UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL ATOMIC ENERGY AGENCY
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

SMR: 962/10

## WORKSHOP ON QUANTUM DISSIPATION AND APPLICATIONS

( 29 July - 9 August 1996 )

---

# *"Non-Universality of Dephasing in Quantum Transport"*

presented by:

**U. WEISS**
Universität Stuttgart
Institut für Theoretische Physik II
Pfaffenwaldring 57/3
D-70550 Stuttgart
Germany

# Non–universality of dephasing
# in
# quantum transport

I. Introduction

II. Classically fluctuating versus
dynamically fluctuating potentials

III. Transversal and longitudinal
interferences

IV. Suppression of interferences

V. Example

VI. Summary

J. Allinger

Z. Phys. B 98, 289 (1995

# I. Introduction

- Quantum features of charge transport in mesoscopic systems:
  - + Aharonov–Bohm–oscillations
  - + Resonant tunneling
  - + Anomalous temperature dependence of magnetoresistivity at low temperatures in disordered systems (weak localization)
  - + Universal Conductance Fluctuations (UCF)
  - + Non–local conductivity tensor

$$j(r,t) \propto \int dr'dt'\,\sigma(r,r',t,t')E(r',t')$$

- Macroscopic description of charge transport:
  - + Drude model (particle picture)

* What determines the crossover between these two regimes?

* How long is the "phase memory" of a particle?

# II. Classically fluctuating versus dynamically fluctuating potentials

<u>Classically fluctuating potential</u>:

Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\psi(q,t) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial q^2} + V(q,t)\right]\psi(q,t)$$

Assume that the potential has a Gaussian stochastic contribution:

$$V(q,t) = V_0(q) + \delta V(t)$$

Propagator:

$$\begin{aligned} K(q_2,q_1;t) &= \left\langle q_2 \middle| \hat{T}\,\exp\left[-\frac{i}{\hbar}\int_0^t d\tau \hat{H}(\tau)\right] \middle| q_1 \right\rangle \\ &= \exp\left[-\frac{i}{\hbar}\int_0^t d\tau\,\delta V(\tau)\right] K_0(q_2,q_1;t) \end{aligned}$$

Statistical average:

$$\boxed{\langle K(q_2,q_1;t)\rangle_{\text{av}} = \exp\left[-\frac{1}{2}\left\langle\delta\varphi^2(t)\right\rangle_{\text{av}}\right] K_0(q_2,q_1;t)}$$

with

$$\begin{aligned} \left\langle\delta\varphi^2(t)\right\rangle_{\text{av}} &\equiv \frac{1}{\hbar^2}\int_0^t d\tau_2 \int_0^t d\tau_1\, \underline{\langle\delta V(\tau_2)\delta V(\tau_1)\rangle_{\text{av}}} \\ &\equiv \frac{1}{\hbar^2}\,\underline{\left\langle\delta V^2\right\rangle_{\text{av}}} \int_0^t d\tau_2 \int_0^t d\tau_1 \underline{\rho(\tau_2-\tau_1)} \end{aligned}$$

If

$$\underline{t \gg t_{\rm corr}} \quad [\rho(t_{\rm corr}) \ll 1]$$

$$\Longrightarrow \qquad \underline{\mu := \int_0^\infty d\tau \rho(\tau)}$$

Then
$$\boxed{\left\langle \exp\left[-\frac{i}{\hbar}\int_0^t d\tau\, \delta V(\tau)\right]\right\rangle_{\rm av} = \exp\left(-\frac{\langle \delta V^2\rangle_{\rm av}}{\hbar^2}\,\mu t\right)}$$

Interpretation:

- *dephasing time*
- *coherence length*

$$\boxed{\tau_{\rm deph} := \frac{\hbar^2}{\langle \delta V^2\rangle_{\rm av}\,\mu}} \boxed{l_{\rm c} = v\,\tau_{\rm deph}}$$

Fully quantized system–plus–reservoir model:

$$\hat{H} = \hat{H}_S + \hat{H}_B + \hat{H}_C$$

How does the environment randomize phase correlations?

Transition probability from $|\psi_i\rangle$ to $|\psi_f\rangle$:

$$P_{fi} = \sum_{\chi'} \left| \left\langle \psi_f \, \chi' \left| e^{-\frac{i}{\hbar}\left(\hat{H}_B + \hat{H}_S + \hat{H}_C\right)t} \right| \chi \, \psi_i \right\rangle \right|^2$$

Weak coupling: $H_C \implies |\chi\rangle$ unchanged

$$P_{fi} = \left| \left\langle \underset{\overset{\uparrow}{\chi}}{\psi_f} \left| \exp\left[ -\frac{i}{\hbar} \left( \epsilon_\chi + \hat{H}_S + \langle \hat{H}_C \rangle_\chi \right) t \right] \right| \underset{\overset{\uparrow}{\chi}}{\psi_i} \right\rangle \right|^2$$

Assign the phase $\exp(-i\epsilon_\chi t/\hbar)$ to the environmental state

$$A_{fi} = \left\langle \psi_f \left| \exp\left[ -\frac{i}{\hbar} \left( \hat{H}_S + \langle \hat{H}_C \rangle_\chi \right) t \right] \right| \psi_i \right\rangle$$

$$\boxed{\left\langle e^{i\varphi} \right\rangle = \frac{\sum_\chi c_\chi \left| A_{fi}(\chi) \right| e^{i\varphi(\chi)}}{\sum_\chi c_\chi \left| A_{fi}(\chi) \right|}}$$

if initially
bath is a mi

$c_\chi$ is the weight of the initial bath state $|\chi\rangle$

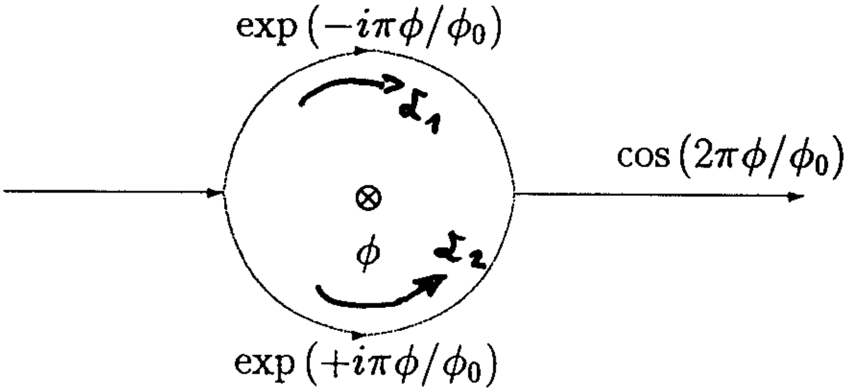
- Works only in the weak coupling limit
- Concept of phase memory fails for arbitrary coupling: "*decoherence*" instead of "*dephasing*"

## III. Transversal and longitudinal interferences

$$\left\langle q_f \left| \exp\left(-\frac{i}{\hbar}\hat{H}t\right) \right| q_i \right\rangle \sim \int_{(q_i)}^{(q_f)} \mathcal{D}q(\tau) \exp\left\{\frac{i}{\hbar} S\left[q(\tau)\right]\right\}$$

Interference occurs between any pair of paths: $e^{iS_1} + e^{iS_2}$

Metallic ring, threaded by magnetic flux $\phi$:



$$\exp\left\{\frac{i}{\hbar} S\left[q(\tau)\right]\right\} = \exp\left\{\frac{i}{\hbar} S_0\left[q(\tau)\right] + \frac{i}{\hbar} e \int_{C} A(q)dq\right\}$$

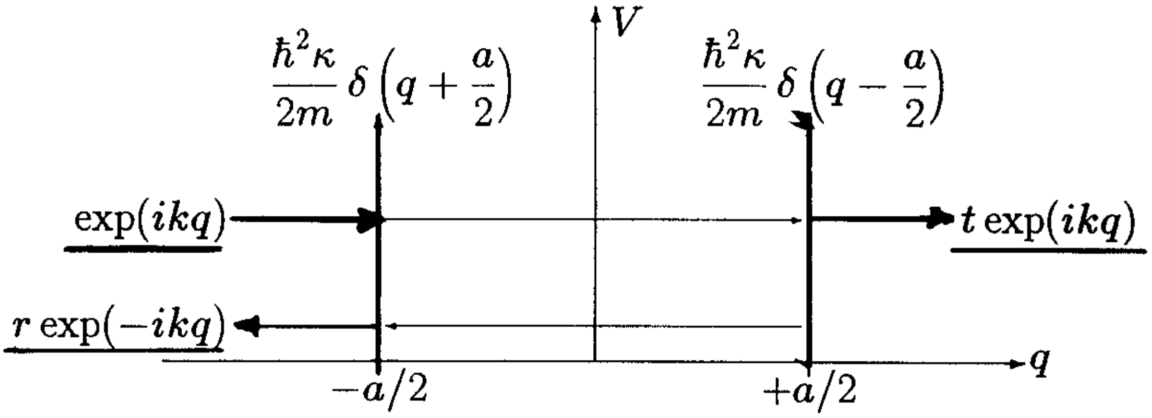
Spatially different contours $\mathcal{C}_1, \mathcal{C}_2$ → closed contour $\mathcal{C}$

Aharonov–Bohm effect:

$$\Delta S = e\int_{C_1} A(q)dq - e\int_{C_2} A(q)dq = e\oint_{C} A(q)dq$$
$$= e\int B(q)\, d^2q = e\,\phi = 2\pi\hbar\frac{\phi}{\phi_0}$$

transversal interference

Resonant tunnelling structure:



$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{\hbar^2\kappa}{2m}\left[\delta\left(\hat{q}+\frac{a}{2}\right) + \delta\left(\hat{q}-\frac{a}{2}\right)\right]$$

Probability of transmission through scattering region:

$$\boxed{T = \frac{1}{1+\gamma\cos^2(ka+\vartheta)}}$$

$$\gamma = (\kappa^2/k^2)(1+\kappa^2/4k^2) \quad , \quad \vartheta = \arctan(\kappa/2k)$$

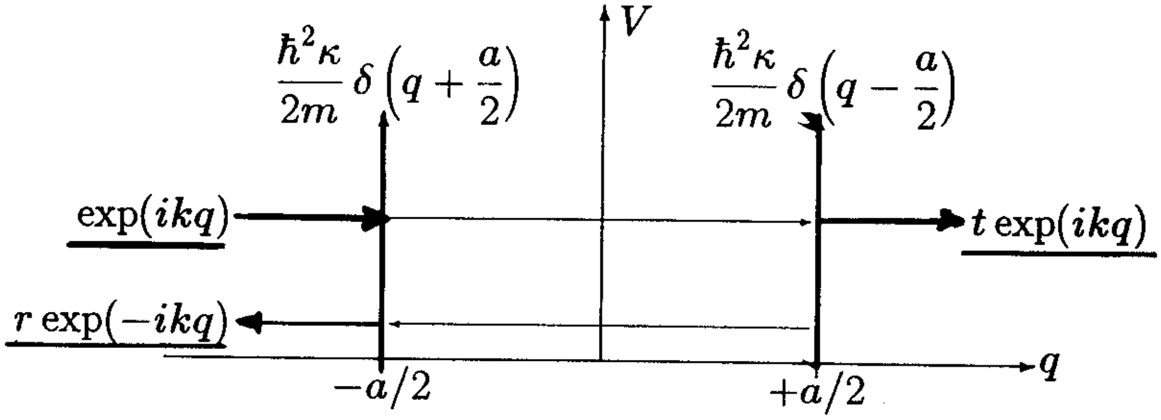
periodic oscillations of $T$ with varying $a$.

Aharonov–Bohm effect:

$$\Delta S = e\int_{C_1} A(q)dq - e\int_{C_2} A(q)dq = e\oint_{C} A(q)dq$$
$$= e\int B(q)\, d^2q = e\,\phi = 2\pi\hbar\frac{\phi}{\phi_0}$$

transversal interference

## Resonant tunnelling structure:



$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{\hbar^2\kappa}{2m}\left[\delta\left(\hat{q}+\frac{a}{2}\right)+\delta\left(\hat{q}-\frac{a}{2}\right)\right]$$

Probability of transmission through scattering region:

$$\boxed{T = \frac{1}{1+\gamma\cos^2(ka+\vartheta)}}$$

$$\gamma = (\kappa^2/k^2)(1+\kappa^2/4k^2) \quad , \quad \vartheta = \arctan(\kappa/2k)$$

periodic oscillations of $T$ with varying $a$.

Consider the transition amplitude

$$A_{fi} = \left\langle \psi_f \left| \exp\left(-\frac{i}{\hbar}\hat{H}t\right) \right| \psi_i \right\rangle$$

This can be expressed in the path integral form

$$A_{fi} = \int_{-\infty}^{-a/2} dq_i \int_{a/2}^{\infty} dq_f \int_{(q_i)}^{(q_f)} \mathcal{D}q(\tau) \exp\left\{\frac{i}{\hbar} S\left[q(\tau)\right]\right\} \exp\left[ik(q_i - q_f)\right]$$

Semi–classical approximation → *classical* paths

Trajectories with $p = \hbar k = \text{const}!$

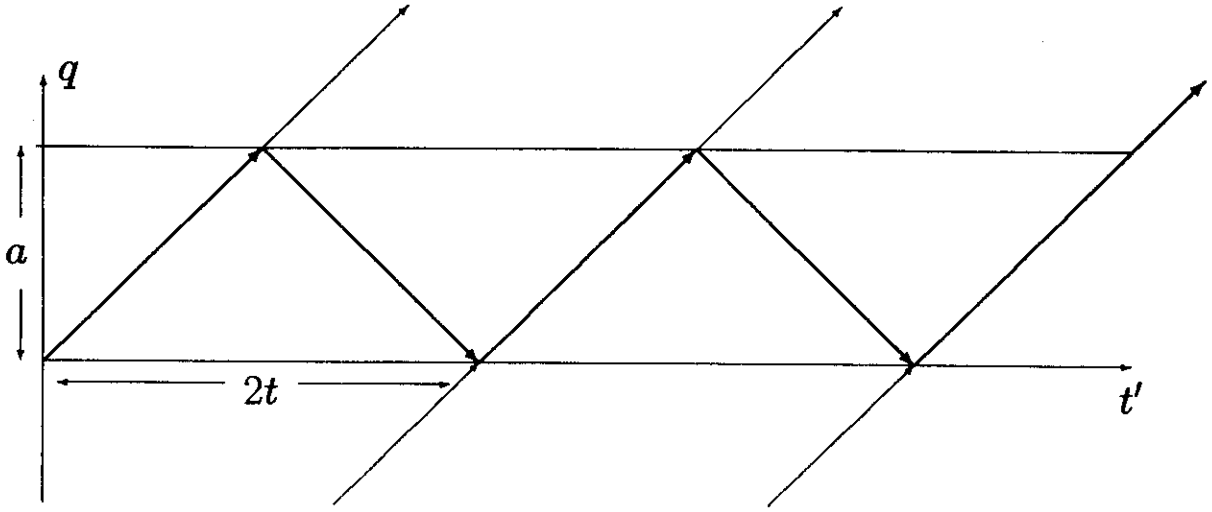
$$A_{fi} \propto \int_{-\infty}^{-a/2} dq_i \int_{a/2}^{\infty} dq_f \sum_{\{q_{\text{cl}}\}} \exp\left\{\frac{i}{\hbar} S\left[q_{\text{cl}}(\tau)\right]\right\} e^{ik(q_i - q_f)}$$

Saddle points are defined by

$$\frac{\partial}{\partial q_i}\left\{kq_i + \frac{1}{\hbar} S\left[q_{\text{cl}}(\tau)\right]\right\} = 0 \quad \text{and}$$

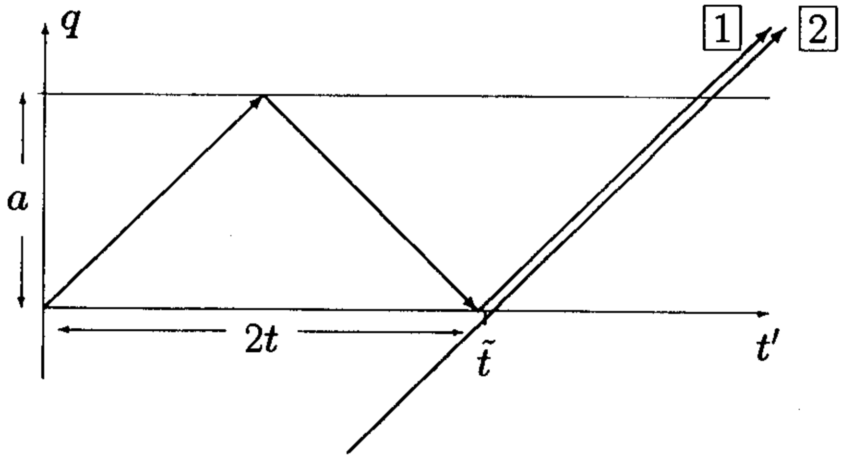
$$\frac{\partial}{\partial q_f}\left\{-kq_f + \frac{1}{\hbar} S\left[q_{\text{cl}}(\tau)\right]\right\} = 0$$

Leads to $k = k_f = k_i$



Phase difference between two "neighbouring" paths:

$$\Delta\varphi = \omega(k)\,\Delta t = \omega(k)\frac{2a}{v(k)} = ka$$



Both paths travel through the same spatial region
$\Longrightarrow$ *longitudinal* interferences

## IV. Suppression of interferences

Caldeira–Leggett–Lagrangian:

$$L = L_S(q,\dot{q}) + \frac{1}{2}\sum_i m_i\left(\dot{x}_i^2 - \omega_i^2 x_i^2\right) - q\sum_i C_i x_i + \frac{1}{2}q^2\sum_i \frac{c_i^2}{m_i\omega_i^2}$$

Transition probability from $q_i$ to $q_f$ in time $t$:

$$P(q_f,q_i;t) = \int \mathcal{D}q^{(2)}(\tau)\int \mathcal{D}q^{(1)}(\tau) \times \exp\left\{\frac{i}{\hbar}S_S\left[q^{(2)}(\tau)\right] - \frac{i}{\hbar}S_S\left[q^{(1)}(\tau)\right] - \Phi\left[q^{(2)}(\tau),q^{(1)}(\tau)\right]\right\}$$

Endpoints: $q^{(1)}(0) = q^{(2)}(0) = q_i$ ; $q^{(1)}(t) = q^{(2)}(t) = q_f$

Feynman–Vernon influence functional:

$$\boxed{\Phi\left[q^{(2)},q^{(1)}\right] = \frac{1}{\hbar}\int_0^t d\tau_2\int_0^{\tau_2} d\tau_1\left[q^{(2)}(\tau_2) - q^{(1)}(\tau_2)\right] \times\left[\gamma(\tau_2-\tau_1)q^{(2)}(\tau_1) - \gamma^*(\tau_2-\tau_1)q^{(1)}(\tau_1)\right]}$$

with

$$\gamma(\tau) = \frac{1}{\pi}\int_0^\infty d\omega\, J(\omega)\left\{\exp(-i\omega\tau) + \frac{2\cos(\omega\tau)}{e^{\hbar\beta\omega}-1}\right\}$$

Ring device: transversal interferences

Cross–terms are absent for statistically independent environments (same spectral density)

$$\Phi\left[q^{(2)},q^{(1)}\right]=\frac{1}{\hbar}\int_0^t d\tau_2\int_0^{\tau_2}d\tau_1\,\Big[q^{(2)}(\tau_2)\gamma(\tau_2-\tau_1)q^{(2)}(\tau_1)\\ +q^{(1)}(\tau_2)\gamma^*(\tau_2-\tau_1)q^{(1)}(\tau_1)\Big]$$

Semiclassical approximation:

- classical paths $\dot{q}_{\mathrm{cl}}=v=\mathrm{const}$

Measure of "*decoherence*":

$$\boxed{D=\left|\frac{P_{fi}^{(\mathrm{int})}}{P_{fi}^{(\mathrm{n-int})}}\right|=\exp\left(-\Phi_{\mathrm{trans}}\right)}$$

with

$$\boxed{\Phi_{\mathrm{trans}}=\left[\frac{2}{\hbar}\int_0^t d\tau_2\int_0^{\tau_2}d\tau_1\,q_{\mathrm{cl}}(\tau_2)\gamma^{(\mathrm{r})}(\tau_2-\tau_1)q_{\mathrm{cl}}(\tau_1)\right]}$$

## *Longitudinal* interferences:

Interfering paths are influenced by the same environment

With $s = (q_2 + q_1)/2, r = (q_2 - q_1)$, the real part of $\Phi$ takes the form

$$\Phi^{(r)}_{\text{long}} = \frac{1}{2\hbar} \int_0^t d\tau_2 \int_0^t d\tau_1 \, r_{\text{cl}}(\tau_2)\gamma^{(r)}(\tau_2 - \tau_1) r_{\text{cl}}(\tau_1)$$

Rough estimation:

$\Phi^{(r)}_{\text{long}}$ depends quadratically on $t$, while $\Phi^{(r)}_{\text{trans}}$ rather increases with a third power of $t$

$\Phi^{(r)}_{\text{long}}$ is translational invariant, $\Phi^{(r)}_{\text{trans}}$ not

$\Longrightarrow$ No *universal* decay of interferences in this model.

New model:

$$\boxed{L_C = -\left(\sum_i C_i x_i\right)\ \lambda\ \underbrace{R\left(\frac{q-q_0}{\lambda}\right)}_{\text{cut–off}}}$$

Particle interacts with the modes centered at $q_0$ only within an interval of length $\lambda$ around $q_0$
Translational invariance restored by continuous distribution of localized modes

$$\boxed{L_C = -\sum_{q_0}\left(\sum_i C_{i,q_0} x_{i,q_0}\right)\lambda\, R\left(\frac{q-q_0}{\lambda}\right)}$$

Influence function $\Phi$ is the sum (integral) over all individual oscillators:

$$\boxed{\begin{aligned}\Phi^{(\mathrm{r})} = \frac{1}{\hbar}\int_0^t d\tau_2 \int_0^{\tau_2} d\tau_1\, \gamma^{(\mathrm{r})}(\tau_2-\tau_1) \\ \times \sum_{i,j=1}^{2}(-1)^{i+j}\, G\left(q^{(i)}(\tau_2)-q^{(j)}(\tau_1)\right)\end{aligned}}$$

with

$$\boxed{\begin{aligned}G(q_1-q_2) &= \int_{-\infty}^{+\infty}\frac{dq_0}{\lambda}\,\lambda R\left(\frac{q_1-q_0}{\lambda}\right)\,\lambda R\left(\frac{q_2-q_0}{\lambda}\right) \\ &:= \lambda^2 g\left(\frac{\Delta q}{\lambda}\right) \qquad \Delta q \equiv q_1 - q_2\end{aligned}}$$

$G(\Delta q)$ decays also on the length scale $\lambda$

## Transversal model:

Only the "self-interactions" of the paths are non-vanishing

$$\Phi^{(\mathrm{r})}_{\mathrm{trans}} = \frac{1}{\hbar}\int_0^t d\tau_2 \int_0^{\tau_2} d\tau_1\, \gamma^{(\mathrm{r})}(\tau_2-\tau_1) \times \sum_{i=1}^{2} G\left(q^{(i)}(\tau_2) - q^{(i)}(\tau_1)\right)$$

## Resonant–tunnelling structure:

Particle is in contact with the environment only when travelling between the barriers

$$\Phi^{(\mathrm{r})}_{\mathrm{long}} = \frac{1}{\hbar}\int_0^{2t} d\tau_2 \int_0^{\tau_2} d\tau_1\, \gamma^{(\mathrm{r})}(\tau_2-\tau_1) \times G\left(q^{(1)}(\tau_2) - q^{(1)}(\tau_1)\right)$$

Particle is affected twice by every oscillator near to its path

→ Interactions between the forward and backward path

However, when $\gamma^{(\mathrm{r})}(\tau)$ decays on a time–scale which is short compared to the traversal time, the correlations between the forward and backward path are negligible

Then:

$$\boxed{\Phi^{(\mathrm{r})}_{\mathrm{long}} = \Phi^{(\mathrm{r})}_{\mathrm{trans}}}$$

Spectral density:

$$J(\omega) = \eta_s \omega_c^{1-s} \boxed{\omega^s} e^{-\omega/\omega_c}$$

Semiclassical: $q_{\text{cl}}(\tau) = v\tau$

Narrow cut–off function $G(q)$: $\lambda \ll a$

$$\begin{aligned}\Phi^{(\text{r})}\left[q_{\text{cl}}\right] &= \frac{1}{2\hbar} \int_0^t d\tau_2 \int_0^t d\tau_1\, \gamma^{(\text{r})}(\tau_2 - \tau_1) G\left[v(\tau_2 - \tau_1)\right] \\ &= \frac{t}{2\hbar} \int_{-\infty}^{+\infty} d\tau\, \gamma^{(\text{r})}(\tau) \lambda^2 g\left(\frac{v\tau}{\lambda}\right)\end{aligned}$$

Final result:

$$\Phi^{(\text{r})} = \frac{1}{2}\left(\frac{a}{\lambda}\right)\left(\lambda^2 \frac{\eta_s}{\hbar}\right)\left(\frac{\lambda}{v/\omega_c}\right)^{1-s} J_s\left(\hbar\beta\omega_b\right)$$

with

$$J_s\left(\hbar\beta\omega_b\right) = \int_0^\infty dx\, x^s \coth\left(\frac{\hbar\beta\omega_b}{2} x\right) I(x), \quad \omega_b = \frac{v}{\lambda}$$

$$I(x) = \int_{-\infty}^{+\infty} dy\; g(y) \cos(yx)$$

$\Phi^{(\text{r})}$ is linear in $a$ $\Longrightarrow$ *coherence length*

$$\boxed{l_c = \frac{a}{\Phi^{(\text{r})}}}$$

Zero temperature limit:

$$\Phi^{(\mathrm{r})} = \frac{\tilde{J}_s}{2} \left(\frac{a}{\lambda}\right) \left(\lambda^2 \frac{\eta_s}{\hbar}\right) \left(\frac{\omega_c}{\omega_b}\right)^{1-s}$$

with

$$\tilde{J}_s = J_s(\hbar\beta\omega_b \to \infty)$$

High–temperature limit ($\hbar\beta\omega_b \ll 1$):

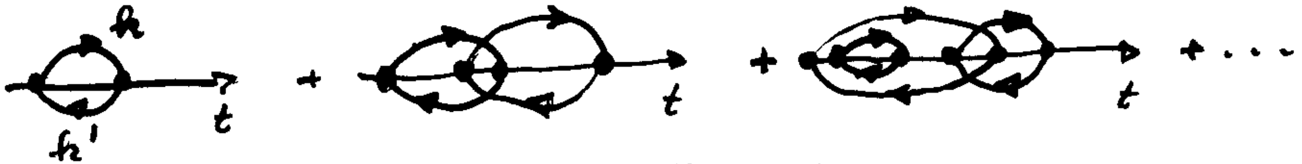
$$\Phi^{(\mathrm{r})} = \tilde{J}_{s-1} \left(\frac{a}{\lambda}\right) \left(\lambda^2 \frac{\eta_s}{\hbar}\right) \left(\frac{\omega_c}{\omega_b}\right)^{1-s} \left(\frac{1}{\hbar\beta\omega_b}\right)$$

Ratio of the coherence length in the high–temperature and zero temperature limit

$$\boxed{\frac{l_c^{(\mathrm{HTL})}}{l_c^{(0)}} = \frac{\tilde{J}_s}{2\tilde{J}_{s-1}} \frac{\hbar v}{k_\mathrm{B} T \lambda}}$$

Ohmic case ($s = 1$):

$$\begin{aligned} l_c^{(0)} &= \text{const.}(v) \\ l_c^{(\mathrm{HTL})} &\propto v \\ t_c^{(\mathrm{HTL})} &:= \frac{l_c^{(\mathrm{HTL})}}{v} = \text{independent of } v \end{aligned}$$



## V. Example: metallic environment

Heavy charged particle moving in a normally conducting metallic environment

$\Longrightarrow$

Electron–hole excitations dragged behind the charge carriers

$$\boxed{\begin{aligned}\Phi^{(\mathrm{r})} = \frac{1}{\hbar^2} \sum_{\vec{k},\vec{k}',\sigma,\sigma'} & \left|\left\langle \vec{k},\sigma \,|\, U \,|\, \vec{k}',\sigma' \right\rangle\right|^2 f_{\vec{k}}\left(1 - f_{\vec{k}'}\right) \\ & \times \mathrm{Re} \int_0^t d\tau_2 \int_0^{\tau_2} d\tau_1 \, \exp\left[-i\left(\omega_{\vec{k}} - \omega_{\vec{k}'}\right)(\tau_2 - \tau_1)\right] \\ & \times \sum_{i,j=1}^{2} (-1)^{i+j} \exp\left[i\left(\vec{k} - \vec{k}'\right)\left(\vec{q}^{(\mathrm{i})}(\tau_2) - \vec{q}^{(\mathrm{j})}(\tau_1)\right)\right]\end{aligned}}$$

$\langle \vec{k},\sigma | \, U \, | \vec{k}', \sigma' \rangle$ is the generally spin–dependent scattering potential and $f_{\vec{k}}$ is the Fermi distribution function

Contact potential: $\langle \vec{k},\sigma | \, U \, | \vec{k}', \sigma' \rangle = \delta_{\sigma\sigma'} U_0$

Ohmic spectral density $\quad J(\omega) = \eta\omega \, e^{-\omega/\omega_c} \quad$ with

$$\eta = \frac{4\hbar}{3\pi} k_{\mathrm{F}}^2 \sin^2\left(\frac{\pi U_0 N_0}{\hbar}\right)$$

$N_0$: density of states of the quasiparticles at the Fermi energy

$$\boxed{G(\Delta q) = \frac{3}{2k_{\mathrm{F}}^2}\left(\frac{\sin(k_{\mathrm{F}}\Delta q)}{k_{\mathrm{F}}\Delta q}\right)^2}$$

Boson–type electron–hole excitations in a fermionic environment can be described in terms of a harmonic bath with localized modes

After few calculations:

$$J_1(\hbar\beta\omega_b) = \frac{3}{2}\int_0^2 dx\, x \coth\left(\frac{\hbar\beta\omega_b}{2}x\right)(2-x)$$

This gives

$$\boxed{\frac{l_c^{(\mathrm{HTL})}}{l_c^{(0)}} = \frac{1}{3}\frac{\hbar v k_{\mathrm{F}}}{k_{\mathrm{B}}T}}$$

# VI. Summary

- Concept of "*phase memory*" cannot be applied to system–plus–reservoir models
  $\Longrightarrow$ Phenomenological models of inelastic free path cannot be justified by a fully quantized calculation

- Two fundamental groups of interfering paths:
  *longitudinal* and *transversal* interferences

- Longitudinal and transversal interferences are subject to different suppression mechanisms
  $\Longrightarrow$ System–plus–environment models do, in general, not lead to universal decay of coherence
  $\Longrightarrow$ Impossibility of defining a *coherence length* or *decoherence time*

- Shortcomings are removed by introducing an environment with localized modes
  $\Longrightarrow$ Universal decay of interferences
  $\Longrightarrow$ *Coherence length* and *decoherence time* can be defined