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"Fermi Condensation of Bosons in the t - J Model"

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g-on Mean Field Theory of the *t*-*J* Model

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Abstract

Implication of our recent proposal [J. Phys. Soc. Jpn. **65** (1996) 687] to treat large-amplitude gauge-field fluctuations around the slave-boson mean-field theory for the *t*-*J* model has been explored in detail. By attaching gauge flux to spinons and holons and then treating them as free *g*-on's which respect the time-reversal symmetry, the optimum exclusion (*g*) and exchange (α) statistics have been determined in the plane of doping rate and temperature. Two different relations between α and *g* have been investigated, namely $g = |\alpha|$ (Case1) and $g = |\alpha|(2 - |\alpha|)$ (Case2). The results indicate that slave fermion is favored at low doping while slave boson at high doping. For two dimension, in Case1 intermediate statistics are found in between, while in Case2 no intermediate statistics are found. The consequences of varying the dimensionality and strength of *J* have been studied also. The latter has no qualitative effect for both cases, while the former has a profound effect in Case1.

KEYWORDS: *t*-*J* model, gauge-field fluctuation, exclusion statistics, *g*-on

1. Introduction

Since the discovery of high- T_c cuprates¹⁾, there are many attempts to understand this interesting class of materials. It has generally been accepted that the essence of electronic properties is in the CuO₂ layers, and

that the low-lying excitations of the layers are described by a single-band *t*-*J* model^{2,3)}. Though the exact results are not known, the slave-boson mean field theory⁴⁻⁷⁾ has played important roles in offering explicit predictions to be compared with experiment as regards the doping dependence and effect of fermi surfaces, and the agreement between the theoretical results⁸⁻⁹⁾ and what have been seen in the experiments on the spin excitations probed by neutron scattering¹⁰⁾ and nuclear magnetic resonance (NMR)¹¹⁾, phonons¹²⁾ and transport properties¹³⁾ are noteworthy. The scheme is based on spin-charge separation and the electron operator is decoupled into spinon and holon, which is assumed to be fermion and boson, respectively. The phase diagram derived by the mean field approximation in this scheme is given by Fig.1. The states realized include antiferromagnetic (AF) state, superconducting (SC) state, "spin gap" (SG) phase, anomalous metallic (AM) state, and electron liquid (EL) state. The AM state, where both spinons and holons move coherently and essentially independent of each other, i.e. spin and charge are separated, is described as the uniform RVB (*u*-RVB) state. In the SG state, where the spin and charge are also separated, the spinons make short range singlet pairs, which is described as the singlet RVB (*s*-RVB) state. On the other hand in the EL state spinons and holons are no longer independent, because of the bose condensation of holons, and coupled together to form electrons. Based on this phase diagram various physical quantities especially, the spin gap phenomenon typically seen in the rate of NMR¹¹⁾ have been discussed⁸⁻⁹⁾ in connection with the SG state in the phase diagram.

It is generally the case, however, there exist fluctuations around the mean field solution. The most important fluctuations, in the present case, are expressed by the phases of the order parameters in the *u*-RVB state, which are viewed as U(1) gauge fields¹⁴⁾. The charge transport properties governed by holons are essentially influenced by these gauge fields because of the small characteristic energy of holons. The linear *T*-dependence of the resistivity has been discussed by treating the gauge field in the Gaussian approximation¹⁵⁾. However there are indications that the gauge-field fluctuations can be large. For example, high temperature expansion on the lattice¹⁶⁾ indicates that for a reasonable value of *t*/*J* the root-mean-square of fluctuations of gauge flux is large ($\sim \pi$ per plaquette). This might lead to quantitative¹⁷⁾ and even qualitative¹⁸⁾ changes in the mean field phase diagram. The existence of such large fluctuations may indicate the necessity

to elaborate the description of the ground state, and there are some efforts towards this target. On this line, there are two approaches both of which try to incorporate the fact that in the limit of vanishing doping rate, i.e. half-filling, the π -flux state is the most stable state. One idea exploits the equivalence of the π -flux state to d -wave due to SU(2) symmetry^{19,20}, while another maps the π -flux state into slave semion scheme²¹ by binding fluxes of half-quanta to both spinons and holons. The latter type of approach to treat strongly correlated electron systems may date back to the work of Kalmeyer and Laughlin²² who mapped the two-dimensional spin system to the fractional quantum Hall system. However, since flux state is stable only at half-filling, it is an open question as regards what will happen in finite doping rate.

Recently, we have proposed a different approach²³ to treat these large gauge-field fluctuations. The spinons and holons may avoid these large gauge-field fluctuations by partly absorbing them, which results in statistical transmutation of these particles. The statistics of holons and spinons will be chosen such that the free energy is minimum, i.e. the statistics are to be optimized. Here the time-reversal symmetry is properly taken into account by considering the fluctuating nature of the gauge field. A similar problem of determining variationally the optimum statistics of spinons and holons has been studied also by Mori²⁴ for the case of the Hubbard model with infinite Coulomb interaction. His formalism is, however, limited to continuum space, breaks time reversal symmetry and it turns out that only slave boson is realized at all doping rates.

In our explicit calculations of the thermodynamic properties the exclusion properties of anyons (particles with flux) has been noted. This exclusion property is based on the idea of Haldane²⁵ which generalizes the concept of Pauli exclusion principle. A particle obeying this exclusion principle is called “ g -on”²⁶ or “exclusionon”²⁷. Its distribution function was determined by Wu.²⁸ Recently, some models are known to realize g -on’s^{29,30}.

This paper has the following outline. Section 2 describes our model and formulation. The results of our numerical calculation are given in Sec.3. In Sec.4 conclusions are given together with discussions on the possible relevance in describing the anomalous metallic states of high- T_c cuprates. A part of the results for two dimensions in this study have been published elsewhere²³.

2. Model and Formulation

2.1) Slave Boson Mean-Field Theory

The t - J model is defined by the Hamiltonian,

$$H = - \sum_{i,j,\sigma} [t_{ij}(1 - n_{i-\sigma})c_{i\sigma}^\dagger c_{j\sigma}(1 - n_{j-\sigma}) + \text{h.c.}] + J \sum_{\langle i,j \rangle} \mathbf{S}_i \mathbf{S}_j - \mu_e \sum_{i,\sigma} n_{i\sigma}. \quad (2.1)$$

where the parameters and operators, especially, $\mathbf{S}_i = \frac{1}{2}c_{i\alpha}^\dagger \boldsymbol{\tau}_{\alpha\beta} c_{i\beta}$ with $\boldsymbol{\tau}$ being Pauli’s spin matrices, have the usual definitions, and \langle, \rangle means the summation over pairs of nearest neighbor sites.

In slave boson scheme the electron operator is decomposed in terms of holon (h_i) and spinon ($s_{i\sigma}$), $c_{i\sigma} = h_i^\dagger s_{i\sigma}$. The holons and spinons are assumed as bosons and fermions, respectively. The Hamiltonian is rewritten as

$$H = - \sum_{i,j,\sigma} [t_{ij}h_i s_{i\sigma}^\dagger s_{j\sigma} h_j^\dagger + \text{h.c.}] + J \sum_{\langle i,j \rangle} \mathbf{S}_i \mathbf{S}_j - \mu_e \sum_{i,\sigma} s_{i\sigma}^\dagger s_{i\sigma} - \sum_i \lambda_i (\sum_\sigma s_{i\sigma}^\dagger s_{i\sigma} + h_i^\dagger - 1) \quad (2.2)$$

where the last term with Lagrange multiplier, λ_i , excludes double occupancy at any sites. In the mean field approximation the local constraint is being relaxed to global one, $\lambda_i \rightarrow \lambda$. It has been shown⁸) that the choice of values of the transfer integrals, $(t/J = 4)$ $t'/t = -\frac{1}{6}$, $t''/t = 0$ and $t'/t = -\frac{1}{6}$, $t''/t = \frac{1}{5}$ simulate the fermi surfaces of LSCO and YBCO, respectively. Here t , t' , and t'' are transfer integrals between the first nearest neighbors, the next nearest neighbors, and the 3rd nearest neighbors, respectively. For simplicity our work will consider only the standard t - J ($t/J = 4$ and $t'/t = t''/t = 0$) since the present conclusions are expected to be insensitive to these choices. Moreover, although the mean field theory predicts various states as shown in Fig.1 we will limit ourselves to the u -RVB state, where $\chi_s = \langle \sum_\sigma s_{j\sigma}^\dagger s_{i\sigma} \rangle$ and $\chi_h = \langle h_i h_j^\dagger \rangle$ are the order parameters. The effect of singlet spinon pairing will be studied in a separate publication.

Then, the mean field hamiltonian, H_{MF} , is

$$H_{MF} = - \sum_{\langle i,j \rangle, \sigma} [t_{ij}(s)s_{i\sigma}^\dagger s_{j\sigma} + \text{h.c.}] - \mu_s \sum_{i,\sigma} s_{i\sigma}^\dagger s_{i\sigma} - \sum_{\langle i,j \rangle} [t_{ij}(h)h_i^\dagger h_j + \text{h.c.}] - \mu_h \sum_i h_i^\dagger h_i \quad (2.3)$$

where $\mu_h \equiv \lambda$, $\mu_s = \mu_c + \mu_h$ are the chemical potentials of holons and spinons, respectively, and $t_{ij}(h) = t_{\lambda h}$, $t_{ij}(s) = t_{\lambda h} + 3J_{\lambda s}/8$ are the effective hopping integrals of holons and spinons.

2.2) Fluctuations around the Mean-Field Solution

The original slave boson scheme without mean field approximation the electron operator, $c_{i\sigma} = h_i^\dagger s_{i\sigma}$, is invariant under local phase transformation which is lost totally in the mean field approximation. As Baskaran and Anderson¹⁴⁾ noted, to recover this local symmetry, the fluctuations around the mean field solutions should be treated as U(1) gauge field and hence the order parameters are not constants but fluctuating in space and time; $t_{ij}(q) = t_q e^{ia_{ij}}$ ($q = h, s$) where t_q are the mean field values. The fluctuation a_{ij} is, indeed, a gauge field that affects the spinons and holons together. This cures the complete decoupling of the electron operator at the mean field level. The Hamiltonian will then be written as

$$H = -t_s \sum_{\langle i,j \rangle, \sigma} [e^{ia_{ij}} s_{i\sigma}^\dagger s_{j\sigma} + \text{h.c.}] - \mu_s \sum_{i,\sigma} s_{i\sigma}^\dagger s_{i\sigma} - t_h \sum_{\langle i,j \rangle} [e^{ia_{ij}} h_i^\dagger h_j + \text{h.c.}] - \mu_h \sum_i h_i^\dagger h_i. \quad (2.4)$$

This is invariant under the following gauge transformation,

$$\begin{aligned} s_{i\sigma}^\dagger &\longrightarrow s_{i\sigma}^\dagger e^{-i\phi_i}, \\ h_i &\longrightarrow h_i e^{i\phi_i}, \\ a_{ij} &\longrightarrow a_{ij} + (\phi_i - \phi_j). \end{aligned} \quad (2.5)$$

In the calculation of transport coefficients¹⁵⁾ the fluctuations have been treated in the Gaussian approximation and it has been indicated that gauge fluctuations are not small.

Here we propose a way to take account of these large gauge-field fluctuations by assuming that the fluctuating gauge fluxes are approximately of Ising type, i.e. the magnitude is nearly constant but its sign is changing spatially and temporally [Fig.2(a)]. In this paper we will determine the magnitude of these Ising-type fluctuations that optimize the free energy by exploiting the ideas of flux binding and exclusion statistics.

2.3) Flux Binding and Exclusion Statistics

In a region where gauge-field fluctuations have some fixed value and then the uniform (internal) magnetic field is present, the fluxes due to gauge-field fluctuations will be attached to spinons and holons [Fig.2(b)]. Now we have a system of anyons whose exchange statistics α (defined as the phase factor $e^{i\alpha\pi}$ accumulated by the total wave function when two particles are interchanged) has approximately a constant absolute value. The new particles are then defined as $\tilde{s}_{i\sigma} = s_{i\sigma} e^{i\alpha Q_i}$ and $\tilde{h}_i = h_i e^{i\alpha Q_i}$. Here $Q_i = \sum_{j \neq i} \theta_i(j)(h_j^\dagger h_j + \sum_{\sigma} s_{j\sigma}^\dagger s_{j\sigma})$ and $\theta_i(j)$ represents the angle of a vector from site j to i with respect to some fixed reference axis. It is straightforward to see

$$\tilde{h}_i \tilde{h}_j^\dagger = e^{i\alpha\pi} \tilde{h}_j^\dagger \tilde{h}_i, \quad \tilde{h}_i \tilde{h}_j = e^{i\alpha\pi} \tilde{h}_j \tilde{h}_i, \quad i \neq j$$

and

$$\tilde{s}_{i\sigma} \tilde{s}_{j\sigma}^\dagger = -e^{i\alpha\pi} \tilde{s}_{j\sigma}^\dagger \tilde{s}_{i\sigma}, \quad \tilde{s}_{i\sigma} \tilde{s}_{j\sigma} = -e^{i\alpha\pi} \tilde{s}_{j\sigma} \tilde{s}_{i\sigma}, \quad i \neq j \quad (2.6)$$

and then the exchange statistics of holons and spinons are α and $\pm 1 + \alpha$, respectively. The onsite commutation relations are simply given by the commutation relations of the undressed particles. The Hamiltonian for \tilde{h}_i and $\tilde{s}_{i\sigma}$ is the same as Eq.(2.4) but we expect that the magnitude of fluctuations $\tilde{a}_{ij} = a_{ij} - \alpha(Q_i - Q_j)$ are much reduced. Then we assume that the fluctuations \tilde{a}_{ij} felt by the new particles \tilde{h}_i and $\tilde{s}_{i\sigma}$ are negligibly small and the mean field approximation is meaningful. If we fix α , it will break the time-reversal symmetry. However, in the present case the time-reversal symmetry is maintained due to the fluctuating nature of α ; the magnitude is fixed but the sign is fluctuating as shown in Fig.2(c), so that the (net) flux will vanish as an average and the thermodynamic properties of the system will be purely described by the exclusion properties by the dressed

holons and spinons. In other words, although the system is composed of domains with fluxes α and $-\alpha$, it will be regarded as homogeneous in its exclusion properties if we neglect domain boundaries [Fig.2(d)], and its thermodynamics may be approximately treated by use of the g -on distribution function²⁸⁾

$$n_g(\varepsilon) = (w_g(\varepsilon) + g)^{-1}.$$

Here $w_g(\varepsilon) \equiv w$ is determined by

$$w^g(1+w)^{1-g} = e^{\beta(\varepsilon-\mu)}, \quad (2.7)$$

with one-particle energy ε , chemical potential μ and inverse temperature β . For a special value of $g = 0$ ($g = 1$) we have the familiar bose (fermi) distribution function. The distribution functions for several choices of g are shown in Fig.3. Here the exclusion statistics is defined as the decrease in the size of one-particle Hilbert space as the number of particles increases²⁵⁾. For boson this size does not change, $g = 0$, and for fermion the decrease is one whenever the number of particles increases by one, $g = 1$. In between boson and fermion we have g -on, $0 < g < 1$.

There are so far two proposed relations between the exclusion statistics $g(\geq 0)$ and exchange statistics α ($-1 \leq \alpha \leq 1$); namely,

$$\text{Case1: } g = |\alpha| \quad (2.8a)$$

and

$$\text{Case2: } g = |\alpha|(2 - |\alpha|). \quad (2.8b)$$

These relations, Eqs.(2.8a) and (2.8b), are the exclusion statistics of system of free-anyon with Dirac-type dispersion³¹⁾ and with quadratic dispersion³²⁾, respectively, which are classified as Case1 and Case2 in the following. In the present study to see overall tendency as the doping rate is varied, the energy spectra of spinons and holons vary in a wide range and the correct assignment is not known and then we simply assume either (2.8a) or (2.8b) and investigate the consequences for both cases. If we assign the exclusion statistics $g_h = g(\alpha)$ for holon, then $g_s = g(1 - \alpha)$ for spinon since they are composite particles of electron. Hence the free energy becomes a function of δ (doping rate), T (temperature) and α (statistics). By treating $\alpha(0 \leq \alpha \leq 1)$ as a variational parameter we can minimize the free energy

and determine the phase diagram in the plane of δ and T . In this study the dimensionality, d , of the system can be arbitrary and we will examine the cases of $d = 1, 2, 3$.

The free energy is given by

$$F = -2T \sum_{\vec{k}} \ln(1 + w_{g_s}^{-1}(\varepsilon_{\vec{k}})) - T \sum_{\vec{k}} \ln(1 + w_{g_h}^{-1}(\omega_{\vec{k}})) \\ + N \left[d(2t\chi_s\chi_h + \frac{3J}{8}\chi_s^2) + (1 - \delta)\mu_s + \delta\mu_h \right] \quad (2.9)$$

and the self-consistent equations are

$$\delta = \frac{1}{N} \sum_{\vec{k}} n_{g_h}(\omega_{\vec{k}}), \quad \delta_s \equiv \frac{1}{2}(1 - \delta) = \frac{1}{N} \sum_{\vec{k}} n_{g_s}(\varepsilon_{\vec{k}}), \\ \chi_h = \frac{1}{dN} \sum_{\vec{k}} \gamma_{\vec{k}} n_{g_h}(\omega_{\vec{k}}), \quad \text{and} \quad \chi_s = \frac{2}{dN} \sum_{\vec{k}} \gamma_{\vec{k}} n_{g_s}(\varepsilon_{\vec{k}}), \quad (2.10)$$

where $\varepsilon_{\vec{k}} = -2t_s\gamma_{\vec{k}} - \mu_s$, $\omega_{\vec{k}} = -2t_h\gamma_{\vec{k}} - \mu_h$ with $\gamma_{\vec{k}} = \sum_{x=1}^d \cos k_x$, n_{g_q} ($q = h, s$) the “ g -on” distribution function and N is the total number of lattice sites.

3. Results

In the following two subsections the dimensionality of the system is fixed to two, $d = 2$, whereas effects of the dimensionality will be discussed afterwards.

3.1) $T = 0$

First we will consider the $T = 0$ where the physics and the overall features of the results are transparent and easy to comprehend. In this case the distribution function is a step function with the “ g -onic fermi energy” μ_q , $n_{g_q} = g_q^{-1}\theta(\mu_q - \varepsilon^q)$ ($q = h, s$) where $\varepsilon^h = \omega_{\vec{k}}$ and $\varepsilon^s = \varepsilon_{\vec{k}}$. The energy is

$$E = -Nd(2t\chi_s\chi_h + \frac{3J}{8}\chi_s^2) \quad (3.1)$$

where the order parameters, χ_q ($q = h, s$), are implicit functions of the statistics, α . In Figs.4 (a) and (b), the energy per lattice site as a function of

α for several values of doping rate are shown for Case1 and Case2. One can easily see that for both cases slave fermion ($\alpha = 1$) is favored for low doping region and slave boson ($\alpha = 0$) for high doping region. However there exists a difference. In Case1 the optimum statistics changes continuously from slave fermion to slave boson through the intermediate statistics ($0 < \alpha < 1$, to be referred to as slave g -on in the following) for $0.351 < \delta < 0.392$, as seen in Fig.5(a). On each boundary of two different statistics the transition is of second order. On the other hand, in Case2, the optimum statistics changes from $\alpha = 1$ to $\alpha = 0$ discontinuously at $\delta = 0.371$, a first order transition as seen in Fig.5(b). This first order transition is accompanied by a discontinuous change of the fermi surfaces of holons and spinons.

The result can be understood as due to bose-condensation of bosons. In the low doping region, spinons are abundant and govern the energetics; the energy gain is maximum if all spinons are bosons and condense into the bottom of the band. Hence, slave fermion ($\alpha = 1$) is favored at low doping as also has been shown in Ref. 33. On the other hand, in high doping region holons are abundant and the maximum energy gain occurs if they become bosons and condense and slave boson ($\alpha = 0$) is favored.

3.2) Finite Temperature

Let us first study high temperature behaviours. The onset temperature of the u -RVB state is

$$T_D = \frac{3}{8} J a_s \left[1 + \sqrt{1 + 2 \frac{a_h}{a_s} \left(\frac{8t}{3J} \right)^2} \right], \quad (3.2)$$

where $a_q = \delta_q(1 - \delta_q g_q)(1 + \delta_q \bar{g}_q)$, $\bar{g}_q = 1 - g_q$, ($q = h, s$), $\delta_s = \frac{1}{2}(1 - \delta)$ and $\delta_h \equiv \delta$. This expression is derived by assuming that the transition is of second order, and does not hold when the transition is of first order (see §3.3, and §3.4).

Above this temperature the effective hopping integral or bandwidth is zero and there is no energy gain due to itineracy. Then, the free energy for $T > T_D$ is determined purely by the entropy,

$$S = N \left\{ \delta \ln \delta + (1 - \delta g_h) \ln(1 - \delta g_h) - (1 + \delta \bar{g}_h) \ln(1 + \delta \bar{g}_h) \right. \\ \left. + 2[\delta_s \ln \delta_s + (1 - \delta_s g_s) \ln(1 - \delta_s g_s) - (1 + \delta_s \bar{g}_s) \ln(1 + \delta_s \bar{g}_s)] \right\}. \quad (3.3)$$

Fig.6(a) and (b) show $-S/N = F/NT$ (free energy per lattice site divided by temperature) for different values of doping rate for Case1 and Case2, respectively. These fix the feature of the phase diagram at $T > T_D$: the intermediate statistics region is possible for $0.371 < \delta < 0.457$ in Case1 but not possible in Case2. We remark that this is independent of spatial dimensionality, since Eq.(3.3) is. As the temperature is lowered through T_D , the transition to u -RVB state is of second order for both cases, except for a small region of doping $0.4043 < \delta < 0.4124$ (from A to B of Fig.7(b)) in Case2 where the transition is of first order. This is because the T_D 's on both sides of slave fermion-slave boson interface do not fall on the same δ , i.e. the T_D from A to B coincides with the interface of slave fermion and slave boson [see inset of Fig.7(b)]. The overall phase diagram for Case1 and Case2 are shown in Fig.7(a) and (b), respectively.

In Case1, the width and the location (in δ -space) of the region where the slave g -on is stabilized, is almost independent of T because $-S/N$, as a function of α , has basically the same features as E/N for the same range of doping δ .

In general, the absence or presence of intermediate statistics can be understood in terms of the total derivative of the free energy with respect to α ,

$$\frac{dF(\alpha)}{d\alpha} = T [g'(\alpha) f_{g_h} - 2g'(1 - \alpha) f_{g_s}] \quad (3.4)$$

where $f_g = \sum_{\vec{k}} n_g \ln(1 + w_g^{-1})$, ($g = g_h, g_s$) and the prime denotes differentiation with respect to its argument. For Case1, at low doping $\frac{dF(\alpha)}{d\alpha}$ is negative while at high doping it is positive for any value of α . This is due to fact $f_{g_s} > f_{g_h}$ ($f_{g_h} > f_{g_s}$) for low (high) doping. At the same time it is seen that $\frac{dF(0)}{d\alpha} < 0$ and $\frac{dF(1)}{d\alpha} > 0$ for some range of δ . With this we note that there exist a region of δ where intermediate statistics is possible.

For Case2, on the other hand, it is straightforward to see that $\frac{dF(\alpha)}{d\alpha}$ has only one zero for $0 \leq \alpha \leq 1$ and $\frac{dF(0)}{d\alpha} > 0$ and $\frac{dF(1)}{d\alpha} < 0$, thus, intermediate statistics is impossible to occur in this case.

We also considered another hypothetical assignment of statistics $g = \alpha^2$ where at $\alpha = 0$ and $\alpha = 1$ reduce properly to boson and fermion, respectively. In this assignment, a wide region of intermediate statistics was found because $\frac{dF(0)}{d\alpha} < 0$ and $\frac{dF(1)}{d\alpha} > 0$.

The possibility of intermediate statistics may generally be understood

by an effective exclusion statistics g_{eff} defined by $g_{eff} = g_h + g_s$. If $g_{eff} > 1$, it is impossible to realize, whereas, if $g_{eff} < 1$, the possibility of its occurrence is very high. The case of $g_{eff} = 1$, which corresponds to Case1, is marginal and elaborate analysis is needed.

3.3) Effect of Dimensionality

The g -on as a generalization of boson and fermion is not limited to two dimensional in contrast to anyon whose realization is restricted to $d = 2$. In this subsection we extend our calculations to $1d$ and $3d$ under the assumption that the free energy given by Eq.(2.9) is still valid in these cases and explores its consequences that may be relevant in understanding more on the 'g-on physics' for the t - J model.

In this subsection only Case1 will be discussed where the role of dimensionality is profound in contrast with Case2 which is insensitive to the change of dimension. The phase diagrams for $1d$ and $3d$ are shown in Fig.8(a) and (b), respectively, where the bose condensation temperature, T_B , is also drawn for $3d$. The main effect of increasing the dimensionality is to reduce the slave g -on region as seen in Figs.8(a), 7(a), and 8(b) for $1d$, $2d$, and $3d$, respectively. This is also seen in Fig.8(d) where, at $T = 0$, the optimum statistics as a function of δ for each dimension is shown. The energy as a function of α for different values of δ in each dimension further supports these understanding as seen in Figs. 9(a)($1d$), 4(a)($2d$) and 9(b)($3d$). Clearly, this suppression of slave g -on-phase as the dimension increases is due to increasing tendency of bose condensation.

In $1d$, at $T = 0$, the entire region of δ favors slave g -on-phase except at the endpoints, $\delta = 0$ and $\delta = 1$ where slave fermion and slave boson, respectively, are realized. On both ends the transitions are of second order. However, the deviation of optimum statistics from $\alpha = 1$ near $\delta = 0$ and from $\alpha = 0$ near $\delta = 1$ are very small as seen in Fig.8(c). At finite temperature, in both regions of doping rate the change of statistics are continuous also. However, even at very low temperature the region of g -on is reduced appreciably compared to that at $T = 0$ as seen in Fig.8(a). This is due to singularity of the density of state at the band bottom which make the effect of entropy important even at very low temperature.

In $3d$, on the other hand slave boson and slave fermion phases are stabilized much compared to $2d$. This is due to bose condensation. The

slave g -on phase is only realized at temperature near and above T_D where the effect of entropy is significant. We note there exists a triple point $P_{tr}(T_{tr} = 1.732J, \delta_{tr}=0.373)$ where the three phases (slave fermion, slave g -on, and slave boson) coexist, at relatively high temperature [Fig.8(b)]. This is because T_B scales with the band width and has the same order of magnitude as T_D . Below P_{tr} , the transition at fixed temperature from slave fermion to slave boson is of first order. At P_{tr} , the slave fermion-slave boson interface branches into slave fermion-slave g -on and slave g -on-slave boson interfaces. It turns out that the transition from slave fermion to slave g -on phases (line $P_{cr}Q$) is of second order, while the transition between the slave g -on and slave boson is of first order on the line $P_{tr}P_{cr}$ but becomes of the second order on $P_{cr}R$. In Fig.8(c), the α -dependence of the free energy are shown at X, Y, Z (inset) where X and Y are points in slave fermion-slave boson boundary and Z is inside the slave g -on region which is chosen such that the free energies of slave fermion and slave boson are the same.

Another consequence of $T_B \lesssim T_D$ is that bose condensation of spinons becomes of first order for $\delta < 0.0530$. This region is indicated by the dotted line in Fig.8(b). Moreover, we found numerically that for very small δ (≤ 0.0062), the transition into the u -RVB state occurs at higher temperature than that given by Eq.(3.2), which has been derived by assuming second-order transition. What actually happens is that u -RVB is driven by bose condensation of spinons, namely, T_D coincides with T_B , and becomes of first order. This is seen from lines 1 and 2 in Fig.10(a), where the Ginzburg-Landau free energy, $\delta F \equiv F(\chi_s) - F(\chi_s = 0)$, as a function of χ_s has two minima. Note that the bose condensation is stabilized for χ_s larger than the value indicated by dot in each line. At slightly larger δ (> 0.0062), the u -RVB transition becomes of second order but bose condensation remains a first order transition (lines 3-7 in Fig.10(a)), until $\delta = 0.0530$ beyond which T_B becomes of second order. The phase diagram very close to half filling is shown in Fig.10(b).

Hence, as an overall view the richness of our phase diagrams discloses the intricate interplay between bose condensation, band edge singularity of the density of states, and entropy at high temperature. In $1d$, entropy is the key factor, except at $T = 0$. However, in $3d$ bose condensation is the key factor. In $2d$, the intermediate statistics are realized on the same range of both high and low doping and then we may infer that the tendency to bose condense and the effect of entropy have the same weight.

3.4) Effect of J

We next study the effect of J . The global features of our results do not depend on the magnitude of J . For example, the phase diagram for $J = 0$ in $2d$ is shown by the dotted lines in Fig.7 and do not deviate much from the results of finite values of J . The same is true for $1d$ and $3d$. However in $3d$ for $J = 0$, the transitions at T_D and T_B are both of second order at low doping in contrast to the case of finite J . Moreover, in Case1 (Case2) the T_B starting from the interface of slave g -on(slave fermion)-slave boson phases up to $\delta = 0.6439$ is of first order transition. Qualitatively, the main effects of $J > 0$ are pronounced at low doping region where the number of spinons are large. For example at half-filling, it removes the degeneracy in the optimum statistics at $T = 0$ and supports finite T_D , as well as T_B in $3d$. Furthermore, it stabilizes the slave fermion-phase and, in general, raises T_D especially at low doping region. In Case1, the former results in the narrowing of the region of the intermediate-statistics phase. All these can be understood as the result of increase in the coherency of spinons, and consequently of holons, due to J .

4. Summary and Discussions

In the slave-boson theory of the t - J model, the electron operator is decomposed into spinons and holons, which are assumed to be fermions and bosons, respectively. Within the mean field approximation, the spinons and holons are assumed to move independently and the characteristic phase diagram is predicted as given by Fig.1. The most important fluctuations around this mean-field solution are the gauge fields, which by themselves couple to spinons and holons leading to the effective interaction between them. Especially once the bose-condensation of holons is realized, i.e. $T < T_B$, these effects of gauge fields are essential resulting in the binding of spinon and holon which implies that the ordinary view based on electrons is recovered. In the region where the spin and charge are separated, i.e. in the region of anomalous metal (AM) and spin gap (SG) in Fig.1, various predictions have been made for spin excitations based on the mean-field approximation without the explicit recourse to the fluctuation effects, which are expected to be relatively minor because of large characteristic energy due to Fermi degeneracy of spinons. Above all the possibility has been pursued^{8,9)} of the identification of the spin gap phe-

nomenon with the onset of the SG phase out of the anomalous metallic (AM) state, the former and the latter are singlet and uniform RVB states, respectively. This identification has been first suggested by Rice³⁴⁾ who payed attention to the quantitatively different temperature dependences of the NMR shift and rate in the optimal and underdoped YBCO, and this fact has been further explored in Ref. 35. This identification has led to the prediction of the anomalous frequency shift of the particular in-plane oxygen phonon modes⁹⁾, which has been confirmed experimentally¹²⁾. The results of the recent angle-resolved photoemission spectroscopy seem to be also in accordance with the prediction³⁶⁾.

In contrast to these spin excitations, the charge transport properties, i.e. the resistivity and the Hall effect coefficient, are very susceptible to the fluctuations around the mean field solutions, since the holons are assumed to be pure boson. Moreover there are theoretical indications that these fluctuations can be very large.

The present study is intended to overcome this apparent difficulty by proposing a new theoretical framework to treat the large gauge fluctuations. By attaching the gauge fluxes to spinons and holons, which are assumed to have a fixed value but with the fluctuating signs (like Ising model), and then treating these particles as free g -on's, the statistics has been optimized by minimizing the free energy on the plane of doping rate and the temperature by use of the distribution functions of g -on's. (The brief report of the present study has been given in Ref.23.) To best of our knowledge, this is the first theory to search for statistics variationally and moreover this theory respects the time-reversal symmetry. The result of the optimization is that the slave fermion, where spinons are bosons, is favored at low doping (near half-filling) while slave boson, where spinons are fermions, is favored at high doping, with the intermediate statistics in between. This general feature of slave-fermions and slave-bosons are easily understood by the existence of the bose condensation of spinons at low doping and holons at high doping. This understanding also clearly indicates that the regions of pure slave-fermions or slave-bosons are overemphasized in the present approximation because the existence of hard cores of spinons and holons is totally ignored. Consequently we expect the region of the intermediate statistics will be enlarged in a more elaborate treatment. In such a region of intermediate statistics the gauge fluctuations are greatly reduced because of the existence of the finite mass in the gauge propagators due to the Chern-Simons term³⁷⁾.

It is possible that such a region of intermediate statistics (but close to the slave-boson) is the one where many interesting and anomalous properties have been observed experimentally.

At the same time we also note the following questions which still remain to be addressed properly. The assignment of statistics, $g = g(\alpha)$, we adopted in the present study is derived for anyons in continuum space and does not reflect the actual dispersion of our spinons and holons. Moreover, the assignment was been made by comparing the second virial coefficients which are appropriate only at high temperature. Hence, further investigations are needed on the lattice effects as well as the temperature dependences.

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Figure Captions

- Fig. 1. The slave boson mean-field phase diagram. The phases realized are antiferromagnetic (AF) state, superconducting (SC) state, "spin gap" (SG) phase, anomalous metallic (AM) state, and electron liquid (EL) state.
- Fig. 2. Temporal view of spatial variation of gauge flux (a), its attachment to particles (b), exchange statistics (c) and exclusion statistics (d).
- Fig. 3. Distribution function of g -on, $n(\varepsilon)$, where ε , μ and β are energy, chemical potential and inverse temperature, respectively.
- Fig. 4. The energy as a function of α for several values of δ for (a) Case1 and (b) Case2. In Case1, the inset shows those at the other values of δ .
- Fig. 5. The optimum values of α at $T = 0$ as a function of δ for (a) Case1 and (b) Case2.
- Fig. 6. The entropy for $T > T_D$ as a function of α for a choice of several values of δ for (a) Case1 and (b) Case2. These are independent of temperature, spatial dimensionality and J .
- Fig. 7. The phase diagrams in $d = 2$ for (a) Case1 and (b) Case2. The inset in (b) shows the region where the transition to u -RVB state is of first order (from $A(C)$ to $B(D)$ for $J = t/4$ ($J = 0$)). The solid and dashed lines are for $J = t/4$ and $J = 0$, respectively.

- Fig. 8. The phase diagrams for (a) $1d$ and (b) $3d$. In (b), the dotted line in T_B and in T_D represents first order transitions. (c) The enlarged version of (b) where slave g -on phase is realized. The inset shows the α -dependence of the free energy measured relative to slave fermion or slave boson on points X, Y, Z. (d) The doping dependences of the optimum statistics are shown for $d = 1, 2, 3$ at $T = 0$.
- Fig. 9. The α -dependence of energy for several values of δ in (a) $1d$ and (b) $3d$ at $T = 0$.
- Fig.10. (a) The Ginzburg-Landau free energy as a function of u -RVB order parameter, χ_s , close to half-filling at temperature chosen such that the two minima have the same value. At the dot mark on each line bose condensation sets in. Lines 1-2 show why T_D coincides with T_B and becomes of first order transition. The series of lines 1-7 shows how the two minima of χ_s smoothly merges into single point, i.e. T_B reduces of second order as δ increases. (b) The δ - T phase diagram or small values of δ where the T_D coincides with T_B . The transition in T_D changes from first order to second order at $\delta = 0.0062$.

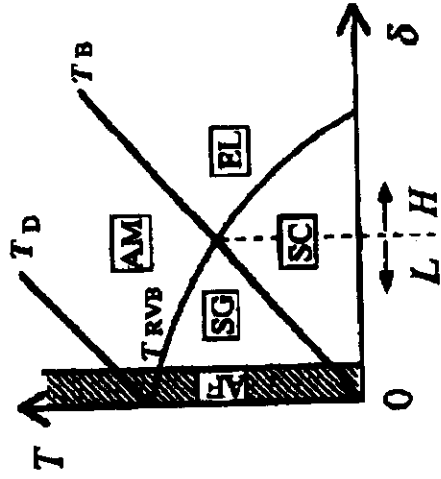
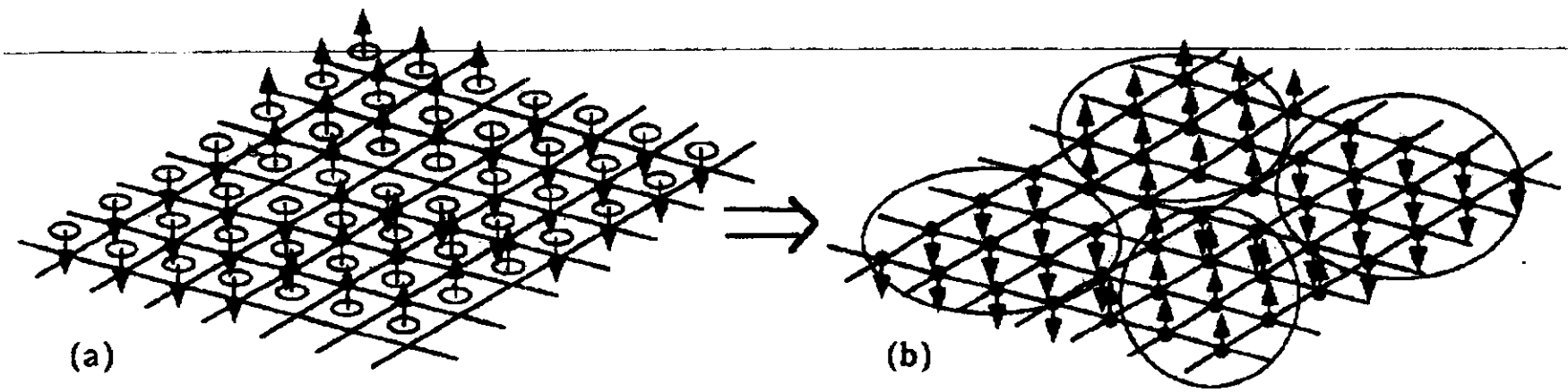


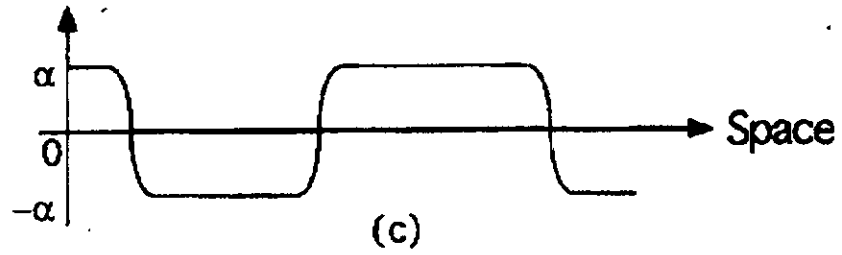
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Exchange Statistics



Exclusion Statistics

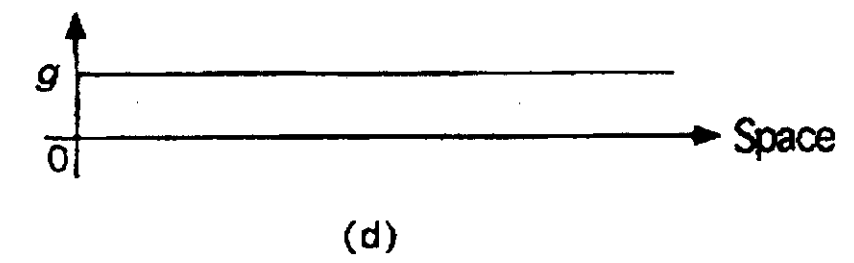


Fig. 2

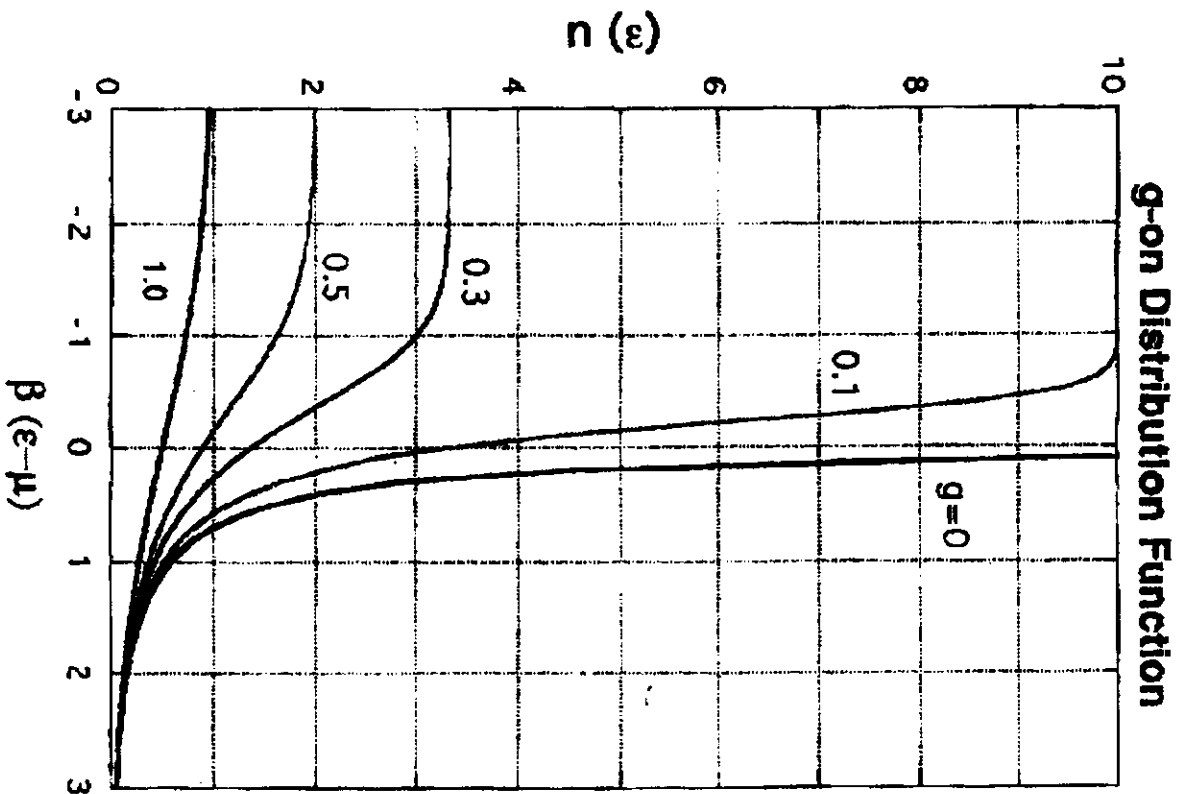


Fig. 3

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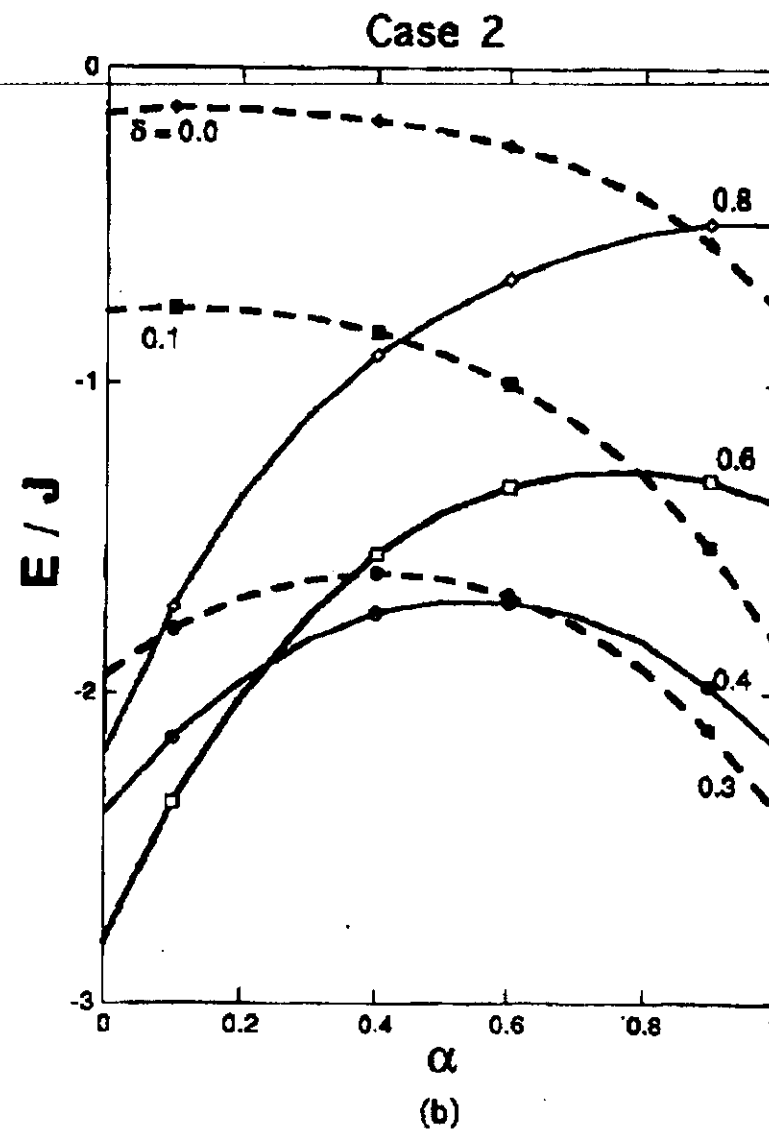
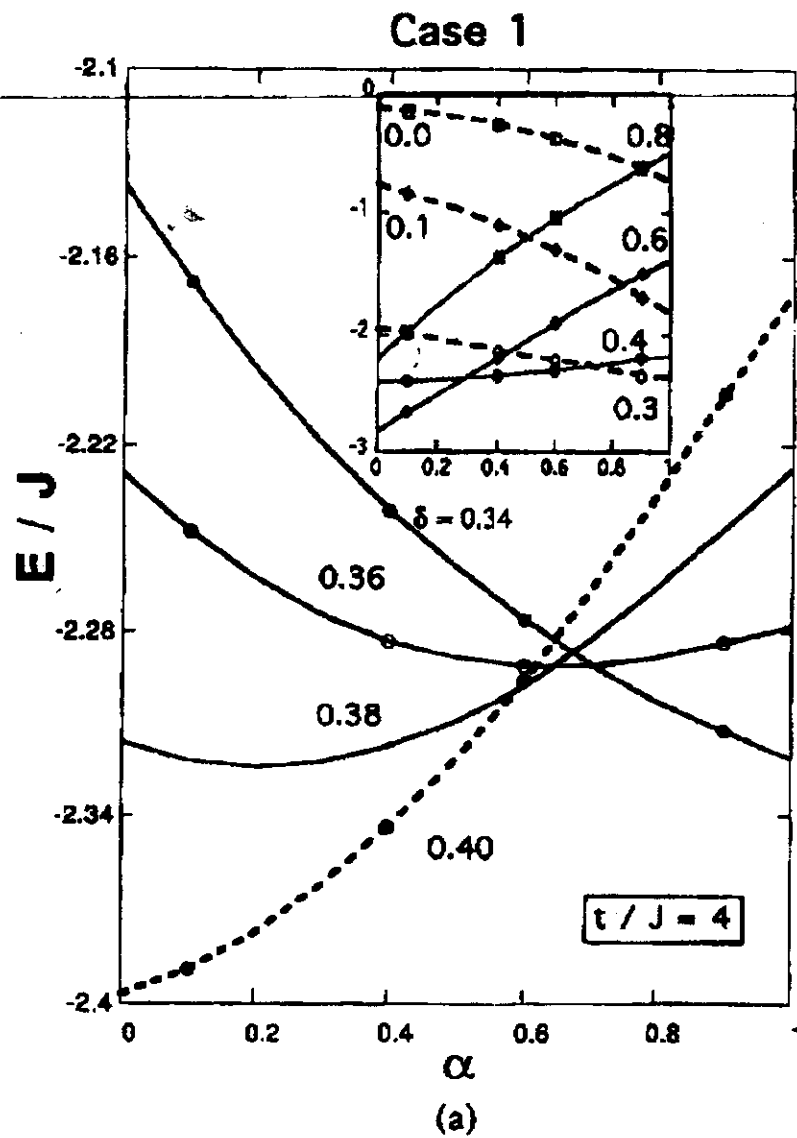


Fig. 4

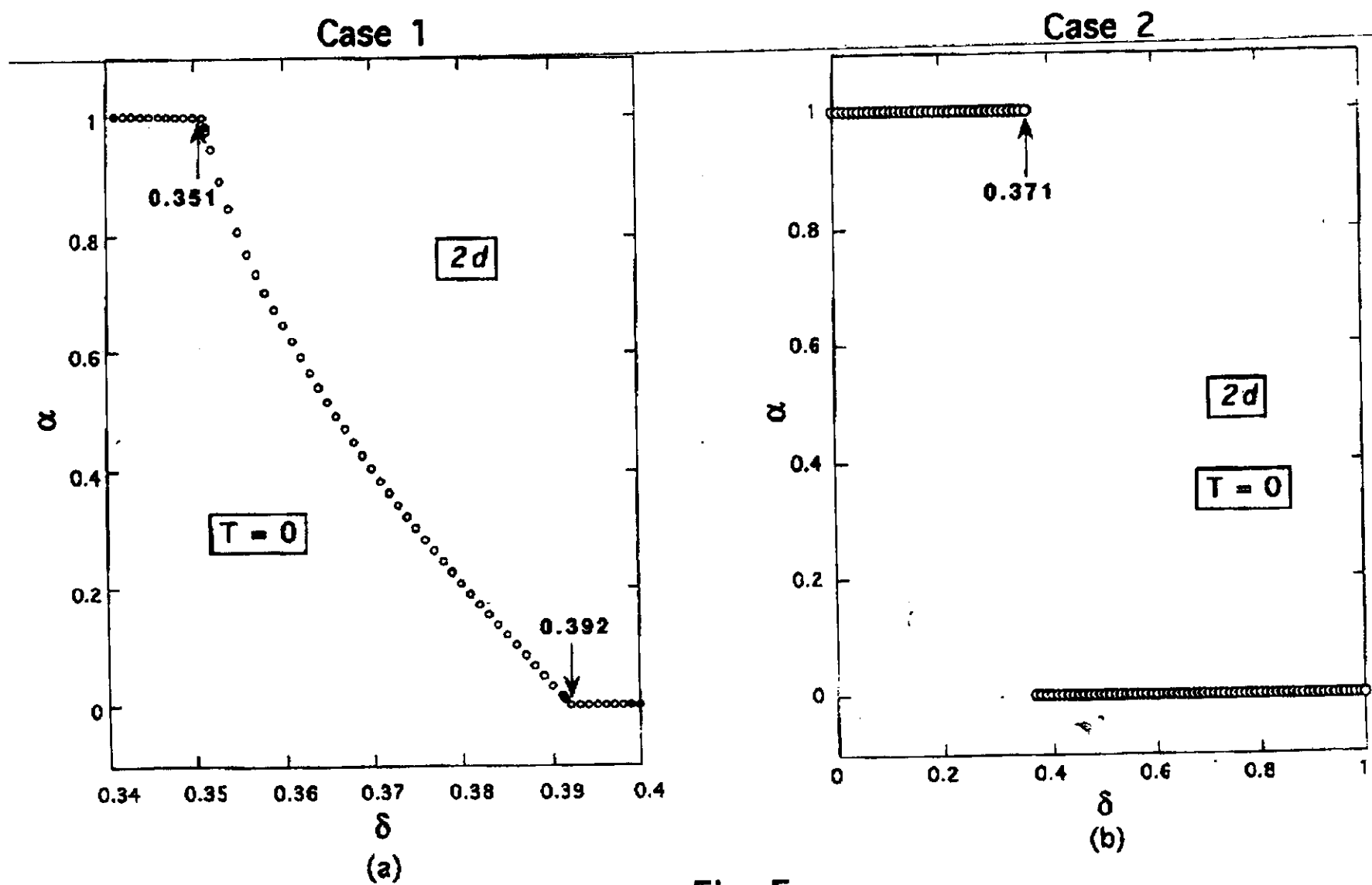


Fig. 5

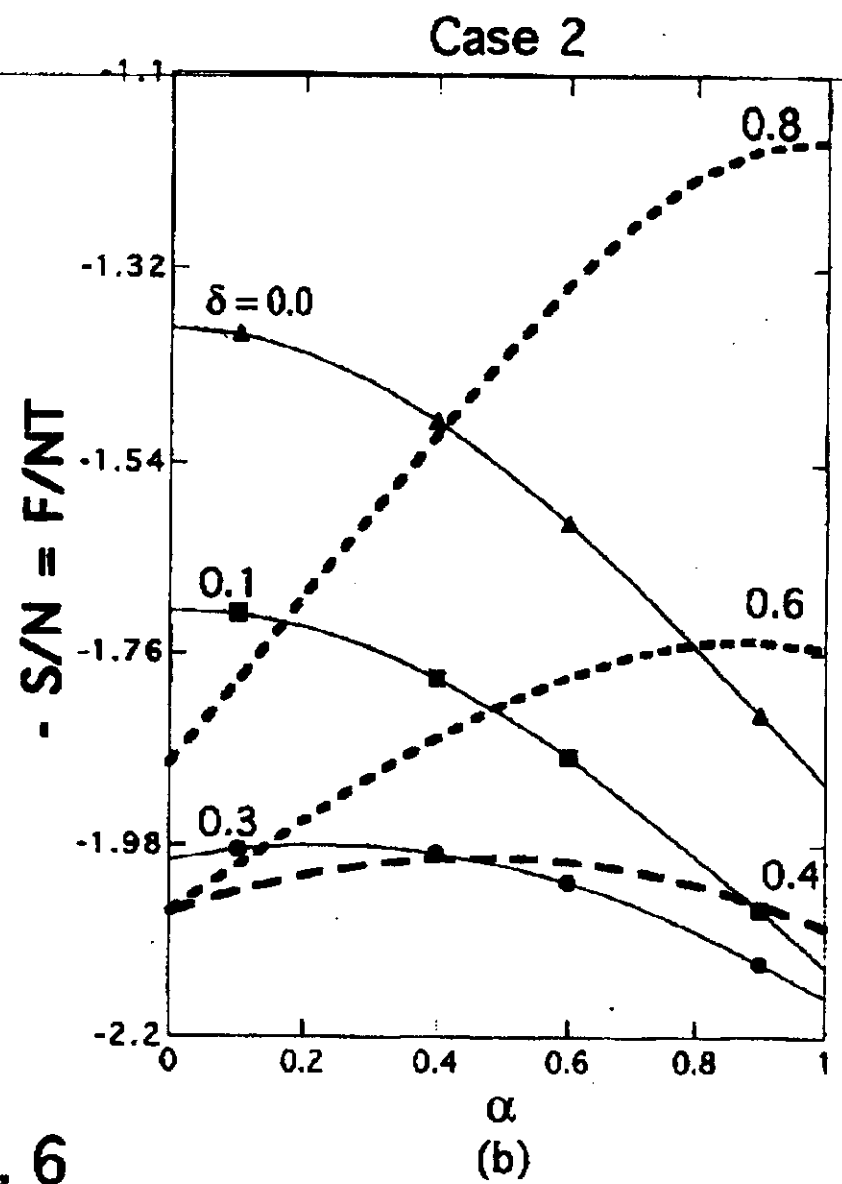
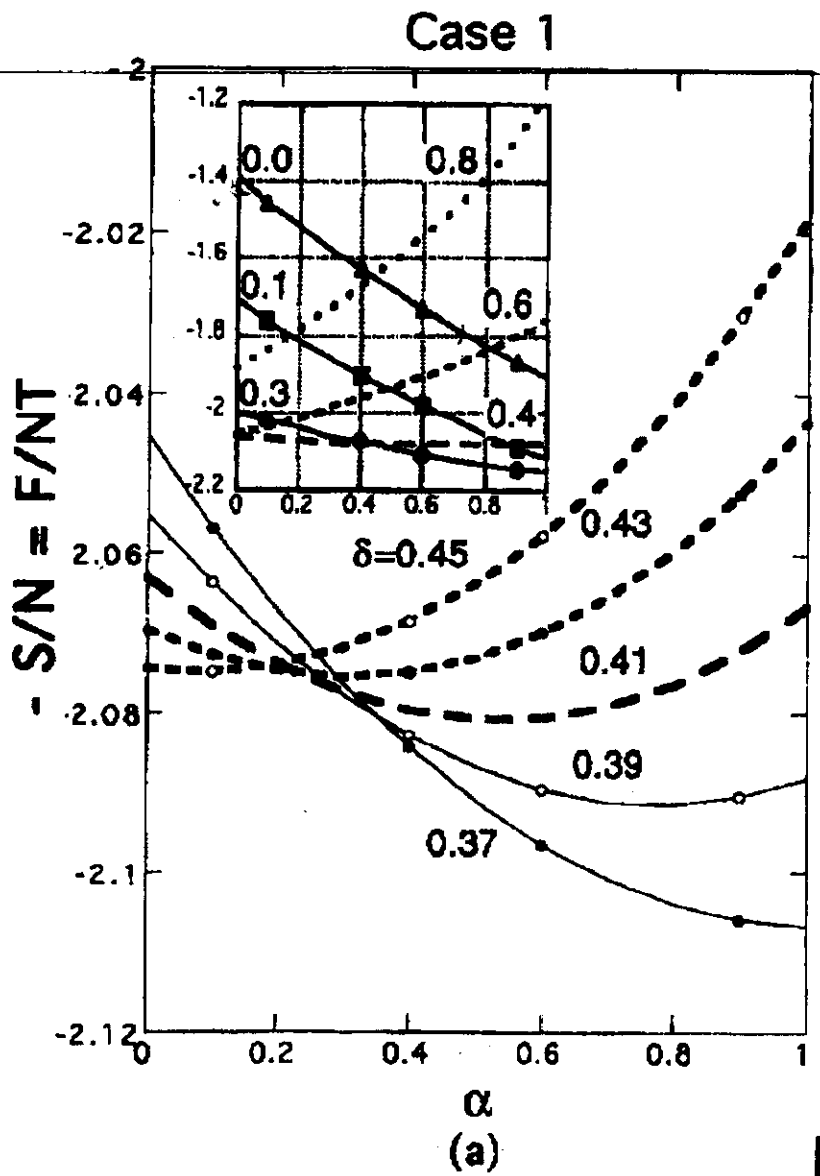


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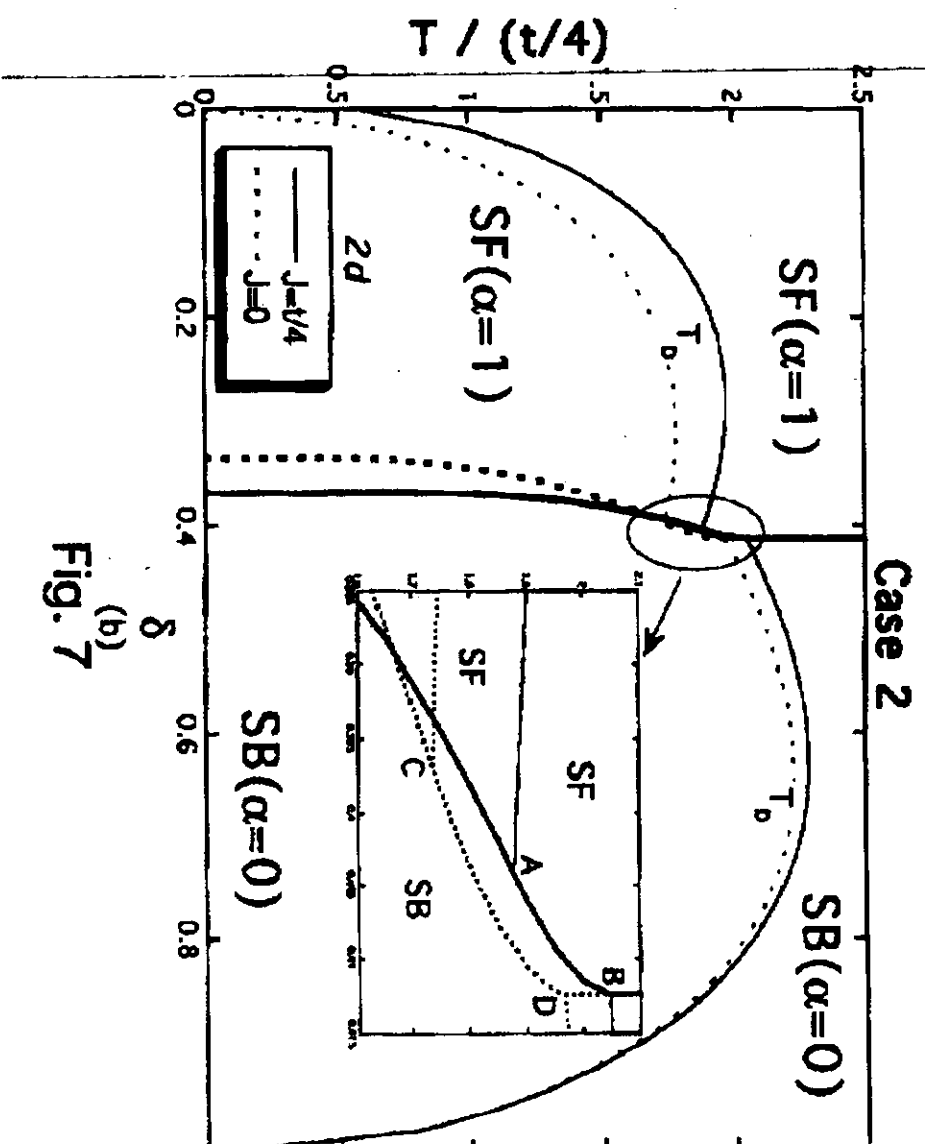
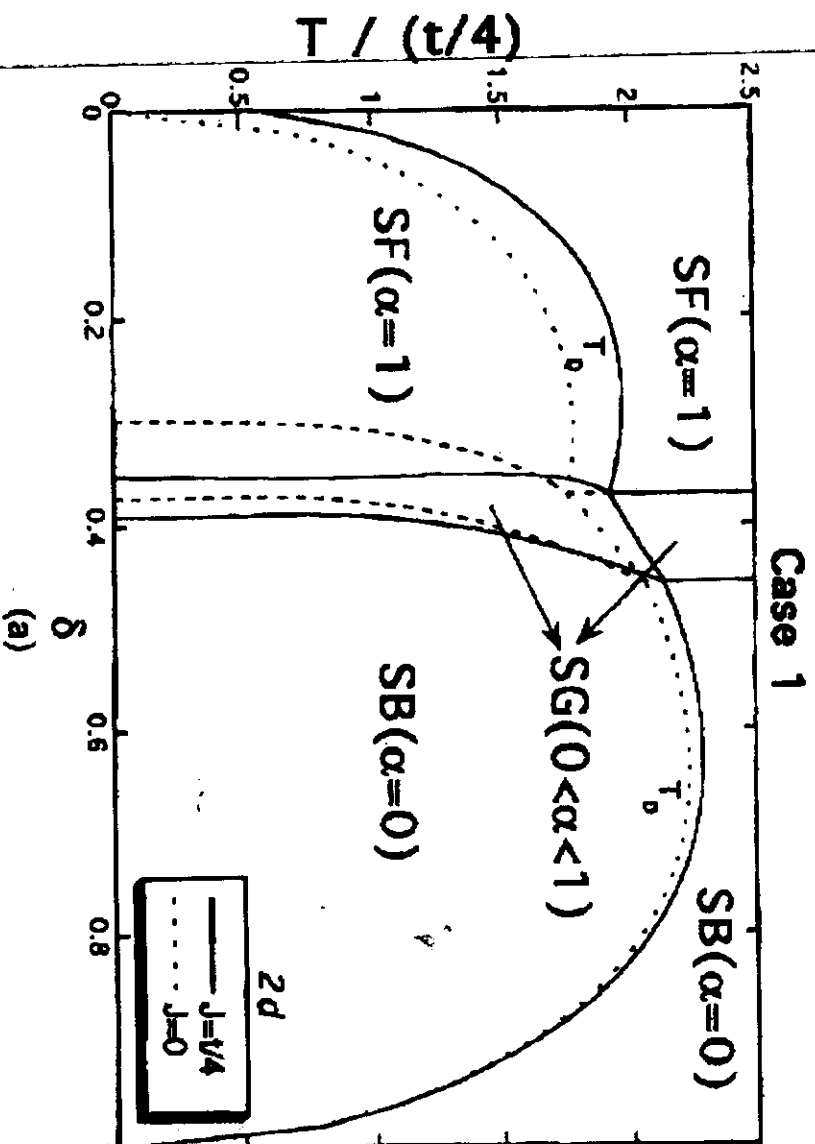


Fig. 7 (b)

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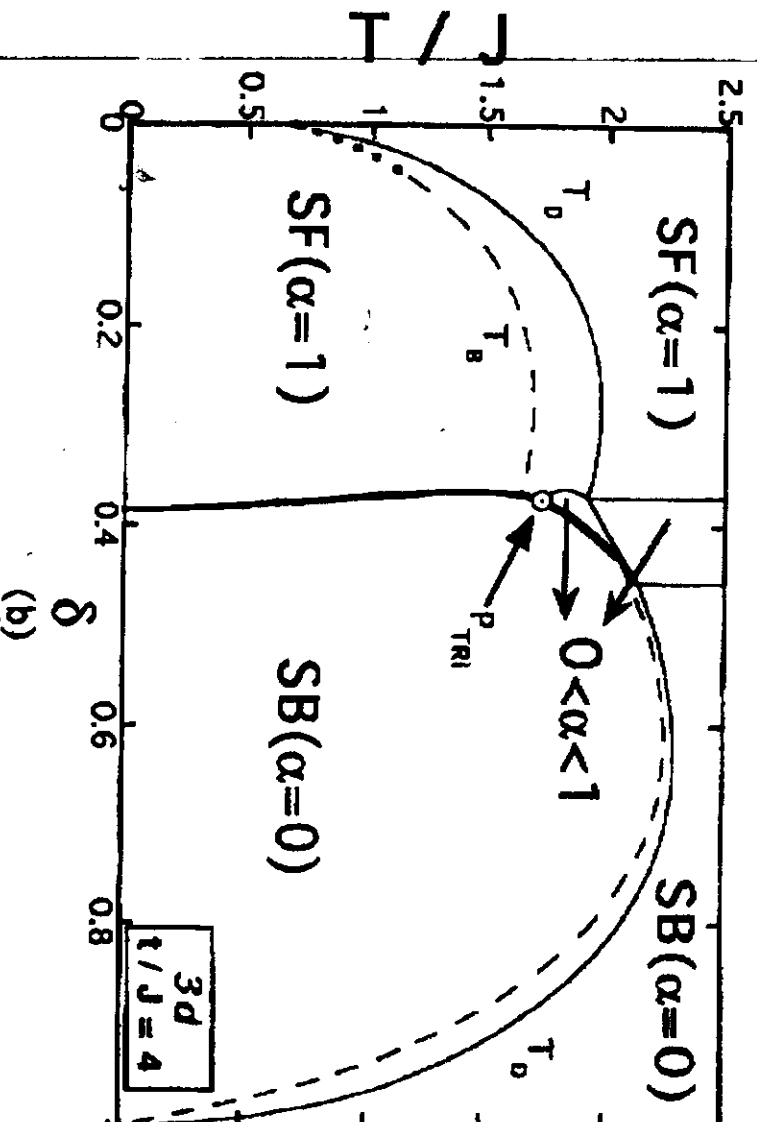
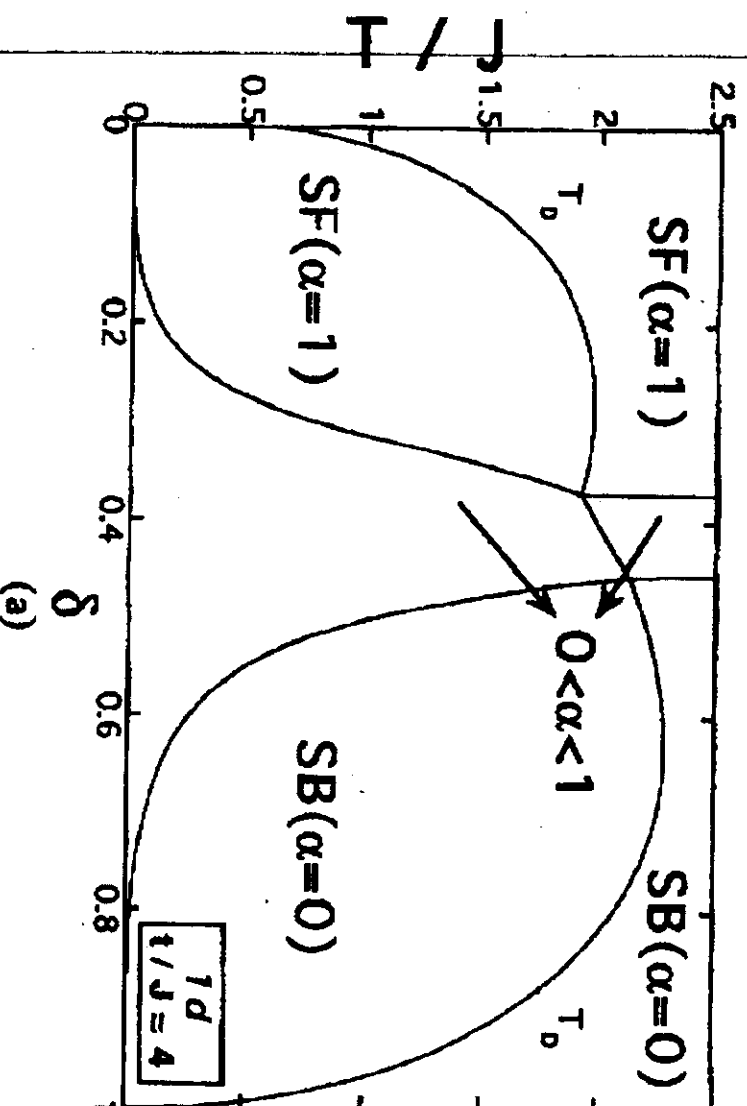
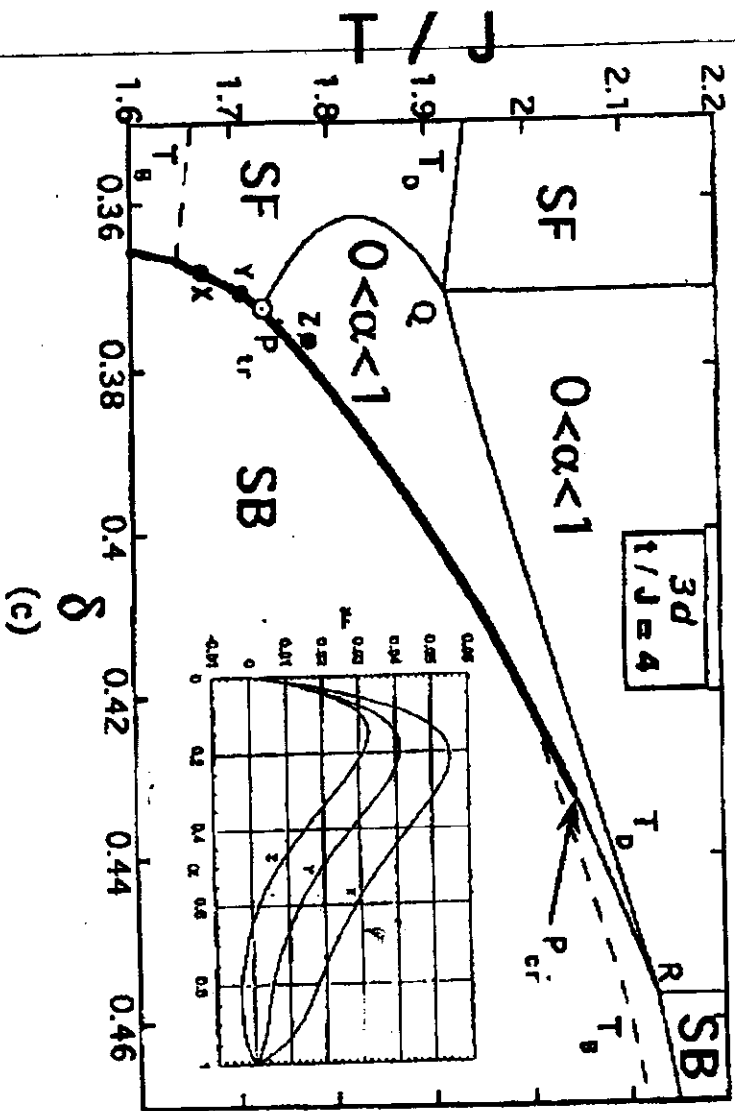
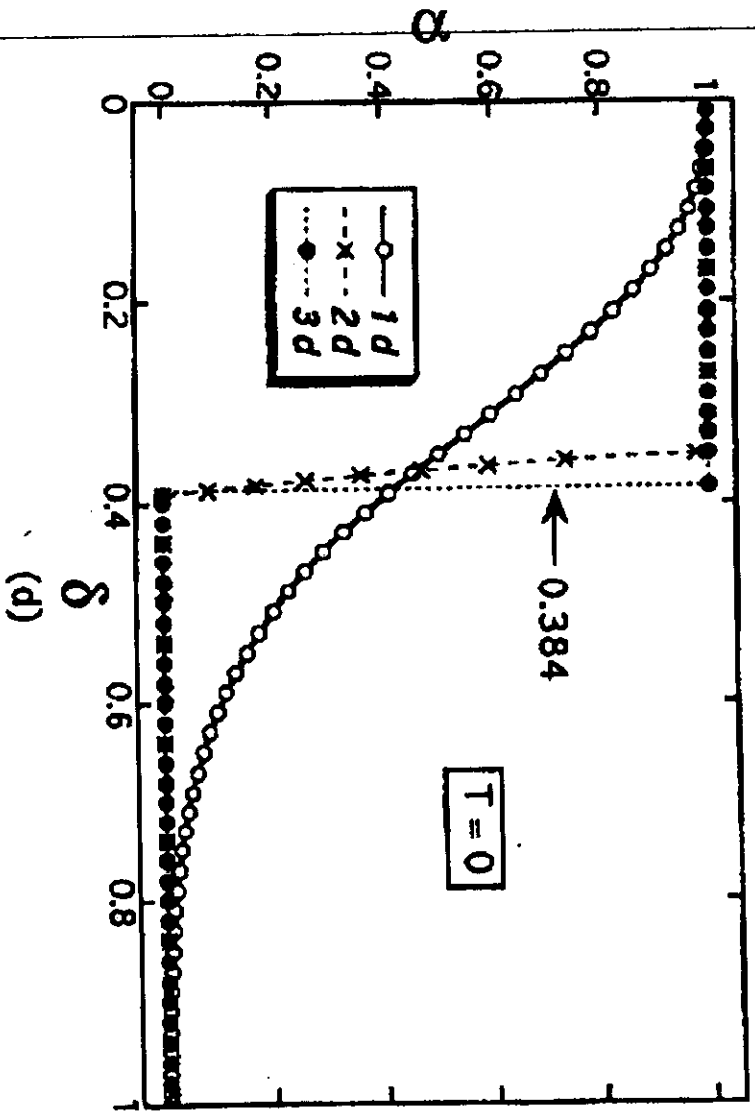


Fig. 8



(c)



(d)

Fig. 8

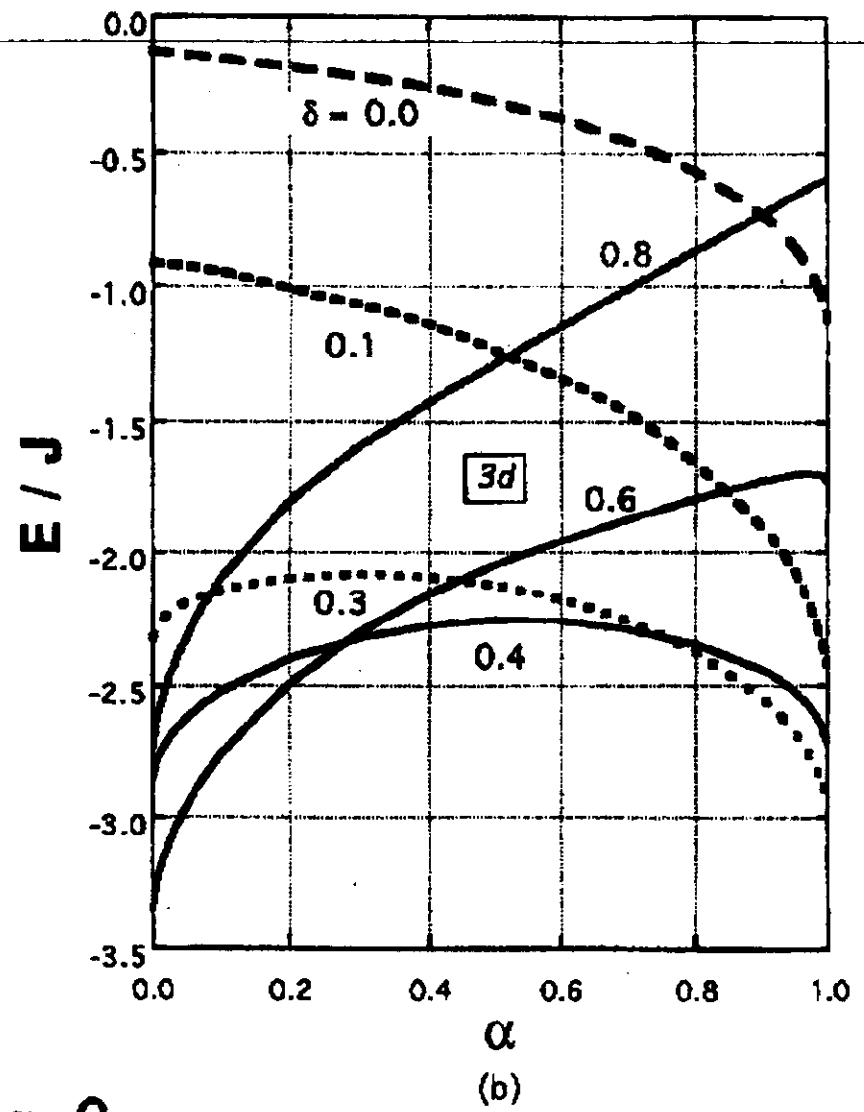
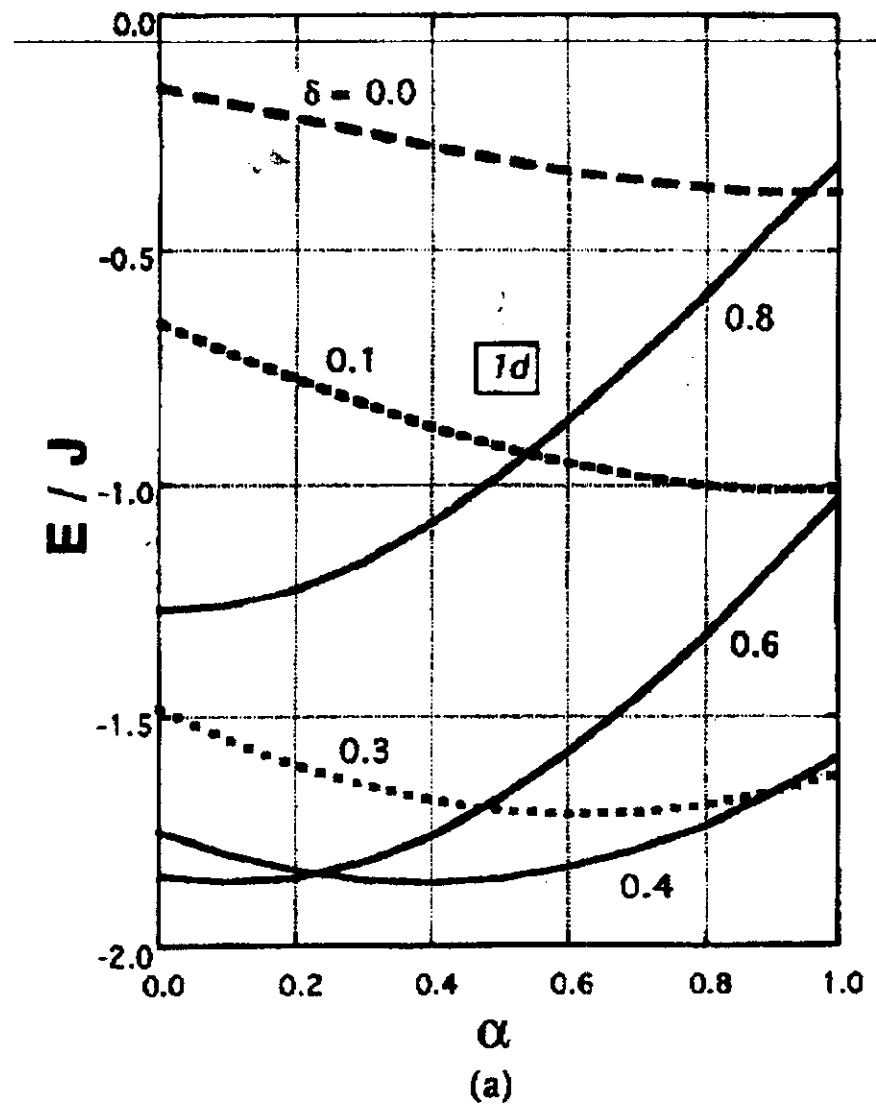
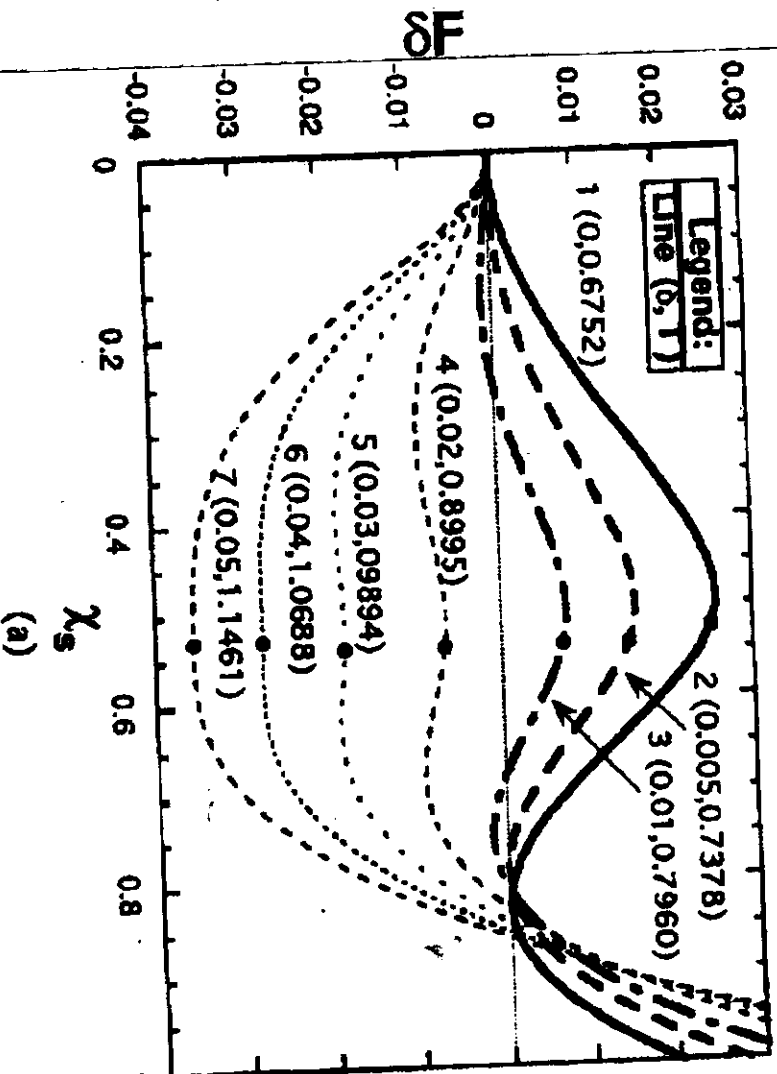
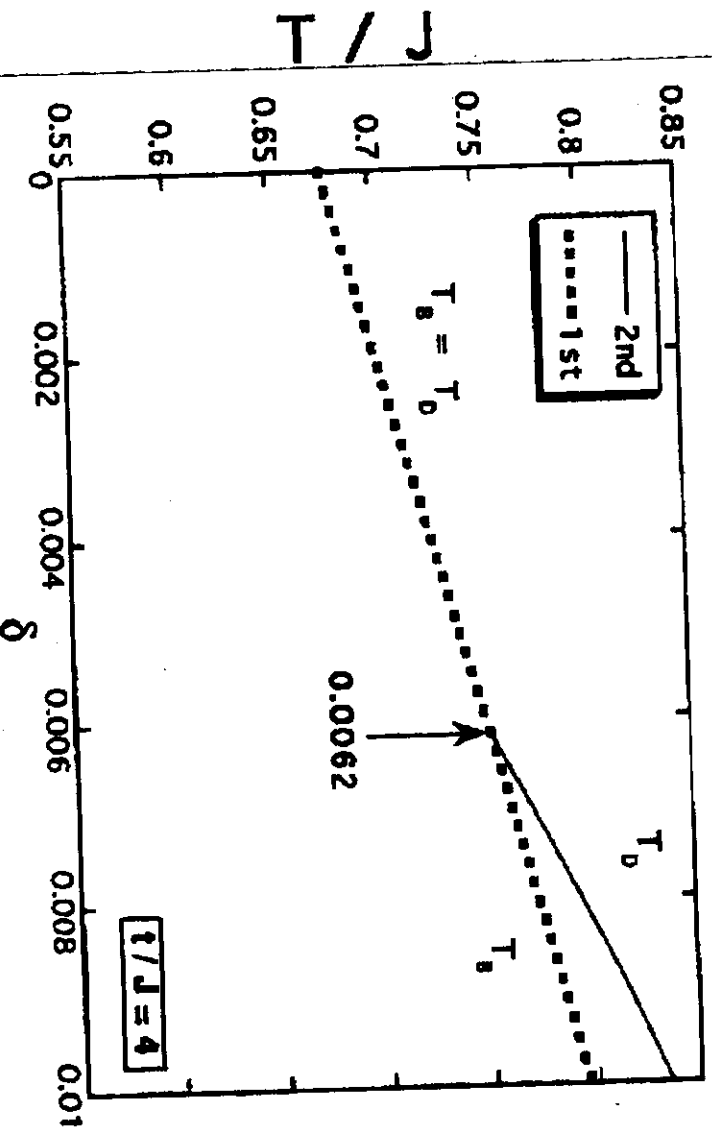


Fig. 9



(a)



(b)

Fig. 10