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AUTUMN COURSE ON GEOMAGNETISM, THE IONOSPHERE
AND MAGNETOSPHERE

(21 September - 12 November 1982)

"ROTATING FLUIDS IN GEOPHYSICS AND PLANETARY PHYSICS"

"THE GIANT PLANETS"

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Rotating Fluids in Geophysics and Planetary Physics*

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(Received 1981 December 22)

1 INTRODUCTION

It was with great pleasure that I accepted the Royal Astronomical Society's invitation to deliver the 1981 Harold Jeffreys Lecture. A general talk about fluid motions that are strongly influenced by Coriolis forces, due to general rotation relative to an inertial frame of reference, seems appropriate to the occasion. Amongst Jeffreys' wide-ranging scientific contributions are several papers on the dynamics of rotating fluids, including a seminal study of the general circulation of the atmosphere published over half a century ago (1). In my own scientific work I have been largely concerned for over 30 years with experimental and theoretical studies of basic hydrodynamic and magnetohydrodynamic processes in rotating fluids and with various applications in Earth and planetary sciences and astronomy.

Several general results concerning 'geostrophic' flows characterized by an approximate balance between dynamic pressure gradients and Coriolis forces follow directly from arguments based on the equations of motion and the findings of certain laboratory experiments with rotating fluids (see Section 2). These results provide considerable insight into the nature of many natural phenomena, such as the highly irregular meandering jet streams that characterize mid-latitude flows in the Earth's atmosphere, the stable closed eddies (including the Great Red Spot and three White Ovals) seen in Jupiter's atmosphere, and the equatorial jet streams and western boundary currents (such as the Gulf Stream in the Atlantic Ocean and the East-African cross-equatorial atmospheric jet stream) found in oceans and atmospheres.

I shall also discuss flows that are strongly influenced not only by Coriolis forces due to general rotation but also by Lorentz forces due to the presence of electric currents in the fluid. It is necessary to investigate a wide range of such flows in the study of planetary and stellar magnetism. Of particular interest are the so-called 'magnetostrophic' flows, for which Coriolis and Lorentz forces can be comparable with each other in magnitude but act in opposite directions (see Section 3). The slow and highly dispersive 'magnetohydrodynamic inertial wave', with a frequency which depends on the square of the Alfvén speed and inversely on the rotation rate of the system, exemplifies magnetostrophic flow. It is very likely that such waves occur in the electrically conducting fluid interiors of the Earth and other magnetic planets, and also in the Sun and other magnetic stars, including pulsars.

*The 1981 Harold Jeffreys Lecture.

Finally I shall outline results of recent observational and theoretical work on the distribution of angular momentum within the Earth, including the recognition from analyses of length-of-day data and meteorological wind and pressure observations of an apparently persistent fluctuation in the angular momentum of the Earth's atmosphere on a time-scale of about two months (see Section 4). Transfer of angular momentum between the Earth's solid parts (inner core, mantle and crust) and fluid parts (outer core, oceans and atmosphere) produces polar motion and changes in the length of the day, which are monitored by astronomers with increasing accuracy. The interpretation of these observations presents many fascinating and novel problems in theoretical geophysics, meteorology and oceanography, including several in the study of the hydrodynamics and magnetohydrodynamics of rotating fluids*.

Hydrodynamics and magnetohydrodynamics are branches of classical physics, but they developed quite slowly until comparatively recently. Useful advances in our knowledge of the behaviour of rotating fluids might have been made in the last century had contemporary scientists appreciated the importance of the discovery, from analyses of meteorological observations, of 'geostrophic' motion in the atmosphere. In the event, they evidently played down the discovery if the attitude of R.H.Scott, as revealed in a lecture delivered at the Royal Institution in 1869, can be regarded as typical. 'A principle has been much before the public of late which was first urged by Professor Buys-Ballot of Utrecht. It may be stated as follows: Stand with your back to the wind and the barometer will be lower on your left than on your right (in the northern hemisphere). No matter how gently the wind blows, the law is found to be true. *This fact, however, is of no use to us in enabling us to judge the coming weather*' [my italics]. Scott, who succeeded Robert Fitzroy as head of the Meteorological Office (where Jeffreys worked for several years in the 1920s), made pioneering contributions to international co-operation in practical meteorology and the present World Meteorological Organisation owes much to Scott's early efforts. But it was left to others to demonstrate, much later, that the concept of geostrophy was of central theoretical importance in dynamical meteorology.

2 GEOSTROPHIC FLOWS

The motion of a fluid of low viscosity (and negligible electrical conductivity, see Section 3) that departs but little from solid body rotation with angular velocity Ω is usually geostrophic nearly everywhere, with the relative

*The lecture was illustrated with many slides and several short films showing the phenomena mentioned in the introduction. It is impossible to reproduce this visual material here and without the material a verbatim account of the whole lecture would be of little value. This written account presents a few basic ideas and results without attempting to provide a comprehensive and critical review of any of the topics mentioned. References to the original work can be found in the bibliographies given in the recent publications cited in the text and listed at the end of this paper.

Eulerian velocity $\mathbf{u}$ (as measured in a frame of reference that rotates with angular velocity $\boldsymbol{\Omega}$ relative to an inertial frame) satisfying

$$2\rho\boldsymbol{\Omega}\times\mathbf{u} = -\nabla p + \rho\mathbf{g}. \qquad (2.1)$$

Here ρ denotes density, p pressure and $\mathbf{g}$ is the acceleration due to gravity and centripetal effects. Equation (2.1) is the leading approximation to the full equation of motion

$$\rho[D\mathbf{u}/Dt + 2\boldsymbol{\Omega}\times\mathbf{u} - \mathbf{r}\times d\boldsymbol{\Omega}/dt] = -\nabla p + \rho\mathbf{g} - \nabla\times(\nu\rho\nabla\times\mathbf{u}) + \mathbf{j}\times\mathbf{B}. \quad (2.2)$$

It is valid in regions where the Coriolis term $2\rho\boldsymbol{\Omega}\times\mathbf{u}$ greatly exceeds the relative acceleration term $\rho D\mathbf{u}/Dt \equiv \rho(\partial\mathbf{u}/\partial t + (\mathbf{u}\cdot\nabla)\mathbf{u})$ (where t denotes time), the precessional term $\rho\mathbf{r}\times d\boldsymbol{\Omega}/dt$ (where $\mathbf{r}$ is the position vector), the viscous term $\nabla\times(\nu\rho\nabla\times\mathbf{u})$ (where ν denotes kinematic viscosity) and the Lorentz term $\mathbf{j}\times\mathbf{B}$ (which vanishes when there is no electric current, density $\mathbf{j}$, or magnetic field $\mathbf{B}$, see Section 3).

Buys-Ballot's law follows immediately from equation (2.1), which shows that the horizontal fluid velocity is directed at right angles to the horizontal component of the pressure gradient, with an average error of the order of the ratio of the neglected (ageostrophic) terms to the Coriolis term. This amounts to about 5 or 10 per cent for large-scale motions in the Earth's atmosphere and much less for the oceans.

Equation (2.1) leads to the important result that (2):

The hydrodynamical motion of a fluid of low viscosity that departs only slightly from steady rigid body rotation will not in general be symmetric about the rotation axis, even when the boundary conditions are axisymmetric. (2.3)

The validity of this result, which provides the most direct explanation of the occurrence of large-scale non-axisymmetric disturbances in the Earth's atmosphere and other natural systems, is readily verified by laboratory experiments. The result can be deduced as follows. In cylindrical coordinates (r, ϕ, z) where $\boldsymbol{\Omega} = (0, 0, \Omega)$ the second component of equation (2.2) (*cf.* equation 2.1) is:

$$u_r = -(2\rho\Omega r)^{-1}\partial p/\partial\phi + A_\phi \qquad (2.4)$$

(since $(\mathbf{g})_\phi = 0$ by the assumption of axial symmetry in the boundary conditions) where A_ϕ denotes the sum of all the ageostrophic terms. Now, over any cylindrical surface of radius r the rate of advective transport $H(r, t; Q)$ of any quantity Q (per unit volume), such as heat, angular momentum, etc., is given by

$$H(r, t; Q) = \int_{z_1}^{z_2}\int_0^{2\pi} u_r Q r d\phi dz = \int_{z_1}^{z_2}\int_0^{2\pi}\left[-\frac{1}{2\rho\Omega}\frac{\partial p}{\partial\phi} + rA_\phi\right] Q d\phi dz. \qquad (2.5)$$

Since the contribution A_ϕ to equation (2.4) decreases rapidly with increasing Ω, advective transport perpendicular to the axis of rotation, as measured by $H(r, t; Q)$, will be negligible unless the flow pattern departs significantly from axial symmetry. In the axisymmetric case we have $\partial p/\partial\phi = 0$ and $H(r, t; Q)$ of the order of the small ageostrophic contribution.

This argument is the basis of (2.3). There may be singular cases when the flow remains axisymmetric and in consequence advective transfer perpen-

dicular to the rotation axis is negligible. Indeed, such cases can be realised in the laboratory by taking certain special precautions, but the general conclusion from laboratory experiments is that (2.3) is a correct inference from the geostrophic equation. Owing to departures from axial symmetry in the properties of the Earth's surface, it is not immediately obvious why large-scale mid-latitude atmospheric motions, which advect heat from tropical to polar regions, are highly non-axisymmetric. But meteorologists now accept that this lack of axial symmetry in atmospheric motions is a direct consequence of the Earth's rapid rotation.

There is a further property of equation (2.1) that leads to a useful general prediction. The equation is mathematically degenerate; being lower in order than the full equation (2.2), to which it is a leading approximation when Ω is large, it cannot be solved under all the necessary boundary conditions. For this to be possible every term in equation (2.2) must be included in the analysis, which implies that (2):

Regions of highly agcostrophic flow occurring not only on the boundaries of the system but also in localized regions (detached shear layers, jet streams, etc.) of the main body of the fluid are necessary concomitants of geostrophic motion. (2.6)

Within these highly ageostrophic regions $\rho D\mathbf{u}/Dt + \nabla\times(\rho\nu\nabla\times\mathbf{u})$ is comparable in magnitude with $2\rho\boldsymbol{\Omega}\times\mathbf{u}$; the corresponding relative vorticity $\nabla\times\mathbf{u}$ can be comparable with or even exceed $2\boldsymbol{\Omega}$ in magnitude. Many examples of such vorticity concentrations are found in the laboratory and in nature. They are often associated with steep gradients of temperature (thermal fronts), as in jet streams and western boundary currents found in the atmosphere and oceans. Western boundary currents such as the Gulf Stream, Kuroshio Current and Somali Current in the oceans and the low-level cross-equatorial atmospheric jet stream off East Africa arise as a result of effects due to the combined influence of the Earth's rotation and curvature. Such effects, which are readily produced in the laboratory, also give rise to strong westerly (i.e. positive) equatorial jet streams in the atmospheres of Jupiter, Saturn and the Sun and in the terrestrial oceans (2, 3).

We have seen that slow relative hydrodynamical flow in a rotating fluid of low viscosity will in general be non-axisymmetric (see 2·3). Laboratory studies show that there are two non-axisymmetric regimes of thermal convection in a rotating fluid annulus subject to differential heating in the horizontal, one highly regular (i.e. spatially and temporally periodic) and the other, which is reminiscent of large-scale flow in the Earth's atmosphere, irregular (2, 4). Thus, when the basic rotation rate Ω of the fluid annulus exceeds a certain value Ω_R Coriolis forces inhibit axisymmetric overturning motions in meridian planes and promote a completely different kind of motion, which has been termed 'sloping convection'. The motion is then non-axisymmetric and largely confined to jet streams, with typical trajectories of individual fluid elements inclined at small but essentially non-zero angles to the horizontal. The kinetic energy of the non-axisymmetric flow derives from the interaction of slight upward and downward motions in these sloping trajectories with the potential energy field produced by the action of gravity on the density variations produced by the applied differential heating. The kinetic energy of the motion is dissipated by friction arising in boundary layers on the walls of the container and in the main body of the fluid. The critical value Ω_R of the rotation speed is of course dependent on many parameters, including the acceleration of gravity, the shape and dimensions of the apparatus, the coefficients of thermal expansion, thermal conductivity and viscosity of the fluid and its mean density, and the distribution and intensity of applied differential heating. This dependence has been determined by extensive laboratory studies and interpreted on the basis of stability theory (**2, 4, 5, 6**).

Provided that Ω, though greater than Ω_R, does not exceed a second critical value Ω_I, the main features of the non-axisymmetric motion are characterized by great regularity and the heat flow is virtually independent of Ω and some 20 per cent less than its value when $\Omega = 0$. This regular flow is either steady (apart from a slow steady drift of the horizontal flow pattern relative to the walls of the container) or it exhibits periodic 'vacillation' in amplitude, shape and other characteristics. The numbers of 'waves' m around the annulus is not uniquely determined by the impressed conditions; the flow is found to be 'intransitive' owing to the occurrence of what are now called 'multiple equilibrium states'. But the most likely value of m tends to increase with increasing Ω and when $\Omega = \Omega_I$, m has that value for which the azimuthal scale of the horizontal flow pattern is about 1·5 times the radial scale and the flow undergoes a transition to irregular flow or 'geostrophic turbulence'. When $\Omega > \Omega_I$ we have the irregular flow regime, for which heat flow decreases with increasing Ω (**2**).

The importance of these findings for theories of large-scale atmospheric motions and their predictability and for certain astrophysical studies (**7, 8**) is now recognized. Many laboratory studies of various aspects of sloping convection have been carried out, including determinations of heat transfer, flow structure and regime transitions over a wide range of mechanical and thermal boundary conditions. Numerical studies based on the governing mathematical equations are now playing an increasingly important rôle in this work and significant if more limited analytical studies have also been made. It is noteworthy that, despite the essential non-linearity of these equations, the main features of the non-axisymmetric flow patterns can be interpreted by straightforward arguments based on general thermodynamic considerations and the requirement that the flow should be quasi-geostrophic nearly everywhere. Thus, when the distribution of applied heating and cooling is such that the corresponding gradient of the impressed temperature field has the same sign at all radii, the most conspicuous feature of the upper-level flow pattern in the regular non-axisymmetric regime is a continuous jet stream meandering in a wavy pattern between the bounding cylinders. When, however, the impressed radial temperature gradient changes sign near mid-radius, as in the case when heat is introduced throughout the body of the fluid (by passing a weak alternating electric current through the fluid) and withdrawn at both side-walls, the corresponding upper surface flow consists of several closed eddies, each circulating 'anticyclonically', with the horizontal motion largely confined in a narrow jet stream at the periphery (**2**).

These general characteristics have their counterparts in atmospheric flows. The meandering jet streams within which the upper-level tropospheric flow of air is mainly concentrated in the Earth's atmosphere are manifestations of sloping convection produced by differential solar heating, which maintains a systematic temperature contrast between tropical and polar regions in each hemisphere. These atmospheric motions are highly irregular (and therefore unpredictable in detail over long periods of time), presumably because the Earth's angular speed of rotation exceeds the critical value Ω_I (which depends, amongst other things, on the depth of fluid), although, as has already been mentioned, non-axisymmetric variations in surface conditions introduce complications which are not yet fully understood (**2**).

Infrared observations of the planet Jupiter indicate that the Jovian atmosphere is heated from below at about the same rate as its upper reaches are heated by solar radiation. Unlike the terrestrial case (where non-solar atmospheric heating is utterly negligible), north-south temperature gradients in Jupiter's atmosphere change sign several times between the equator and pole and there is no evidence of any significant temperature contrast between equatorial and polar regions (**2, 9, 10**). There are abundant observations of Jovian atmospheric motions at upper cloud level, some of which go back many decades and even longer (**9**), and the *Pioneer* and *Voyager* spaceprobes have added further details (**11**). But our knowledge of what goes on below the cloud level is meagre and this produces difficulties with the interpretation of observations of upper-level atmospheric motions. Indeed, it can be argued that the main task of the 'Jovian meteorologist' should perhaps be to use these observations to improve our knowledge of the vertical structure of the planet. But here is not the place to discuss these observations of Jupiter in detail and review the many interesting though largely controversial issues being debated by those of us who take an interest in the interpretation of the observations in terms of basic dynamical processes (**12**). There is, however, one striking phenomenon upon which the above-mentioned laboratory experiments on sloping convection in a fluid annulus subject to internal heating might have some bearing (**2, 13**). The highly stable closed anticyclonic eddies, with the mean motion concentrated in a jet stream at the periphery of each eddy, that are found in the experiments when the temperature gradient changes sign at mid-radius are remarkably similar dynamically to the long-lived eddies to be seen in Jupiter's atmosphere in the southern hemisphere. Of these Jovian eddies, the largest, oldest and most conspicuous is the Great Red Spot in the South Tropical Zone, which may be at least 300 years old. Next in size and age are the three White Ovals that formed in 1939 at the boundary between the South Temperate Belt and the South Temperate Zone, apparently as the residue of the highly irregular South Tropical Disturbance that was first seen in 1901. The smallest of the long-lived eddies are clearly seen in the magnificent *Voyager* pictures as about a dozen oval markings somewhat closer to the pole (**11**). The motion in each of these Jovian eddies is anticyclonic and in the case of the Great Red Spot it is largely confined to a narrow region at the edge of the eddy. It has been proposed that the eddies are manifestations of sloping convection in Jupiter's atmosphere, implying that they derive their kinetic energy directly from the potential energy due to the action of gravity acting on density variations produced by internal and solar heating, and that they transport heat from the middle parts towards the edges of the latitudinal bands in which they arise (2, 13).

Preliminary calculations indicate that there is nothing unreasonable about this hypothesis so far as its implications for the vertical structure and other properties of Jupiter's atmosphere are concerned, and a detailed examination of these implications and a critical comparison of the hypothesis with other proposals as to the nature of the long-lived anticyclonic eddies is now being prepared. The hypothesis (which also has implications for motions in the atmosphere of Saturn) raises a number of fluid-dynamical questions to be resolved by further laboratory and numerical work and some of this is now in hand. Amongst these questions is that of the instability of the strongly-sheared flow in the jet stream itself. There is experimental evidence that when viscous effects are sufficiently small the jet stream develops local instabilities on one side but not on the other. Pictures of Jupiter show highly irregular flow on a comparatively small scale just outside the Great Red Spot (and other long-lived eddies), but not on the inside (11). It will be important to establish by experiment and theory whether or not this highly irregular flow arises as a result of a 'one-sided' instability of the jet-stream at the edge of the main eddy*.

3 MAGNETOSTROPHIC FLOWS

The magnetic fields of the Earth and Sun, and of other magnetic planets and stars, are thought to be due to electric currents flowing within their interiors. It is now accepted that these currents are largely maintained against the effects of ohmic dissipation by electromotive forces due to motional induction, as Larmor first pointed out in his pioneering paper on self-exciting fluid dynamos (14, 15, 16). The fluid motions involved are produced in most (if not all) cases by the action of gravity on density variations (17, 18).

Application of the laws of Gauss, Faraday and Ampère, together with Ohm's law, to a moving medium leads to the following differential equation

*The so-called Taylor column theory of the Great Red Spot (9) was put forward by the writer in 1960 when Jeffreys' proposal that the planet was partially solid was still in vogue. According to that theory the Great Red Spot was the upper end of a columnar disturbance (associated with the geostrophic constraint) in Jupiter's atmosphere, anchored to some very shallow topographic feature of the underlying solid surface. A key element in the Taylor column theory was the discussion of the extensive east–west motion of the Spot (relative to its mean position) and its negligible north–south motion, in terms of angular momentum transfer between the fluid and solid parts of the 'model' planet. Indeed, one of the main attractions of the Taylor column theory was its ability to account for the motion of the Great Red Spot in a fairly unforced way. It is therefore somewhat puzzling to find in reviews and popular articles an often-repeated but thoroughly erroneous statement that the Taylor column theory is unacceptable because it fails to take the extensive east–west motion and the negligible north–south motion of the Spot into account. The best models of Jupiter's internal constitution now indicate that the planet is probably fluid throughout and *this* is the main reason why the Taylor column theory is no longer tenable. The writer's more recent suggestion (13) as outlined here is compatible with modern knowledge of Jupiter's atmosphere and interior and (like the Taylor column theory) is capable of being tested by experiment and observation, with the joint objectives of deepening our understanding of the structure and dynamics of Jupiter's atmosphere and extending our knowledge of basic hydrodynamical processes in rotating fluids.

for the magnetic field $\mathbf{B}$ at a general point at time t:

$$\partial\mathbf{B}/\partial t = -\nabla\times(\sigma^{-1}\nabla\times(\mu^{-1}\mathbf{B}))+\nabla\times(\mathbf{u}\times\mathbf{B}). \quad (3.1)$$

Here σ is the electrical conductivity and μ the magnetic permeability. This equation shows that a necessary (but not sufficient) condition for dynamo action is that the magnetic Reynolds number

$$R \equiv UL\bar{\mu}\bar{\sigma} \quad (3.2)$$

should exceed a certain critical value R^* (say), where R^* depends on the boundary conditions, etc. but is typically about 10 or 10^2. Here U is a typical flow speed, L a characteristic linear dimension of the system and $\bar{\mu}$ and $\bar{\sigma}$ are typical values of μ and σ respectively (15, 16).

The mathematical analysis of theoretical dynamo models is greatly complicated by the finding that suitable departures from axial symmetry seem to be required for dynamo action to occur. This follows from the existence theorems for self-exciting dynamos (15, 16) and the recent extension of Cowling's theorem (19) showing quite generally that:

The ohmic decay of a magnetic field that retains an axis of symmetry cannot be prevented by fluid motions. (3.3)

This result (which suggests, incidentally, that palaeomagnetic and archaeomagnetic data might be expected to show evidence that departures from axial symmetry are systematically less during the decay phase of a geomagnetic polarity 'reversal' or 'excursion' than during the growth or recovery phase (20)) when combined with (2.3) provides a useful starting point for discussing how rotation affects the generation of magnetic fields by the dynamo process. By promoting departures from axial symmetry when the magnetic field $\mathbf{B}$ is so weak that the magnitude of the Coriolis force per unit volume $2\rho\mathbf{\Omega}\times\mathbf{u}$ far exceeds that of the Lorentz force $\mathbf{j}\times\mathbf{B}$ (see equation (2.1) earlier and (3.3) above) – where $\mathbf{j}$ denotes the current density equal, by Ampère's law, to $\nabla\times(\mathbf{B}/\mu)$ – the field will be amplified by dynamo action (provided, of course, that $R > R^*$, see equation (3.2) above). As $\mathbf{B}$ increases in strength, the Lorentz force $\mathbf{j}\times\mathbf{B}$ grows, thereby raising the magnitude of the ageostrophic term A_ϕ in equation (2.4). If Lorentz forces eventually match Coriolis forces in magnitude the flow becomes approximately magnetostrophic, satisfying

$$2\rho\mathbf{\Omega}\times\mathbf{u} \doteqdot -\nabla p+\rho\mathbf{g}+\nabla\times(\mu^{-1}\mathbf{B})\times\mathbf{B}. \quad (3.4)$$

The study of magnetostrophic flows, particularly of certain classes of wave motions, sheds considerable light on the dynamics of the fluid interiors of planets and stars, including pulsars, and on the origin of planetary and stellar magnetism (5, 8, 15, 21, 22).

The near alignment between the axes of rotation and magnetic dipole moment of the Earth, Jupiter and Saturn is almost certainly a direct consequence of the strong influence of Coriolis forces on the fluid motions producing dynamo action, as several workers have suggested, but much remains to be done in the way of detailed studies of dynamos based on the equations of hydrodynamics and electrodynamics. In an attempt to set useful limits on the strength of the magnetic field produced by dynamo action the writer has argued on the basis of general considerations (2.2) and (3.1) that the *magnetic field is unlikely to build up beyond that value for which the Lorentz torque acting on an individual fluid element balances the appropriate acceleration term* (9). Whence, the average strength B_i of the magnetic field within the fluid satisfies

$$B_i \lesssim B_s R^{\frac{1}{2}}, \tag{3.5}$$

where B_s is the 'scale magnetic field strength' given by

$$B_s \equiv (\rho(\Omega + UL^{-1})/\bar{\sigma})^{\frac{1}{2}}. \tag{3.6}$$

Observe that the ratio of the magnetic to kinetic energy implied by equation (3.5) is $\lesssim (1+L\Omega/U)$ and may, therefore, be very large when the basic rotation rate is so rapid that the so-called Rossby number

$$Y \equiv U/L\Omega \tag{3.7}$$

is much less than unity. Equipartition between magnetic energy and kinetic energy is to be expected only when Y is very large; for the Earth's core the precise value of Y is not known but (as for the fluid interiors of other planets and satellites) it is certainly very much *less* than unity, possibly $\sim 10^{-6}$.

Now, although the rate of generation of total magnetic energy by the dynamo mechanism, and hence B_i, is expected to *increase* with increasing electrical conductivity σ, the strength B_e of the *external* magnetic field produced by the dynamo (and this is the only part of the field we are able to observe) should *decrease* with increasing σ when σ is large, with B_e vanishing altogether when σ is infinite, for it is impossible to change the magnetic flux linkage of a perfect conductor (23). It can therefore be supposed that

$$B_e/B_i \doteqdot R^{-q} \tag{3.8}$$

where the index q is essentially positive and possibly close to unity. Hence

$$B_e \lesssim B_s R^{-q+\frac{1}{2}} \equiv \hat{B}_e. \tag{3.9}$$

$\hat{B}_e$ is thus an upper limit to the strength of the magnetic field just above the fluid region where dynamo action is taking place. If $q = 1$ then $\hat{B}_e$ satisfies

$$B_s R^{-\frac{1}{2}} \lesssim \hat{B}_e \ll B_s R^{\frac{1}{2}}. \tag{3.10}$$

Corresponding expressions for the equivalent magnetic moment can be obtained from the equations (3.10) and (3.6) by multiplying by the cube of an appropriate length. Taking as typical values for the core of the Earth

$$\Omega \doteqdot 10^{-4}\ \text{rad s}^{-1},\quad \rho \doteqdot 10^4\ \text{kg m}^{-3},\quad \bar{\sigma} \doteqdot 3\times 10^5\ \text{Sm}^{-1},$$
$$U \doteqdot 10^{-4}\ \text{ms}^{-1},\quad L \doteqdot 10^6 \text{m}$$

we find that $Y \equiv U/L\Omega \doteqdot 10^{-6}$ and $B_s \doteqdot (\rho\Omega/\bar{\sigma})^{\frac{1}{2}} = 2\times 10^{-3}$T (20 Gauss). Accordingly, by equations (3.5) and (3.10) we have

$$B_i \lesssim 10^{-2}\text{T (100 Gauss)};\quad B_e \lesssim 4\times 10^{-4}\text{T (4 Gauss)}$$

if for the magnetic Reynolds number R we take a value suggested by various studies of kinematic dynamos of the Bullard–Gellman type (15, 16) namely $R \doteqdot 25$. Now, the mainly dipolar field of 5×10^{-5}T (0·5 Gauss) at the surface of the Earth implies that the average field strength just outside the core, B_e, is $\lesssim 10^{-3}$T (10 Gauss). This falls within the preferred range of B_e implied by the above discussion. So far as the value of B_i for the Earth's core is concerned, this cannot be inferred directly from geomagnetic observations. But various lines of evidence indicate that the magnetic field throughout the main body of the core might be largely toroidal in configuration, with lines of force lying approximately on horizontal surfaces, and $\sim 10^{-2}$T (100 Gauss) in strength, which is also concordant with the above calculation (9).

Other approaches to the problem of finding a useful expression for the magnetic moment produced by a dynamo in a rotating fluid have been taken by various writers. Levy and Roberts, for example, exploited a specific dynamo model due to Parker in which a key parameter is a 'turbulence correlation length' λ. Because the dependence of λ on Ω and other quantities is unknown it is hard to compare this approach with the one described above. Superficially equation (3.10) indicates a weaker $\Omega-$dependence for B_e than the expressions given by Levy and Roberts, but because they involve λ the difference between the two approaches may turn out to be more apparent than real. By *assuming* that B_i is no bigger than B_e, Busse was able to exploit a particular dynamo model based on the assumption that Lorentz forces are small in comparison with Coriolis forces, and thus obtain the result that $B_e \doteqdot B_i \doteqdot 10^{-2.5} L\Omega(\bar{\kappa}\bar{\mu}\bar{\sigma})^{\frac{1}{2}}$ where $\bar{\kappa}$ denotes the thermal diffusivity of the fluid (9).

By Faraday's law, the magnetic flux linkage of a perfect conductor cannot change (23), so that effects due to ohmic dissipation are central to dynamo theories of the generation of the external magnetic field (15, 16). On the other hand, such effects may not be of primary importance when dealing with secular changes, which occur on much shorter time-scales than the time-scale of free decay, and their neglect leads to a considerable simplification of the governing equations. The mathematical difficulties involved are still severe, especially when realistic boundary conditions are taken into account, and their discussion lies beyond the scope of this article (5, 21, 22). Fortunately, some of the main processes are exemplified by the properties of plane waves of angular frequency ω and vector wavenumber **k** propagating relative to a fluid which rotates uniformly with steady angular velocity **Ω** and is pervaded by a uniform magnetic field $\mathbf{V}(\mu\rho)^{\frac{1}{2}}$ where **V** is the Alfvén velocity. The dispersion relationship for such waves is the biquadratic equation

$$\omega^4 - \omega^2(2\omega_V^2 + \omega_\Omega^2) + \omega_V^4 = 0 \tag{3.11}$$

where $\omega_V^2 \equiv (\mathbf{V}\,.\,\mathbf{k})^2$ and $\omega_\Omega^2 \equiv (2\mathbf{\Omega}\,.\,\mathbf{k})^2/|\,\mathbf{k}\,|^2$,

with solutions

$$\omega^2 = \omega_V^2 + \tfrac{1}{2}\omega_\Omega^2 \pm [\tfrac{1}{4}\omega_\Omega^4 + \omega_V^2\omega_\Omega^2]^{\frac{1}{2}} \equiv \omega_+^2, \omega_-^2 \tag{3.12}$$

according as the upper or lower sign is taken. For wavelengths so short that $2\omega_V^2 \gg \omega_\Omega^2$ – which for the Earth's core (where $V \lesssim 10^{-1}$m s^{-1}) corresponds to motions on scales much less than 0·1 times the core radius – equation (3.12) reduces to the dispersion relationship for ordinary Alfvén waves, $\omega^2 = \omega_V^2$, which are non-dispersive, linearly polarized and characterized by equipartition between magnetic and kinetic energy. At the other extreme, when $2\omega_V^2 \ll \omega_\Omega^2$ – and this is the case of interest when dealing with waves in the core on scales $\doteqdot 10^6$m – Coriolis forces are so strong that the two roots of equation (3.12) have quite different values:

$$\omega_+^2 = \omega_\Omega^2; \quad \omega_-^2 = \omega_V^4/\omega_\Omega^2. \tag{3.13}$$

This extreme 'frequency splitting' due to rotation is accompanied by other effects, notably wave dispersion, circular or elliptical polarization of the trajectories of individual fluid elements, and imbalance of kinetic energy (the whole of which is now associated with the fast 'inertial mode') and magnetic energy (now entirely in the slow magnetostrophic 'magnetic mode').

When equations (3.13) are satisfied, the period of the *inertial mode* $2\pi/\omega_+$ is then typically $\gtrsim\pi/\Omega$ (i.e. a few days), whilst that of the *magnetic mode* $2\pi/\omega_-$ is $\sim 2\pi\Omega L^2/V^2$, which for the Earth's core when $L \sim 10^6$m is 10^{10}s (300 yr) and therefore comparable with the time-scale of the geomagnetic secular variation. This is the quantitative basis of the theory of the geomagnetic secular variation that interprets its general time-scale and westward drift in terms of magnetohydrodynamic oscillations of the liquid core (22). The electrical conductivity of the overlying 'solid' mantle, though weak, could be sufficient to prevent magnetic changes in the core on the short time-scale of the inertial modes from penetrating to the Earth's surface.

4 THE EARTH'S NON-UNIFORM ROTATION

In the absence of internal energy sources and mechanical, gravitational or radiative interactions with other bodies, the solid parts of the Earth (i.e. crust, mantle and inner core) and the fluid parts (atmosphere, hydrosphere and outer core) would rotate together as a rigid body at a constant rate about its fixed axis of maximum moment of inertia through its centre of mass. Positional astronomers equipped with perfect telescopes and clocks (and the necessary life-support systems all, together with the astronomers, of zero mass!) would find no variation in the latitude of any station and the longitude of all fixed stars would change at a constant rate. .

The successful use of the Earth's rotation as the basis of the earliest attempts to provide a practical unit of time attests to the validity of the above picture as a good 'zeroth approximation' to the truth (24). But over the years, as clocks based on other physical phenomena were invented and methods of positional astronomy improved, tiny fluctuations in the 'length of the day' (up to a few milliseconds) and very slight movements of the Earth's pole of rotation (up to a few metres) came to light. The interpretation of these variations in the magnitude and direction of the Earth's rotation vector in terms of energetic processes and angular momentum transfer within the Earth–Moon system is one of the most fascinating problems in the whole of the Earth sciences, for it brings together several otherwise diverse areas of study, including solid Earth geophysics, meteorology, oceanography, hydrology, glaciology and geomagnetism.

Such studies of the Earth's rotation go back to the last century. The first thorough review of the subject was presented by Munk and MacDonald in 1960 and Lambeck (25) has given an up-to-date discussion taking into account recent improvements in instrumentation, international cooperation and basic ideas in geodynamics.

The magnitude of the principal moment of inertia of the atmosphere I_a is about $1{\cdot}42\times10^{32}$ kg m², which should be compared with that of the whole Earth $I = 8{\cdot}04\times10^{37}$ kg m². During the development of the three-dimensional and non-axisymmetric global atmospheric circulation following the imaginary 'switching on' of the main source of energy for atmospheric motions, solar heating, the angular momentum of the atmosphere $\mathbf{M}_a$ (say) would increase by an amount $\delta\mathbf{M}_a$ (magnitude $\doteqdot 1{\cdot}5\times10^{26}$ kg m²s⁻¹). This increase is a measure of the 'super-rotation' of the densest parts of the atmosphere, the troposphere and lower stratosphere, at an average angular speed

$$|\delta\mathbf{M}_a|/I_a \doteqdot 10^{-6}\ \text{rad s}^{-1} \doteqdot 1{\cdot}5\times10^{-2}\,|\mathbf{\Omega}| \tag{4.1}$$

relative to the underlying planet, the corresponding linear speed being about 10 m s⁻¹. Here $\mathbf{\Omega}$ is the angular velocity of the solid Earth, which is very close to the total angular momentum $\mathbf{M}$ (magnitude $5{\cdot}85\times10^{33}$ kg m²s⁻¹) divided by I; $|\mathbf{\Omega}| \doteqdot 7{\cdot}28\times10^{-5}$ rad s⁻¹.

Solar radiation provides the kinetic energy of atmospheric motions but it cannot provide the angular momentum. This comes largely from the underlying planet which, to conserve angular momentum of the whole Earth, would undergo a change $-\delta\mathbf{M}_a$, largely manifested as a change

$$|\delta\mathbf{\Omega}| \doteqdot -|\delta\mathbf{M}_a|/I \doteqdot -3\times10^{-8}\,|\mathbf{\Omega}|$$

in the magnitude of $\mathbf{\Omega}$; the corresponding increase in the length of the day (lod) would be about 3×10^{-3}s. This simple calculation shows, as Munk and Miller were evidently the first to realise (25), that fluctuations in the general atmospheric circulation could produce changes in the length of the day of up to about 10^{-3}s. It also implies that since the biggest of the observed irregular lod changes – the so-called 'decade variations' – are about 5×10^{-3}s, they cannot originate in the atmosphere. The decade variations are now generally supposed to originate largely in the Earth's liquid metallic core, where the main geomagnetic field is produced by the magnetohydrodynamic dynamo process (see Section 3).

Length-of-day fluctuations of meteorological origin are largely associated with displacements in the mean latitude of the major jet streams in the troposphere. The concomitant vertical transfer of angular momentum at the Earth's surface is due to a slight imbalance between the positive couple exerted on the underlying planet by the average surface westerlies in mid-latitudes and the negative couple exerted by the surface easterlies at low and high latitudes, the net couple being $\mathbf{K}$. Monitoring the angular momentum 'budget' of the atmosphere with the aid of improved data on lod is now beginning to interest meteorologists concerned with fluctuations of the atmospheric circulation on time-scales upwards of a few days.

Direct determinations of $\mathbf{K}$ cannot yet be made with acceptable accuracy. Even with a complete set of surface wind and pressure observations, an understanding of boundary layer processes and particularly of airflow over and around mountain ranges that goes well beyond present knowledge would be required (3). The value of $\mathbf{K}$ is best obtained indirectly from the rate of change of the total angular momentum of the atmosphere, which depends only on the distribution of velocity and pressure throughout the atmosphere. But even this quantity is subject to large uncertainties owing to the paucity of regular meteorological data from the southern hemisphere and tropical regions.

During 1979, meteorologists made a special effort to monitor the whole atmosphere, providing for the first time data sets which could be used with confidence in the assessment of fluctuations in $\mathbf{M}_a$. It has been shown from these data sets that observed short-term changes in the lod during 1979 can be fully accounted for on the basis of angular momentum exchange with the atmosphere (26). This demonstration improves the meteorological usefulness of the data set on rapid changes in the lod since 1955, when 'atomic' clocks were introduced. This data set reveals a pronounced fluctuation on a time-scale of about 50 day with an amplitude comparable with the better known seasonal variations (27, 28). The elucidation of this fluctuation should greatly advance our understanding of the general circulation of the atmosphere and might lead to improvements in long-range weather forecasting by a combination of statistical and dynamical methods.

All components of $\mathbf{M}_a$ exhibit fluctuations. The non-axial components give rise to polar motions of a few metres, including a contribution to the so-called Chandler wobble with a period of 14 month (25). The excitation and damping of the Chandler wobble are still controversial issues amongst geophysicists but any attempts to infer mantle motions preceding earthquakes from Chandler wobble observations will require in the first instance an assessment of atmospheric effects. This might be possible soon when a new collaborative programme involving a number of institutions is fully operational.

We have seen that, at several milliseconds in amplitude, the so-called 'decade fluctuations' are too great to be accounted for in terms of atmospheric processes and that geophysicists generally agree that, in the absence of a reasonable alternative, the fluctuations must be largely due to angular momentum transfer between the Earth's 'solid' mantle of thickness $\doteqdot$ 2900 km and underlying liquid outer core of thickness $\doteqdot$ 2200 km. The distortion and displacement of the geomagnetic field pattern at the Earth's surface, including the well-known westward drift of the non-dipole field at some 3×10^{-4}m s^{-1}, can be taken as manifestations of core motions, but the accurate determination of all but the broadest features of these motions from geomagnetic observations is an impossible task (29, 30). Nevertheless, the quantitative requirement that the time-scale and rms value of fluctuations in zonal speed of core motions be generally compatible with the amplitude of the decade variations in the length of the day is not particularly restrictive (25).

The principal quantitative difficulties emerge when the nature of the horizontal stresses that couple the core to the mantle, across the core-mantle interface, are considered (25, 29). These stresses must suffice both quantitatively and qualitatively to account for the fluctuating couple at the core-mantle interface implied by foregoing interpretation of the decade variations in the length of the day. We can write

$$F = F_V + F_E + F_T \doteqdot 0{\cdot}04 \text{ N m}^{-2} \qquad (4.2)$$

where F is the average magnitude of these horizontal stresses, the corresponding magnitude of the unbalanced couple between core and mantle being 10^{19}–10^{20} kg m^{2}s^{-2}. F_V, F_E and F_T are the contributions to F associated with viscous coupling, electromagnetic coupling and topographic coupling respectively.

If δ is the thickness of the viscous boundary layer at the surface of the core then

$$F_V = C_V \nu \rho \Delta U/\delta \qquad (4.3)$$

where ρ is the density of the core ($\doteqdot 10^4$ kg m^{-3}), ν is the coefficient of molecular or eddy kinematic viscosity, depending on whether the flow is laminar or turbulent, ΔU is a typical magnitude of fluctuations in the azimuthal flow speed of the core material relative to the mantle and C_V is a 'viscous drag coefficient' which on general grounds should be around unity.

The electric currents responsible for the main geomagnetic field leak out of the metallic core into the weakly conducting lower mantle and these, together with currents induced in the lower mantle by fluctuations in the main geomagnetic field, give rise to electromagnetic coupling between the core and mantle. If B_1 is the fluctuating horizontal component of the magnetic field at the core-mantle interface and B_2 the vertical component, and μ denotes the magnetic permeability (not significantly different from $4\pi\times10^{-7}$ H m^{-1}, that of free space) then the term F_E in equation (4.2) is given by

$$F_E = C_E B_1 B_2/\mu \qquad (4.4)$$

where C_E is the 'electromagnetic drag coefficient' which, like C_V, should be around unity.

The third term in equation (4.2), F_T, arises if irregular topographic features are present on the core-mantle interface and h; a typical value of the vertical dimensions of these features, exceeds the thickness of the viscous boundary layer δ. F_T is related to h by the expression

$$F_T = C_T \Omega \rho \Delta U (h - \delta) \qquad (4.5)$$

where Ω is the angular speed of rotation of the mantle ($\doteqdot 10^{-4}$ rad s^{-1}) and C_T is the 'topographic drag coefficient'. C_T is unlikely to exceed unity but (unlike C_V and C_E) it may in some circumstances take on very small values.

Viscous coupling is almost certainly inadequate but electromagnetic coupling (proposed by Bullard) might suffice and topographic coupling (proposed by the writer) could be important with bumps no more than about 1 km in height (14, 29, 31). But high topography does not necessarily imply strong coupling owing to the complicated dependence of C_T on h, with C_T decreasing rapidly with increasing h when h exceeds a certain critical value h^*, dropping to values at $h \gg h^*$ that are typically very much smaller than those attained when $h < h^*$. The determination of h^* and the investigation of the detailed dependence of C_T on h and other parameters are matters of considerable importance in the study of the dynamics of atmospheres, oceans and planetary cores, where the influence of topography on fluid flow can in some circumstances be considerable. Preliminary studies of magnetohydrodynamic flow over and around topography in a rapidly rotating fluid have now been made. They indicate that, owing to the presence of magnetohydrodynamic inertial waves produced by interactions involving the magnetic field at the core-mantle interface, the appropriate value of C_T might be comparatively high, around unity (32). The corresponding topographic coupling would be adequate to account for the observed 'decade' variations in the length of the day.

5 CONCLUDING REMARKS

Developments in many areas of physics have been stimulated by the demands made upon them by planetary sciences and astronomy. An important weapon required in the theoretical armoury of these subjects is a deep and extensive understanding of basic fluid dynamical processes in rapidly rotating fluids, pioneering contributions to which have been made by Sir Harold Jeffreys, Professor S.Chandrasekhar and other leading members of this Society. Our knowledge of hydrodynamic processes, though far from adequate, still greatly exceeds that of magnetohydrodynamic processes. Whilst the former can be studied in the laboratory under controlled and reproducible conditions, which enable crucial experiments to be carried out, laboratory studies of the latter are inevitably much more limited in their scope owing to the difficulty of achieving high values of the magnetic Reynolds number (see equation 3.2). Moreover, whilst geostrophic flows can be observed directly in the terrestrial oceans and atmosphere, magnetostrophic flows occur in inaccessible regions, the nearest being the Earth's liquid metallic core, where the main geomagnetic field originates. But magnetostrophic flows probably underlie a very wide range of phenomena encountered in the study of stars and planets, the successful interpretation of which will require in the first instance further systematic theoretical and numerical work on the magnetohydrodynamics of rotating fluids.

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(To appear in "Physics Bulletin" and "Bulletin de la Société Française de Physique")

HOLWECK PRIZE LECTURE 1982

THE GIANT PLANETS*

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*Lecture delivered on 11 June 1982 at the École Polytechnique in Paris following the presentation of the 1982 Holweck Medal and Prize of the Société Française de Physique and Institute of Physics.

1. INTRODUCTION

The study of planets as physical objects goes back to the early seventeenth century, when Galileo discovered Jupiter's four main satellites and initiated the observations that led to Huyghens's discovery of Saturn's rings. But systematic work in planetary science is a comparatively recent development which has accompanied the rise of modern geophysics and astrophysics. Planetary research is based on a variety of observations. Many are made with ground-based instruments at wavelengths in the electromagnetic spectrum where the Earth's atmosphere is transparent or only weakly absorbing, namely the visible range from 0.4 to 0.8 microns, various "windows" in the infrared and the radio spectrum from about 1 millimetre to the ionospheric cutoff at about 100 metres. Other important observations come from instruments carried on rockets, aircraft and balloons, operating at levels where effects due to ozone, which absorbs ultraviolet radiation, and water vapour, which absorbs in the infrared, are much weaker than at ground level. The most recent data include measurements made with instruments mounted on space probes which, in the case of the giant planets Jupiter and Saturn, were the Pioneer 10 fly-by, which encountered Jupiter in 1973 December before heading out of the Solar System, Pioneer 11, which encountered Jupiter in 1974 December and Saturn in 1979 September, Voyager 1, which encountered Jupiter in 1979 March and Saturn in 1980 November, and Voyager 2, which encountered Jupiter in 1979 July, Saturn in 1981 August and should reach Uranus in 1986 January, eight years after being launched from Earth.

My talk* outlined the main characteristics of the giant

(Footnote)* Parts of the paper presented here are based on the written version of the 1980 Halley lecture at Oxford University (Hide 1980), brought up to date by the inclusion of recent material. The talk was illustrated with slides and several short films describing (a) the Voyager missions, (b) motions of Jupiter's visible surface of dense cloud, (c) a numerical simulation of motions in Jupiter's atmosphere and (d) laboratory experiments on stable vortices in thermally-driven rotating fluids. It is impossible to reproduce most of this visual material here and inappropriate to give a verbatim account of the lecture. (End of footnote).

planets and discussed aspects of their study in which I have been interested for a number of years, namely the circulation of their atmospheres, the structure of their interiors and the origin of their magnetic fields. Progress with the physical interpretation of the observational data involves not only the application of a wide range of available knowledge in physics and chemistry but it also requires the development of new ideas and insights. The findings of laboratory studies of hydrodynamical flows in rapidly rotating fluids play an important direct role in the study of large-scale atmospheric motions, but this is unusual in planetary science and astronomy, where basic processes can rarely be simulated on the scale of the terrestrial laboratory. In the study of planetary magnetic fields, for example, the theoretician has to work directly with the observations and the basic laws of physics.

2. JUPITER'S MAIN CHARACTERISTICS

Jupiter is the fifth planet in order of distance from the Sun, which it orbits in 11.9 years at a mean distance of 5.20 astronomical units. The largest and most massive of the nine planets, Jupiter has a mean radius of 6.97×10^7 metres, more than ten times the radius of Earth, but its mean density is only about one quarter of that of the 'terrestrial' planets, indicating that its main chemical constituent is hydrogen, the lightest and most abundant of all the elements, with helium as the main "impurity". The rapid spin of the planet on its axis, in a period of just under 10 hours, produces a noticeable oblateness in the appearance of the visible disk in a telescope, and the concomitant distortion of the gravitational field of the planet perturbs the orbits of the innermost satellites. These rotational effects are nevertheless somewhat weaker than those expected from a gravitating body of <u>uniform</u> density, implying that the density of Jupiter increases with depth to a central value of about three times the mean density. At 30×10^{11} pascals, Jupiter's central pressure is eight times that of the Earth and sufficient to ensure that throughout most of the planet the main constituent hydrogen takes the metallic form, to which ordinary molecular hydrogen changes when subject to pressures in excess of about 2×10^{11} pascals.

Jupiter is enveloped in several layers of cloud suspended in a deep and well-stirred atmosphere of hydrogen, helium, methane, ammonia and other gases. At some wavelengths it is possible to see below the upper layer of white ammonia cirrus, through a clear region to a more substantial and colourful cloud deck which might consist mainly of particles of ammonium hydrosulphide (NH_4SH). Jupiter's striking and variable colours are probably due to the presence of products of the photolysis of NH_3, H_2S, PH_3 and other compounds by solar ultraviolet light. Jupiter's clouds reflect back into space most of sunlight incident upon them, the remainder being absorbed at various levels in the atmosphere. Absorbed sunlight accounts for about one-half of Jupiter's thermal (infrared) emission, the balance being due to internal heat sources associated with slow gravitational contraction of the planet and primordial heat. The outward heat flow should maintain the temperature gradient close to its adiabatic value throughout most of the planet.

From their studies of Jupiter's strong emission of non-thermal radiation at decimetre and decametre wavelengths, radio-astronomers established that Jupiter possesses a strong dipole magnetic field inclined at about 10 degree to the rotation axis, and an associated system of van-Allen-type 'radiation' belts of electrically-charged particles with relativistic energies, extending beyond and interacting with the innermost Galilean satellite Io. The decametric radiation, which comes in bursts and is modulated by Io, is still not fully explained. More detailed information on the structure of Jupiter's magnetic field and magnetosphere and their interactions with the solar wind came later from in situ measurements made with instruments on the Pioneer and Voyager fly-by space probes.

Jupiter rotates non-uniformly. This property is reflected in the early use by optical astronomers of two longitude systems when studying spots and other markings on the planet's visible disk namely System I with rotation period 9 hours 50 minutes 30.003

seconds for features within 7 degrees of the equator and System II with rotation period 9 hours 55 minutes 40.632 seconds for higher-latitude features — and in the need to introduce yet another system — System III with period 9 hours 55 minutes 29.710 seconds (formerly 29.390 seconds) — for the convenience of radio astronomers. Jovian decametric and decimetric sources, which are nearly fixed in System III, are presumably tied to Jupiter's magnetic field which, in turn, is intimately linked to the electrically-conducting part of the interior of the planet, but not with the overlying non-conducting layers, including the atmosphere. The interpretation of Jupiter's non-uniform rotation involves the consideration of the dynamics magnetohydrodynamics of Jupiter's atmosphere and deep interior, just as the theoretical interpretation of the general eastward motion (on average) of the terrestrial atmosphere relative to the solid Earth, the general westward motion of the geomagnetic field relative to the solid Earth, and fluctuations in the rotation of the solid Earth, involves the discussion of fluid motions in the Earth's atmosphere, oceans and liquid core.

3. JUPITER'S ATMOSPHERIC MOTIONS

Prominent markings on Jupiter's visible surface are the bright cloud zones, of which there are usually about seven or eight, which run parallel to the equator and are separated by darker belts. The zones and belts are not entirely regular: dark patches often appear on the brighter regions and bright patches on the darker regions, and the boundaries between belts and zones often take on a serrated shape. The most striking feature of all is the long-lived Great Red Spot (see figure 1) which is certainly 150 years old at least and may have been seen by Hooke and Cassini in the seventeenth century. The Great Red Spot is elliptical in shape, having its long axis along 'zenocentric' latitude 22 degrees South, and occupies about 30 degrees of longitude and 10 degrees of latitude.

FIGURE 1 NEAR HERE

The movement of irregular markings seen on the visible disk yields information about the rotation of the planet at the upper cloud level as a function of latitude. As we have seen, rotation periods of markings within about 7 degrees of the equator are typically about 5 minutes less than typical rotation periods of most higher-latitude features (and of radio sources), showing the presence of a sharply-bounded westerly equatorial jet-stream moving at about 100 ms^{-1} relative to the underlying planet. During the past 100 years the period of rotation of the Great Red Spot has varied irregularly by about 13 seconds (or 360 parts in a million).

Motions occur in planetary atmospheres because the special conditions for stable or neutral hydrostatic equilibrium cannot in general be met when heat sources are present. These conditions for a fluid of low viscosity in a steady gravitational field are, first,

that the density field should have no horizontal gradients, and, secondly, that vertical density gradients should nowhere exceed to the adiabatic value. Owing to the general intractability of the equations of hydrodynamics and uncertainties in several important parameters such as the atmospheric depth, theoretical work on the circulation of Jupiter's atmosphere has been largely confined to the construction of simple models of the main phenomena indicated by the observations, notably the general arrangement of clouds in bands parallel to the equator, the equatorial jet stream and the Great Red Spot. There is no generally accepted explanation of any of these phenomena, but their study has stimulated important investigations in basic hydrodynamics. Recent theoretical work includes studies based on numerical models and the application of modern ideas on solitons and two-dimensional turbulence.

The very existence and durability of the Great Red Spot present a strong challenge to theoreticians, particularly those concerned with the important practical question of atmospheric predictability. Several incomplete suggestions have been made as to the origin of the Great Red Spot and other long-lived eddies in Jupiter's atmosphere, such as the three White Ovals that were seen to form in 1939 and are clearly visible in the recent Voyager pictures. I argued in my Halley lecture two years ago that these long-lived eddies might be dynamically similar to the closed stable baroclinic eddies found in the regular non-axisymmetric flow regime investigated in experiments on thermal convection in a rotating fluid annulus subject to horizontal temperature gradients produced by internal heating and side-wall cooling, where they are largely responsible for horizontal and vertical heat transfer. In some new laboratory and numerical studies of the structure, energetics and stability of such eddies, my colleague Dr Peter L Read and I find results confirming that there are strong dynamical similarities between these atmospheric and laboratory flows (see figures 2 and 3). In particular, the horizontal upper level motion in each eddy is largely confined to a peripheral jet stream, where there is strong shear, enclosing a comparatively quiescent region in which slow upwelling occurs. Surrounding the peripheral jet stream is a narrow region of downwelling, a feature which could account for the "collar" of enhanced infrared emission surrounding the Great Red Spot.

In this baroclinic eddy hypothesis, the energy required to produce and maintain the kinetic energy of the eddy against the dissipative action of viscosity derives largely from the potential energy due to gravity acting upon the density field associated with local horizontal temperature gradients. Coriolis forces due to the rotation of the planet play a crucial role in the hypothesis, but the so-called "beta-effect" associated with the latitudinal variation of the vertical component of the rotation of the planet produces no direct influence. The beta-effect is also unimportant in the hypothesis that long-lived Jovian atmospheric eddies are analogous to terrestrial hurricanes, where the release of latent heat of condensation, in motions involving small-scale moist convection organized by Coriolis forces and friction, provides the energy source. It is in the so-called "soliton" and "modon" theories where the beta-effect plays a crucial direct role. These theories differ from one another in the assumption made as to the source of energy required

to overcome viscous dissipation, with the "soliton" drawing its energy directly from the kinetic energy of the background zonal shear flow, and the "modon" being produced by the coalescence of smaller eddies.

4. JUPITER'S MAGNETISM AND INTERIOR

The existence of a strong Jovian magnetic field of internal origin and nearly dipolar form, with a dipole moment about 10^4 times the magnitude and opposite in sign to that of the Earth's present field, was, as we have already noted, first inferred from radio-astronomical observations of non-thermal radiation at decimetre wavelengths, produced by magnetically-trapped relativistic electrons. Such an electron typically spends most of its time near the mirror points of its orbit, moving in a flat spiral around a magnetic field line, and this accounts for the significant degree (about 25 per cent) of linear polarization of the total decimetre emission. The direction of polarization, which is roughly parallel to Jupiter's equator and fluctuates with the System III rotation period, indicates that the dipole axis is inclined at about 10 degrees to the rotation axis. The observed time variations of the direction of polarization differ slightly from those expected from a purely dipole field, showing that Jupiter's magnetic field possesses a weak non-dipolar component. The decimetre radiation also shows some degree of circular polarization, as would be expected if some of the radiation emerges in directions parallel to lines of magnetic force. Indeed, it was from the observed sense of polarization that it was first concluded that Jupiter's dipole moment is roughly parallel to the rotation axis (unlike the present alignment of the Earth's magnetic dipole, which is roughly anti-parallel to the rotation axis). Such a polarity is evidently consistent with the sense of elliptical polarization of decametre bursts, which is almost invariably right-handed.

Direct measurements of the Jovian magnetic field generally confirm inferences drawn from radioastronomical observations and add many important details. Instruments on board the Pioneer 10 and Pioneer 11 spacecraft provided the first such measurements of the field structure and charged-particle density in the Jovian magnetosphere. The magnetosphere is closed and blunt in shape, with a well-defined magnetopause. Its outer regions are strongly influenced by the fluctuating pressure of the solar wind, which produces changes of up to a factor of two in the size of the dayside magnetosphere. A prominent feature of the mid-magnetosphere is a thin annular current sheet which co-rotates with the planet like a rigid body. The sheet lies almost parallel to the Jovian equator, but is distorted, so that it lies above the equatorial plane on one side of the planet and below on the other. Voyager 1 and Voyager 2 provided further information about the highly complex behaviour of Jupiter's magnetosphere.

Theories of the observed modulation of Jovian decametre bursts by the innermost Galilean satellite Io suppose that Io's motion through the Jovian magnetic field induces electric currents of about 5×10^6 amps in the plasma occupying the magnetic flux tube connecting Io to Jupiter's ionosphere. Direct evidence for this current was obtained during the Voyager mission. The passage of the current through the surface layers of Io would produce 10^{12} watts of ohmic heating there. (This is somewhat less than the heating produced by gravitational tides, the calculation of which led to the prediction that violent eruptions of material should occur from Io's surface, published in a paper that appeared just a few days before Voyager TV pictures provided confirmatory evidence!)

The only serious suggestion as to the origin of Jupiter's magnetic field closely parallels ideas developed by geophysicists over the past thirty years towards an explanation of the Earth's magnetism. The Jovian magnetic field is due to ordinary electric currents circulating in conducting regions within the planet. The currents are produced by the self-exciting magnetohydrodynamic (MHD) dynamo process — first suggested by Larmor — involving inductive interactions between fluid motions and the magnetic field. Whether dynamo action is possible depends _inter alia_ on the value of the so-called 'magnetic Reynolds number' $\sigma\mu LU$, where σ is the electrical conductivity, μ the magnetic permeability, L a typical length scale and U a typical speed of relative fluid motion. Such action cannot occur when $\sigma\mu LU$ is very small, for then motional induction cannot overcome effects of ohmic dissipation. It is also impossible or inefficient when $\sigma\mu LU$ is very large indeed, because dynamo action involves not only the amplification of magnetic energy but also the diffusion of field lines from the conducting region into the surrounding insulator, which is impossible when the conductor is perfect. Efficient dynamo action should occur in those parts of the planet where $\sigma\mu LU$ has some optimum value R, which might be around 10 or 100.

The broad features of the internal structure of Jupiter have now been elucidated. Theoretical models are sensitive to the assumed equations of state of the main constituents, hydrogen and helium. Recent work indicates that the planet is probably fluid throughout, with outward heat flow taking place largely by convection. Dynamo action should occur in those regions well below the visible surface of the planet, radius r_s, where the electrical conductivity σ is

FIGURE 4 NEAR HERE

about equal to $R/\mu LU$. Denote by r_c the outer radius of the electrically-conducting region in which dynamo action is occurring and by r_m the mean radius of the boundary where the pressure attains the value (about 2×10^{11} pascals) at which molecular hydrogen changes to the metallic form. Estimates of r_m range from 0.7 to 0.8 and it is possible but not certain that r_c is equal to r_m. Impurities and high temperature and pressure might render the molecular hydrogen 'mantle' sufficiently conducting in its lower reaches to support dynamo action there, and if this is so r_c could be significantly greater than r_m.

I have ——> introduced a method for finding r_c which exploits the fact that over short intervals of time the magnetic flux linkage of the surface of the core cannot change significantly (see figure 4). This method makes use of observations of secular changes in the magnetic field in the accessible region above the surface of the planet. When applied to the Earth the method gives r_c to within 2% of the more accurate "seismological" value, but even the best Jovian magnetic field determinations currently available for this purpose are not good enough to give a reliable value of r_c for Jupiter. The rough determinations that have been made could be greatly improved on in the future if detailed magnetic measurements can be carried out with the aid of the Jupiter orbiter on the proposed "Galileo" mission. The determination of r_c will be important in the study of the structure and dynamics of Jupiter's interior. The rotation period of System III is close to that of material at a depth $(r_s - r_c)$ below the visible surface of the planet. The best we can say at present is that this depth is probably around 20,000 km but it might be as small as 7000 km, one tenth of the mean radius of the planet.

5. SATURN

Saturn is the sixth planet in order of distance from the Sun, which it orbits in 29.5 years at a mean distance of 9.55 astronomical units. The outermost of the planets known in remote antiquity (before the discovery of Uranus, Neptune and Pluto from telescopic observations), Saturn is second only to Jupiter in size (mean radius 5.74 x 10^7 metres) and mass and has the low mean density expected of a planet composed mainly of hydrogen and helium. Saturn possesses a rich satellite system, which includes Titan, the only satellite in the solar system showing evidence on an atmosphere. It also includes the beautiful rings of Saturn, which comprise an enormous number of discrete rocky fragments which may have been produced by the tidal disruption of a larger object (see figure 5).

FIGURE 5 NEAR HERE

Saturn is comparable with Jupiter as a reflector of sunlight and, like Jupiter, is also enveloped in dense clouds of ammonia crystals suspended in an atmosphere of hydrogen, helium, methane and other gases and arranged in bands parallel to the equator. These bands appear to be more regular than those on Jupiter, irregular markings being comparatively rare in ground-based observations. Colour variations on Saturn are much less pronounced than on Jupiter and nothing comparable with Jupiter's Great Red Spot has ever been seen on Saturn, but Voyager TV pictures of Saturn show a variety of interesting atmospheric disturbances, which are now providing the basis for dynamical studies. As with Jupiter, in order to account for Saturn's infrared emission it is necessary to invoke a substantial source of internal heating, with the gravitational settling of helium playing a more significant rôle than in the case of Jupiter.

¶ Transits of long-lived spots on Saturn (including one investigated in the early 1930's by the well-known English comedian Will Hay) yield rotation periods of 10 hours 13 minutes within about 20° of

the equator and 10 hours 40 minutes at higher latitudes. Thus, Saturn shows evidence of an equatorial current moving eastward at about 400 metres per second, four times the speed of Jupiter's equatorial current. Hydrodynamical theory indicates that the width of an equatorial current should be roughly proportional to the square root of its speed, and this accords with the observations.

Saturn's equatorial diameter exceeds its polar diameter by more than 10 per cent, the corresponding degree of oblateness being bigger than Jupiter's but, again, rather less than that of a rotating gravitating body of uniform density. Saturn does not differ greatly from the Earth in its surface gravity and central pressure. On recent ideas concerning its internal structure, the central pressure of Saturn might be too low for helium to dissolve at any level within metallic fluid hydrogen core.

The first evidence that Saturn possesses a general magnetic field came with the detection of weak non-thermal radio emission at hectametre wavelengths about seven years ago. Pioneer 11 encountered Saturn in 1979 September and its magnetometers showed that the planet has a dipole moment inclined by as little as 2° to the rotation axis with a strength about 0.1 times that of Jupiter. General dynamical considerations indicate that the rotation rate of Saturn's interior should be much closer to that of the atmosphere in middle and high latitudes (10 hours 40 minutes) than to that of atmospheric markings near the equator (10 hours 13 minutes). This expectation was confirmed by determinations of the rotation period of Saturn's hectametre radio sources, 10 hours 40 minutes.

Saturn's magnetic field is attributable, presumably, to a magnetohydrodynamic dynamo driven by motion in the deep interior. A basic tenet of dynamo theory (which has recently been proved quite rigorously) is that no magnetic field that everywhere retains an axis of symmetry can be supported by dynamo action. But some theoreticians have been misled into questioning this result on the basis of the near coincidence of Saturn's magnetic and rotation axes and the predominance of the dipole component of the field. These features of the field are more readily explained in terms of the comparative smallness and other properties of Saturn's electrically-conducting fluid core.

Information about Saturn, though still much less abundant than for Jupiter, now includes new data from recent Pioneer and Voyager encounters, which are transforming all aspects of the study of Saturn's interior, magnetic field, atmosphere, magnetosphere and satellites. Particularly noteworthy is the resurgence of interest in the dynamics of the rings following the acquisition of high resolution pictures showing fine detail and other features of their structure. Indeed, the study of the giant planets Jupiter and Saturn is now one of the most active areas of planetary astronomy, involving the investigation of a wide range of phenomena, many of which I was unable even to mention in the space of a single talk.

6. CONCLUDING REMARKS

It remains for me to thank the Société Française de Physique and Institute of Physics for inviting me to give this lecture on the occasion of the presentation of the Holweck Medal and Prize for 1982, which recognizes developments in geophysical fluid dynamics and planetary science. I am also indebted to Professor Jean Coulomb of the Institut de Physique du Globe de Paris, for kindly sending me a very moving account of the scientific career of Fernand Holweck, his activity during the Second World War and his tragic death at the hands of the Gestapo in the Santé prison in 1941. Holweck was a distinguished and prolific scientist, a man of great physical and moral courage and an outstanding patriot. Scientists of my age group were too young to serve in the war, but we were old enough to benefit from the great prestige enjoyed by science when we started our careers soon after the war. Our research has been well supported and we have been able to pursue it without interruption and interference. This freedom we can never take for granted with the example of Fernand Holweck before us.

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LEGENDS FOR DIAGRAMS

Figure 1. Long-lived anticyclonic eddies in Jupiter's atmosphere, namely the Great Red Spot, which is about 20,000 km long and at least 150 years old and may have first been seen by Robert Hooke in 1664, and one of the three White Ovals that were seen to form 43 years ago. The arrows give the sense of relative motion. The "wind" speeds, as determined by Dr R F Beebe of the New Mexico State University, are given in kilometres per hour. For comparison note that points on Jupiter's equator rotate at about 35,000 km per hour. (See NASA 1979 for further Voyager pictures of Jupiter.) The very existence of these long-lived eddies has important implications for theories of atmospheric predictability. There is no generally accepted explanation of the eddies but the experiments illustrated in Figures 2 and 3 indicate one possible line of attack.

Figure 2. Streak photographs giving one example of the top-surface flow pattern in each of the three main modes of thermal convection in a rotating fluid annulus subject to axisymmetric heating at the outer side-wall and cooling at the inner side wall, according to experiments first carried out by the author in 1950. The character of the motion depends on a number of parameters involving the acceleration of gravity, dimensions of the apparatus, thermal coefficient of cubical expansion, viscosity and thermal conductivity of the working fluid, etc. In an experiment in which Ω, the angular speed of rotation of the whole apparatus about its vertical axis of symmetry, is the only quantity varied, axisymmetric flow is found at low values of Ω (see case (I)). At intermediate and high values of Ω the flow is non-axisymmetric, with well-developed jet streams (cases II and III). At intermediate values the flow is regular — ie. spatially and temporally periodic, with wavenumber ranging from 2 to 5 as Ω increases for the geometry shown — as in case II. The wavenumber is 3 and there is little tendency for the flow to "vacillate", so that the pattern is effectively steady apart from an angular drift at constant speed relative →

(continued)

to the walls of the apparatus. At high values of Ω (see case III) the flow is highly irregular; this chaotic motion is an example of what meteorologists concerned with large-scale motions in the Earth's atmosphere have termed "geostrophic turbulence". A wide variety of fluid dynamical and other non-linear systems are now known to exhibit either ordered or chaotic behaviour depending on the impressed conditions and important unifying theoretical concepts are beginning to emerge. (For further details and references see Hide 1981a and 1982.)

Figure 3. Numerical simulation of to top-surface flow pattern (figure 3b) of thermal convection in a rotating fluid annulus subject to internal heating, with cooling taking place at both side-walls, as produced in laboratory apparatus (figure 3(a)). Conditions are such that the flow is non-axisymmetric and regular with azimuthal wavenumber 4 (cf. figure 2 case II) and little vacillation; that is to say the flow consists of 4 virtually identical steady eddies each circulating in an anticyclonic sense at the upper level, with horizontal motion largely confined to a peripheral jet stream. Recent studies of the structure, stability and energetics of such eddies strengthen the proposal that they are dynamically similar to the long-lived eddies in the atmospheres of Jupiter and Saturn, such as the Great Red Spot and three White Ovals illustrated in figure 1. (For details and references see Hide 1980, 1981a; Read and Hide 1982.)

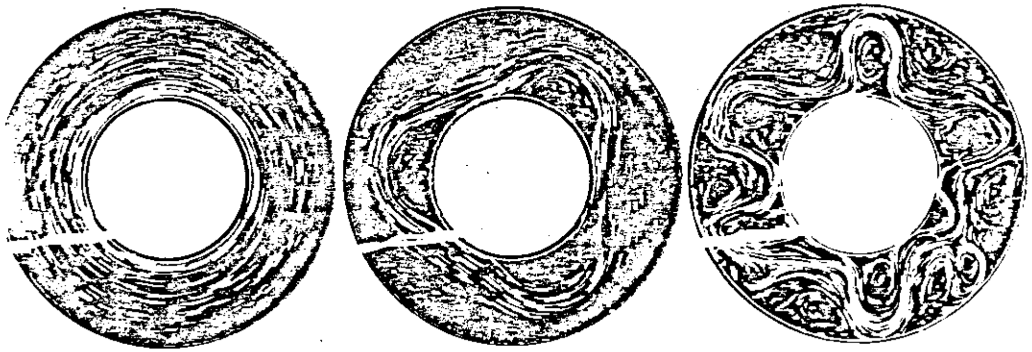
Figure 4. Illustrating the principle of the method introduced by the author in 1978 (for details and references see Hide 1981b) for finding the radius of the electrically-conducting fluid core of a planet. $N(S;t)$ is the number of intersections of magnetic lines of force with the outer surface of the planet at time t, and $N(S_0;t)$ is the corresponding quantity evaluated at the

(continued)

surface of the core. Whilst $N(S;t)$ varies on the time-scale of motions in the core, $N(S_0;t)$ varies on the very much longer time scale of Ohmic decay, more than 10^4 years for the Earth and 10^6 years for Jupiter. The value of t_c can be determined from detailed measurements of secular changes in the magnetic field in the accessible region near S by finding from the measurements, by downward extrapolation, that surface S_0 for which temporal changes in $N(S_0;t)$ are effectively negligible. When applied to the Earth the method gives to within 2% of the value determined by the more accurate methods of seismology. In the case of Jupiter and Saturn at the present time there is no alternative to the magnetic method and the observations not yet adequate.

Figure 5. View of Saturn from Voyager 1 from a distance of more than 5x 10^6 km, four days after the encounter in November 1980 (see Science 1982, 215 Number 4532 or Beatty et al 1981).

FIGURE 1



(I) symmetric (Ω = 0·341 rad s^{-1})

(II) steady waves (Ω = 1·19 rad s^{-1})

(III) irregular (Ω = 5·02 rad s^{-1})

FIGURE 2

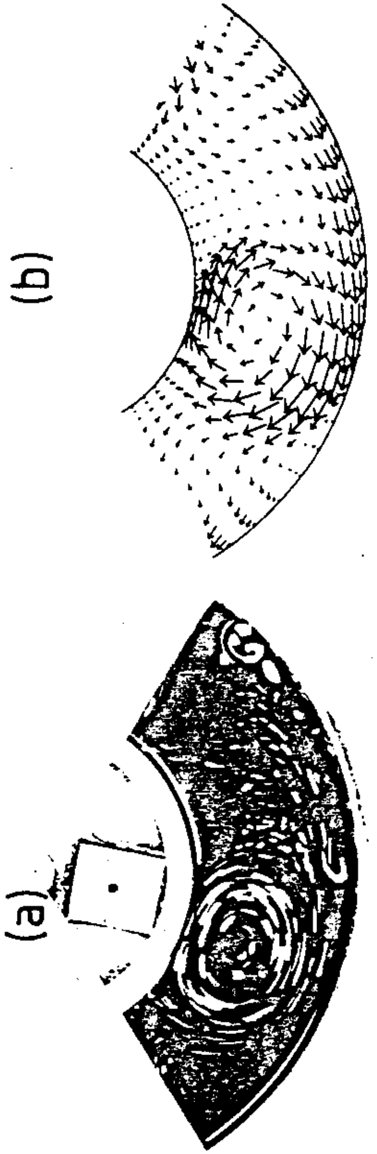


FIGURE 3

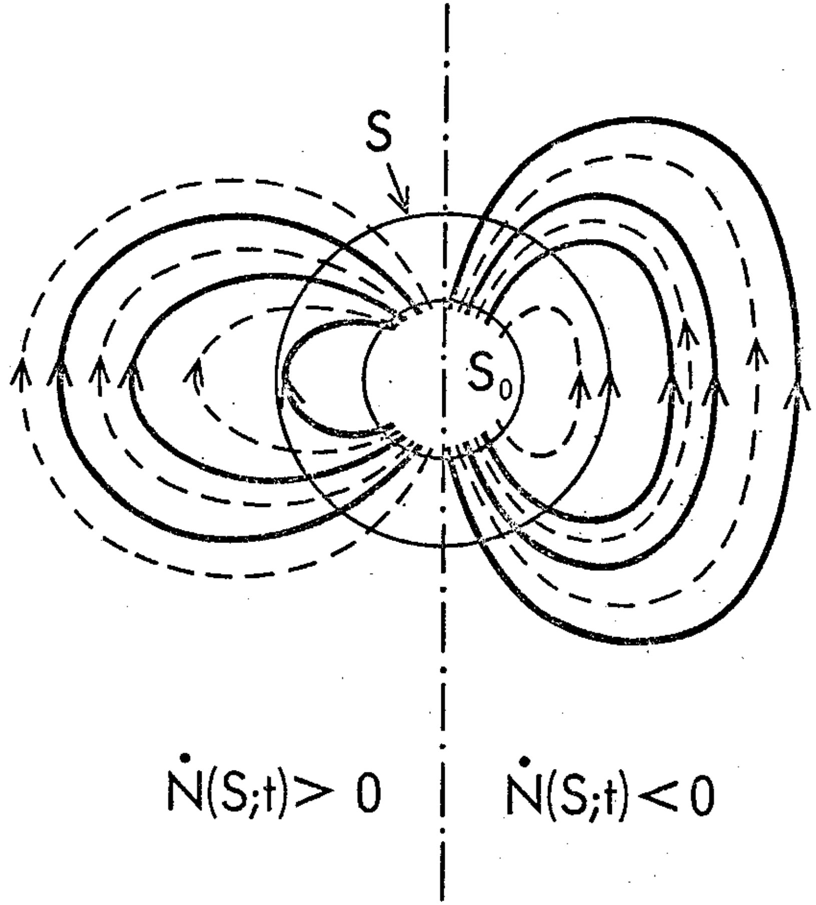


FIGURE 4

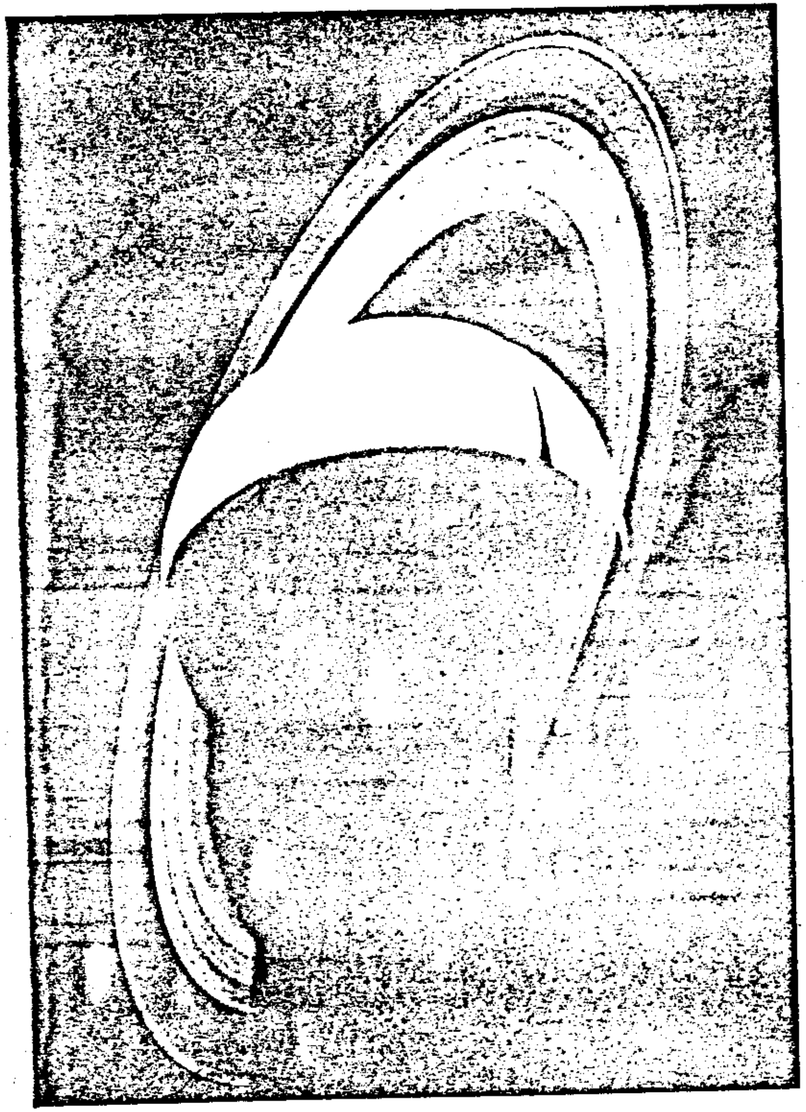


FIGURE 5

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NOTE

The following papers, also by Professor Hide, are available in the ICTP Library:

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