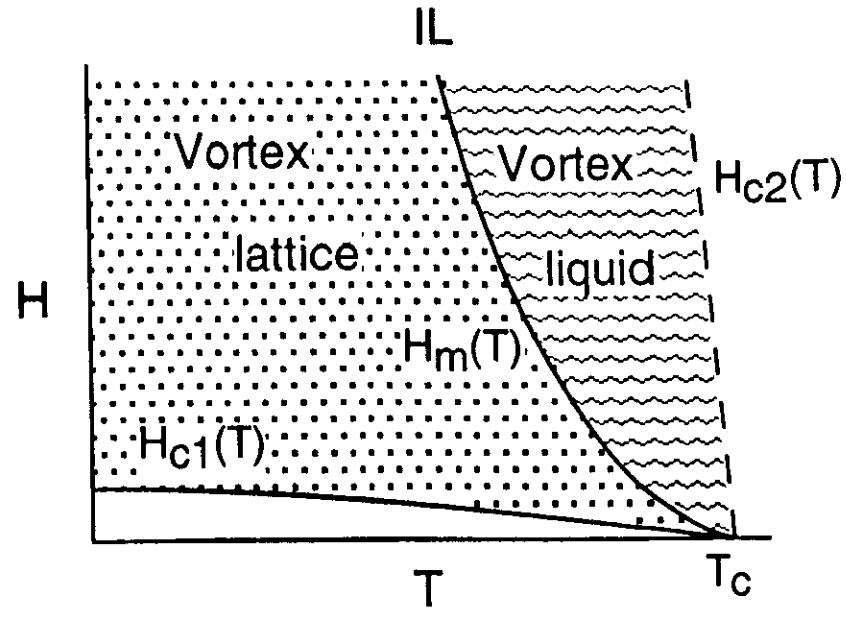
Conventional superconductors:



High-temperature superconductors:



dirty system



clean system

VORTEX / 2D-BOSON ANALOGY

Model free energy for N flux lines

$$F_N = \int_0^L dz \left\{ \sum_{i=1}^{N} \left[\frac{\varepsilon_\ell}{2} \left| \frac{d\vec{r}_i(z)}{dz} \right|^2 + V_D(\vec{r}_i(z)) \right] + \frac{1}{2} \sum_{i \neq j} V\left[|\vec{r}_i(z) - \vec{r}_j(z)| \right] \right\}$$

$\{\vec{r}_i(z)\}$: vortex trajectories (magnetic field aligned with the z axis)

$V(|\vec{r}_i - r_j|)$: the interaction potential between vortices

$\varepsilon_\ell =$: the vortex linear tension

$= \varepsilon^2 \varepsilon_0$ $\qquad \varepsilon_0 = \left(\frac{\Phi_0}{4\pi\lambda} \right)^2 \qquad \lambda \equiv \lambda_{ab}$

ε : the material anisotropy

$V_D(\vec{r})$: the pinning potential modeled by a random array of identical cylindrical traps of average spacing d and radius r_0

a potential well depth: U_0

The partition function:

$$Z = \sum_{n=0}^{\infty} \frac{1}{N!} \sum_{P} e^{\mu N L / T} \int \mathcal{D}\vec{r}_1(z) \ldots \int \mathcal{D}\vec{r}_N(z)\, e^{-F_N / T}$$

$\mu = \varepsilon_l - H\Phi_0 / 4\pi$: the chemical potential

Periodic boundary conditions are imposed on the system in the z direction

$\sum_P$: goes over all possible permutations of boundary conditions such that

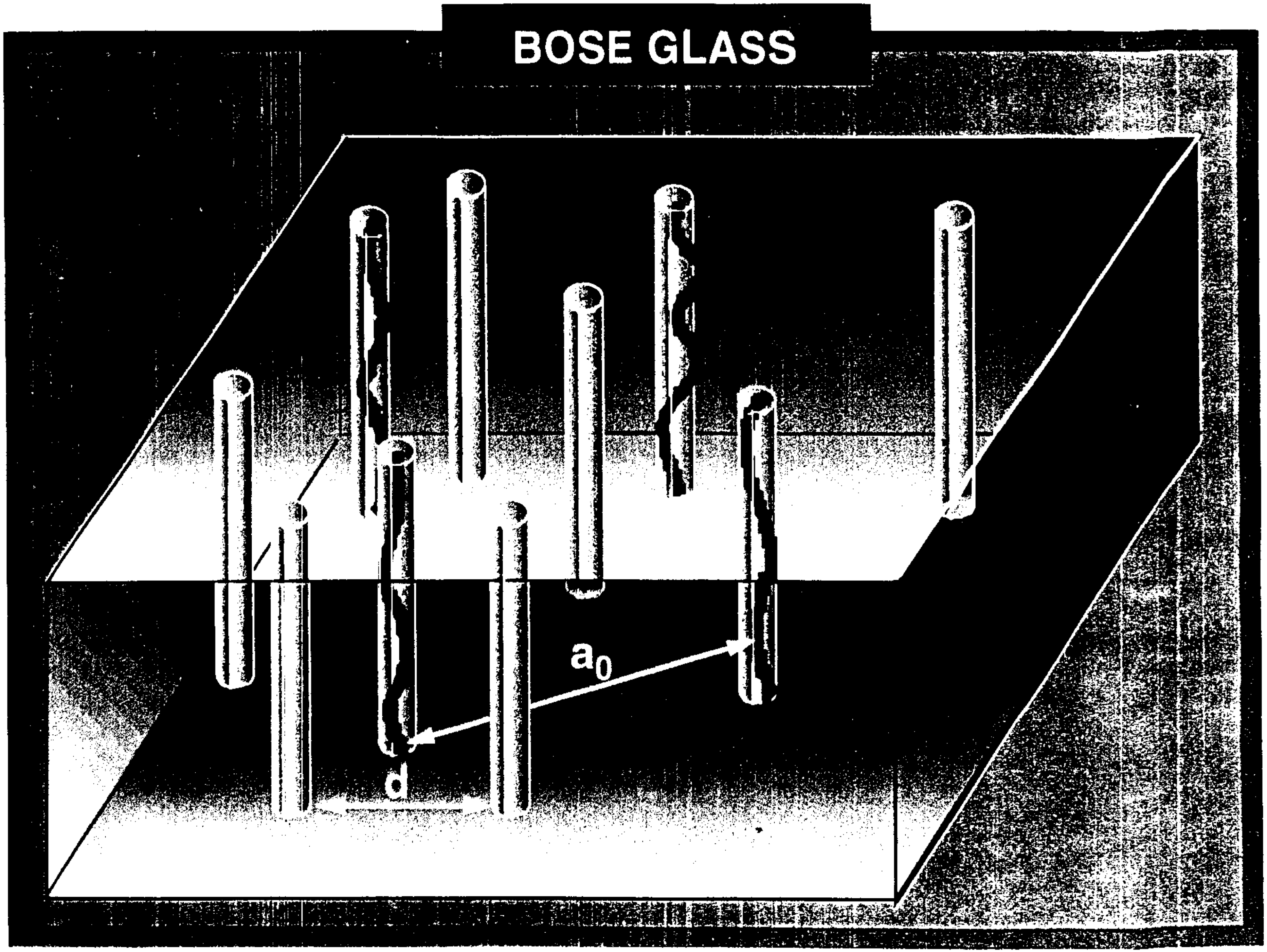
$$\vec{r}_i(0) = \vec{r}_j(L)$$

Z is identical to an imaginary time Feynman path integral of a system of 2D bosons.

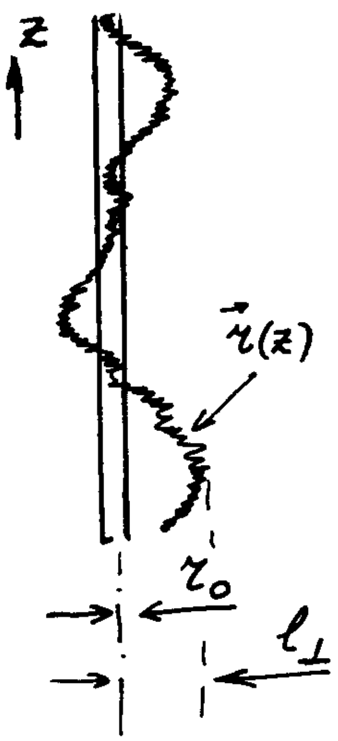
D.R. Nelson, PRL 60, 1973 (1988)
D.R. Nelson and H. S. Seung PR B 39, 9153 (1989)

Boson mapping

2D bosons	$\longleftrightarrow$	Vortices
Mass	$\longleftrightarrow$	ε_l
$\hbar$	$\longleftrightarrow$	T
$\hbar/T$	$\longleftrightarrow$	L
Charge	$\longleftrightarrow$	ϕ_0
$\vec{E}$	$\longleftrightarrow$	$\hat{z}\times\vec{J}/c$
J/c	$\longleftrightarrow$	$\vec{E}$
σ	$\longleftrightarrow$	ρ


BOSE GLASS
a0
d

CRITICAL CURRENTS



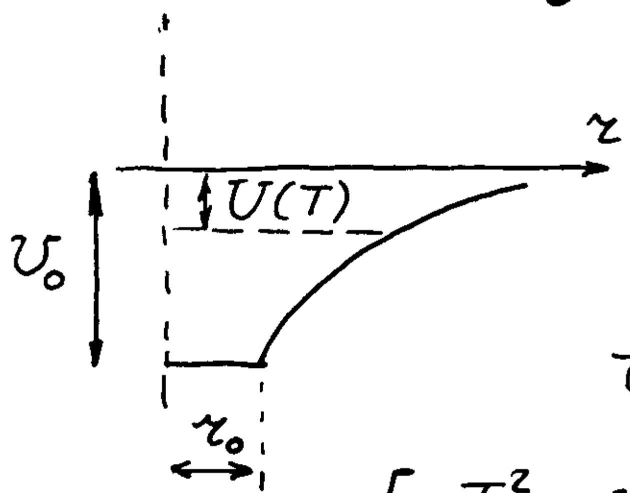
a vortex line trapped near a single columnar pin $\parallel z$

The binding free energy

$$U(T) = U_0 - TS$$

$$\exp\left[U(T)L/T\right] =$$

$$= \frac{\int \mathcal{D}\vec{r}(z) \exp\left[-\frac{1}{T}\int_0^L dz \left(\frac{\varepsilon_l}{2}\left(\frac{d\vec{r}}{dz}\right)^2 + V_D(\vec{r}(z))\right)\right]}{\int \mathcal{D}\vec{r}(z) \exp\left[-\frac{1}{T}\int_0^L dz \frac{\varepsilon_l}{2}\left(\frac{d\vec{r}}{dz}\right)^2\right]}$$



In the limit $L \to \infty$ the smallest eigenvalue dominates and

$$U(T) = -E_0$$

$$\left[-\frac{T^2}{2\varepsilon_l}\nabla^2 + V_D(\vec{r})\right]\Psi_0(\vec{r}) = E_0 \Psi_0(\vec{r})$$

$$V_D(r) \cong \frac{U_0 r_0^2}{r_0^2 + r^2}$$

(G.S. Mkrtchyan and V.V. Schmidt, Sov. Phys. JETP 34, 195 (1972))

$$U(T) \simeq U_0 \, e^{-T/T^*}$$

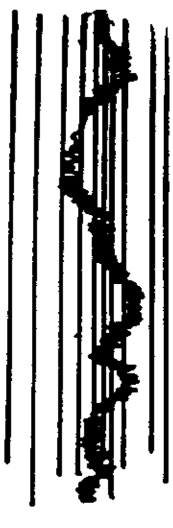
$$T^* \simeq \frac{\sqrt{2}}{\pi} r_0 \sqrt{\varepsilon_\ell U_0}$$: 'depinning' energy

$$\ell_\perp \simeq r_0 \frac{T}{T^*} e^{T/2T^*}$$

$$j_c \simeq j_0 \, \frac{r_0^2}{\xi^2} \, \frac{T^*}{T} \exp\left(-\frac{3}{2} \frac{T}{T^*}\right)$$

pair breaking current

$$T \gg T_1 = T^* \ln(d^2/r_0^2)$$



$\ell_\perp > d$

$$\ell_\perp \simeq d \left(\frac{T}{T_1}\right)^2$$

$$U(T) \simeq U_0 \left(\frac{r_0}{d}\right)^2 \left(\frac{T_1}{T}\right)^2$$

$$j_c \simeq j_0 \left(\frac{r_0}{d}\right)^2 \left(\frac{T_1}{T}\right)^4 \frac{\xi}{d}$$

Point defects

Let us assume that vortex lines are bound by some effective '2D' potential.

Binding energy:

$$|E| \sim \exp\left(-\frac{\hbar^2}{m|U_{eff}|\xi^2}\right) \to \exp\left(-\frac{T^2}{\varepsilon_\ell |U_{eff}|\xi^2}\right)$$

Find $|U_{eff}|$:

$$\sqrt{\gamma L_z} \sim T \qquad |U_{eff}| \sim \frac{T}{L_z} \sim \frac{\gamma}{T}$$

strength of the pinning potential

$$E \sim \exp\left(-\frac{T^3}{\varepsilon_\ell \gamma \xi^2}\right)$$

$$\left[\ell_\perp : \ \varepsilon_\ell \frac{\ell_\perp^2}{L_z} \sim T\right]$$

cf. $$j_c \sim \exp\left[-\left(\frac{T}{T_{dp}}\right)^3\right]$$

$$T_{dp} \simeq \left(\varepsilon_\ell \gamma \xi^2\right)^{1/3}$$

[M. V. Feigel'man & Vinokur, PR B 41, 8986 (1990)]

BOSE GLASS TRANSITION

BOSE GLASS PHASE:
VORTICES LOCALIZED NEAR COLUMNAR DEFECTS

$\underset{\text{MAPPING}}{\overset{\text{QM}}{\longleftrightarrow}}$

2D BOSONS are LOCALIZED BY THE QUENCHED 2D DISORDER

THE GLASS TRANSITION LINE IS OBTAINED AS THE TRANSITION FROM THE LOCALIZED INTO THE SUPERFLUID STATE IN THE RELATED BOZE SYSTEM AT ZERO TEMPERATURE

DILUTE LIMIT $a \gg \lambda$ $\quad (a \simeq \sqrt{\Phi_0/B})$

$r \simeq a\sqrt{N}$

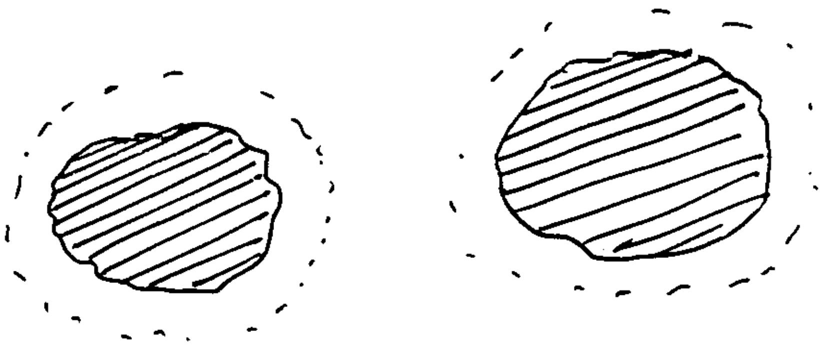
N is the number of the particles in the each well

GLASS-TO-LIQUID TRANSITION:
SEQUENTIAL FILLING OF THE POTENTIAL WELLS GENERATED BY DISORDER

THE RELATED QM SYSTEM IS a weakly NONIDEAL GAS OF HARD CORE 2D BOSONS WITH INTERACTIONS DESRIBED BY

$$g = \frac{1}{2.9\pi}\,\frac{g_0}{1 + g_0 \ln(1/\lambda^2 n_v)}$$

$$g_0 \simeq \frac{\varepsilon_\ell}{2\pi T^2}\int dr\, r\, V(r) = \frac{\varepsilon_1 \varepsilon_0 \lambda^2}{2\pi T^2}$$



THE TRANSFER OF A PARTICLE FROM ONE DROPLET TO ANOTHER INCREASES THE TOTAL ENERGY BY $2gE$

HOWEVER, SUCH AN EXCHANGE INCREASES THE POSITIONAL ENTROPY AND THE CORRESPONDING GAIN IN THE FREE ENERGY IS $E\exp(-r/\ell)$

$$[\ \ell = T^2/(\Delta_1)^{1/2}\varepsilon_\ell\ ]$$

$$\Delta_1 = \frac{U_0^2 r_0^4}{d^2}$$

UPON ICREASING THE DEGREE OF FILLING THE DISTANCE BETWEEN THE DROPLETS DECREASES, THE VORTEX STATES BECOME MORE EXTENDED, AND THE PERCOLATION-LIKE TRANSITION INTO A SUPERFLUID STATE (VORTEX LIQUID) OCCURS.

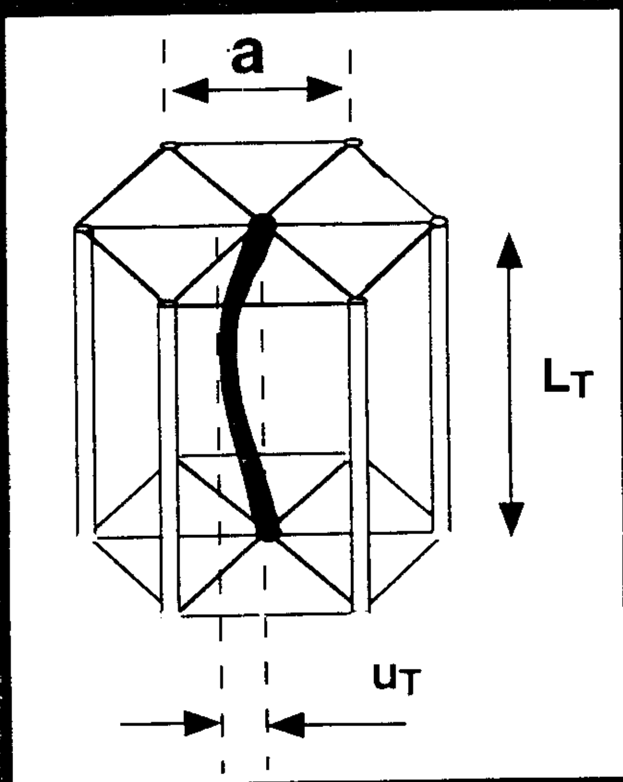
THE TRANSITION LINE: $$E\exp\left(-\frac{r}{\ell}\right) = gE$$

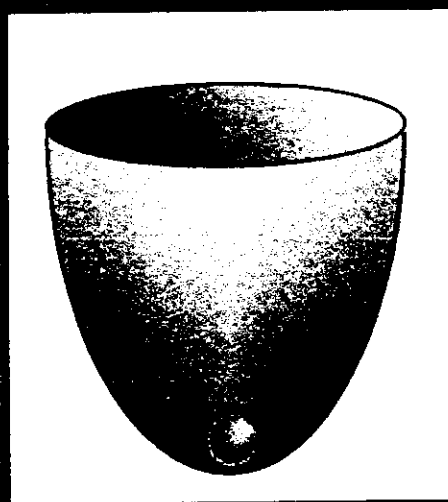
$$\Rightarrow B_{BG} \simeq \frac{\Phi_0}{\ell^2}\mathcal{L} \cong B_\Phi\left(\frac{T^*}{T}\right)^4 \propto (T_c - T)^4$$

$$\mathcal{L} = \frac{\ln(1/\lambda^2 n_v)}{[\ln\ln(1/\lambda^2 n_v)]^2}\quad (g \ll 1),\quad \mathcal{L} \simeq 1\quad (g \simeq 1)$$

g is the coupling const.

BOSE GLASS TRANSITION



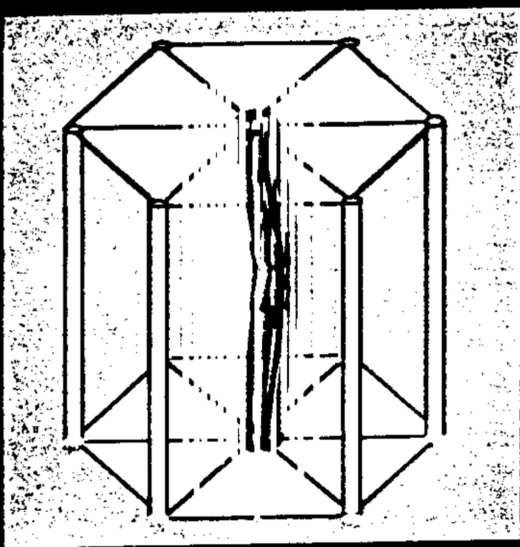


$$E_{el} = c_{66}u^2L + \varepsilon_\ell(u^2/L)$$

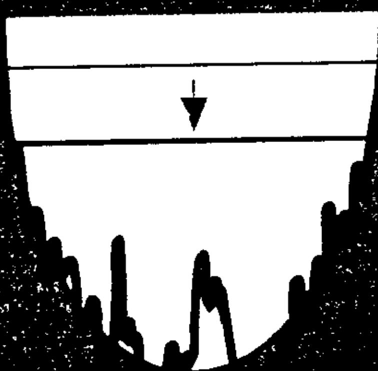
$$L_T = \sqrt{\varepsilon_\ell / c_{66}} \quad u_T^2 = T/\sqrt{\varepsilon_\ell c_{66}}$$

MELTING:

$$T \cong E_b = c_L^2 c_{66} a^2 L_T$$

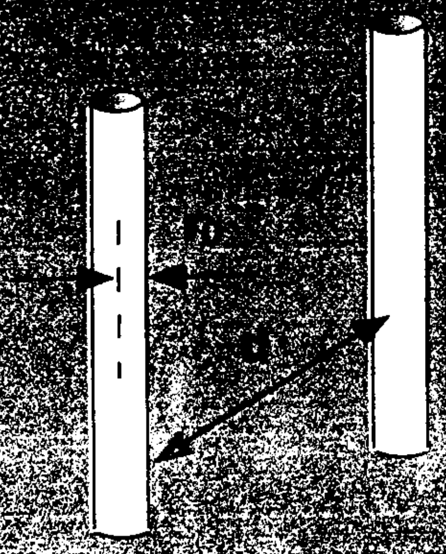


$$E_p = L_T\sqrt{\Delta/(u_T^2+\xi^2)}$$



$$T = c_L^2 c_{66} a^2 L_T + L_T\sqrt{\Delta/(u_T^2+\xi^2)}$$

$$T_{BG} = T_m\left[1+\frac{\pi^2}{2}\frac{c_L^2 a}{d}\frac{\left(T^*\right)^2}{T_m^2}\right]$$



$$T_{BG} = L_T\sqrt{\Delta/(u_T^2+\xi^2)}$$

$$B_{BG} \cong 4\Phi_0\varepsilon_\ell^2\Delta/T^6 \propto (T_c - T)^6$$

$$T^* = (2/\sqrt{\pi})r_0\sqrt{\varepsilon_\ell U_0}$$

ALTERNATIVE APPROACH:

→ FROM THE LIQUID

DISORDER-INDUCED RENORMALISATION OF THE TILT MODULUS:

$$\frac{1}{C_{44}^{vR}} = \frac{1}{C_{44}^{v}} - \frac{T^4 n_v \Delta_1}{(C_{44}^{v})^2 \varepsilon_1} \int \frac{d^2 q\, q^4}{\varepsilon^4(q)}$$

[T. Hwa et al., PRL 71, 3545 (1993); and to be published]

where $\varepsilon(q)$ represents the spectrum of the 2D related bosons

vortex related part of the tilt modulus becomes infinite at the Bose glass transition line

Test:

in the dilute limit, $\lambda < a$,

$$\varepsilon(q) = \left[\frac{T^4 q^4}{(2\varepsilon_1)^2} + \frac{4\pi T^4 n_v q^2}{\varepsilon_1^2 \ln(n_v \lambda^2)} \right]^{1/2}$$

$$\Rightarrow \quad B_{BG} \simeq B_\phi \left(\frac{T^*}{T}\right)^4 \mathcal{L} \qquad \mathcal{L} = \frac{\ln(1/\lambda^2 n_v)}{[\ln \ln (1/\lambda^2 n_v)]^2}$$

which coinsides up to a log factor with the transition found from the condition $\ell_\perp \simeq a$

DENSE LIMIT, $a < \lambda$,

interpolation formula

$$\varepsilon^2(q) = T^2\omega_p^2 - \frac{2\epsilon_0 T^2 q^2}{\pi \varepsilon_1} + \frac{(Tq)^4}{(2\varepsilon_1)^2}$$

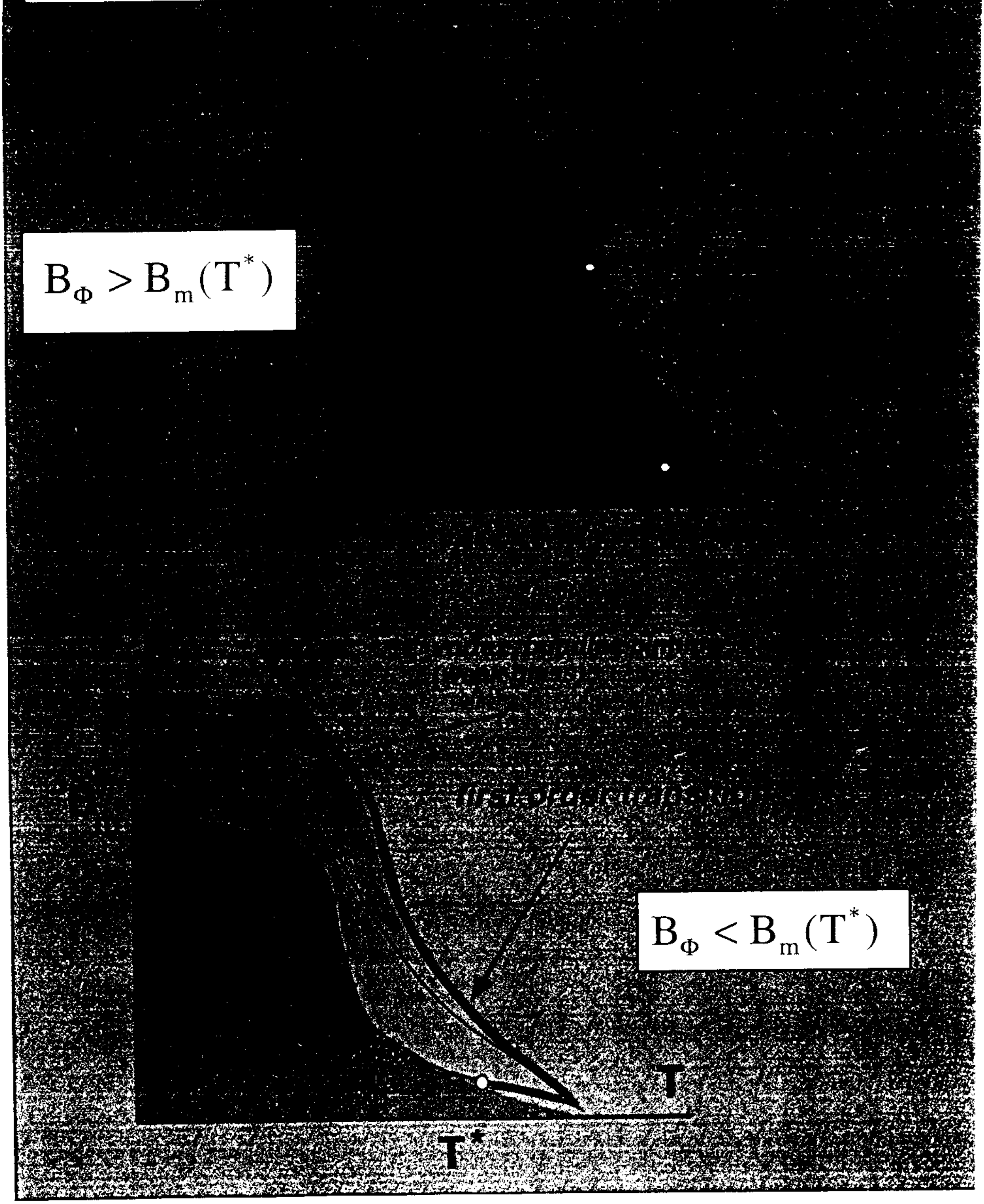
[cf M.V. Feigelman et al PRB 48, 16641 (1993)]

$$\omega_p^2 = \frac{4\pi n \epsilon_0}{\varepsilon_1} \qquad (\varepsilon_1 \leftrightarrow m \,,\; e^2 \leftrightarrow \epsilon_0)$$

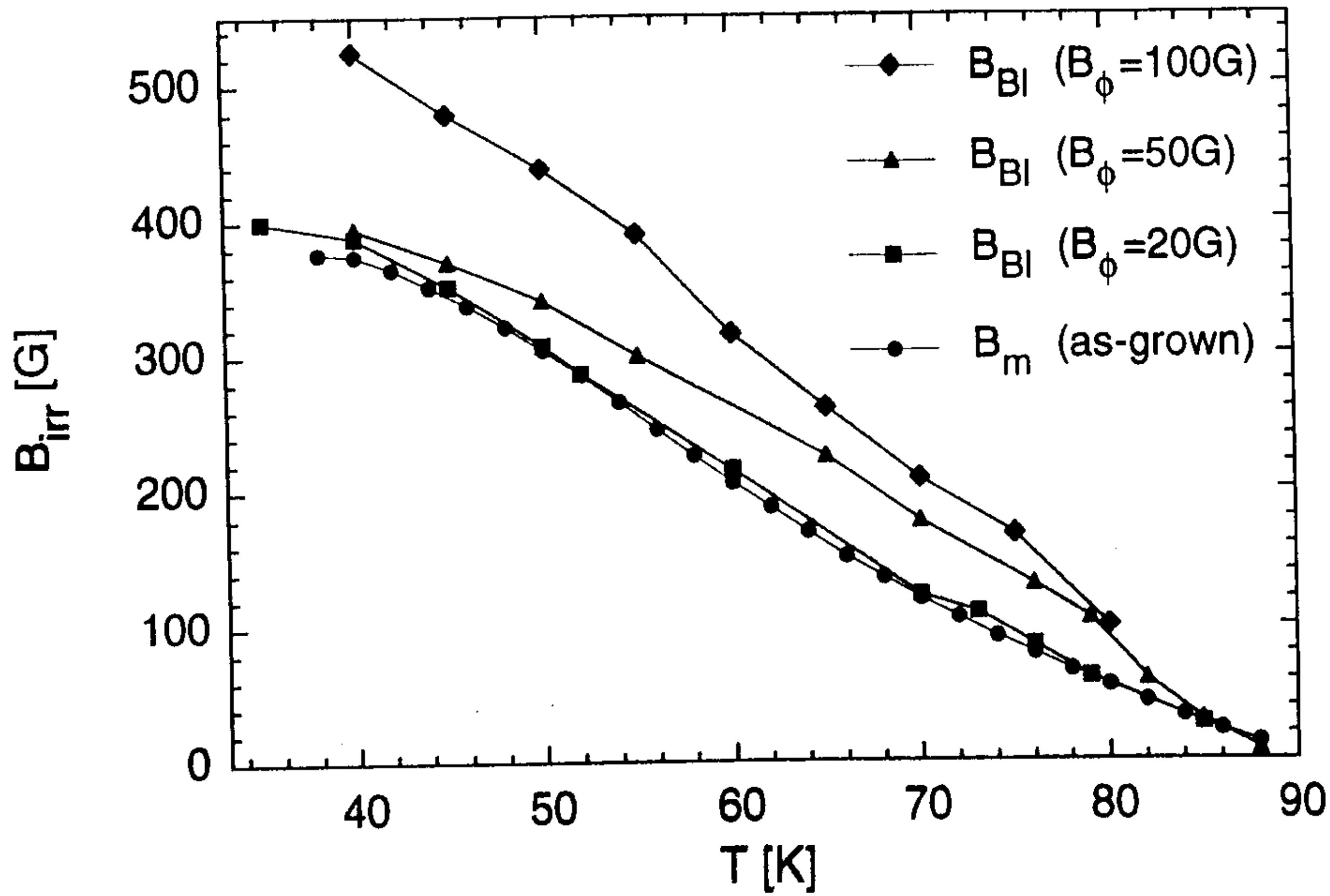
$$\varepsilon_0 = \frac{\Phi_0^2}{(4\pi\lambda)^2} \qquad \varepsilon_1 = \varepsilon^2 \epsilon_0$$

$$\Rightarrow \quad B_{BG} \simeq \frac{4\Phi_0 \varepsilon_1^2 \Delta}{T^6}$$

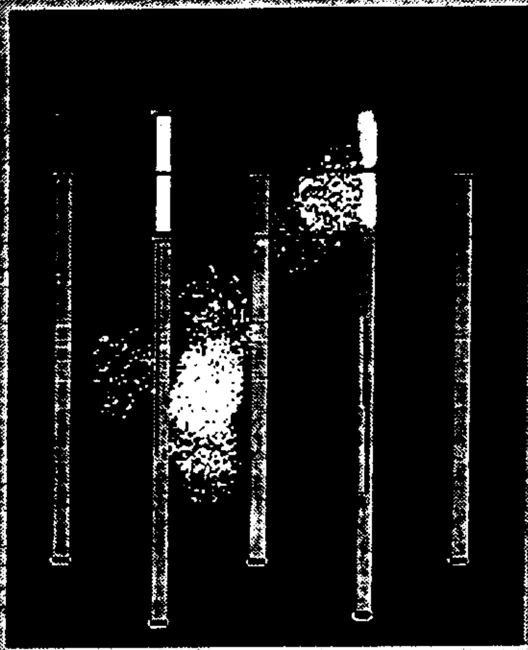
SCHEMATIC PHASE DIAGRAM FOR SUPERCONDUCTORS WITH COLUMNAR DEFECTS



Onset of bulk pinning in BSCCO crystal irradiated by heavy ions with different doses



DISORDER-FIELD-DRIVEN TRANSITION



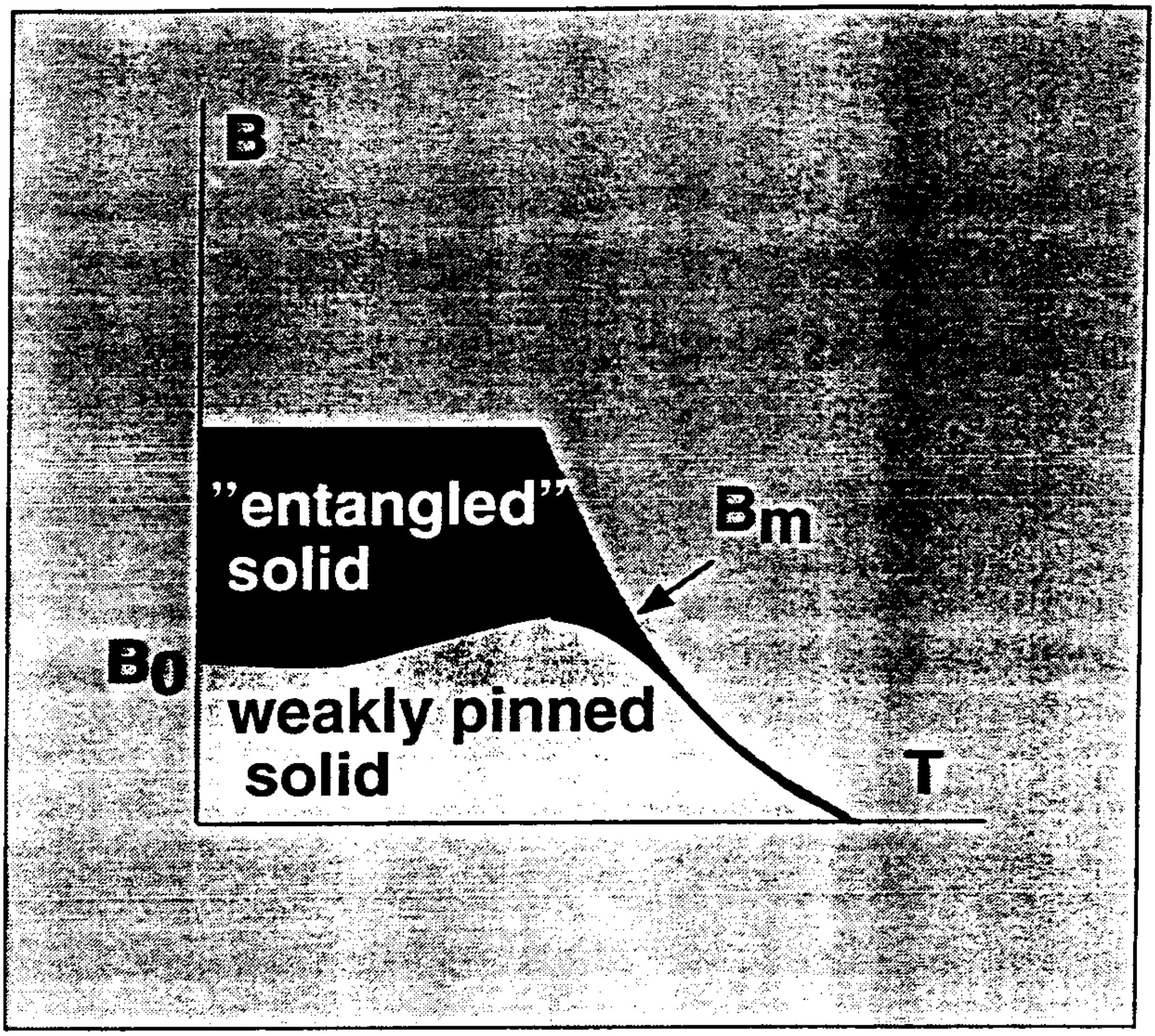
$$c_L^2 C_{66} a^2 L_T = \sqrt{\gamma} L_T$$

$$L_T = 2\varepsilon a$$

$$\frac{B_0}{H_{c2}} = \frac{c_L^8 \varepsilon^6}{2\pi}\left(\frac{\varepsilon_0 \xi}{T_{dp}}\right)^6$$

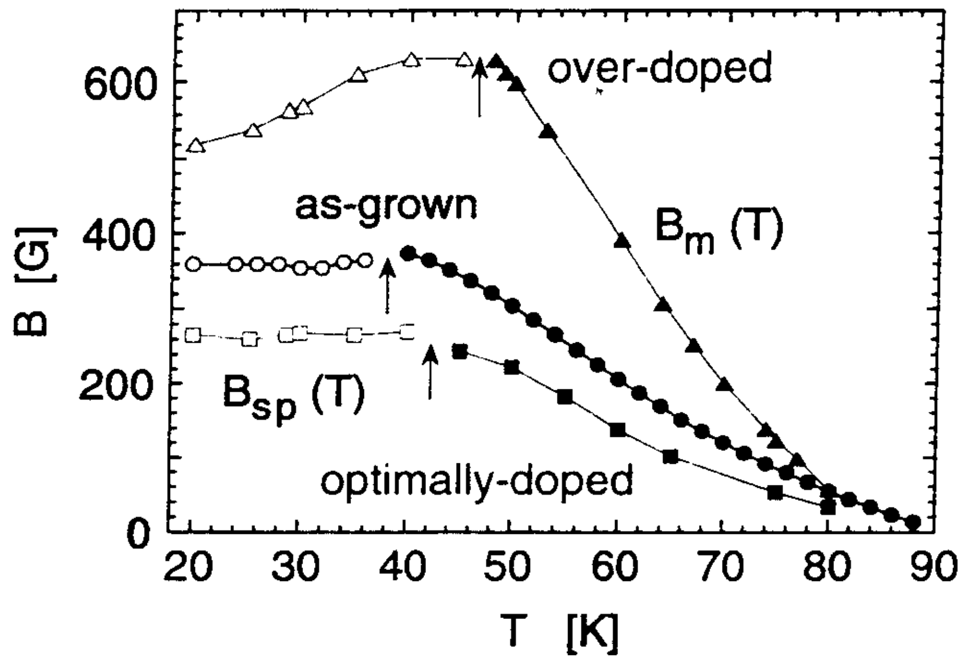
$$B_0 \approx 1.5\ 10^5 \left(\frac{4}{T_{dp}}\right)^6$$

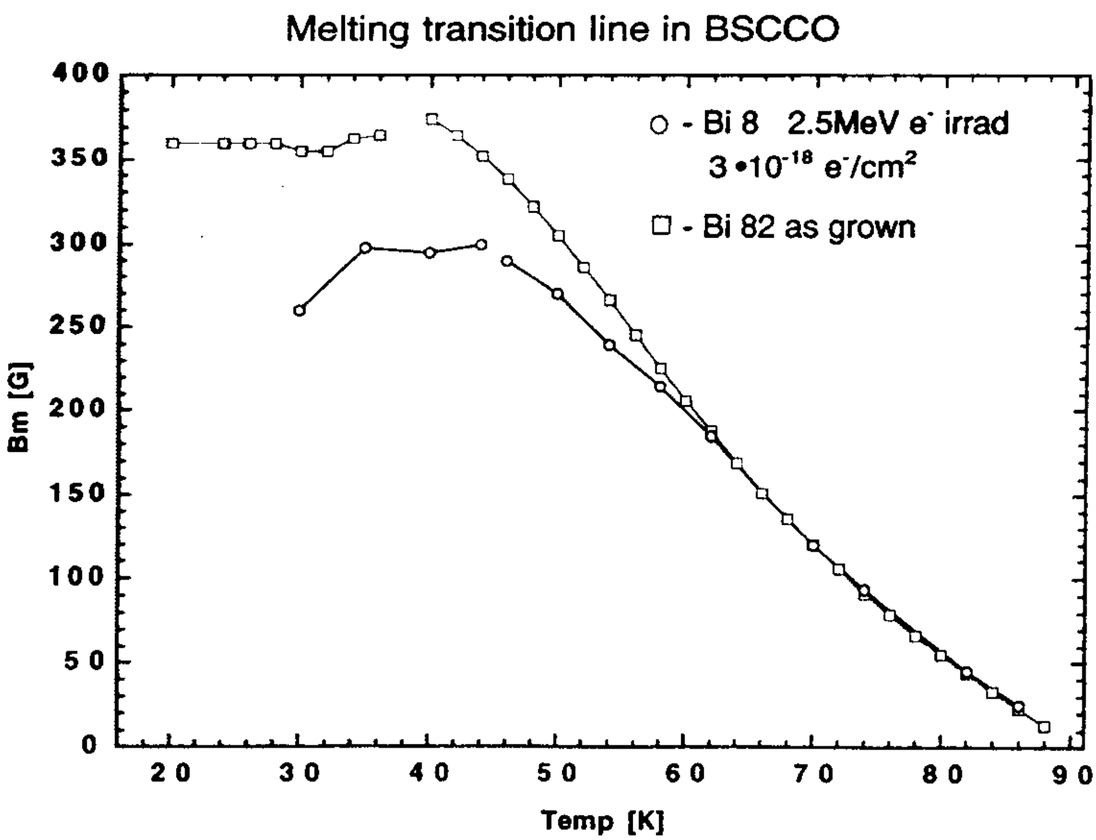
D. Ertaş & D. Nelson



$$c_L^2 C_{66} a^2 L_T = \sqrt{\frac{\gamma L_T}{1+\langle u_T^2\rangle/\xi^2}} + T$$

BSCCO melting and second peak lines different anisotropy crystals



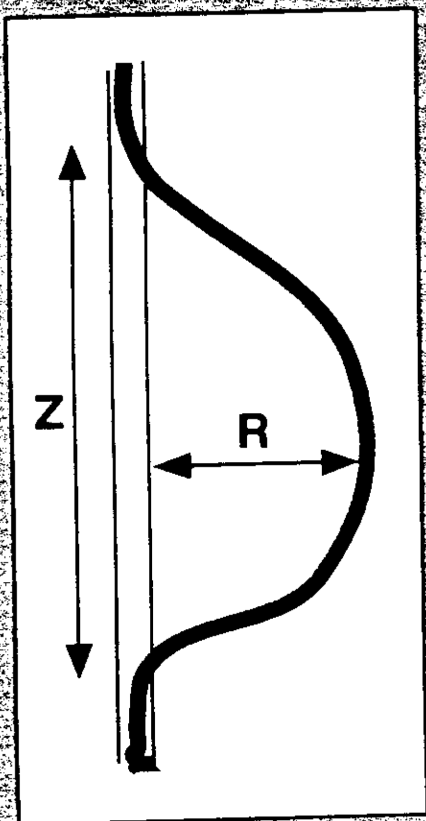


CREEP FROM THE SINGLE WELL

One flux-line free energy: $$F = \int dz \left[\frac{\varepsilon_\ell}{2} \left(\frac{d\mathbf{R}}{dz} \right)^2 + V_D(\mathbf{R}(z)) - \mathbf{f}_L \mathbf{R}(z) \right]$$

$$f_L = \frac{1}{c} J\Phi_0 \; ; \; J_1 < J < J_c \; ; \; J_1 = \frac{cU_0}{\Phi_0 d}$$

d is the distance between pins



The energy change (due to formation of the half loop) relative to the case f_L=0 :

$$\delta F \approx \frac{\varepsilon_\ell R^2}{Z} + U_0 Z - f_L R Z$$

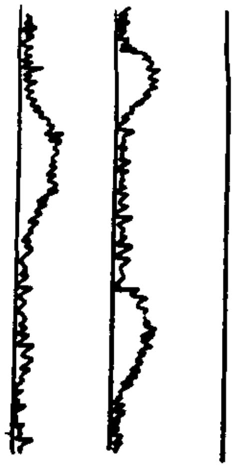
cost in elastic energy

pinning energy

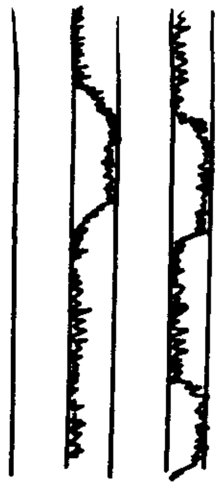
gain in energy due to applied current

Optimization with respect to R and Z: $$R^* = \sqrt{\frac{U_0}{\varepsilon_\ell}} Z^* = \frac{cU_0}{J\Phi_0}$$

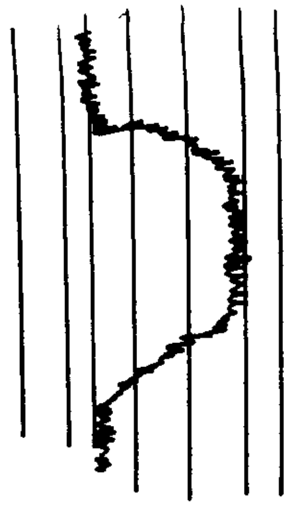
$$\delta F^* = \frac{c\varepsilon_\ell^{1/2} U_0^{3/2}}{J\Phi_0}$$



The 'half loop' excitations are relevant at high current densities where the critical nucleus does not yet reach out to the neighboring columns



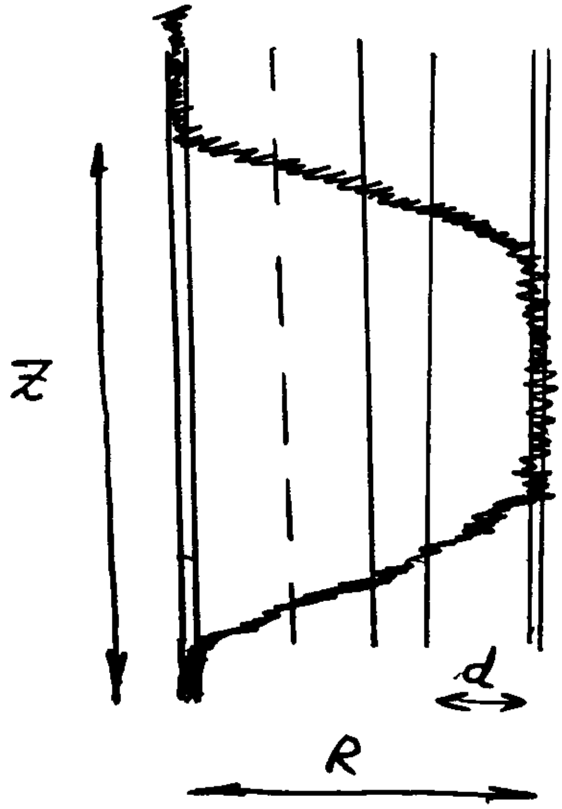
Current decreases, and double-kink excitations to the neighboring track become relevant



Dispersion in the pinning energies becomes important for small current densities (VRH)

$$\mathcal{E} \propto \exp\left(- \mathrm{const}/J^{1/3}\right)$$

VRH REGIME



$$\delta F(R,Z) \simeq 2E_K \frac{R}{d} + \frac{Z}{gR^2} - f_L R Z$$

Kink energy:

$$E_K \simeq d\sqrt{\varepsilon_\ell U_0}$$

$$f_L = \frac{1}{c}\Phi_0 J$$

The characteristic energy difference between the initial and final vortex states: the total number of states with energy within $\Delta\varepsilon$ of this flux line at distances less than or equal to R is $g\Delta\varepsilon R^2$ (g is the density of states)

By setting $g\Delta\varepsilon R^2 \simeq 1$, find $\Delta\varepsilon \simeq 1/gR^2$

$$\delta F^* \simeq \frac{E_K}{d}\left(\frac{c}{g\Phi_0 J}\right)^{1/3} \rightarrow \mathcal{E} \propto \exp\left[-\frac{E_K}{T}\left(\frac{J_{VRH}}{J}\right)^{1/3}\right]$$

$$J_{VRH} \simeq \frac{c}{\Phi_0 g d^3}$$

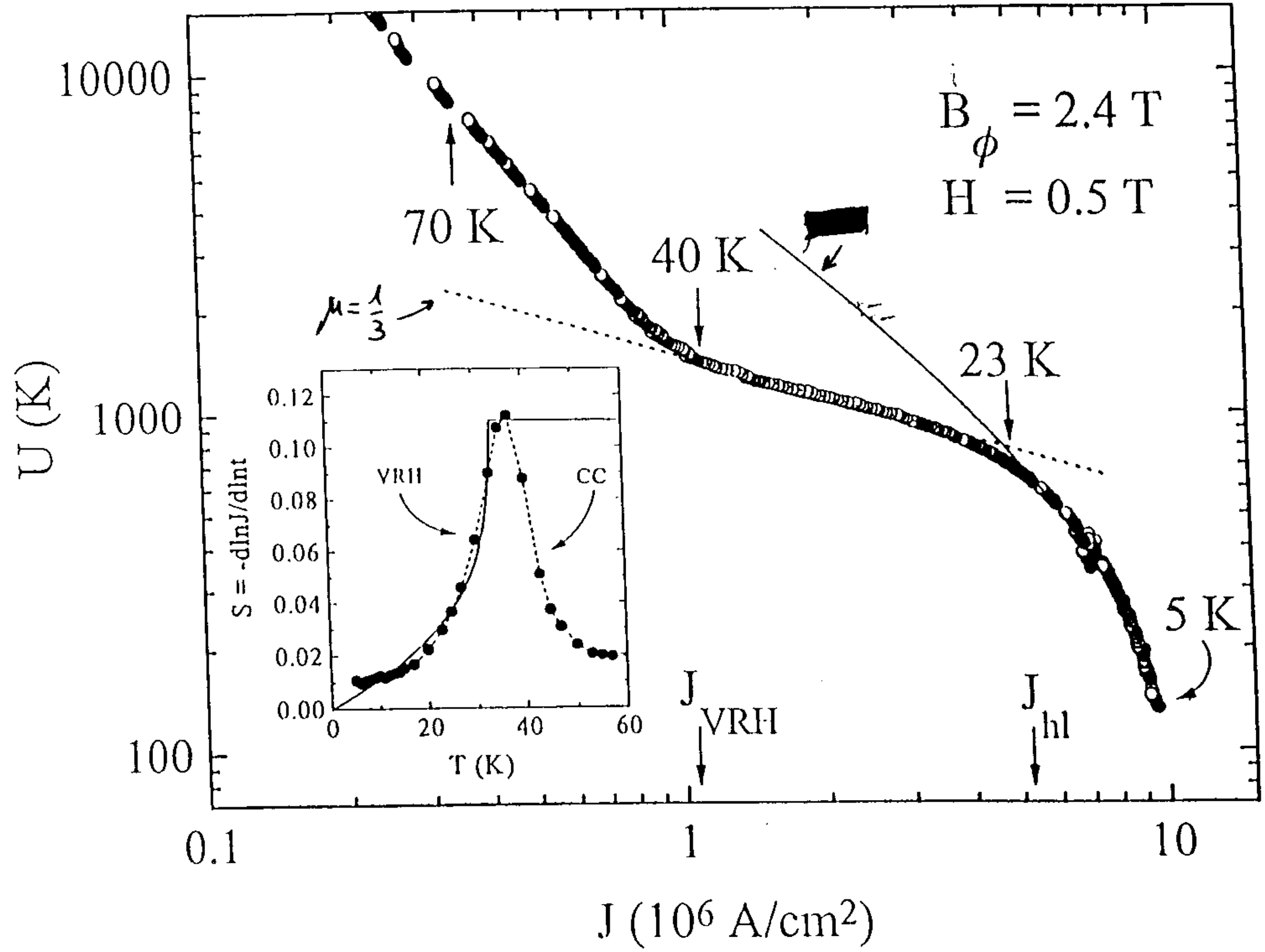
vortex velocity:

$$v \sim \exp\left(-\frac{c\,\varepsilon_l^{1/2}\,U_0^{3/2}}{J\,\Phi_0\,T}\right)$$

Cold emission:

$$w \simeq \exp\left[-\frac{4\sqrt{2}}{3}\,\frac{U_0^{3/2}\,m^{1/2}}{\hbar\,\mathcal{E}e}\right]$$

Electric field



B_ϕ = 2.4 T
H = 0.5 T
70 K
40 K
23 K
5 K
$\mu = \frac{1}{3}$
J_{VRH}
J_{hl}
U (K)
10000
1000
100
0.1
1
10
J (10^6 A/cm^2)
S = -dlnJ/dlnt
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0
20
40
60
T (K)
VRH
CC

Fig 4

CONCLUSIONS

- PHYSICS OF FLUX LINES IS MAPPED ONTO THE PROBLEM OF LOCALIZATION OF QUANTUM MECHANICAL BOSONS IN TWO DIMENSIONS

- PHASE DIAGRAM FOR THE BOSE- AND VORTEX GLASSES IS CONSTRUCTED

- FIRST ORDER TRANSITION LINES TERMINATE AT CRITICAL POINTS

- THE IRREVERSIBILITY LINE DUE TO COLUMNAR DEFECTS SHIFTS UPWARDS AS COMPARED TO THE MELTING LINE IN CLEAN SISTEMS

- TEMPERATURE DEPENDENCE OF THE CRITICAL CURRENTS IS FOUND

- NON-TRIVIAL CREEP REGIME : VARIABLE RANGE VORTEX HOPPING IS PREDICTED