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MINIWORKSHOP ON STRONG ELECTRON CORRELATIONS
"Disorder and Interaction in Quantum Systems and Their Classical Analogs"

(1 - 19 July 1996)

"Superconductivity from doping a spin liquid (or also a magnetic) insulator: a simple 1D example"

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These are preliminary lecture notes, intended only for distribution to participants.

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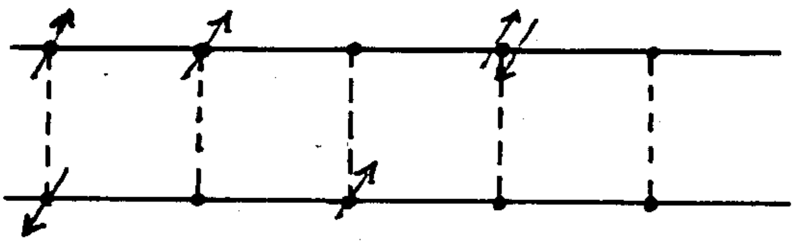
SUPERCONDUCTIVITY FROM DOPING A SPIN LIQUID (OR ALSO A MAGNETIC) INSULATOR : A SIMPLE 1D EXAMPLE

M.F. (SISSA - TRIESTE)

- MOTIVATION
- THE MODEL
- THE PHASE DIAGRAM
- CONCLUSIONS

• MOTIVATIONS •

• TWO LEG LADDER COMPOUNDS $(SrCu_2O_3,\ (VO)_2P_2O_7)$

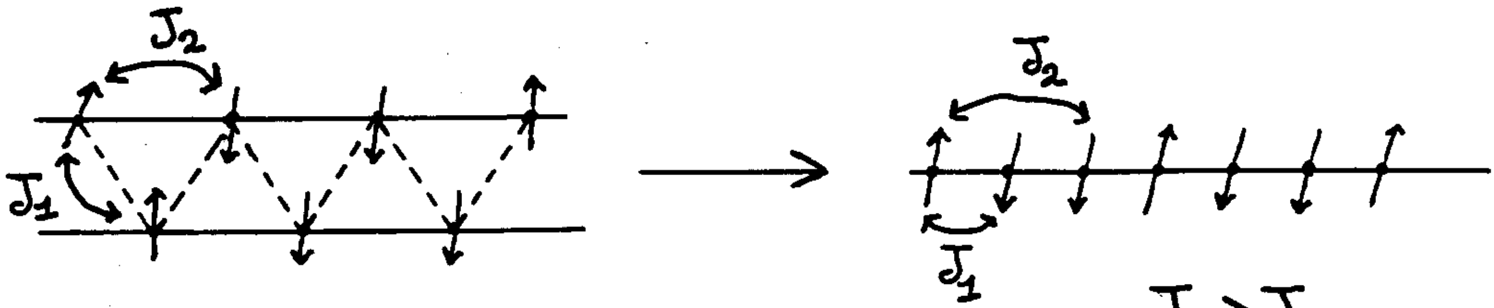


- SPIN LIQUID INSULATORS AT HALF FILLING
- CONDUCTING BEHAVIOR UPON DOPING ALTHOUGH THE SPIN GAP SEEMS TO PERSIST
- SUPERCONDUCTIVITY UNDER PRESSURE

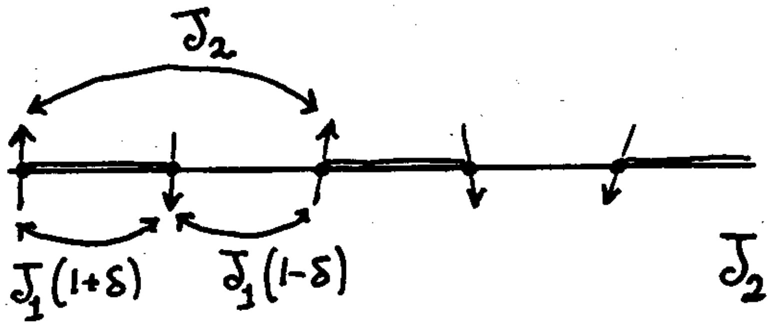
THE THEORETICAL ANALYSES SEEM TO SUGGES THAT

THE EXISTENCE OF A SPIN GAP IN THE INSULATING PHASE WHICH REMAINS FINITE UPON DOPING FAVORS SUPERCONDUCTIVITY IN SPITE OF REPULSIVE INTERACTION

- ZIG-ZAG CHAINS ($SrCuO_2$) AND SPIN-PEIERLS INORGANIC COMPOUNDS ($CuGeO_3$)



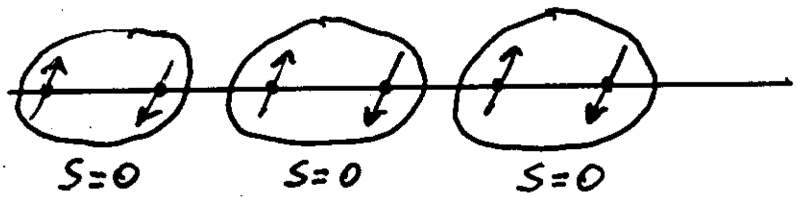
$J_2 > J_1$



$J_2 = 0.2 \div 0.24\, J_1$ $\delta \sim 0.03$

- THE $J_1 - J_2$ IS THE SIMPLEST SINGLE CHAIN MODEL BEING A SPIN-LIQUID INSULATOR AT HALF FILLING
  (FOR $J_2 \gtrsim 0.25\, J_1$)

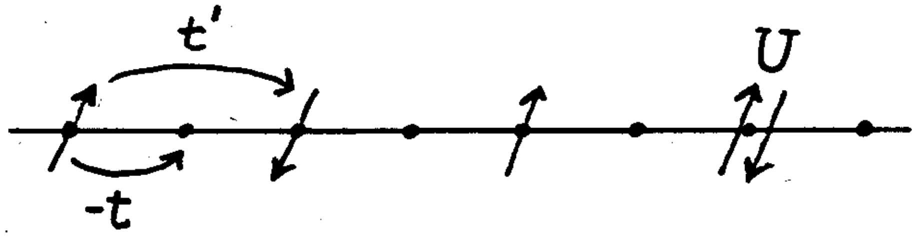
  - DIMERIZED STATE (EXACT AT $J_2 = J_1/2$) -



- WHAT HAPPENS UPON DOPING ?
- AND UNDER PRESSURE ?

- THE MODEL -

- I CONSIDER A SIMPLE ELECTRONIC MODEL WHICH MAPS ONTO THE J_1-J_2 SPIN MODEL AT HALF FILLING AND FOR STRONG ON-SITE REPULSION



IF $U \gg t, t'$ THIS MODEL INDEED MAPS AT HALF FILLING ONTO THE J_1-J_2 MODEL WITH

$$\frac{J_1}{J_2} = \left(\frac{t}{t'}\right)^2$$

- THIS MODEL CAN BE ANALYSED BY STANDARD ONE-DIMENSIONAL WEAK COUPLING TECHNIQUES

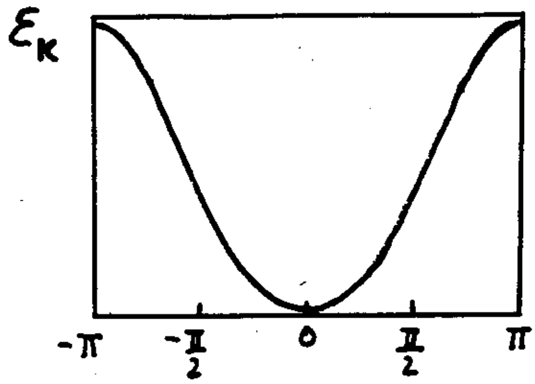
(g-OLOGY, BOSONIZATION, ...)

I WILL PRESENT THE PHASE DIAGRAM OF THIS MODEL FOR DENSITIES CLOSE TO HALF FILLING

- WEAK COUPLING ANALYSIS $(U \ll t, t')$ -

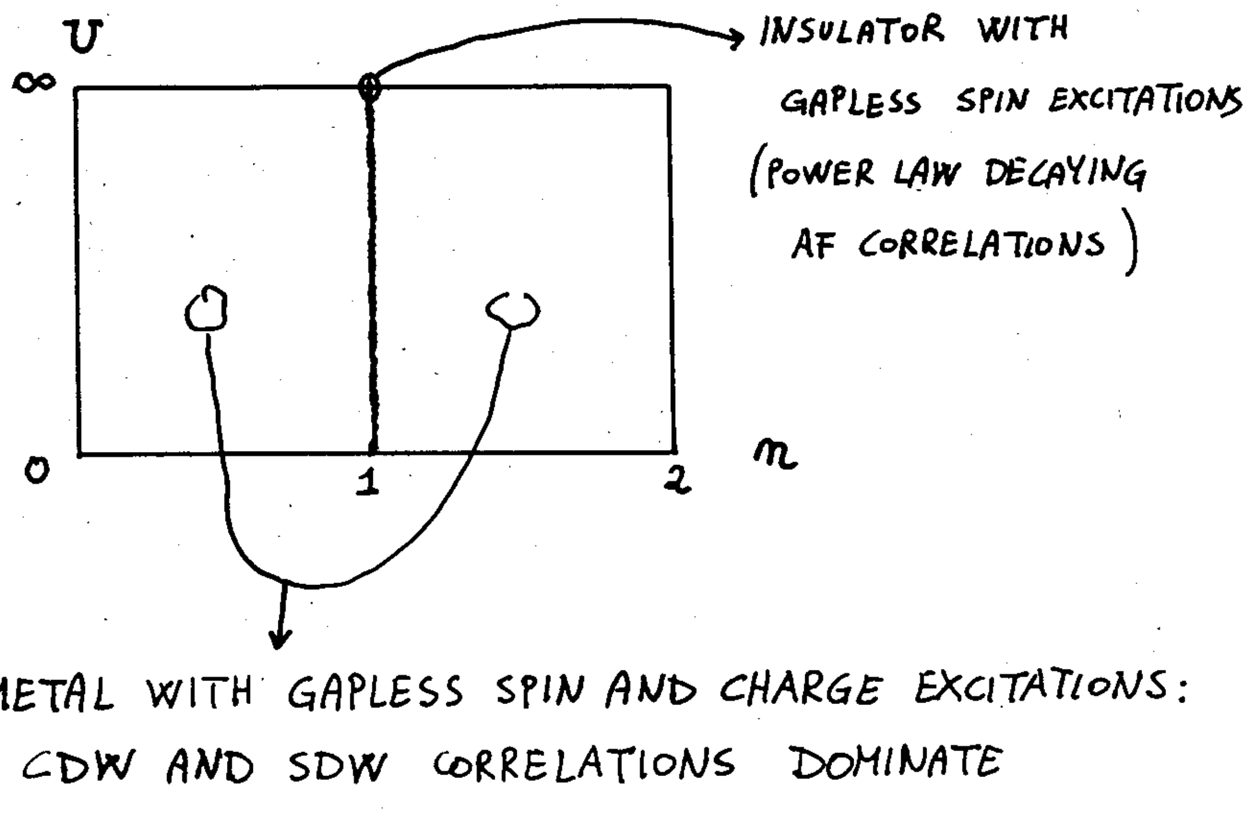
- ENERGY DISPERSION RELATION: $\varepsilon_K = -2t\cos K + 2t'\cos 2K$

- $t' < t/4$



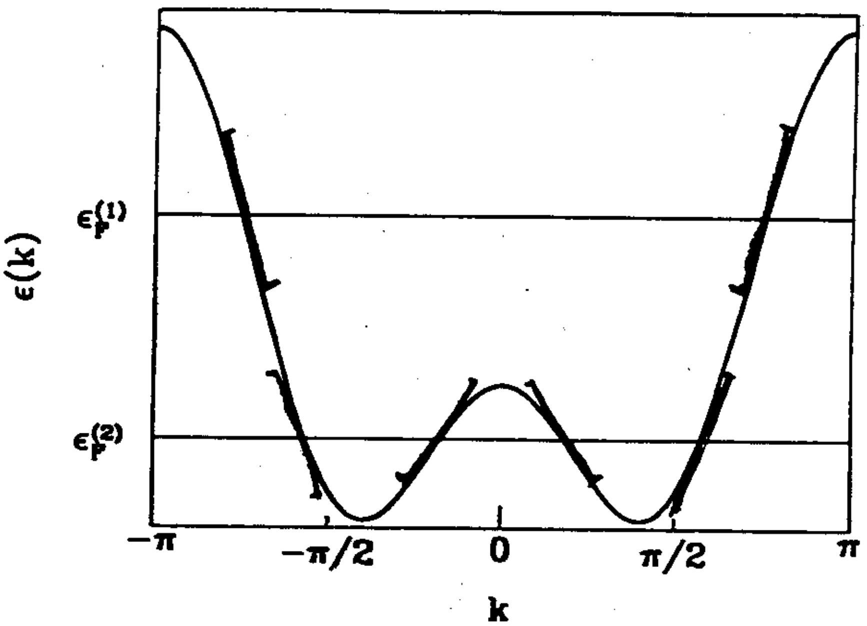
IT IS A SINGLE BAND MODEL, SIMILAR TO THE STANDARD HUBBARD MODEL

THEREFORE WE EXPECT THAT



- WEAK COUPLING ANALYSIS -

• $t' > t/4$



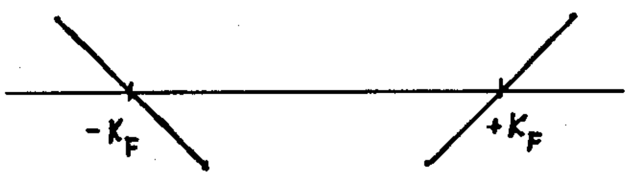
LINEARIZE THE BANDS CLOSE TO THE FERMI POINTS

CONSIDER ALL THE POSSIBLE COUPLINGS COMPATIBLE WITH SYMMETRIES

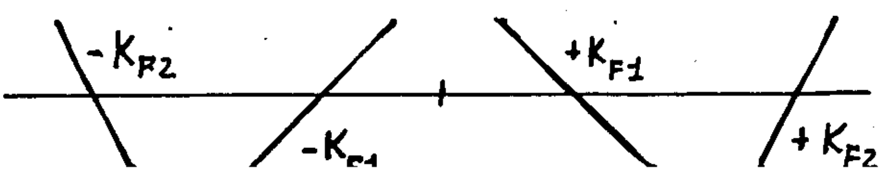
PERFORM A PERTURBATION EXPANSION + RG, TO CURE THE LOG-SINGULARITIES

• DEPENDING ON THE DENSITY AS WELL AS ON t/t' THE CHEMICAL POTENTIAL CAN CROSS TWICE $(\epsilon_F^{(1)})$ OR FOUR TIMES $(\epsilon_F^{(2)})$ THE BAND

• WHEN THE FERMI SURFACE IS MADE BY TWO POINTS THE STANDARD SINGLE BAND ANALYSES CAN BE PERFORMED

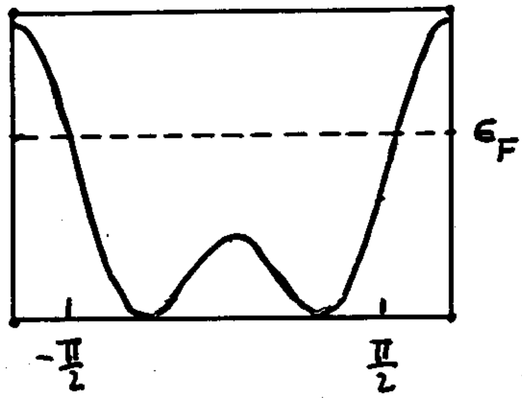


• WHEN FOUR FERMI POINTS ARE INVOLVED, A MORE COMPLICATED TWO BAND MODEL ANALYSIS IS NEEDED



– HALF FILLING $t' > t/4$ –

- $t/4 < t' < t/2$



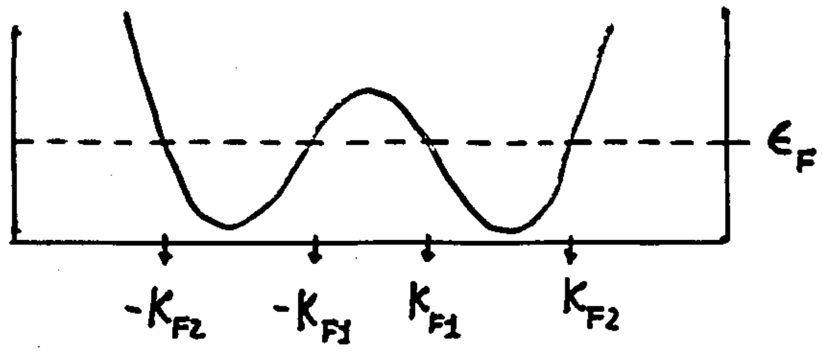
AT LOW ENERGY IS LIKE A SINGLE BAND HUBBARD MODEL AT HALF-FILLING, I.E. IT IS AN INSULATOR WITH GAPLESS SPIN EXCITATIONS

$$\langle S^i(x) S^i(0) \rangle \simeq \frac{1}{4\pi^2} \frac{1}{x^2} + \text{Const.} \frac{(-1)^x}{x}$$

WE FIND THEREFORE A MAGNETIC INSULATOR FOR

$$0 < t' < t/2$$

- HALF FILLING $t' > t/2$ -



- THE FERMI POINTS SATISFY THE RELATION

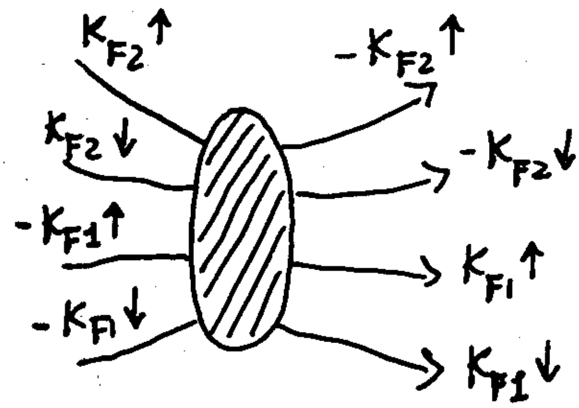
$$4K_{F2} - 4K_{F1} = 2\pi$$

AS A CONSEQUENCE THERE IS NO SIMPLE UMKLAPP

$\Longrightarrow \quad P = -P + 2\pi$

→ DON'T EXIST AT THE FERMI SURFACE

BUT THERE ARE HIGHER ORDER UMKLAPP



- THE CONSEQUENCE OF THE PRESENCE OF ONLY HIGHER ORDER UMKLAPP PROCESSES IS THAT

A CHARGE GAP, I.E. AN INSULATING BEHAVIOR, OCCURS ONLY IF THE INTERACTION IS BIGGER THAN A CRITICAL VALUE

$$\Downarrow$$

"A METAL-INSULATOR TRANSITION OCCURS AT HALF-FILLING AS U INCREASES, WHEN $t' > t/2$"

$$\Downarrow$$

IF THE ZIG-ZAG CHAINS ARE NOT FAR FROM THIS POINT, IT COULD BE POSSIBLE TO INDUCE THE INSULATOR-TO-METAL TRANSITION UNDER PRESSURE

WHAT ARE THE PROPERTIES OF THE INSULATING AND OF THE METALLIC PHASES ?

- PROPERTIES OF THE INSULATOR -

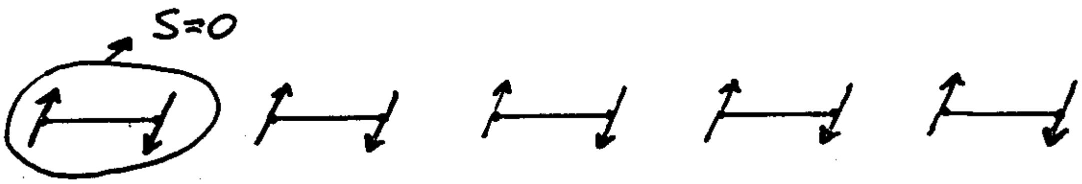
• DIMER ORDER PARAMETER:

$$O_D(x) = \vec{S}(x)\cdot\vec{S}(x+a) - \vec{S}(x)\cdot\vec{S}(x-a)$$

AS A CONSEQUENCE OF THE RELEVANT UMKLAPP

$$\langle O_D(x)\rangle = \text{CONST.}\cdot(-1)^x$$

THE INSULATING PHASE FOR $t' > t/2$ CORRESPONDS TO A DIMERIZED PHASE



WITH A GAP ALSO IN THE SPIN EXCITATION SPECTRUM

THE TRANSITION AT LARGE U FROM THE MAGNETIC TO THE SPIN LIQUID INSULATOR OCCURS WITHIN THIS APPROACH AT $t' = t/2$, I.E. AT

$$J_2/J_1 = (t'/t)^2 = 1/4 \quad !!$$

↓

CLOSE TO THE NUMERICAL RESULT!
(AS WELL AS TO THE CLASSICAL RESULT!)

– PROPERTIES OF THE METAL –

- BELOW A CRITICAL U_c, THE UMKLAPP IS IRRELEVANT AND THE DIMER ORDER PARAMETER

$$O_D(x) = \vec{S}(x)\cdot\vec{S}(x+a) - \vec{S}(x)\cdot\vec{S}(x-a)$$

HAS NOMORE A FINITE AVERAGE $\propto (-1)^x$ OVER THE GROUND STATE.

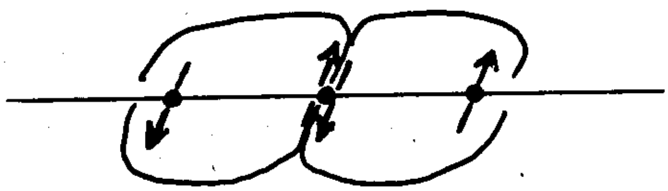
- HOWEVER THE DIMER-DIMER CORRELATION FUNCTION SHOWS AN ASYMPTOTIC POWER LAW DECAY

$$\chi_{DW}(x) = \langle O_D(x)\, O_D^{+}(0) \rangle \simeq$$

$$\simeq \frac{\cos\left[2(k_{F2}-k_{F1})x\right]}{x^{2k_\rho}} = \frac{\cos(\pi x)}{x^{2K_\rho}}$$

- THE SPIN GAP IS STILL FINITE : THE ELECTRONS ARE PAIRED INTO SINGLETS, AS χ_{DW} SHOWS

HOWEVER SINCE U IS NOT EXTREMELY LARGE, TWO DIMERS CAN OVERLAP



THEREFORE THE DIMERS CAN MOVE $\Rightarrow$ NO CHARGE GAP

• PROPERTIES OF THE METAL •

- THE SPIN GAP ALSO SUGGESTS THE EXISTENCE OF RELEVANT PAIRING FLUCTUATIONS

  - PAIRING ORDER PARAMETER

$$\Delta(x) = \sum_{\sigma} \sigma \Psi_{K_{F2}\sigma}(x) \Psi_{-K_{F2}-\sigma}(x) - \sigma \Psi_{K_{F1}\sigma}(x) \Psi_{-K_{F1}-\sigma}(x)$$

$\downarrow$

COMPATIBLE WITH REPULSIVE INTERACTION

$$\chi_{SC}(x) = \langle \Delta(x) \Delta^{+}(0) \rangle \simeq \frac{1}{x^{1/2K_{\rho}}}$$

- K_{ρ} IS A LUTTINGER LIQUID PARAMETER PROPORTIONAL TO THE CHARGE COMPRESSIBILITY

(Δ^{+} CREATES A PAIR, WHILE O_D IS IN SOME SENSE THE PAIR DENSITY OPERATOR)

- IF $K_{\rho} > 1/2$ THE PAIRING CORRELATION FUNCTION DECAYS SLOWER : SUPERCONDUCTIVITY DOMINATES

- IF $K_{\rho} < 1/2$ THE DIMER-DIMER CORRELATION DECAYS SLOWER : DENSITY WAVES DOMINATE.

- NOTICE THAT THE UMKLAPP TERM AT HALF FILLING IS RELEVANT JUST WHEN $K_\rho < 1/2$
- MOREOVER K_ρ LIKE THE COMPRESSIBILITY IS A DECREASING FUNCTION OF THE INTERACTION, AND FOR SMALL U

$$K_\rho \simeq 1 + \mathcal{O}(U)$$

- ON THE OTHER HAND WE KNOW THAT FOR $U \gg t, t'$ THE MODEL IS INSULATING

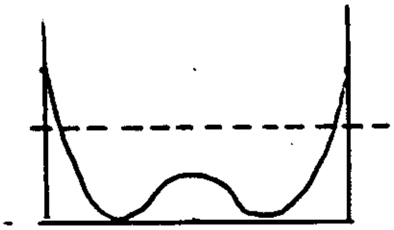
THEREFORE

THIS MODEL AT HALF FILLING HAS A DIRECT TRANSITION FROM A METAL WITH DOMINANT SUPERCONDUCTING INSTABILITY TO A SPIN LIQUID INSULATOR

– AWAY FROM HALF FILLING –

- THE UMKLAPP SCATTERING IS NOT ANYMORE RELEVANT

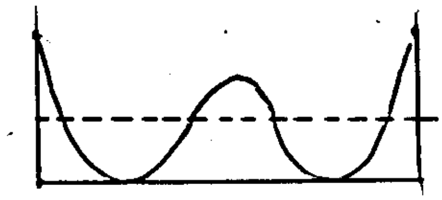
- IF



I.E., IF THE FERMI SURFACE IS MADE BY TWO POINTS, THE SITUATION IS LIKE THAT OF THE HUBBARD MODEL:

A METAL WITH GAPLESS CHARGE AND SPIN MODES, AND DOMINANT CDW AND SDW FLUCTUATIONS

- IF

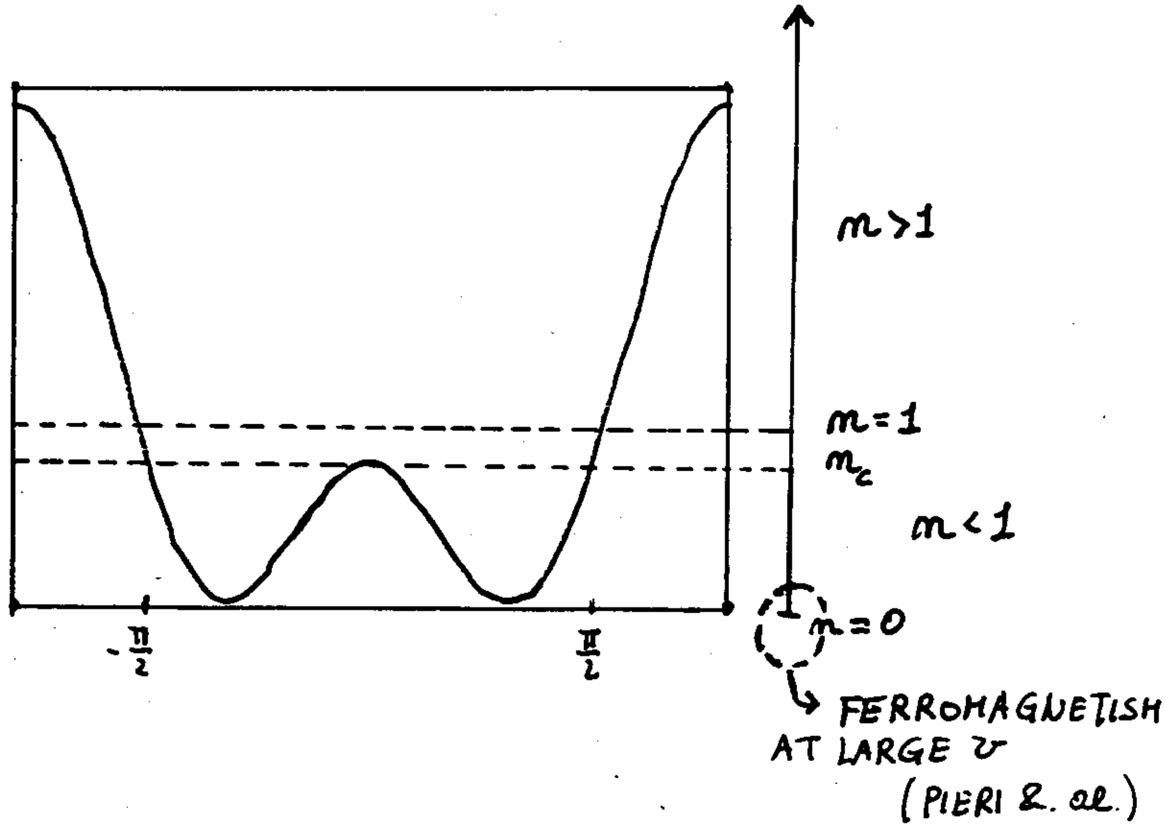


I.E. FOUR FERMI POINTS ARE INVOLVED, THEN THE METAL HAS THE SAME PROPERTIES OF THAT WE FOUND AT HALF FILLING, THAT IS

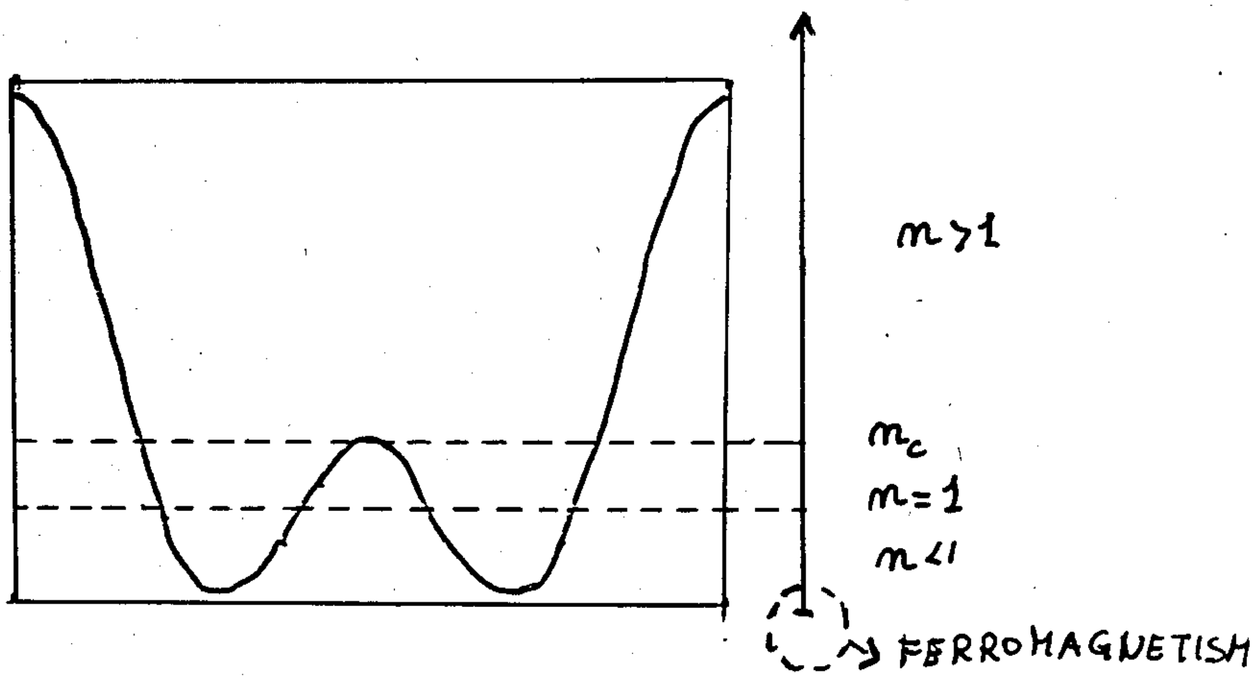
A METAL WITH A SPIN GAP AND DOMINANT DIMER-DIMER (LARGE U, OR SPECIFICALLY $K_\varsigma < 1/2$) OR SUPERCONDUCTING FLUCTUATIONS (SMALL U, OR $K_\varsigma > 1/2$)

• TWO BEHAVIORS, DEPENDING WHETHER AT HALF FILLING TWO OR FOUR FERMI POINTS ARE INVOLVED

$\frac{t}{4} < t' < \frac{t}{2}$



$t' > t/2$



CONCLUSIONS

- THE t-t'-U MODEL MIGHT BE RELEVANT FOR THE ZIG-ZAG CHAIN COMPOUNDS

- ANYWAY IT HAS QUITE INTERESTING FEATURES INCLUDING

  A SUPERCONDUCTOR-INSULATOR TRANSITION AT HALF FILLING FOR $t' > t/2$ AS A FUNCTION OF U

  A SPIN GAP REGION WHICH APPEARS UPON DOPING THE MAGNETIC (SPIN GAPLESS) INSULATOR. THIS REGION MAY SHOW EITHER DIMER WAVE OR SUPERCONDUCTING DOMINANT CORRELATIONS

- FROM OUR WEAK COUPLING ANALYSIS IT TURNS OUT THAT THE EXISTENCE OF MORE THAN TWO FERMI POINTS SEEMS TO BE AN ESSENTIAL INGREDIENT FOR THE OCCURRENCE OF METALLIC BEHAVIOR IN THE PRESENCE OF SPIN GAPS (WITH OR WITHOUT DOMINANT SUPERCONDUCTIVITY)
  [THE SAME OCCURS FOR THE TWO LEG LADDER]