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WORKSHOP ON QUANTUM DISSIPATIQN AND APPLICATIONS

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# "*Quantum Dissipation, An Overview*"

presented by:

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# QUANTUM DISSIPATION and APPLICATIONS

a) The fundamental problem and the experimental motivation

b) The phenomenological approach and the problems of interest

c) Experimental realizations and their consequences.

d) Microscopic models: motion of particles and topological defects

e) Comments and conclusions.

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## a) The fundamental problem

The problem we are going to deal with, is the quantum mechanics of a "particle" that, in the classical limit, obeys the Langevin equation

$m\ddot{q} + \eta\dot{q} + V'(q) = f(t)$ where $\langle f(t)\rangle = 0$

and $\langle f(t) f(t')\rangle = 2\eta kT\,\delta(t-t')$

The quantum limit of this dynamics has always presented a challenge because the Langevin equation cannot be derived from any $H(q,p)$ or $L(q,\dot{q})$

During many years, several alternative schemes of quantization have been proposed such as:

i) $L(q,\dot{q},t)$ or $H(q,p,t)$ $\rightarrow$ problems with the uncertainty principle.

ii) Non-linear Schrödinger equations problems with the superposition principle.

Speculative explanations and lack of examples where these effects could take place, except for corrections $O(\hbar)$ in the transport coefficients. For example, where to study dissipative tunnelling?

The main goal is to study the quantum limit of the Langevin equation

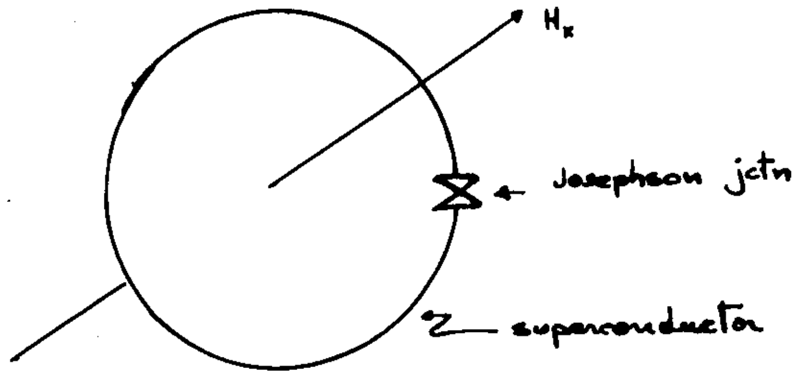
$$m\ddot{q} + \eta\dot{q} + V'(q) = f(t)$$

for general potentials.

i) Where to find the appropriate system?

ii) How to treat the specific problem?

b1) SQUIDs (superconducting quantum interference devices)

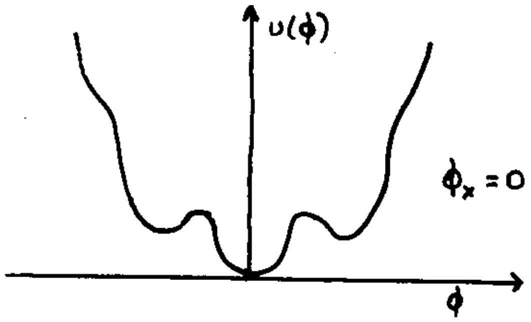


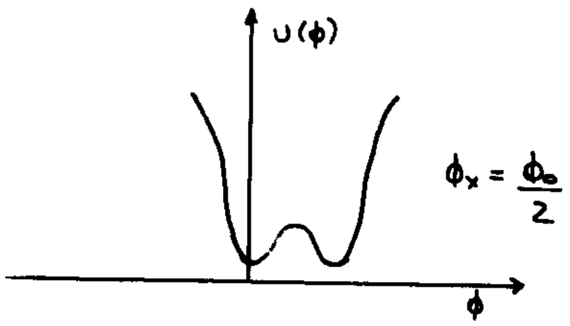

The equation of motion for the total flux $\phi$ (external field + currents) is given by

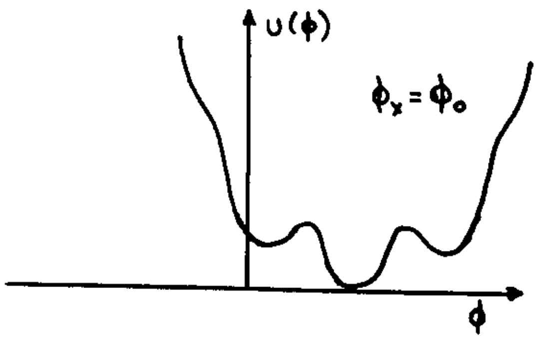
$$C\ddot{\phi} + \frac{1}{R}\dot{\phi} + \frac{dU}{d\phi} = 0 \qquad \text{where}$$

$$U(\phi) = \frac{(\phi - \phi_x)^2}{2L} - \frac{i_c \phi_0}{2\pi}\cos\frac{2\pi\phi}{\phi_0} \qquad \text{and}$$

$C$ - capacitance of the junction
$R$ - resistance in the normal state
$i_c$ - critical current
$\phi_x$ - external flux
$\phi_0$ - flux quantum




$$U(\phi) = \frac{(\phi - \phi_x)^2}{2L} - \frac{\phi_0 i_c}{2\pi} \cos \frac{2\pi\phi}{\phi_0}$$

if $2\pi \frac{L i_c}{\phi_0} > 1 \Rightarrow$

$\Rightarrow$ several minima

if $2\pi \frac{L i_c}{\phi_0} \leq 1 \Rightarrow$

$\Rightarrow$ only one minimum

In a SQUID:

$L \sim 10^{-10}$ H

$C \lesssim 10^{-12}$ F

$i_c \sim 10^{-5}$ A

$\Downarrow$

$$\omega^2 \sim \frac{1}{LC} \sim 10^{22}\ s^{-2}$$

$$\Rightarrow \frac{\hbar\omega}{k} \sim 1^\circ K$$

$\Downarrow$

$T \lesssim 1^\circ K$

quantum effects of the electromagnetic field

$$\phi = \oint \vec{A} \cdot d\vec{l}$$

b) The phenomenological approach

Dissipative systems are such that

$$H = H_S + H_{int} + H_R$$

S is NOT an isolated system

TWO CHOICES:

i) Simple model for $H_R$ obeying the classical constraint

ii) Realistic model for $H_R$ (classical result?)

Let's adopt (i)

$$H = H_S + H_{int} + H_R + H_{CT}$$

$$H_S = \frac{p^2}{2m} + V(q) \qquad H_{int} = \sum_k C_k q q_k$$

$$H_R = \sum_k \frac{p_k^2}{2m_k} + \frac{1}{2} m_k \omega_k^2 q_k^2 \qquad \text{and}$$

$$H_{CT} = -\sum_k \frac{C_k^2 q^2}{2 m_k \omega_k^2}$$

we still have to define

$$J(\omega) \equiv \sum_k \frac{\pi C_k^2}{2 m_k \omega_k} \, \delta(\omega - \omega_k)$$

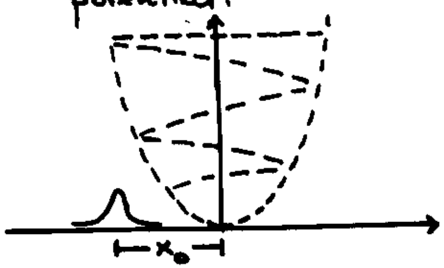
In the case of ohmic dissipation $(\eta \dot{q})$

$$J(\omega) = \begin{cases} \eta \omega & \text{if } \omega < \Omega \\ 0 & \text{if } \omega > \Omega \end{cases}$$

Strategy: trace out the variables of R.
$\Downarrow$
effective dynamics depends only on $\eta$

Some problems of interest

i) Motion of a wave packet in a harmonic potential.



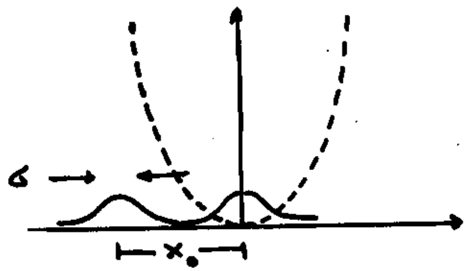
$$m \langle \ddot{x}(t) \rangle + \eta \langle \dot{x}(t) \rangle + m \omega_0^2 \langle x(t) \rangle = 0$$

and

$$\langle x^2(t) \rangle = \frac{\hbar}{\pi} \int_0^\infty \chi''(\omega) \coth \frac{\hbar \omega}{2kT} \cos \omega t \, d\omega$$

$$\chi''(\omega) = \frac{1}{m} \frac{2 \gamma \omega}{(\omega^2 - \omega_0^2)^2 + 4 \gamma^2 \omega^2} \qquad \gamma \equiv \frac{\eta}{2m}$$

ii) Interference of wave packets in a harmonic potential.



$$\psi(x,t) = \psi_0(x,t) + \psi_1(x,t)$$

$$\rho(x,t) = \psi^*(x,t) \psi(x,t)$$

a) $\gamma = 0$ ; $\rho(x,t) = \rho_1(x,t) + \rho_2(x,t) + 2\sqrt{\rho_1(x,t)\rho_2(x,t)} \cos \varphi(x,t)$

b) $\gamma \neq 0$ ; $\tilde{\rho}(x,t) = \tilde{\rho}_1(x,t) + \tilde{\rho}_2(x,t) + 2\sqrt{\tilde{\rho}_1(x,t)\tilde{\rho}_2(x,t)} \cos \tilde{\varphi}(x,t) \, e^{-\Gamma t}$

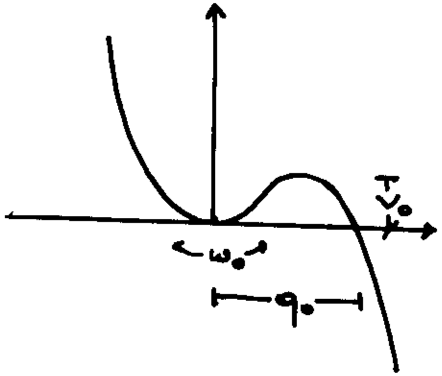
Term of interference 

center of $\tilde{\rho}_1(x,t) \sim e^{-\gamma t}$ but

$$\Gamma = \begin{cases} N \gamma & \text{if } T = 0 \\ \dfrac{2 N \gamma k T}{\hbar \omega_0} & \text{if } kT \gg \hbar \omega_0 \end{cases}$$

$$N = \frac{x_0}{\sigma}$$

## c) Decay by quantum tunnelling




$$V(q) = \frac{1}{2} m\omega_0^2 q^2 - \lambda q^3$$

Probability of decay

$$P(t) = e^{-\Gamma t} \quad ; \quad \Gamma = A \exp - \frac{B}{\hbar}$$

$$\Gamma_{cl} \simeq \omega_0 \exp - V_0/kT$$

i) $T = 0$ and $\gamma = 0$

$$\Gamma = \Gamma_0 = A_0 \exp - \frac{B_0}{\hbar}$$

where $A_0 = \omega_0 \left( \frac{30 B_0}{\pi \hbar} \right)^{\frac{1}{2}}$ and

$$B_0 = \frac{36}{5} \frac{V_0}{\omega_0}$$

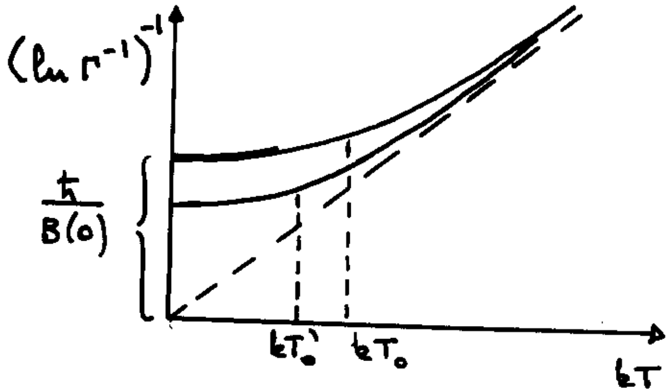
ii) $T = 0$ and $\gamma \neq 0$

$A(\gamma) > A_0$ and $B(\gamma) > B_0$

but: $\Gamma(\gamma) < \Gamma_0$

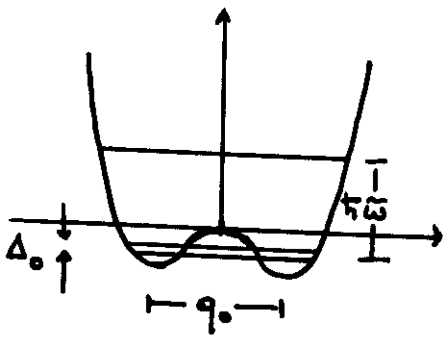
iii) $T \neq 0$ and $\gamma \neq 0$

$$B(\gamma, T) \approx \begin{cases} B(\gamma, 0) & \text{if } T < T_0 \\ -\frac{\hbar V_0}{kT} & \text{if } T > T_0 \end{cases}$$




$\eta_2 > \eta_1$

## d) Coherent Tunnelling



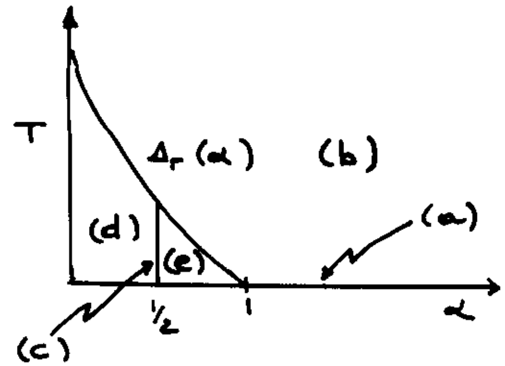

$$V(q) = -\frac{1}{2} m\omega_0^2 q^2 + \frac{\lambda}{4} q^4$$

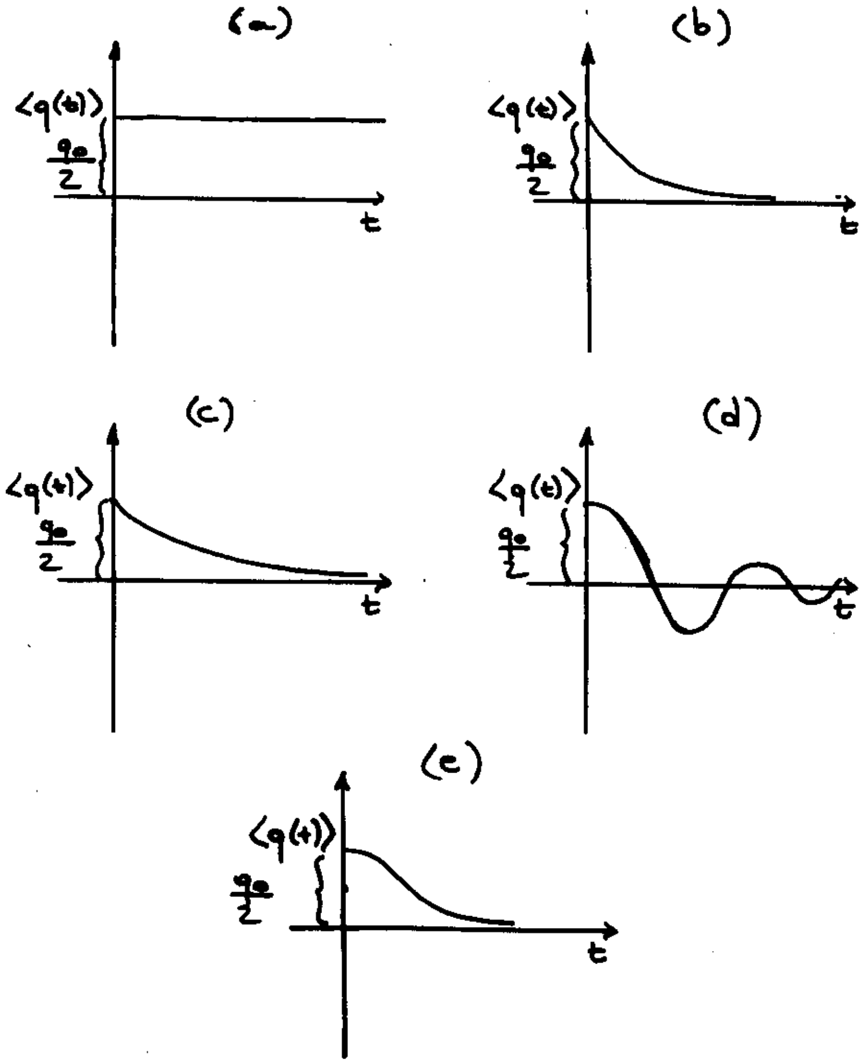
$kT \ll \hbar\tilde{\omega} \Rightarrow$ Two level system + dissipation.

$$\langle q(t) \rangle = \frac{q_0}{2} \cos \frac{\Delta_0}{\hbar} t \quad \text{se } \eta = 0$$

Phase diagram

$$\alpha \equiv \frac{\eta q_0^2}{2\pi\hbar}$$




c) Experimental realizations

SQUID's can be used to:

i) Test the structure of levels

ii) Measure quantum decay

iii) Measure coherent tunnelling.

But... what tunnels and what is the physics involved in the process?

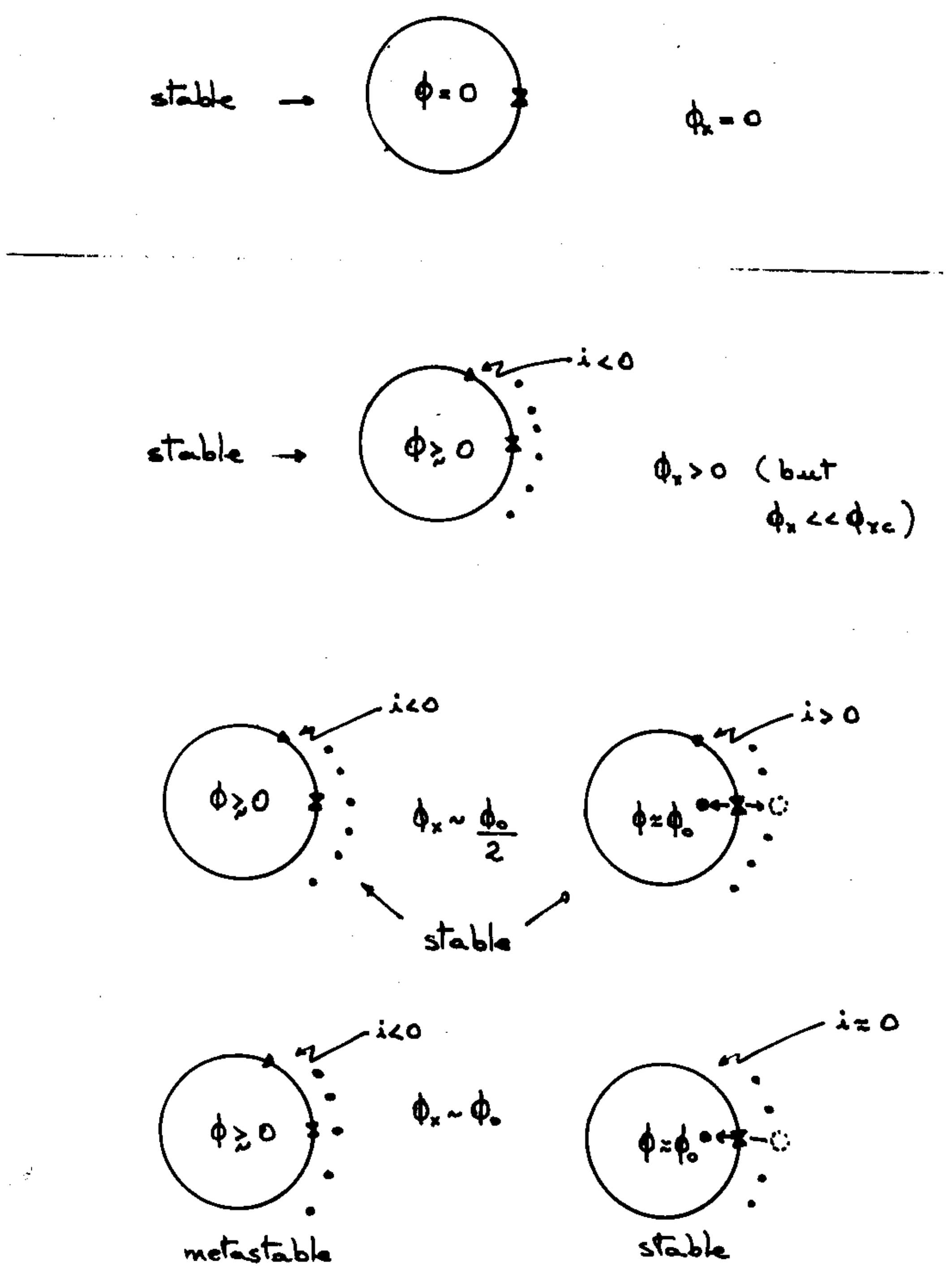
We shall see that effects of superposition of macroscopically distinguishable states (N-body states) take place.

$$a\,\psi_A(x_1, x_2, \dots x_N) + b\,\psi_B(x_1, x_2, \dots, x_N)$$

$$\Downarrow$$

i) Experimental realization of the Schrödinger's cat

ii) Fundamental questions in the quantum theory of measurement.






The state of the condensate

i) Coherent tunnelling

$$\psi(\vec{r}_1 \ldots \vec{r}_N, t) = a(t)\,\psi_1(\vec{r}_1 \ldots \vec{r}_N) + b(t)\,\psi_2(\vec{r}_1 \ldots \vec{r}_N)$$

where $|a|^2 + |b|^2 = 1$ and $\begin{cases} \psi_1 \to \text{state with } i < 0 \\ \psi_2 \to \text{state with } i > 0 \end{cases}$

ii) Quantum decay

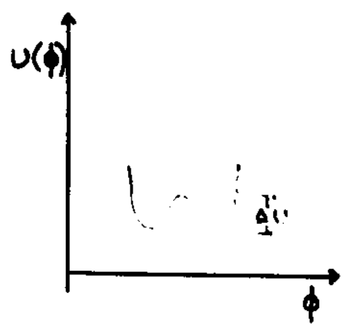
$$\psi(\vec{r}_1 \ldots \vec{r}_N, t) = e^{-\gamma t/2}\,\psi_1(\vec{r}_1 \ldots \vec{r}_N) + \sqrt{1 - e^{-\gamma t}}\,\psi_2(\vec{r}_1 \ldots \vec{r}_N)$$

$\psi_1 \to$ state with $i < 0$

$\psi_2 \to$ state with $i \simeq 0$

$\psi_1$ and $\psi_2$ are states corresponding to macroscopically distinct values of the Josephson current.

Order of magnitude of the relevant parameters




$$\Delta U \sim \frac{6\,(N-1)^2}{8\pi^2}\,\frac{\phi_0^2}{L}$$

$$N = \frac{2\pi L i_c}{\phi_0} = 1 + \epsilon$$

if $\epsilon \sim 0.1 \Rightarrow \Delta U \sim 10^{-23}$ joules b2) CBJJ (current biased Josephson junctions)


The same as a SQUID but:

$L \to \infty$ , $\phi_x \to \infty$ with $\frac{\phi_x}{L} \equiv I_x$ (finite)

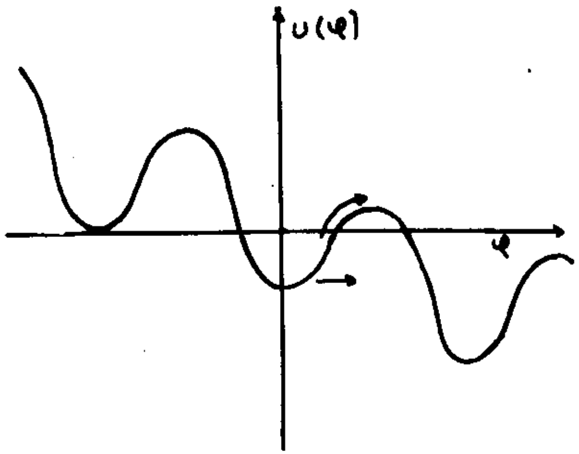
and $\phi \leftrightarrow \frac{\phi_0}{2\pi} \Delta\theta \equiv \frac{\phi_0 \varphi}{2\pi}$

Superconducting wave function: $\begin{cases} \psi_2 = \sqrt{\rho_2}\, e^{i\theta_2} \\ \psi_1 = \sqrt{\rho_1}\, e^{i\theta_1} \end{cases}$

and $\Delta\theta \equiv \theta_2 - \theta_1$

Equation of motion:

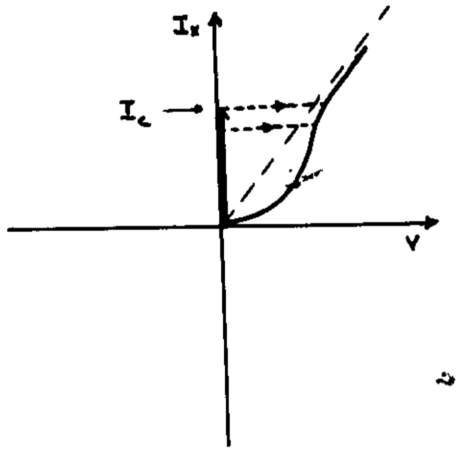
$$\frac{\phi_0}{2\pi} C \ddot{\varphi} + \frac{\phi_0}{2\pi R} \dot{\varphi} + U'(\varphi) = 0 \quad ; \; U(\varphi) = -I_x \varphi - i_c \cos\varphi$$



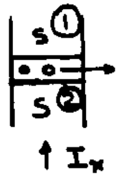



Remark: Here, it must be noticed that $\varphi$ is a phase
$\Rightarrow \varphi \to \varphi + 2n\pi$
$\varphi$ is not uniquely defined as $\phi$ in SQUIDS
Tunnelling does not involve "distances" $\sim 2\pi$
($I_x \lesssim I_c$ is appropriate)

One measures




Tunnelling in $\varphi$ $\Rightarrow$ quantum "phase slippage"




motion of vortex lines

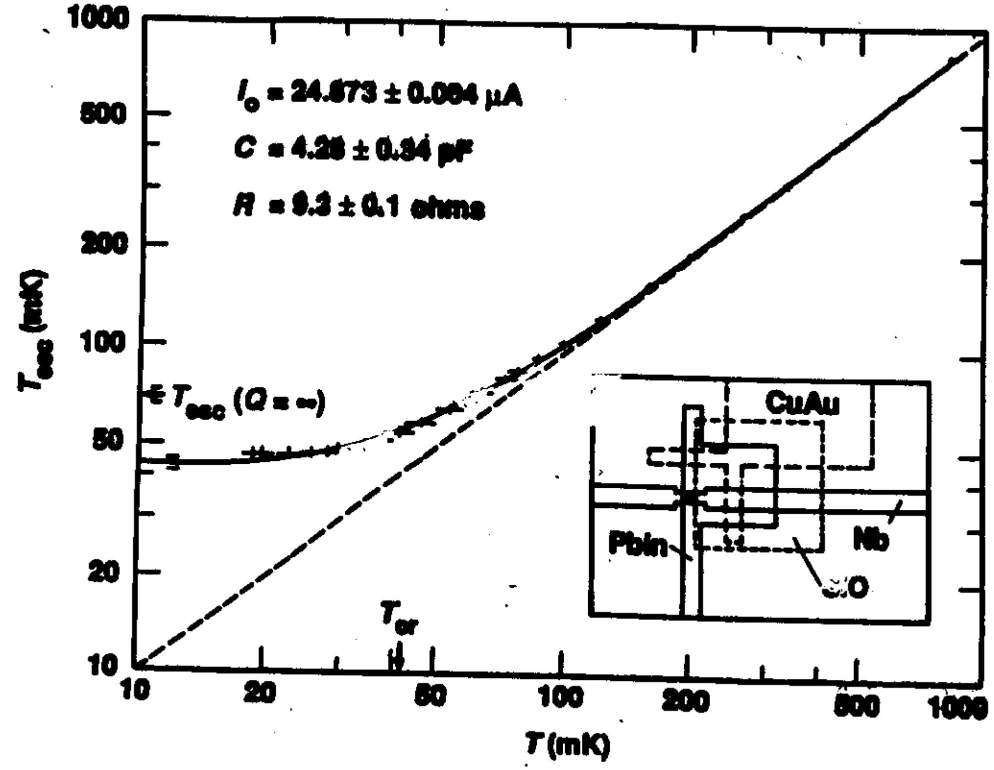
$$\langle \mu_1 - \mu_2 \rangle = h \left\langle \frac{dn}{dt} \right\rangle$$

Once again, the electronic condensate is in macroscopically distinct quantum states ($V \neq 0$ and $V = 0$)

Remark: In CBJJ's there is only quantum decay

"Bloch oscillations" ???

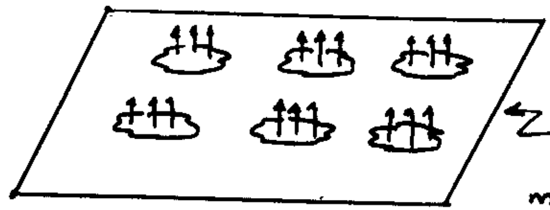
c3) Magnetic Systems



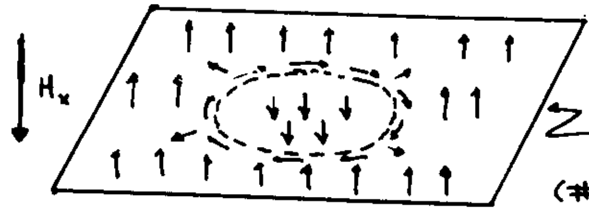

J. Clarke et. al.

Science 239 (1988) 992

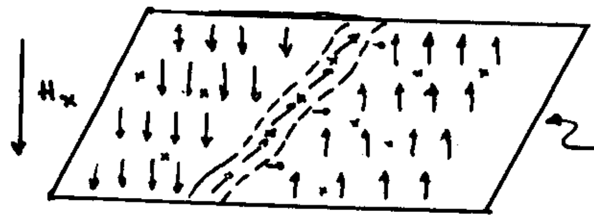
Three possibilities



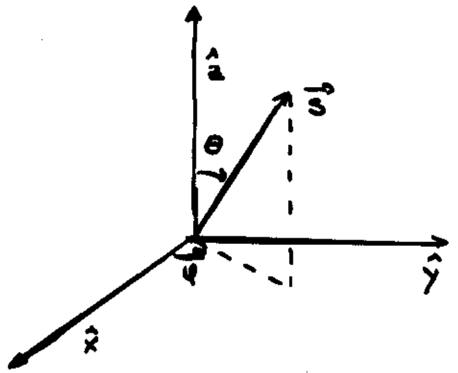
coherent tunnelling of magnetic particles ($10^3 - 10^4$ spins)




quantum nucleation (# of spins involved depends on $(1 - \frac{H_x}{H_c})N$




depinning of Bloch walls.



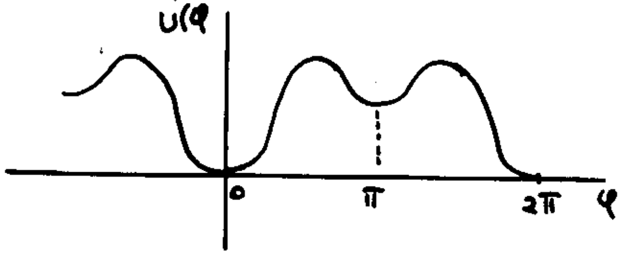

Totally anisotropic system

$\hat{x} \rightarrow$ easy axis

$$\Downarrow \qquad \varphi = \varphi(y, z)$$

$$\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} - \nabla^2\varphi = -A_1 \sin\varphi - A_2 \sin 2\varphi = U'(\varphi)$$

$A_1 \propto$ external field ; $A_2 \propto$ anisotropy

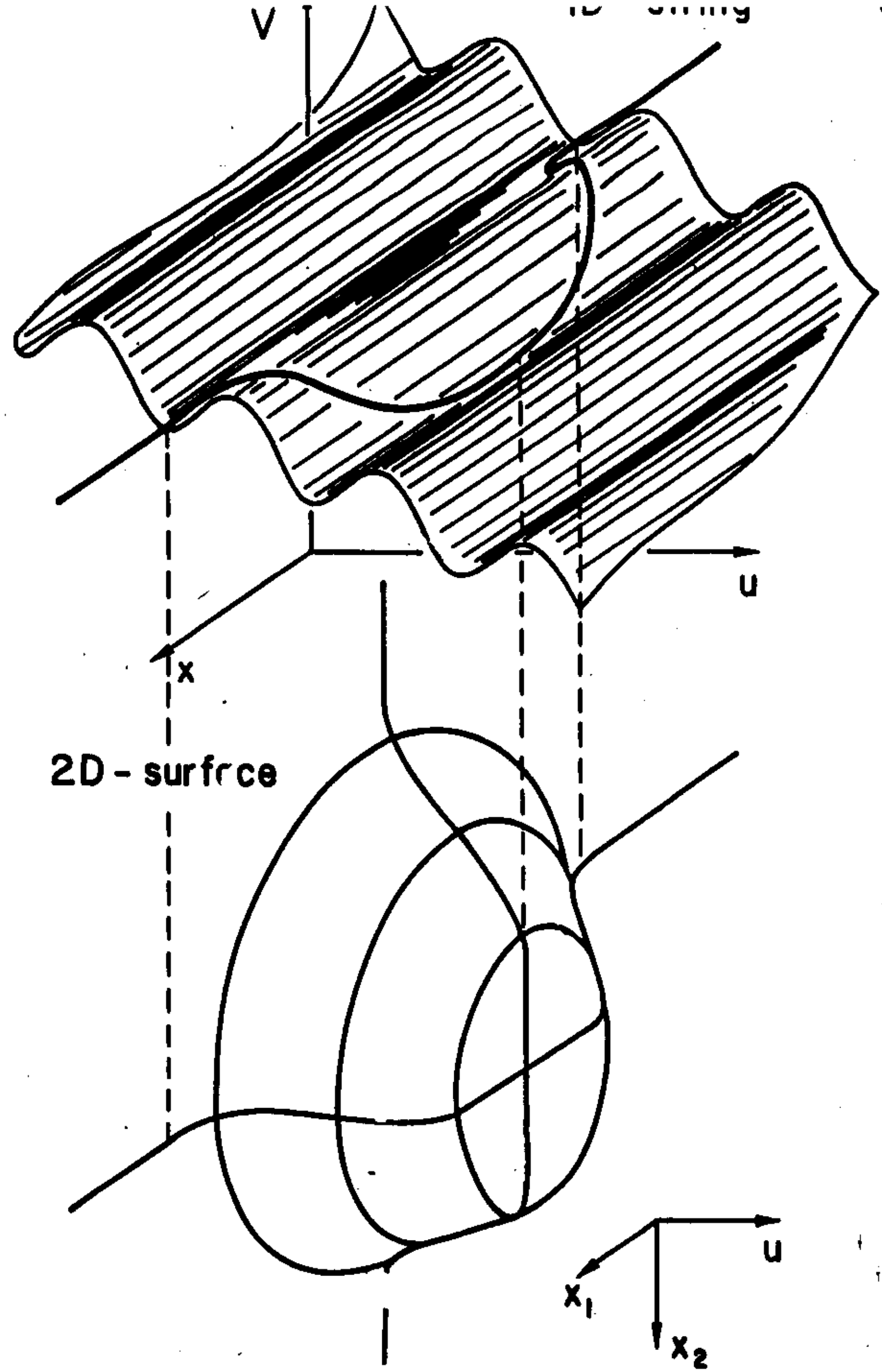



$1^{st}$ case: $A_1 = 0$ and $\nabla^2\varphi = 0 \Rightarrow$ bistable potential

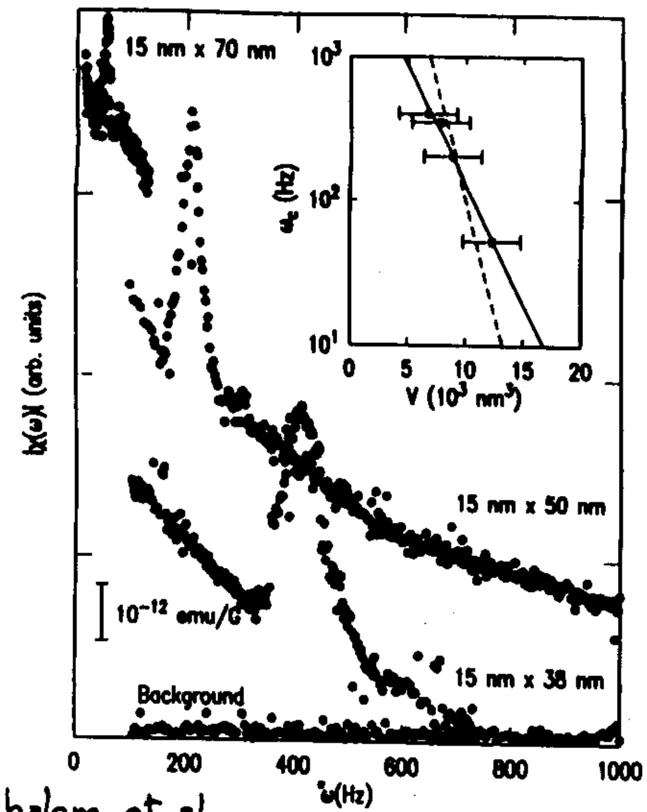
$2^{nd}$ case: $\nabla^2\varphi$ in cylindrical coordinates (bubbles)

$3^{rd}$ case: equation of motion + impurity potential (∗)

When dissipation is present $+\ \eta \dfrac{\partial\varphi}{\partial t}$ (∗∗)

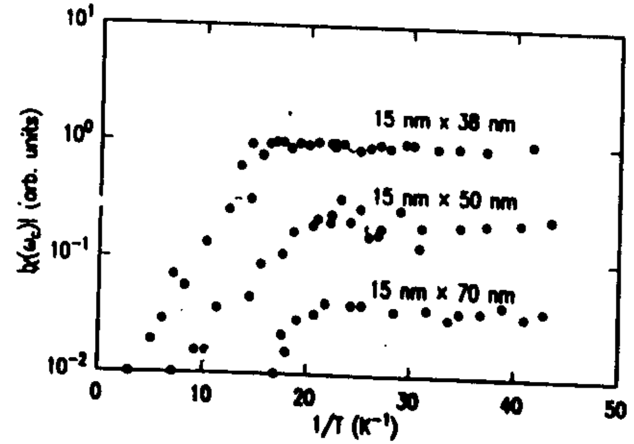


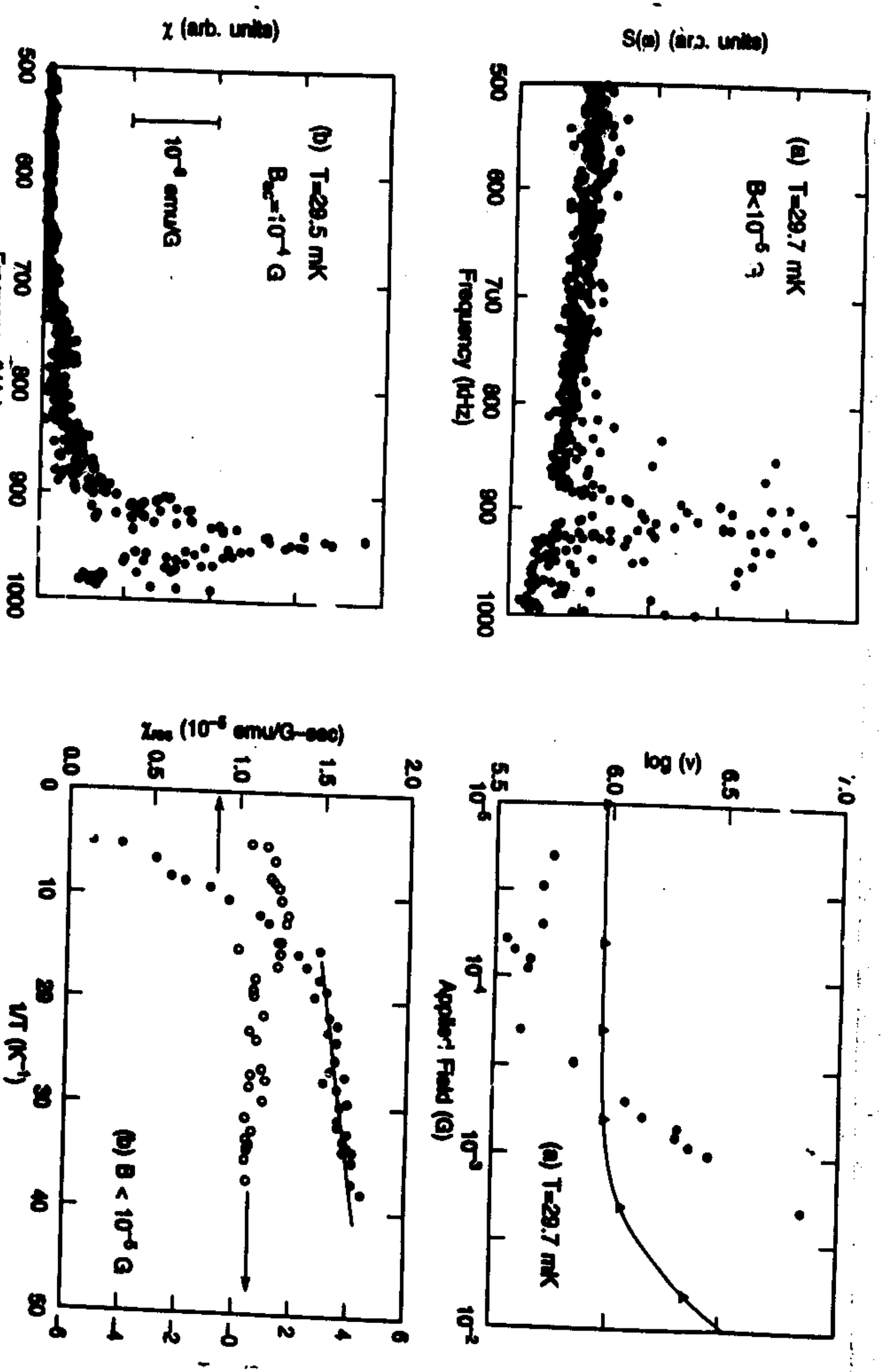

G. Blatter et. al. Rev. Mod. Phys. 66 (1994) 1125




D. D. Awschalom et. al.
PRL 65 (1990) 783

$Fe(CO)_5$



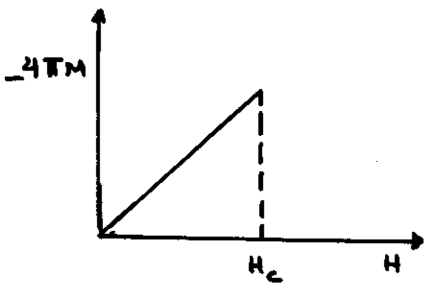




D. D. Awschalom et. al
PRL, 68 (1992) 3092

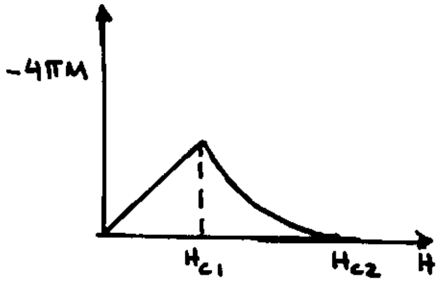
## c4) Vortices in superconductors

SUPERCONDUCTORS

TYPE I




TYPE II

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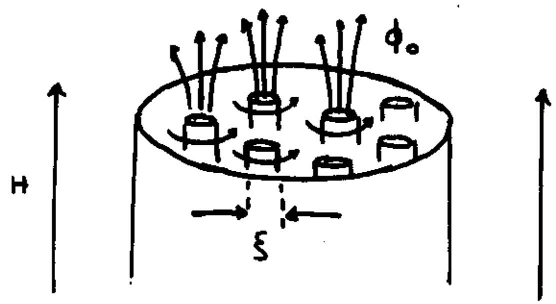


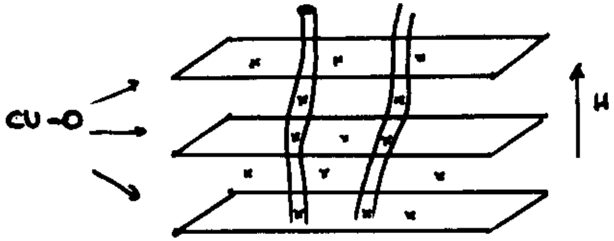

High $T_c$ superconductors

i) Type II

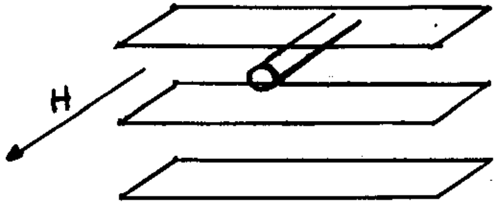
ii) $\lambda >> \xi$ $\begin{cases} \lambda \text{ - penetration depth} \\ \xi \text{ - coherence length} \end{cases}$

iii) quantum fluctuations are important.

## Pinning of vortices (high $T_c$)



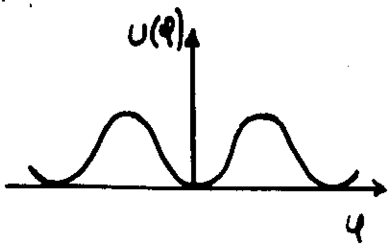

by impurities

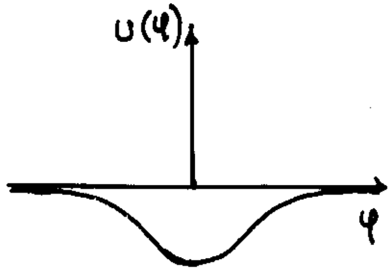



intrinsic

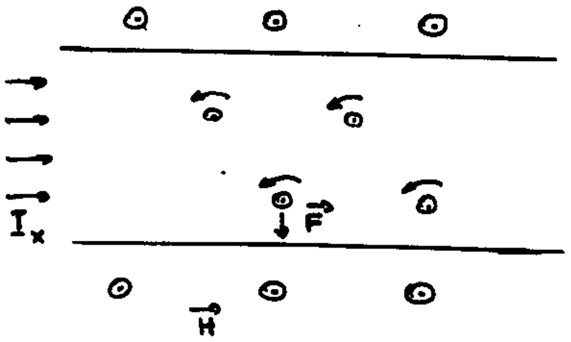

by columnar defects

Displacement of vortices, $\vec{\varphi}(\vec{r},t)$, subject to potentials such as



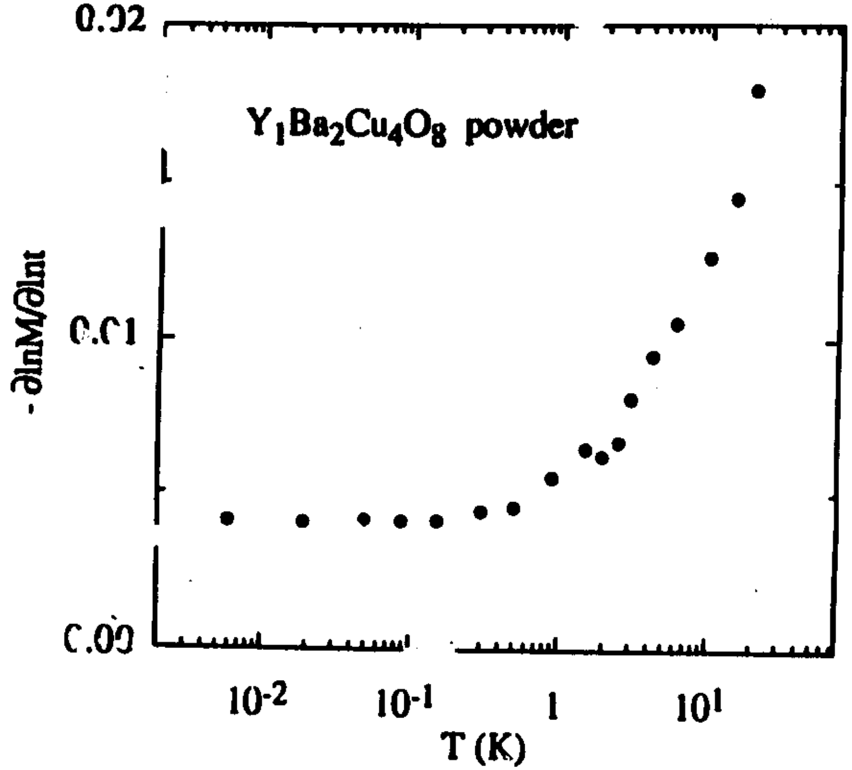



The effect of an external current $I_x$

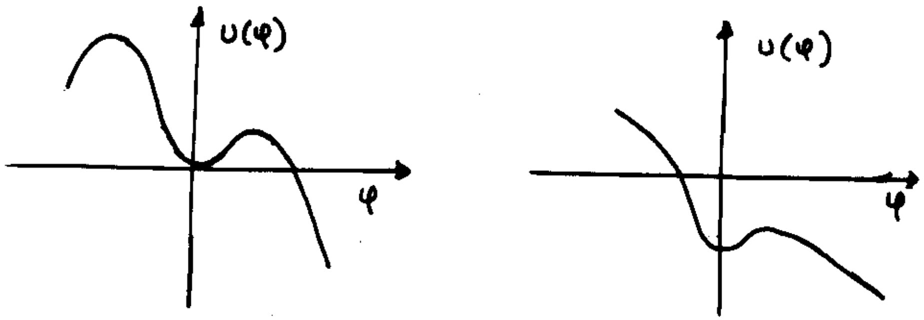



upper view of the system

A.C. Mota et. al.
Physica C 165-185 (1991) 343




New potentials




$$\frac{1}{c^2}\frac{\partial^2 \vec{\varphi}}{\partial t^2} - \nabla^2 \vec{\varphi} = -U'(\varphi)$$

$$+\ \eta \frac{\partial \vec{\varphi}}{\partial t} \quad \text{(generalization)}$$

Analogy with the displacement of Bloch walls

d) Microscopic models

Bloch walls $\leftrightarrow$ Topological excitations in spin models ( dissipation: eq$^s$ Landau - Lifshitz or Bloch)

Vortices $\leftrightarrow$ topological excitations in superconductor (dissipation: Time dependent Ginzburg - Landau

Phenomenological approach requires knowledge of $\eta(T)$ or $J(\omega, T)$

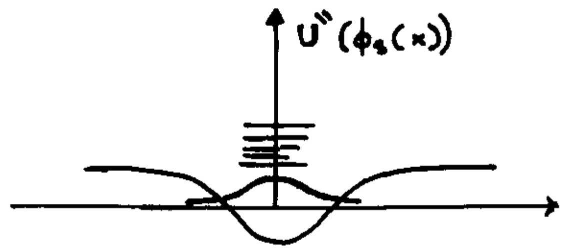
These objects can be treated quantum mechanically with collective coordinates.

$$\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} = -U'(\phi)$$

admits localized solutions $\phi_s(x - X_0)$

$$\Downarrow$$

$$-\frac{\partial^2\varphi_n}{\partial x^2} + U''(\phi_s)\varphi_n = \lambda_n\varphi_n \quad \text{(stationary case)}$$




$$\Downarrow$$

if $X_0 \neq 0$; $\phi(x,t) = \varphi_0(x - X_0(t)) + \sum_{n\neq 1}^{\infty} a_n \varphi_n(x,t)$

$$\Downarrow$$

$$H = \frac{1}{2M}\left(P - \sum_n P_n\right)^2 + V_e(X_0) + \sum_n \frac{P_n^2}{2m} + V_i(q_n)$$

$$\Updownarrow \quad W \equiv \exp -\frac{i}{\hbar} X_0 \sum_n P_n$$

$$H = \frac{P^2}{2M} + \sum_n V_i(q_n - X_0) + \sum_n \frac{P_n^2}{2m} + V_e(X_0)$$

$V_{i(e)} \longrightarrow$ internal (external) potential

$p_n$ and $q_n$ come from the expansion of $\varphi_n(x,t)$

The effective dynamics of $X_0(t)$ allows us to compute $\mu(T)$ and $D(T)$ of external particles or topological excitations in general media.

Some problems of interest

i) Optical and acoustical polarons in 1-D

ii) External particles in 1-D systems of massive bosons or fermions ($\delta$-repulsive)

iii) Motion of Bloch walls in 1-D magnets (solitons in non-linear media)

iv) Motion of vortices in superconductors (2-D)

v) Motion of skyrmions in 2-D electronic systems (QHE; $\nu = 1$)

e) Comments and conclusions

1) Dissipation inhibts quantum effects but does not fully destroy them.

— weak dissipation

- Possibility to realize experiments presenting quantum effects on a macroscopic scale
- Preservation of quantum coherence for long times.

— strong dissipation

- More efficient pinning of vortices
- Understanding of a measurement apparatus from a quantum-mechanical point of view.

2) Vast range of applicability of the phenomenological model once $J(\omega, T)$ is known

3) Other examples

— CDW's
— dislocations
— biological applications

4) Open problems