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MINIWORKSHOP ON STRONG ELECTRON CORRELATIONS
"Disorder and Interaction in Quantum Systems and Their Classical Analogs"

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"The Anderson-Mott Transition as a Quantum Glass Problem"

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These are preliminary lecture notes, intended only for distribution to participants.

The Anderson-Mott Transition as a Quantum Glass Problem

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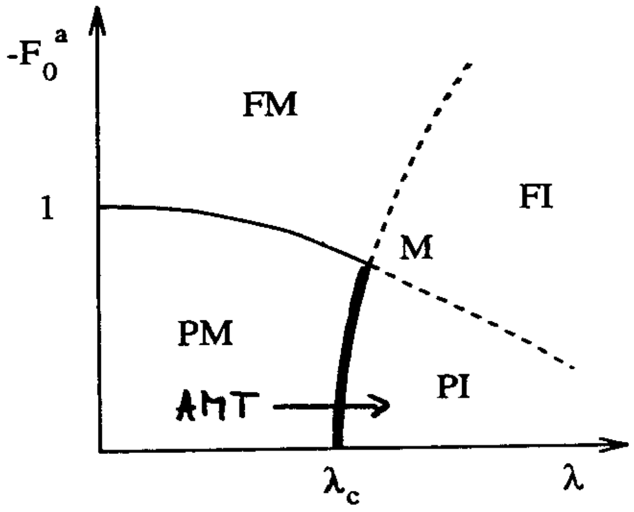
I. The Anderson-Mott Transition

- Consider **disordered**, interacting electrons



$T = 0$

FIG. 1.

F_0^a : exchange interaction parameter λ : disorder

PM : paramagnetic metal FM : ferromagnetic metal

PI : paramagnetic insulator FI : ferromagnetic insulator

$0 = F_0^a$: Anderson transition at $\lambda = \lambda_c$

$0 \neq F_0^a$: Anderson-Mott transition (AMT)

- AMT believed to be realized in doped semiconductors
  e.g. Si:P $\lambda \sim n$ (P concentration)

1. Review of Conventional Theory

- AMT is continuous quantum phase transition at
  $t = |n - n_c| = 0,\ T = 0$
  conductivity $\sigma(t,T)$ vanishes continuously at $T = 0$, $t \to 0$

- Theories based on Goldstone modes in metallic phase F. Wegner
  Technical implementation: Nonlinear sigma-model A.M. Finkel'stein

- Not order parameter theories → No Landau theory,
  No mean-field theory

- Homogeneity law for the conductivity:

$$\boxed{\sigma(t,T) = b^{-s/\nu}\,\sigma(t\,b^{1/\nu}, T b^{z})}$$

ν correlation length exponent
z dynamical exponent b RG rescaling factor

- Exponent values in $d = 3$ not known

- Constraints on exponent values:
  - Harris criterion: $\sqrt{\delta n_c^2} \sim \xi^{-d/2} < t \quad \to \quad t^{d\nu/2} < t$

$$\to \boxed{\nu \geq 2/d} \qquad \underline{(\nu \geq 2/3 \ \ in \ \ d = 3)}$$

  - Wegner's scaling law: $\sigma \sim \xi^{2-d} \sim t^{\nu(d-2)} \sim t^{s}$

$$\boxed{s = \nu(d-2) \qquad \to \qquad (s \geq 2/3 \ \ in \ \ d = 3)}$$

1. Review of Experiments

- Two precision experiments gave mutually incompatible results

- Bell Group (Thomas, Rosenbaum, et al. 1980 - 82) stress tuning experiment, lowest $T \sim$ [illegible] 3 mK
  - $n_C = 3.7 \times 10^{18}$ cm^{-3} (Fig.2)
  - $s = 0.5$ ($< 2/3$!) (Fig.3)
  - $\nu z = 2.7$ (Fig.4)

- Karlsruhe Group (von Löhneysen et al. 1993) separate samples, lowest $T \sim$ [illegible] 40 mK
  - $n_C = 3.5 \times 10^{18}$ cm^{-3} (Figs.5)
  - $s = 1.3$ (Fig.6)

- Both experiments encountered problems at very low temperatures:
  - Bell group discarded 'tail' at $n < n_C$ due to sample-to-sample fluctuations (Fig.7)
  - Karlsruhe group took the 'tail' seriously, but had equilibration problems at $T < 40\,\mathrm{mK}$ (Fig.5b)

- Conventional scaling analysis:
  - Bell data do not scale unless $s \approx 0.3$ (Figs. 8 - 11)
  - Karlsruhe data give equally good scaling plot with very different parameter values (Fig.12)

Bell Labs

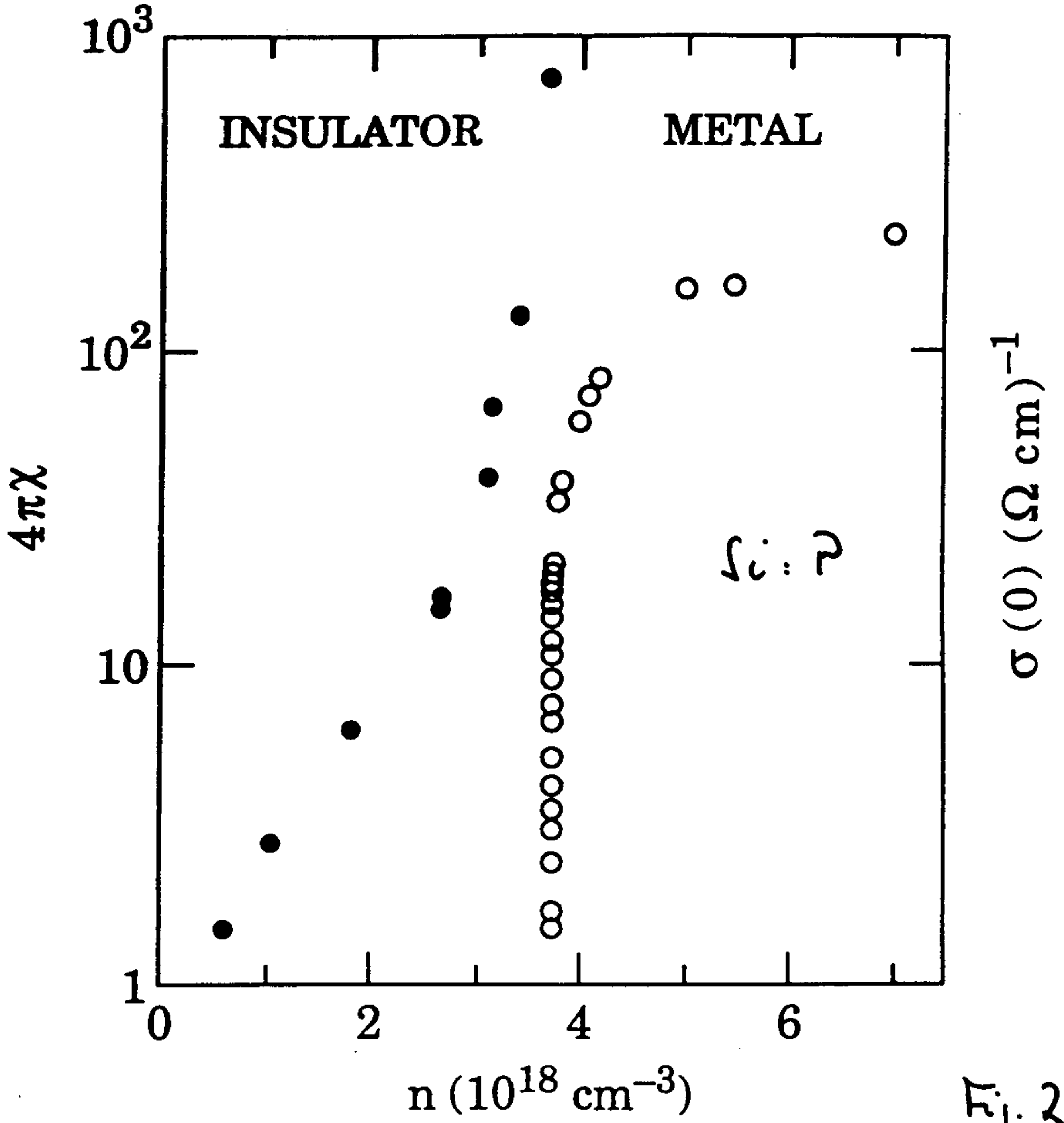


Fig. 2

Bell Labs

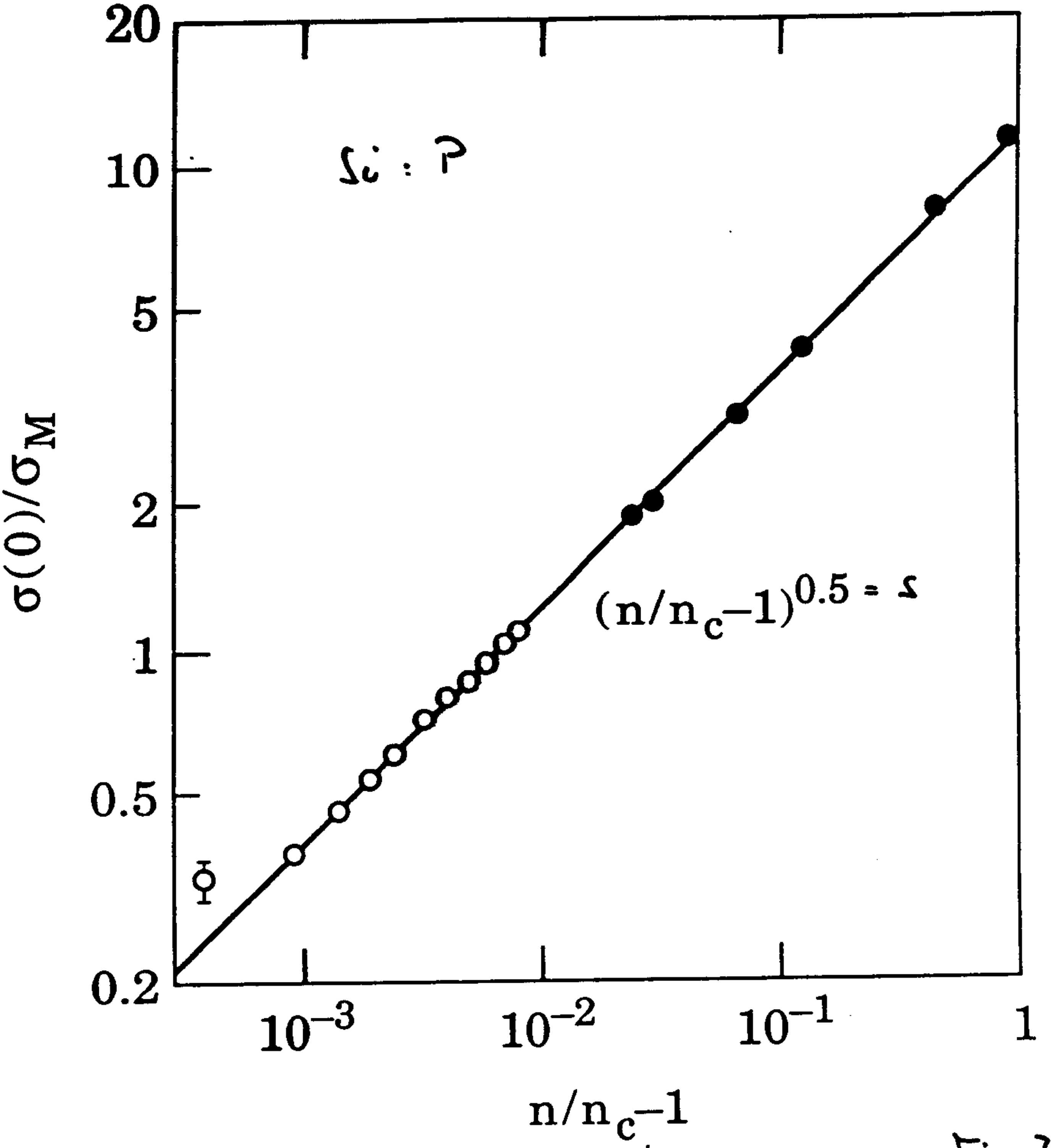


Fig. 3

Bell Labs

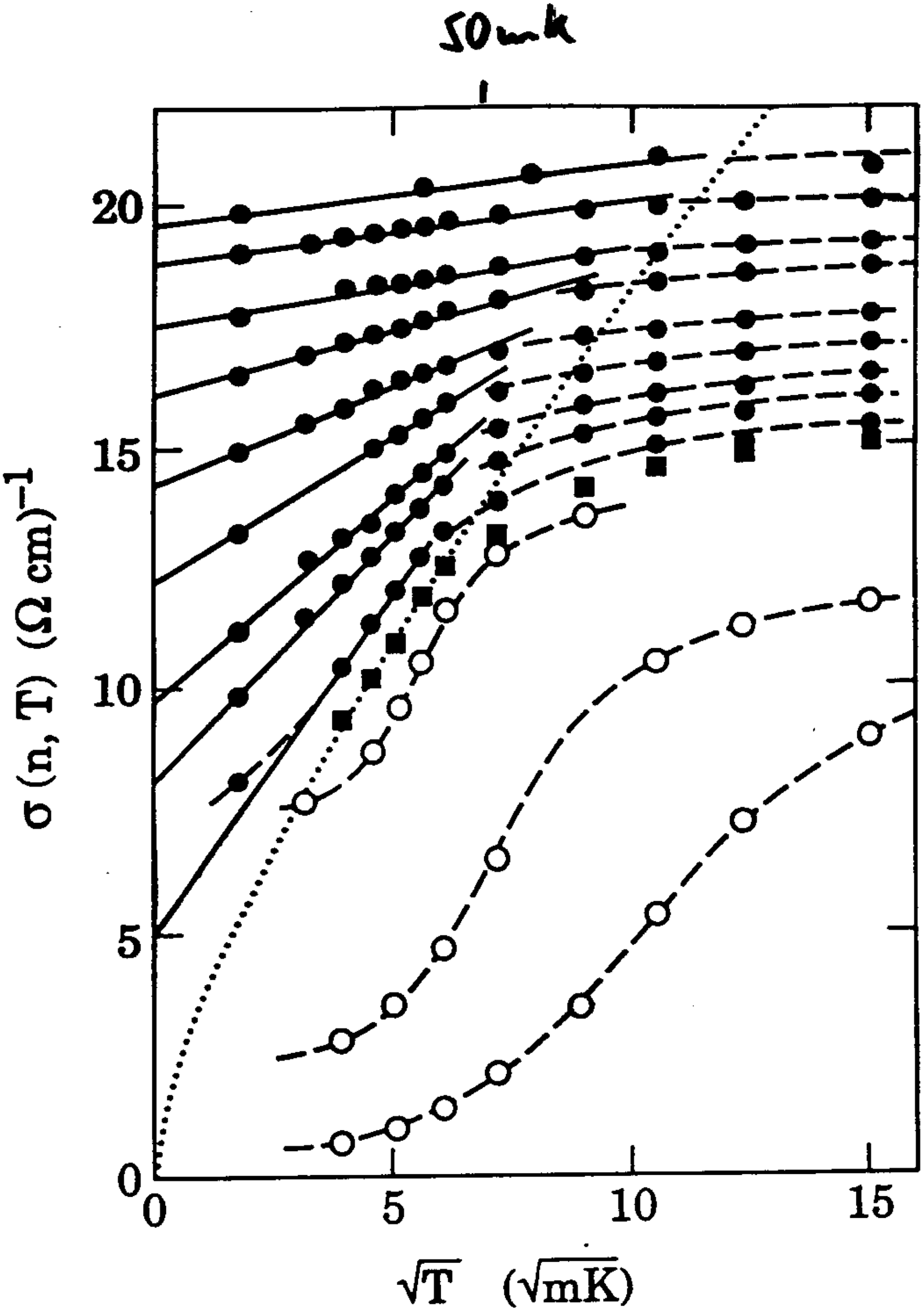


Fig. 4

Karlsruhe

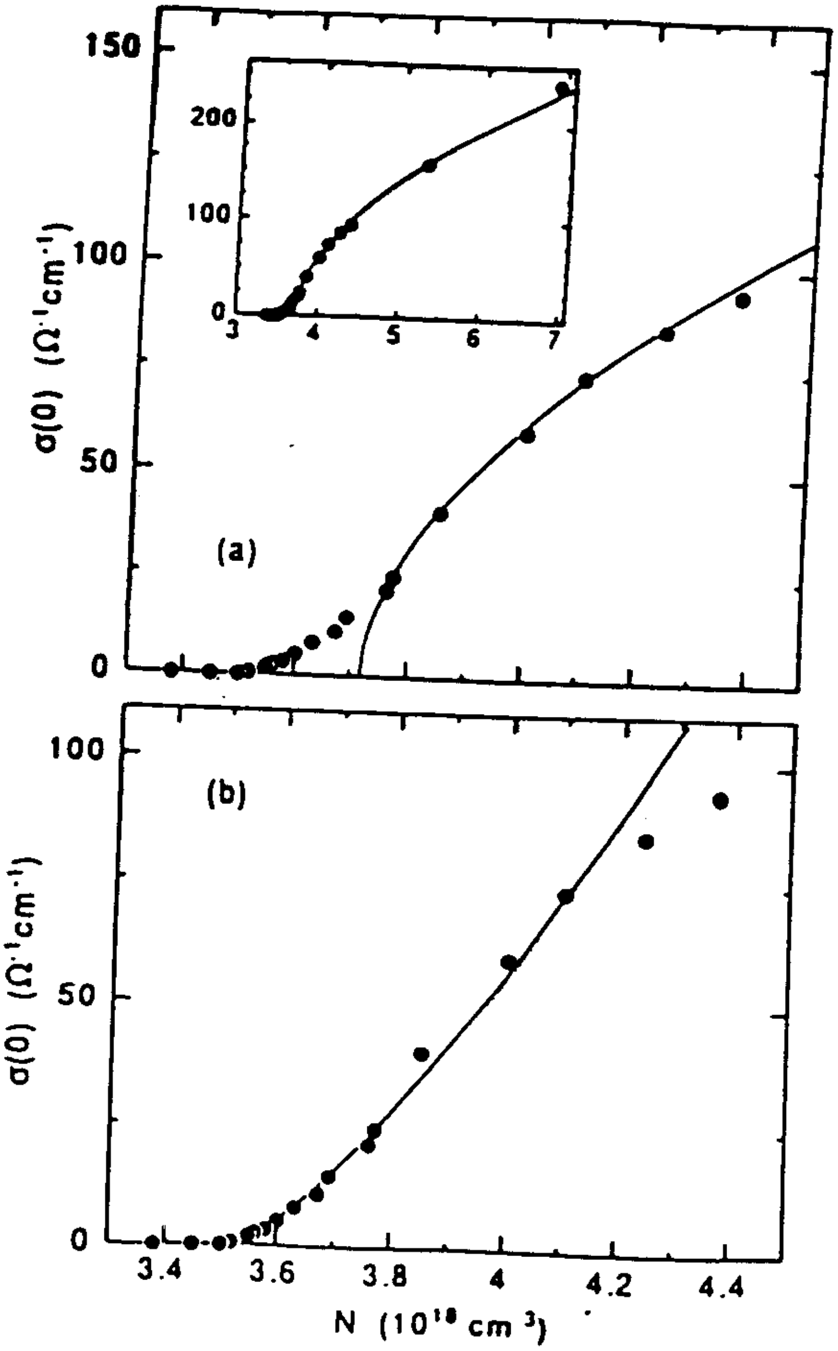


Fig. 5a

Karlsruhe

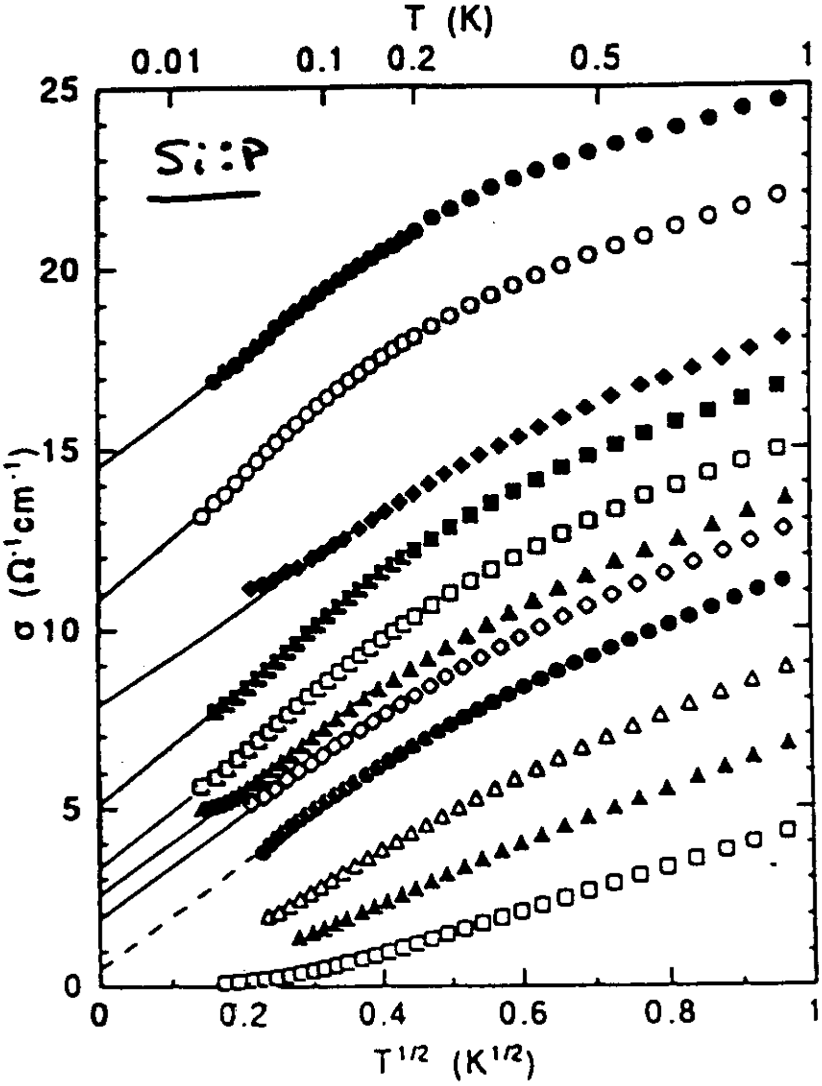


Fig. 5b

Karlsruhe

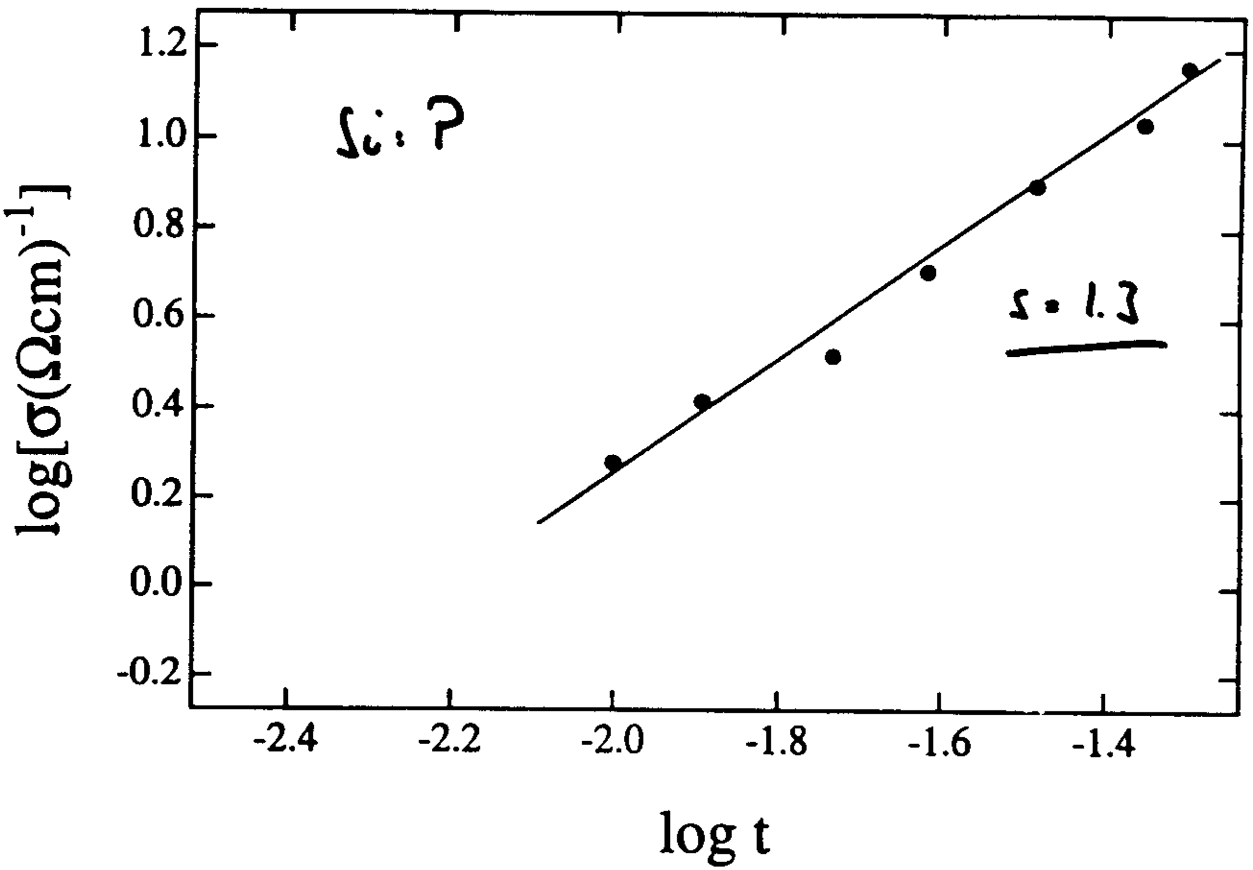


Fig. 6

Bell Labs

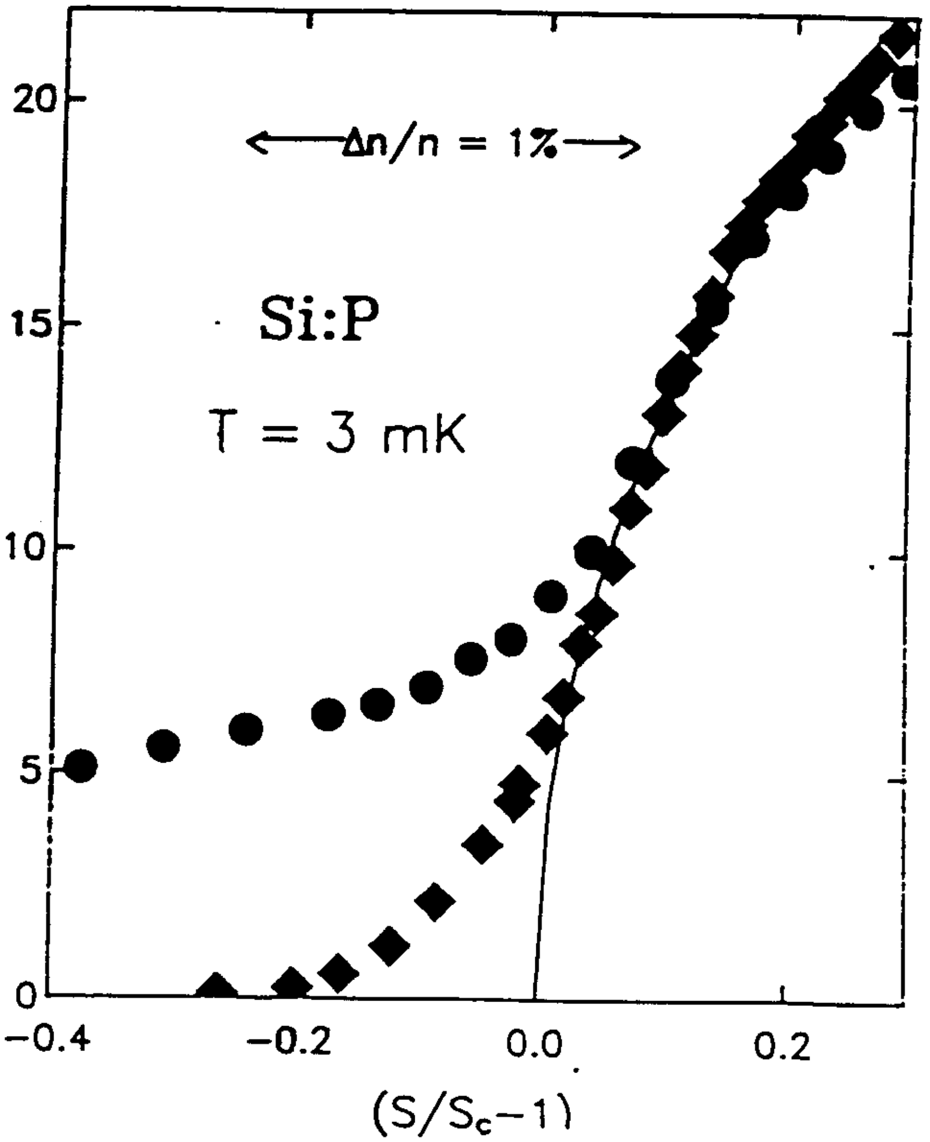


Fig. 7

Bell Labs data

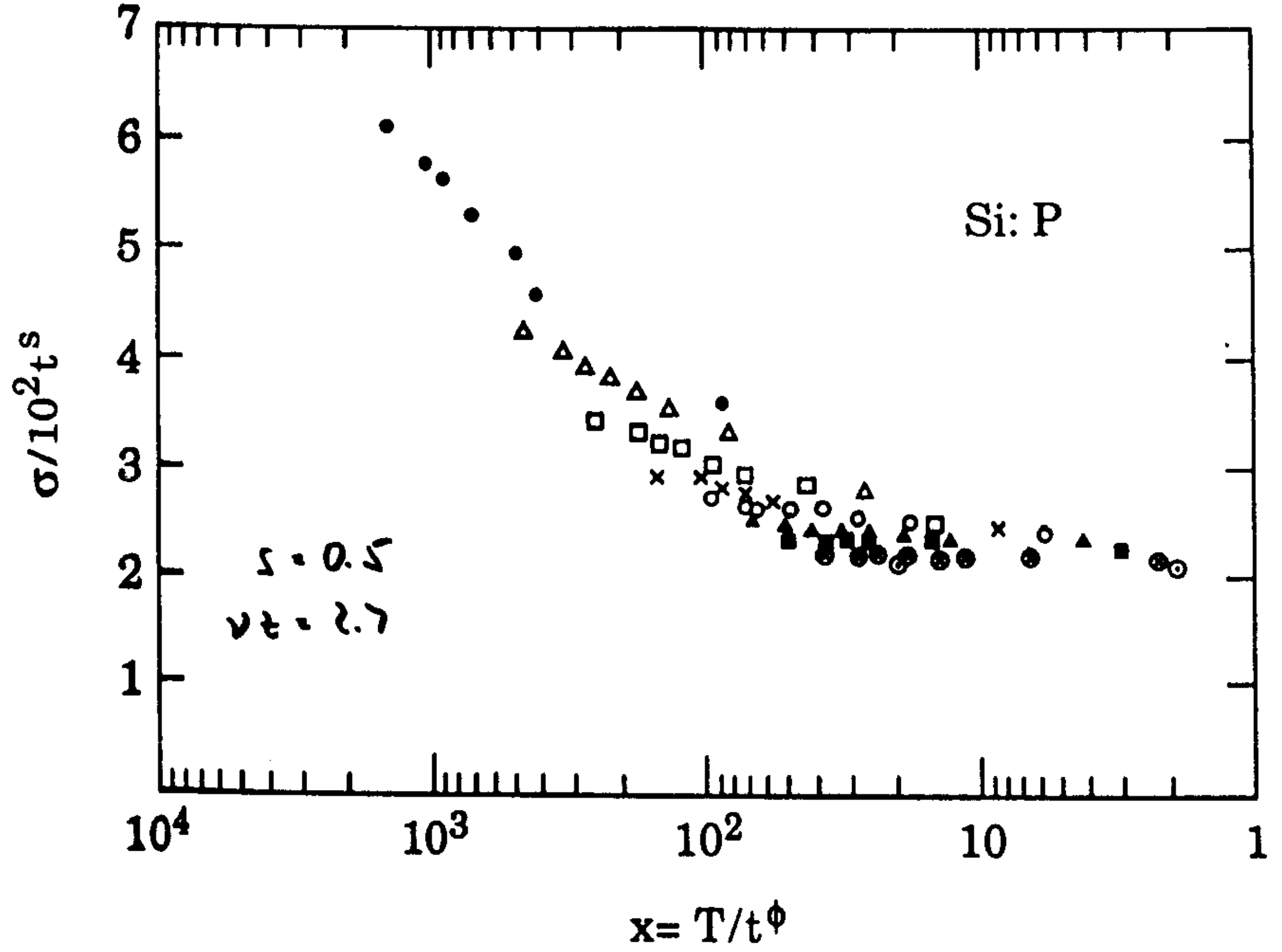


Fig. 8

Bell Labs data

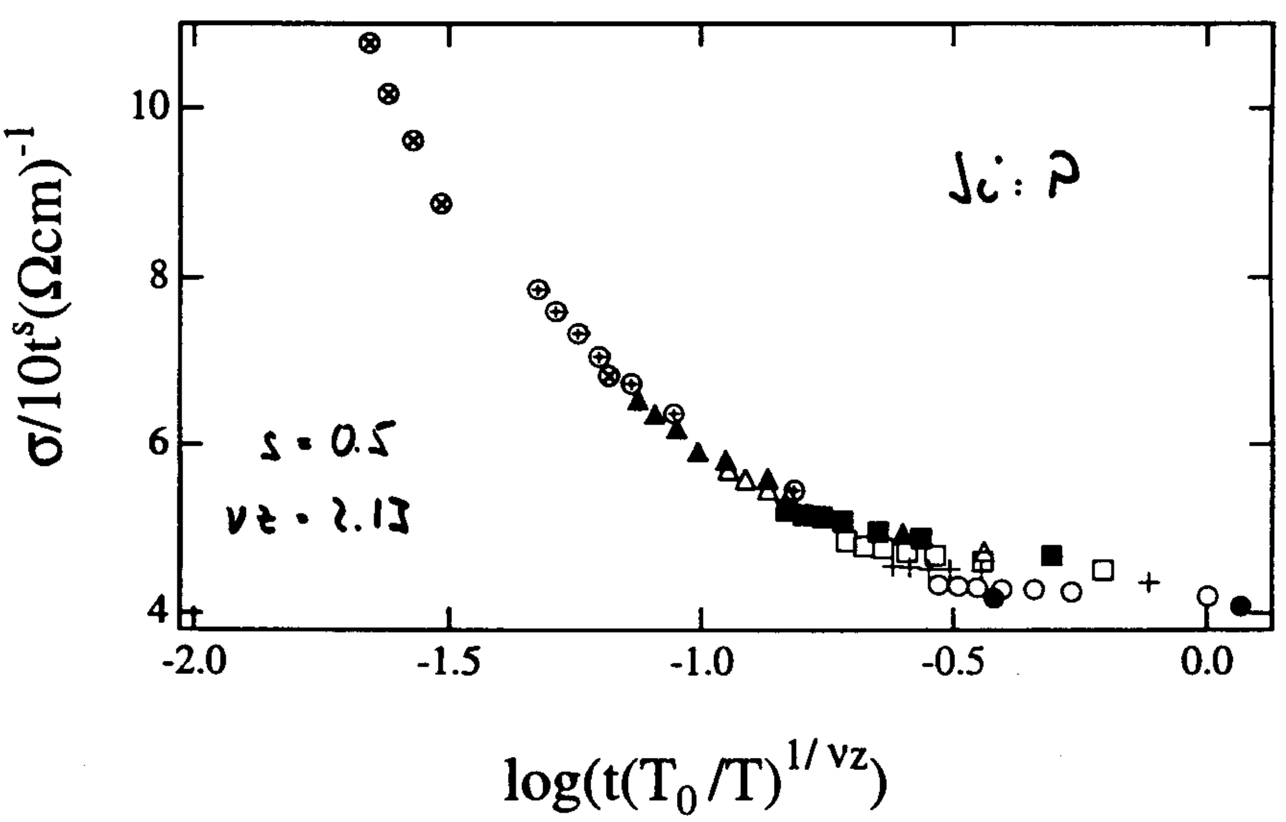


Fig. 9

Bell Labs data

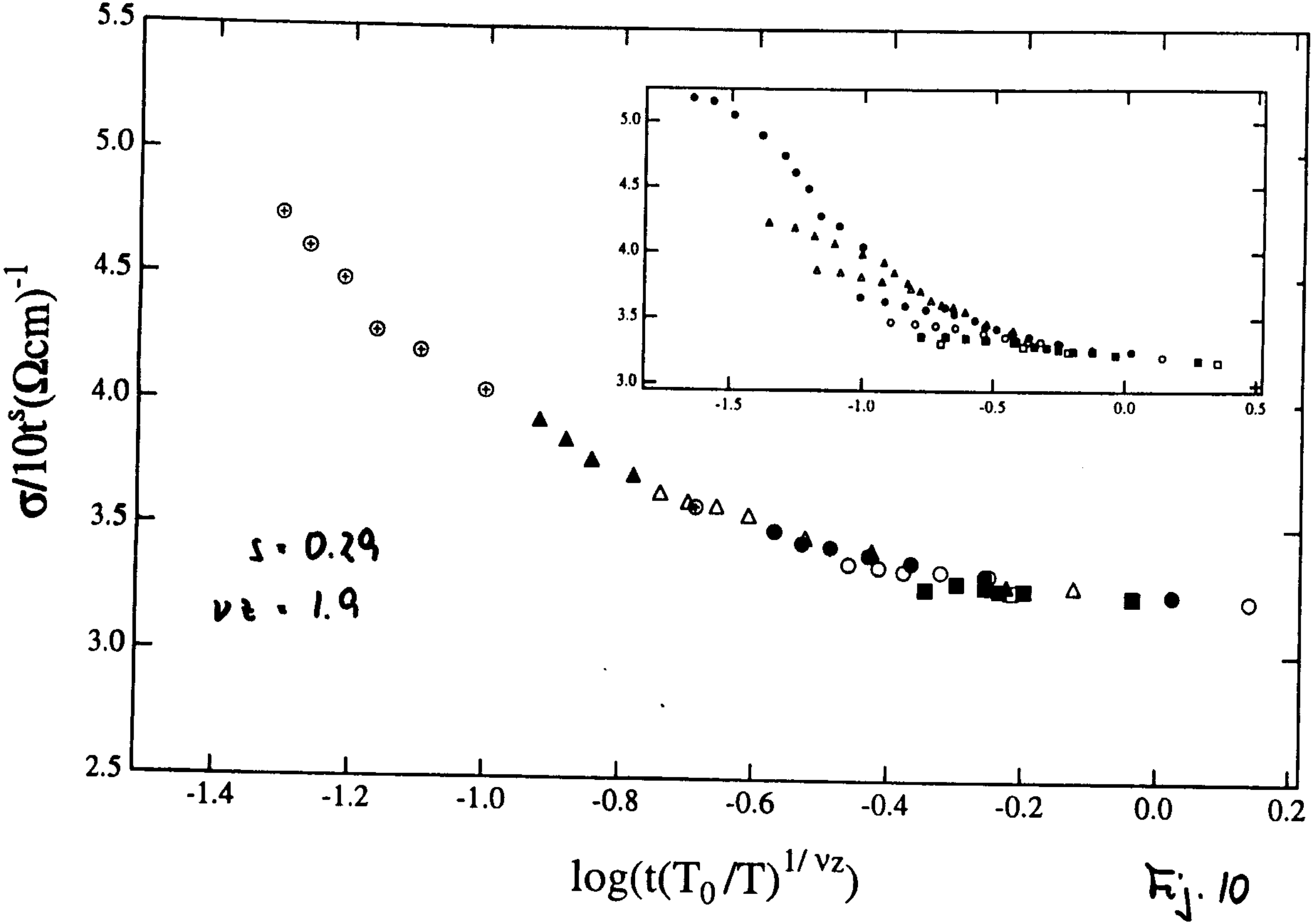


Fig. 10

Bell Labs data

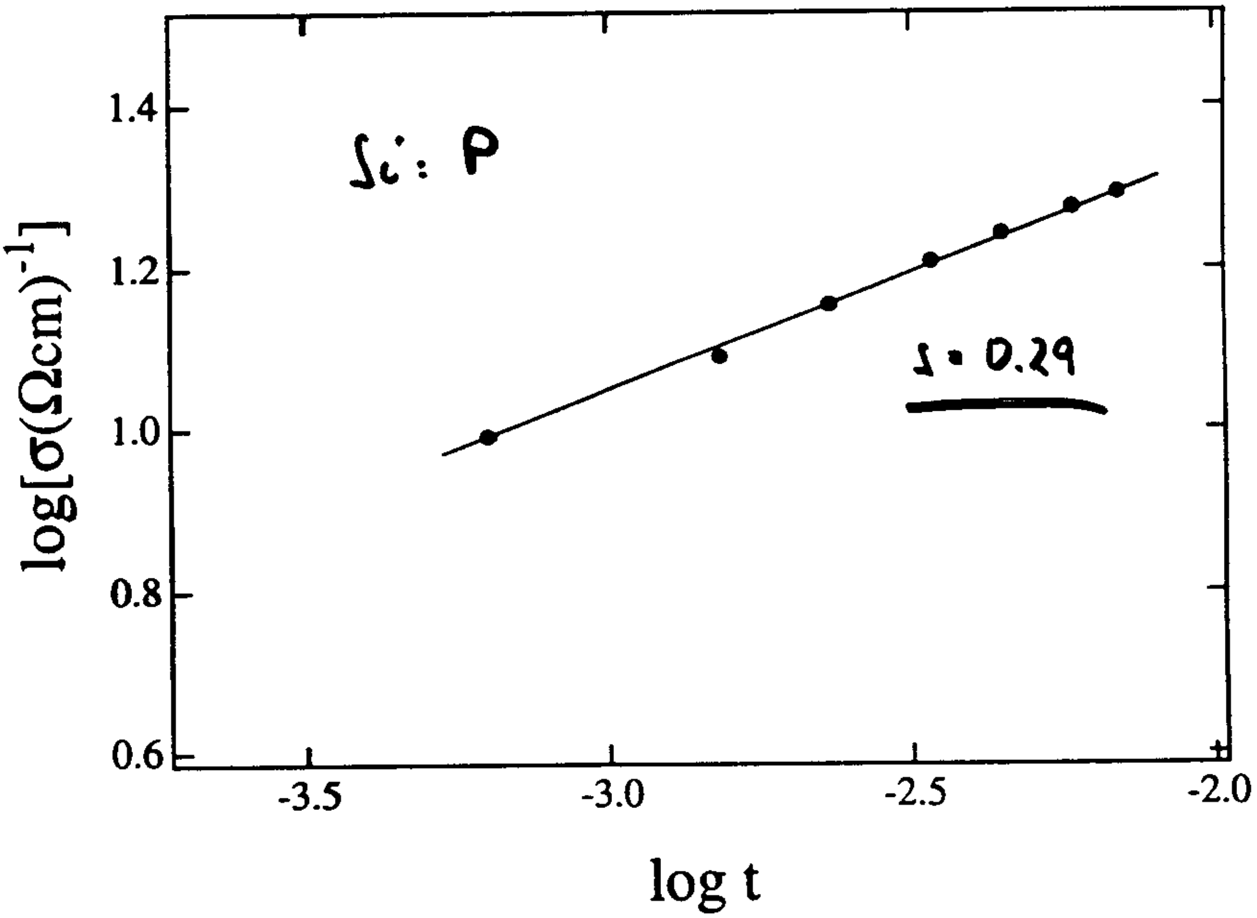


Fig. 11

Karlsruhe data

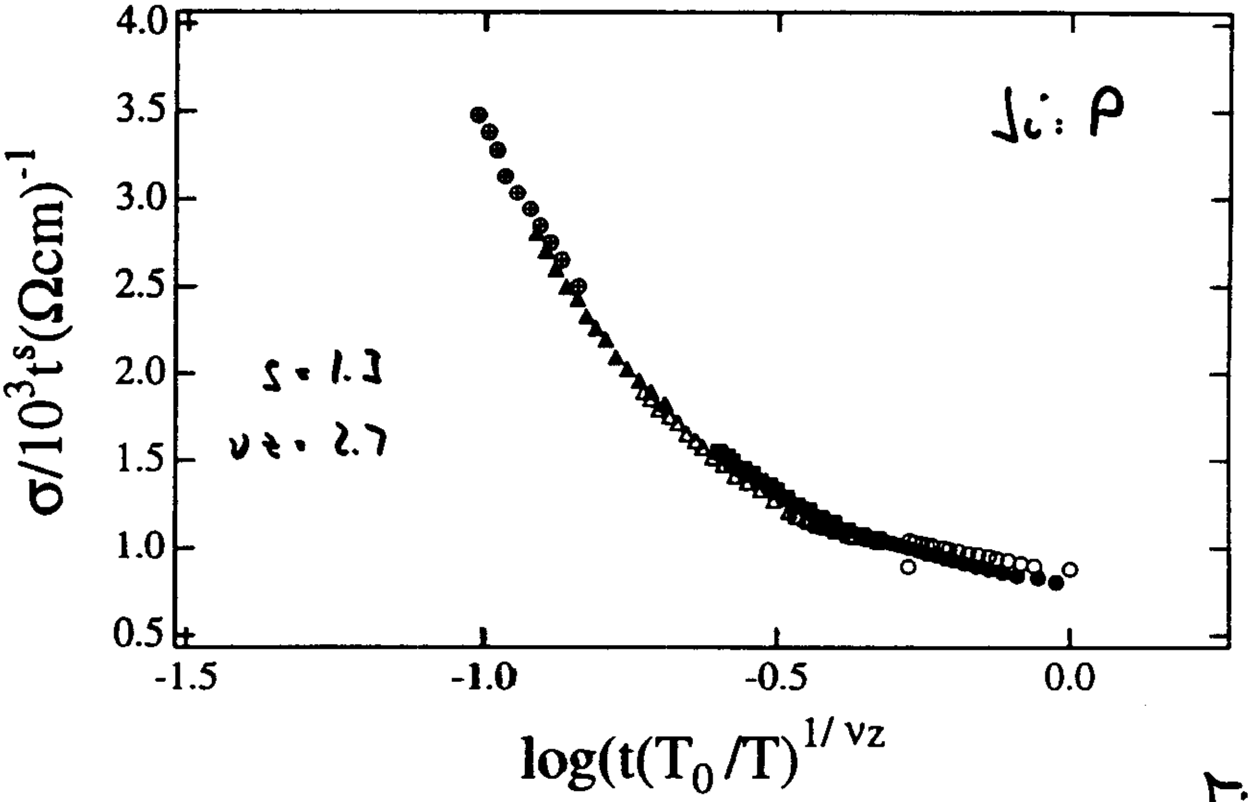


Fig. 12

Conclusions:

- Conventional scaling analysis cannot discriminate between the two experiments
- Irreproducibility problems reminiscent of glassy systems (e.g., magnets in random magnetic fields)

  NB: Recent experiments on Ni(S,Se) saw hysteresis effects near the AMT (Rosenbaum et al.)

II. The AMT as a Random-Field Problem

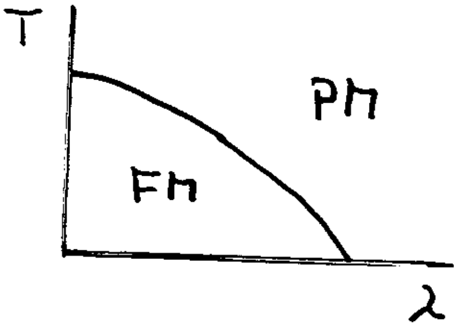
1. Physical Arguments

- Recall Random-Field Ising Model

$$\boxed{H = -J \sum_{i,j} S_i S_j + \sum_i h_i S_i}$$

$$\{h_i\}_{dis} = 0, \qquad \{h_i h_j\}_{dis} = \lambda \, \delta_{ij}$$

- λ large → No LRO at any T
- For small enough λ and T, J wins, resulting in LR ferromagnetic order



- Near criticality, the local RF results in clusters of spins pointing 'the wrong way' in otherwise ordered state

  large clusters hard to flip → glassy behavior (exponentially large time scales, activated scaling)

- Most important features:

  Disorder couples linearly to order parameter

  Disorder frustrates the interaction

- Analogy to AMT:
  - Order parameter for AMT: Density of states at the Fermi level $N(t,T)$
  - Experiments + all existing theories $\rightarrow N(t,T=0)$ vanishes for $t \rightarrow 0$ with some exponent β (Fig. 13)
  - Disorder term in the action:

$$\boxed{S_{dis} = \int d\vec{x} \int d\tau \; u(\vec{x}) \; \bar{\psi}(\vec{x},\tau) \, \psi(\vec{x},\tau)}$$

$$\{u(\vec{x}) \, u(\vec{y})\}_{dis} = \lambda \; \delta(\vec{x}-\vec{y})$$

But

$$N(\vec{x},t,T) \sim \int d\tau \; \langle \bar{\psi}(\vec{x},\tau) \, \psi(\vec{x},\tau) \rangle$$

→ Disorder couples linearly to the order parameter, like in RFIM

- Negative potential fluctuations increase DOS

  Interaction always decreases DOS

  → Interaction and Disorder leads to frustration, like in RFIM

Bishop, Spencer, Dynes

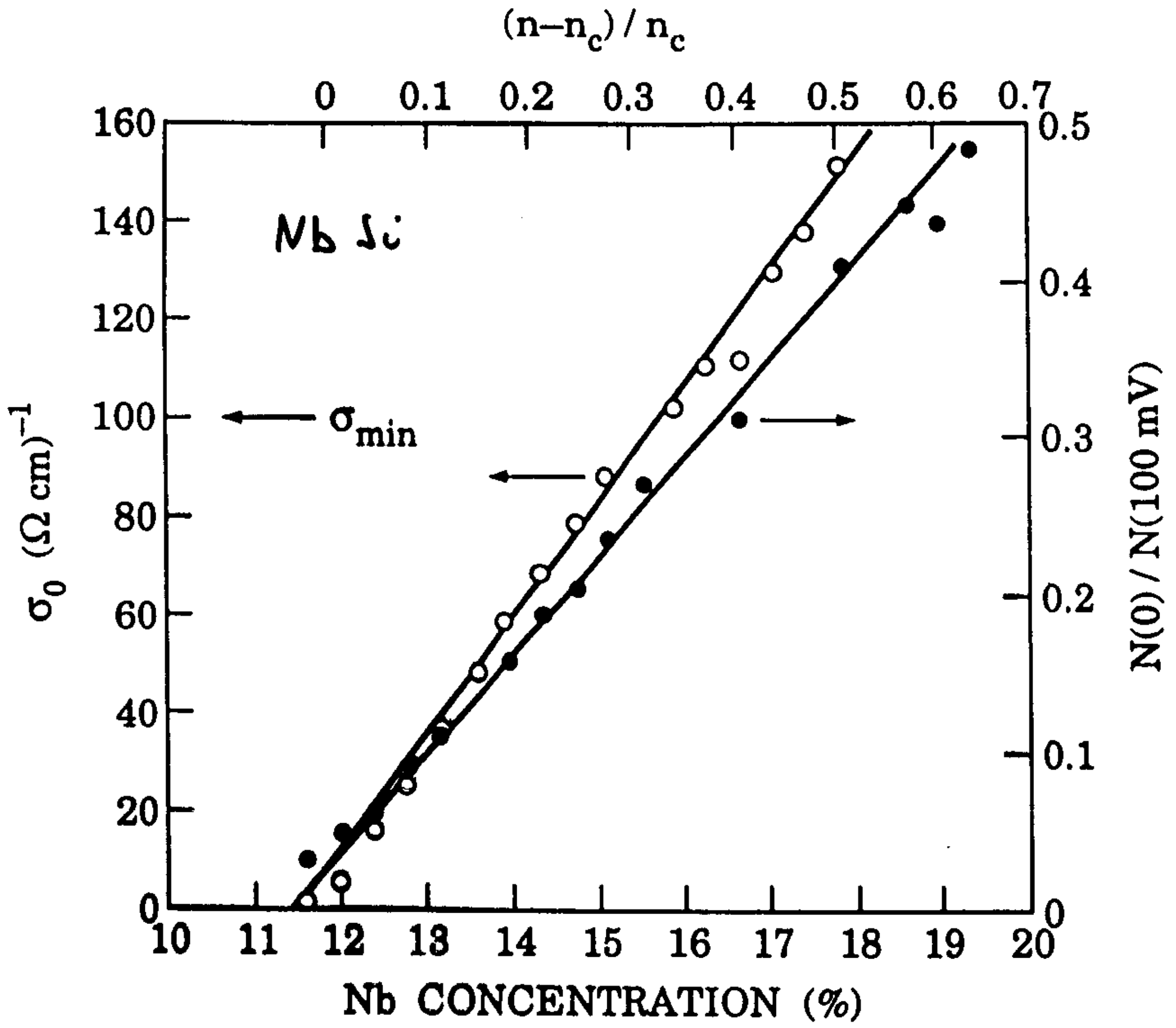


Fig. 13

2. Technical Arguments

- **An order parameter description of the AMT has been constructed with** $N(t,T)$ as the order parameter (DB & TRK 1995)

- Saddle-point solution → Landau or mean-field theory with exponents

$$\boxed{\nu = 1/2 \qquad \beta = 1/2 \qquad z = 3}$$

- Expect **mean-field critical behavior** to be exact for d larger than some d_c^+

  RFIM: Disorder fluctuations stronger **than thermal** fluctuations → $d_c^+ = 6$

  AMT: Disorder fluctuations stronger than quantum fluctuations → $d_c^+ = 6$

- → Critical behavior at the AMT known exactly for $d > 6$

- Analogy still holds in $\epsilon = 6 - d$ expansion: Static exponents to $O(\epsilon)$ are the same as in RFIM:

$$\boxed{\nu = 1/2 + \epsilon/12, \qquad \beta = 1/2 - \epsilon/6, \qquad z = 3 - \epsilon/2}$$

III. Implications

1. Breakdown of Wegner Scaling

- RFIM: Disorder fluctuations dominate thermal fluctuations
  - → T is dangerous irrelevant variable
  - → third independent exponent θ, and breakdown of hyperscaling: $d \to d - \theta$
- AMT: Disorder fluctuations dominate quantum fluctuations
  - → $\hbar$ is dangerous irrelevant variable with exponent $-\theta < 0$
  - → Modification of Wegner's scaling law:

$$\boxed{\begin{array}{l} \qquad\qquad s = \nu \left(d - 2 - \theta\right) \\ \text{in } d = 3 \quad \to \quad s \leq \nu \end{array}}$$

2. Activated Scaling

- In the RFIM, frustration leads to broadly distributed free energy barriers that diverge at criticality (Villain, D.S. Fisher)
  - → dynamical scaling changes from power-law to exponential ('activated scaling'):

$$\tau \sim \xi^z \qquad \to \qquad \ln \tau \sim \xi^{\psi}$$

    with ψ the 'barrier exponent' ($= \theta$?)
  - This has been observed in RF magnets (Jaccarino)

- **Assumption (based on physical and technical analogies between RFIM and AFM):** Activated scaling also at at the AFM

$\rightarrow$ **Replace $T\,b^{z}$ in homogeneity laws by** $b^{\psi}/\ln(T_0/T)$

- **Density of states:**

$$N(t,T) = b^{-\beta/\nu}\, F_N\left(t\, b^{1/\nu}, b^{\psi}/\ln(T_0/T)\right)$$

$$\rightarrow \quad \boxed{N(t,T) = \frac{1}{[\ln(T_0/T)]^{\beta/\nu\psi}}\, G_N\left[t^{\nu\psi}\,\ln(T_0/T)\right]}$$

Anomalously slow T dependence at $t=0$!

- Specific heat:

$$\boxed{c_V(t,T) = \frac{T^{\,const\times t^{\nu\psi}}}{[\ln(T_0/T)]^{\,1+(d-\theta)/\psi}}}$$

Continuously varying exponent (Griffiths phase) !

- Conductivity:

$\{\sigma\}_{dis} \sim \tau$ (relaxation time) is broadly distributed

$\rightarrow$ $\{\sigma\}_{dis}$ not self-averaging, not a scaling quantity !

$\{\ln \sigma\}_{dis}$ scales instead

III. Summary, and Conclusions

- Order parameter description and mean-field theory for AMT → <u>Critical behavior known exactly for $d > 6$</u>

- Physical and technical evidence for random-field aspects of the AMT

- At the very least, Wegner scaling is modified → <u>No bound on conductivity exponent s !</u>

- If analogy between RFIM and ~~AFM~~ AMT still holds in $d = 3$, then expect activated scaling and glassy behavior
  - Exponentially long equilibration times
  - Exponentially low T needed to observe static exponents
  - Averaged conductivity is not a scaling quantity
  - <u>Static critical behavior unobservable for all practical purposes</u> → What is actually seen depends on details of the experiment

- Mean-field transition shifts to strong interaction for weak disorder → AMT likely to be preempted by some other transition (Anderson transition ?)

  Possible phase diagram: