UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL ATOMIC ENERGY AGENCY
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

SMR: 962/12

WORKSHOP ON QUANTUM DISSIPATION AND APPLICATIONS

(29 July - 9 August 1996)

"Transport Properties of a Particle in 1-D Quantum Fluid"

presented by:

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Transport properties of a particle in 1-D quantum fluid

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Coll: Matthew P.A. Fisher
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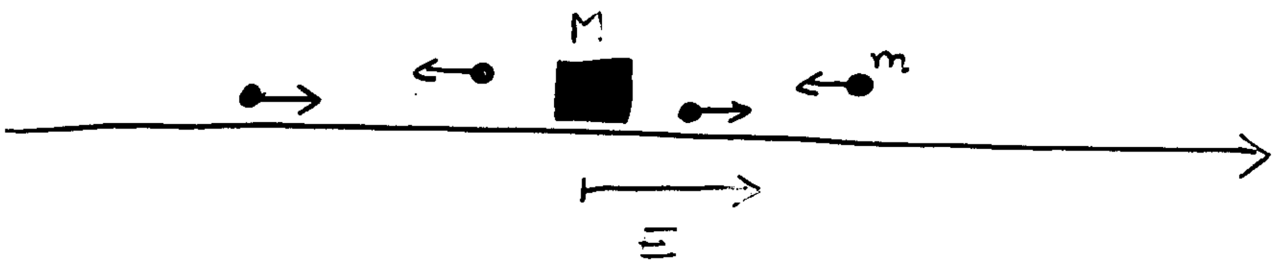
- 1D Systems :
  - Field theory
  - Bethe ansatz
  - Conformal field theory
  - ⋮

$\downarrow$

Transport ??

Puzzle: Integrability !



Classical mechanics:

$$M \frac{d\vartheta}{dt} = -\gamma \vartheta + eE$$

$t \to \infty \qquad \langle \vartheta \rangle = \frac{eE}{\gamma} \qquad \mu \propto \frac{1}{\gamma}$

Quantum mechanics $\Longrightarrow$ Caldeira-Leggett

General Problem:

$$H = \frac{P^2}{2M} + \sum_{i=1}^{N} U(X - x_i) + \sum_{i=1}^{N} \frac{p_i^2}{2m} + \sum_{i \neq j} V(x_i - x_j)$$

$$U = e^{iX\sum_{i=1}^{N} p_i} \qquad \begin{cases} P \to P - \sum_{i=1}^{N} p_i \\ X \to X \\ p_i \to p_i \\ x_i \to x_i + X \end{cases}$$

$$H' = UHU^{-1} = \frac{1}{2M}\left(P - \sum_{i=1}^{N} p_i\right)^2 \longrightarrow \text{Recoil}$$

$$\underbrace{+ \sum_{i=1}^{N} \frac{p_i^2}{2m} + U(x_i) + \sum_{i \neq j} V(x_i - x_j)}_{\text{Quantum impurity problem}}$$

$\longrightarrow$ Kane and Fisher

1-D interacting electronic gas
$\Rightarrow$ Luttinger liquid
_Haldane

$M = \infty$

$T = 0$ $\begin{cases} \text{Repulsive interactions} \Rightarrow t = 0 \\ \text{Attractive interactions} \Rightarrow t = 1 \end{cases}$

Partition function

$$Z = \int DP\,DX\,Dx_i\,DP_i\; e^{-\int_0^\beta d\tau\, L}$$

$$L = \frac{1}{2M}\Big(P - \sum_i P_i\Big)^2 + i\dot{X}P$$
$$+ \sum_i \left[\frac{P_i^2}{2m} - U(x_i) + i\,\dot{x}_i P_i\right] - \sum_{i \neq j} V(x_i - x_j)$$

Integ[illegible]

$$L = \frac{M\dot{X}^2}{2} + i\,\underbrace{\dot{X}}_{\text{Recoil}}\sum_i P_i + \sum_i\left(\frac{P_i^2}{2m} - U(x_i) + i\,\dot{x}_i P_i\right)$$
$$+ \sum_{i \neq j} V(x_i - x_j)$$

[illegible] 4

Transport properties of a particle in 1-D quantum fluids

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w/:

Matthew Fisher

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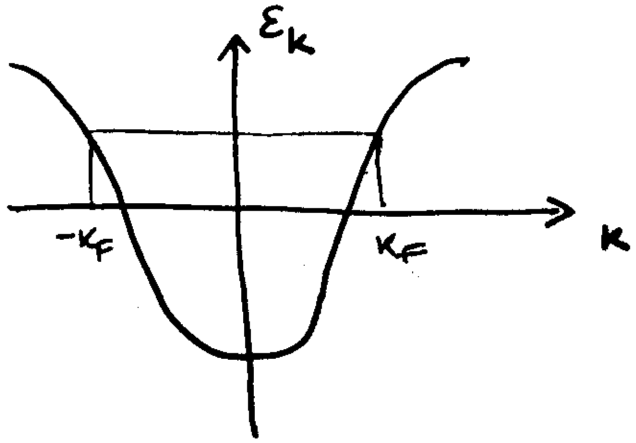
Luttinger liquid problem → Bosonization



$$\psi(x) = e^{ik_F x}\psi_R(x) + e^{-ik_F x}\psi_L(x)$$

$$\psi_{R,L}(x) = \eta\, e^{i\sqrt{\pi}(\phi \pm \theta)}$$

$$H_{LL} = \frac{v_F}{2}\int dx \left(\frac{1}{g}(\partial_x\theta)^2 + g(\partial_x\phi)^2\right)$$

$g<1 \Rightarrow$ Repulsive
$g>1 \Rightarrow$ Attractive

$N_R + N_L = \frac{1}{\sqrt{\pi}}\partial_x\theta \rightarrow$ Density
$N_R - N_L = \frac{1}{\sqrt{\pi}}\partial_x\phi \rightarrow$ Current

Reco : $\dot{X}\sum_i F_i = \dot{X}\int dx\,(N_R - N_L)\,k_F$

$= \dot{X}\,\frac{k_F}{\sqrt{\pi}}\int dx\,\partial_x\phi$

$U(x_i)$?

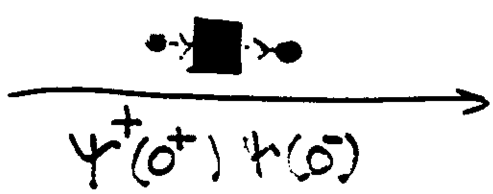
Weak link $\rightarrow$ $t=0$ (Opaque!)

$$\Phi(t) = \frac{1}{2}\left(\phi(0^+,\tau) - \phi(0^-,\tau)\right)$$

Integrate [illegible]
except [illegible] $x=0$. $\rightarrow$ [illegible]

$$S_0 = \sum_n \left\{ \frac{M\omega_n^2}{2}|X_n|^2 + \frac{2K_F\omega_n}{\sqrt{\pi}} X_n \Phi_n + g\,|\omega_n|\,|\Phi_n|^2 \right\}$$

Include transmission



$$S_T = -t\int d\tau\, \cos\left[\sqrt{\pi}\,\Phi(\tau)\right]$$

Extreme case $t=0$

Integrate out Φ_n !

$$S_{eff} = \sum_n \left(\frac{M\omega_n^2}{2} + \frac{k_F^2 |\omega_n|}{\pi g} \right) |X_n|^2$$

$\rightarrow$ Caldeira - Leggett

$$\gamma = \frac{k_F^2}{\pi g}$$

$t \neq 0 \rightarrow$ Non-linear problem !

Integrate out X_n !

$$S = \sum_n \left(\frac{2k_F^2}{\pi M} + g|\omega_n| \right) \Phi_n^2 - t \int dz \cos \sqrt{\pi}\Phi$$

$\uparrow$

Mass term.

Renormalization group (T=0)

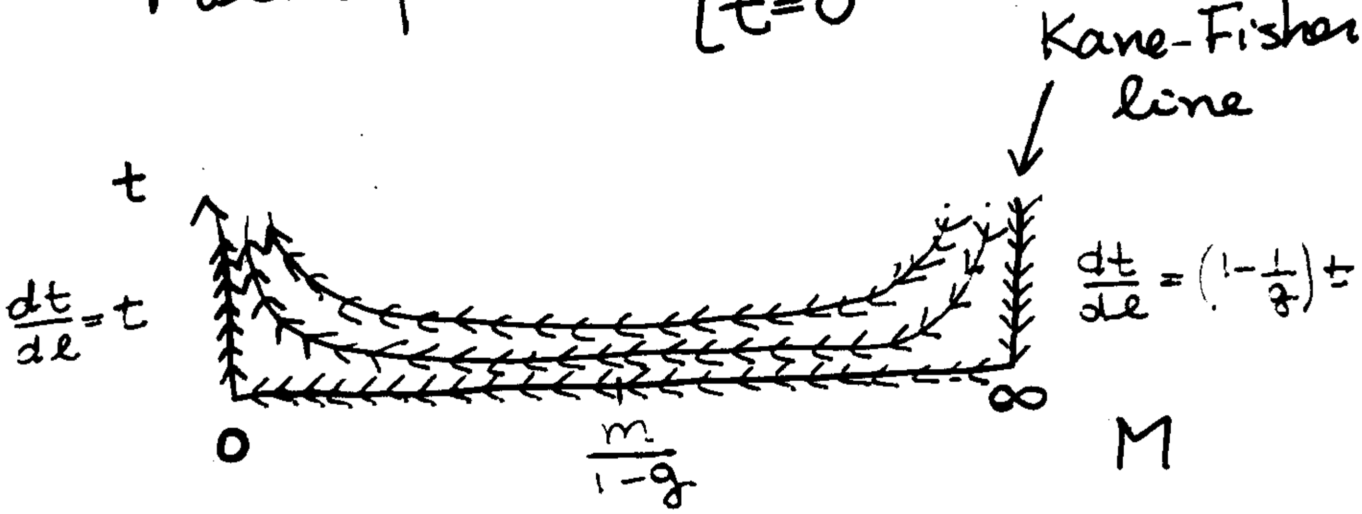
Integrating modes: $\Lambda/b < \omega < \Lambda$

$b = e^{dl}$

$$\begin{cases} \dfrac{dM}{dl} = -M \\[2ex] \dfrac{dt}{dl} = \left(1 - \dfrac{1}{g + \frac{m}{M}}\right) t + \Theta(t^2) \end{cases}$$

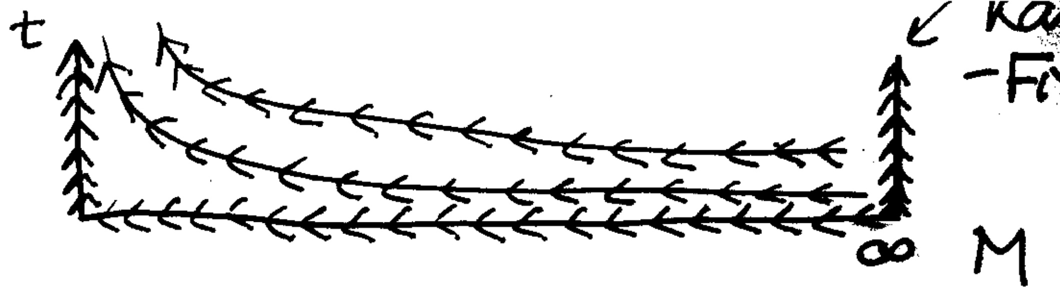
$l \mapsto \infty \quad 0 \to \infty \Rightarrow$ low energies

Fixed point: $\begin{cases} M=0 \\ t=0 \end{cases}$



$g < 1$

Repulsive



$g > 1$
Attractive

$T \neq 0 \longrightarrow$ Flow is cut-off

$$e^{\ell^*} \sim \frac{\Lambda}{T} \qquad \Lambda \sim E_F$$

$$M(\ell^*) = M_0\, e^{-\ell^*} = \frac{m}{1-g}$$

$$T^* = \frac{1}{1-g}\,\frac{m}{M_0}\,T_F$$

$\Rightarrow$ Below $T^* \longrightarrow$ t grows
$\downarrow$
Particle becomes transparent!!!

$T > T^* \longrightarrow$ t decreases $g<1$
$\downarrow$
Particle is opaque $\Rightarrow$ γ

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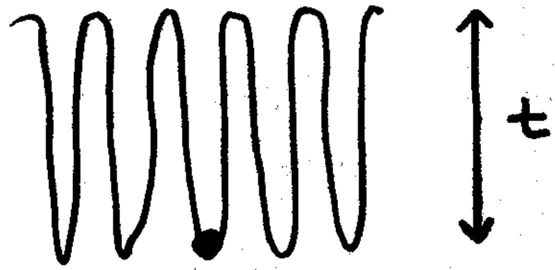
$T < T^*$



$$-t\cos 2\sqrt{\pi}\,\Phi$$

$$\sim T=0$$

$$-t + 2\pi t\,\Phi^2$$

$\downarrow$

Symmetry is spontaneously broken!

$$\Phi \to \Phi + 2\pi$$

$$S \doteq \sum_n \left(\frac{M\omega_n^2}{2} |X_n|^2 + \frac{2k_F\omega_n}{\sqrt{\pi}} X_n \Phi_{-n} + \left(g|\omega_n| + 2\pi t \right) |\Phi_n|^2 \right)$$

Trace over Φ

$$S = \sum_n \left(\frac{M\omega_n^2}{2}|X_n|^2 + \frac{k_F^2\omega_n^2}{\pi(g|\omega_n| + 2\pi t)} |X_n|^2 \right)$$

$\omega_n \gg 2\pi t/g \longrightarrow$ Caldeira-Leggett γ $\quad T > T^*$

$\omega_n \ll 2\pi t/g \rightarrow \left(M + \dfrac{k_F^2}{2\pi^2 t} \right) \dfrac{|X_n|^2}{2} \quad T < T^*$

$\Longrightarrow$ Mass renormalization !

K

$T \neq 0$? $\rightarrow$ Symmetry is restored!

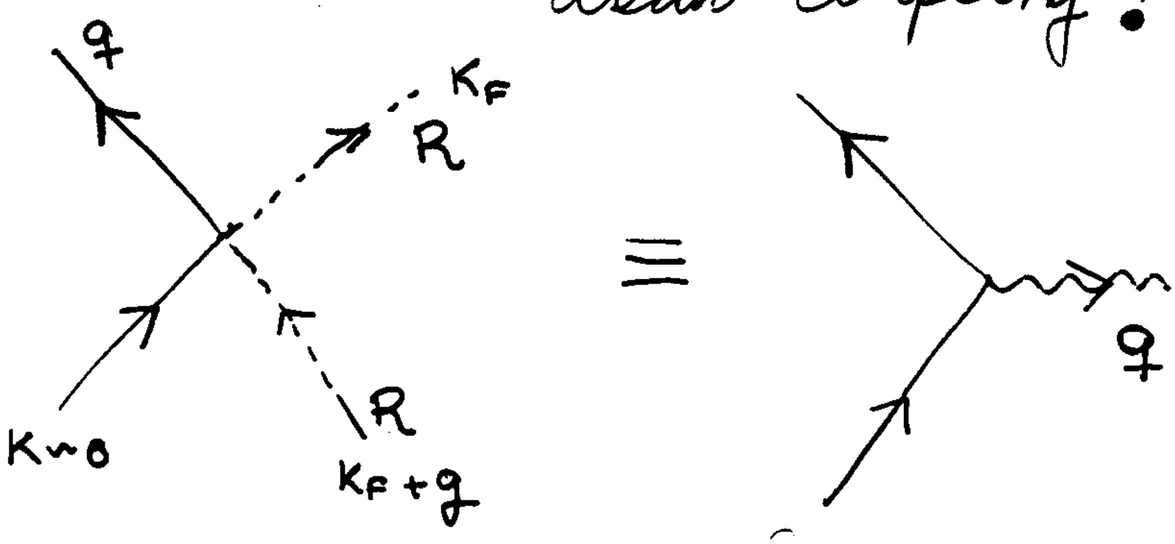
$T < T^*$ $\quad$ t grows

$\downarrow$

Particle becomes transparent

$\Downarrow$

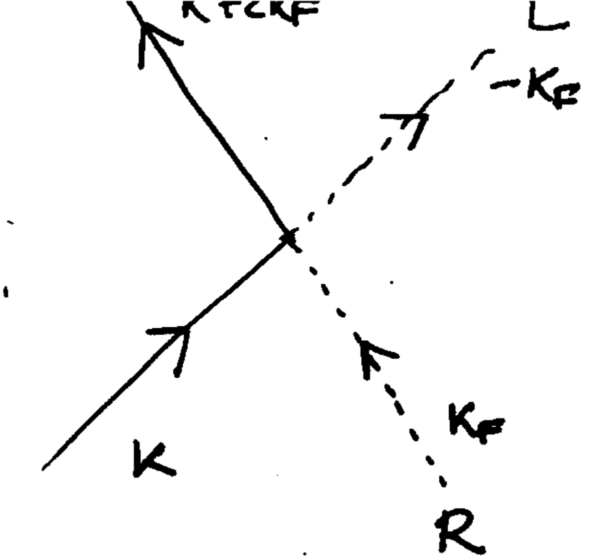
Weak coupling!



$\rightarrow$ Minimum energy

$$= \frac{4M}{m} T_F$$

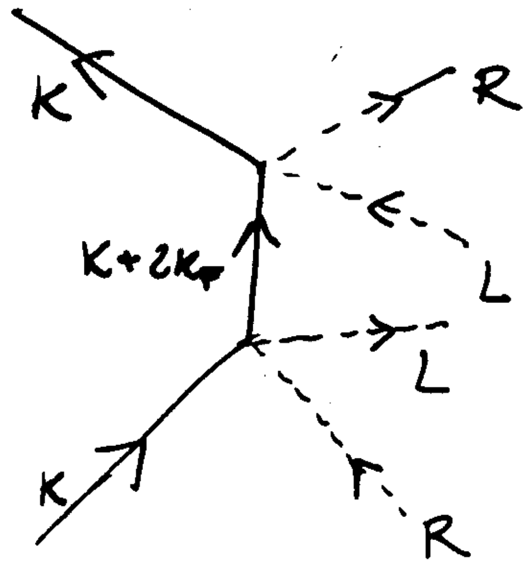
$\rightarrow$ Frozen



Minimum energy

$= \frac{m}{M} T_F$

$\Longrightarrow$ Frozen!



$$H_I = \int dx\ c^\dagger(x) c(x)\ \psi_R^\dagger \psi_R \psi_L^\dagger \psi_L$$

$\downarrow$ Bosonize

$$\int dx\ c^\dagger(x) c(x) \left\{ (\partial_x \theta)^2 - (\partial_x \phi)^2 \right\}$$

$\swarrow$

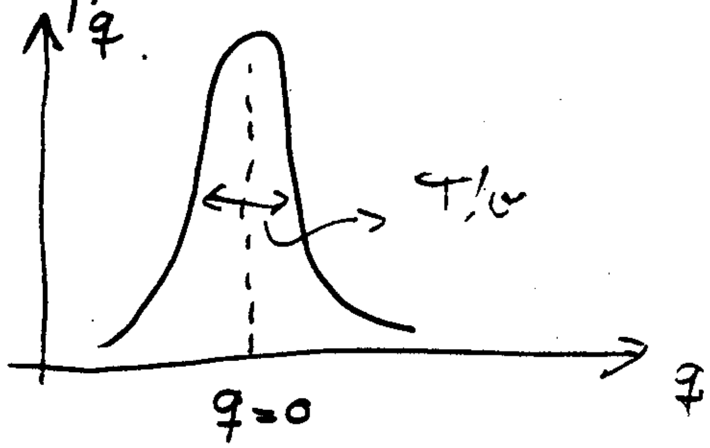
$\Gamma_{k \to k+q} \to$ Transition Probability

Fermi's Golden rule

Boltzmann Equation

$$\partial_t f(p,t) + E\,\partial_p f(p,t) = I(p,t)$$

$$I = \sum_k \left\{ f(k,t)\,\Gamma_{k\to p} - f(p,t)\,\Gamma_{p\to k} \right\}$$



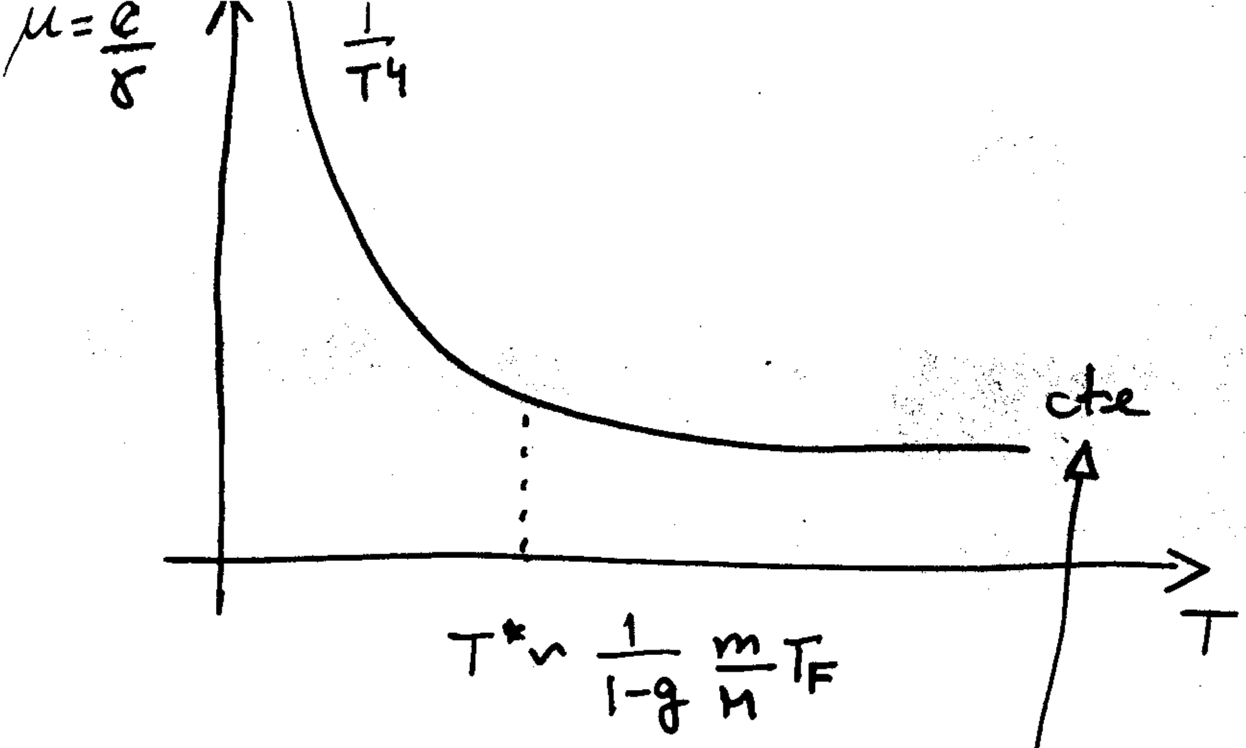
$$f(k) = e^{-\beta\varepsilon_k}\,G(k)$$

$$I = \sum_q \Gamma_{p+q\to p}\, e^{-\beta\varepsilon_{p+q}} \left[G(p+q) - G(p)\right]$$

$$\sim A\,\partial_p e^{-\beta\varepsilon}\partial_p G + \frac{A}{2}\, e^{-\beta\varepsilon}\partial_p^2 G$$

$$A = \sum_q q^2\Gamma_q \sim T^5$$

$$\mu = \frac{\langle v\rangle}{E} = \frac{1}{ME}\int dp\, p\, f(p) = \frac{T}{A} \sim \frac{1}{T^4}$$



With Amir Caldeira

TOMORROW!

- RG + Bosonization

$T < T^* \Longrightarrow$ Free particle +
Mass renormalization
$\mu \sim \frac{1}{T^4}$

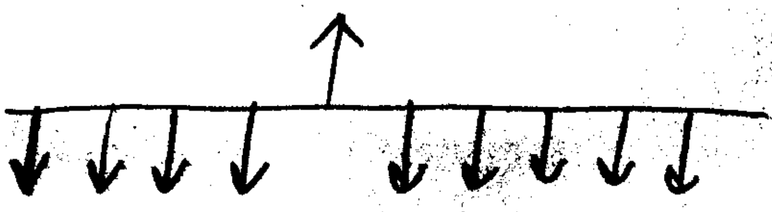
$T > T^* \Longrightarrow$ Brownian Motion
Caldeira-Leggett

Strong-coupling $\Rightarrow$ Soliton Dynamics.

Puzzle:

- McGuire

  $\underline{\underline{M=m}}$



$T=0 \Rightarrow$ Mass renormalizati

- Castella, Zotos, Prelovsek

  $M=m \longrightarrow \mu(T)=\infty$ all T !!

  - Charge stiffness!

    $\updownarrow$?

    mobility

  - Fermi Golden's rule is violated?
  - Kubo formula?

    $\updownarrow$

    ?