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PLANE HYDROMAGNETIC WAVES IN A STRATIFIED ROTATING INCOMPRESSIBLE FLUID

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Appendix: Plane hydromagnetic waves in a stratified rotating incompressible fluid. (Based on reference [29].) In [25] (see also pages 278 and 327–328) an approximate dispersion relationship for free hydromagnetic oscillations of the fluid core of the Earth was proposed, and it was conjectured that when

$$|N|^2 \ll |2\boldsymbol{\Omega}|^2 \tag{1}$$

effects due to vertical gradients of density, ρ, can safely be ignored. Here $\boldsymbol{\Omega}$ is the angular velocity of the Earth's rotation and

$$\boldsymbol{N} \equiv \frac{\boldsymbol{g}}{g}\left(\frac{g}{\rho}\frac{\partial\rho}{\partial z}\right)^{1/2} \tag{2}$$

$|N|$ is the Brunt–Väisälä frequency (effects due to compressibility being negligible in the theoretical model) and $\boldsymbol{g} = (0, 0, g)$ is the acceleration due to gravity and centrifugal effects. The purpose of this appendix is to investigate the validity of the criterion expressed by Equation (1).

The most acceptable procedure would be to find an accurate dispersion relationship for the case $N \neq 0$, but this has not yet proved feasible. Owing to the (nearly) spherical geometry of the system, even when $N = 0$ the problem is mathematically intractable (see [25], [21], [29], [45], [63], [65]), except in very special cases. For this reason we shall examine an elementary

[4] See pages 327–328.

but related problem whose solution should indicate, in part at least, how effects due to rotation, density stratification and magnetic fields interact with one another. Thus, we shall consider plane, small amplitude, harmonic waves propagating in an inviscid, perfectly conducting, incompressible, rotating fluid of indefinite extent in all directions when both N (based on the undisturbed density field $\rho = \rho_0(z)$) and

$$\boldsymbol{V} \equiv \boldsymbol{B}_0/(\mu\rho_0)^{1/2}, \tag{3}$$

the Alfvén velocity, are uniform, where $\boldsymbol{B}_0$ is the undisturbed magnetic field vector and μ is the magnetic permeability.

The equations of the problem referred to a frame which rotates with steady angular velocity $\boldsymbol{\Omega}$ relative to an inertial frame are, when all transport processes (viscosity, electrical resistivity, thermal conduction, etc.) are negligible, the following (in rationalized MKS units):

$$\partial\boldsymbol{u}/\partial t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + 2\boldsymbol{\Omega}\times\boldsymbol{u} = -(1/\rho)\nabla p + (\boldsymbol{j}\times\boldsymbol{B})/\rho + \boldsymbol{g}, \tag{4}$$

$$\nabla\cdot\boldsymbol{u} = 0, \tag{5}$$

$$\partial\rho/\partial t + (\boldsymbol{u}\cdot\nabla)\rho = 0, \tag{6}$$

$$\nabla\cdot\boldsymbol{B} = 0, \tag{7}$$

$$\nabla\times\boldsymbol{B} = \mu\boldsymbol{j}, \tag{8}$$

$$\nabla\times\boldsymbol{E} = -\partial\boldsymbol{B}/\partial t, \tag{9}$$

$$\boldsymbol{E} + \boldsymbol{u}\times\boldsymbol{B} = 0. \tag{10}$$

Here $\boldsymbol{u}$ denotes the Eulerian flow velocity, p pressure, $\boldsymbol{j}$ current density, $\boldsymbol{E}$ electric field, and t time. Equations (4)–(6) express conservation of momentum, matter and density of individual fluid elements, respectively; Equations (7)–(9) are the laws of Gauss, Ampère and Faraday, respectively; Equation (10) states that in a perfect conductor of electricity, the electric field acting on a moving element must vanish because otherwise, by Ohm's law, electric currents of infinite strength would be implied.

If we write

$$\begin{aligned} \rho &= \rho_0(z) + \rho_1(x, y, z, t), & \boldsymbol{u} &= \boldsymbol{u}_0 + \boldsymbol{u}_1(x, y, z, t), \\ p &= p_0(z) + p_1(x, y, z, t), & \boldsymbol{B} &= \boldsymbol{B}_0 + \boldsymbol{B}_1(x, y, z, t) \end{aligned} \tag{11}$$

where $\boldsymbol{u}_0 = 0$ and $\nabla p_0 = \boldsymbol{g}\rho_0$ and we assume that $|p_1| \ll |p_0|$, $|\boldsymbol{B}_1| \ll |\boldsymbol{B}_0|$ $|\rho_1| \ll \rho_0$ and spatial variations in ρ_0 are small in comparison with ρ_0, then to first order of small quantities p_1, $\boldsymbol{B}_1$, ρ_1 and $\boldsymbol{u}_1 = (u_1, v_1, w_1)$ Equations (4)–(7) become

$$\partial\boldsymbol{u}_1/\partial t + 2\boldsymbol{\Omega}\times\boldsymbol{u}_1 = -(1/\bar{\rho}_0)\nabla p_1 + ((\nabla\times\boldsymbol{B}_1)\times\boldsymbol{B}_0)/\mu\rho_0 + \boldsymbol{g}(\rho_1/\bar{\rho}_0) \tag{12}$$

(where $\bar{\rho}_0$ is the mean density of the fluid),

(13) $$\nabla \cdot \boldsymbol{u}_1 = 0,$$

(14) $$\partial \rho_1/\partial t + w_1(d\rho_0/dz) = 0,$$

(15) $$\nabla \cdot \boldsymbol{B}_1 = 0,$$

Equation (8) having been used to eliminate $\boldsymbol{j}$ from Equation (4)—and the equation that results when $\boldsymbol{E}$ is eliminated between Equations (9) and (10) becomes

(16) $$\partial \boldsymbol{B}_1/\partial t - (\boldsymbol{B}_0 \cdot \nabla)\boldsymbol{u}_1 = 0.$$

Eliminate p_1, $\boldsymbol{B}_1$ and ρ_1 between Equations (9)–(16) and show that

(17) $$[\partial^2/\partial t^2 - (\boldsymbol{V} \cdot \nabla)^2]\nabla \times \boldsymbol{u}_1 - 2(\boldsymbol{\Omega} \cdot \nabla)\partial \boldsymbol{u}_1/\partial t - \boldsymbol{N} \times \nabla(\boldsymbol{N} \cdot \boldsymbol{u}_1) = 0$$

(cf. Equations (2) and (3)).

Substitute a plane wave solution

(18) $$\boldsymbol{u}_1 \propto \exp\{i(\omega t - \boldsymbol{K} \cdot \boldsymbol{r})\}$$

(where ω is the angular frequency, $\boldsymbol{K} = (k, l, m)$ is the wavenumber vector and $\boldsymbol{r} = (x, y, z)$), in Equations (13) and (17) and thus find, after a little manipulation, the dispersion relationship

(19) $$\omega^2 = (\boldsymbol{V} \cdot \boldsymbol{K})^2 + \frac{1}{2}\left\{\frac{(\boldsymbol{N} \times \boldsymbol{K})^2 + (2\boldsymbol{\Omega} \cdot \boldsymbol{K})^2}{K^2} \pm \left[\left(\frac{(\boldsymbol{N} \times \boldsymbol{K})^2 + (2\boldsymbol{\Omega} \cdot \boldsymbol{K})^2}{K^2}\right)^2 + \frac{4(\boldsymbol{V} \cdot \boldsymbol{K})^2(2\boldsymbol{\Omega} \cdot \boldsymbol{K})^2}{K^2}\right]^{1/2}\right\}.$$

The phase velocity, $\omega \boldsymbol{K}/K^2$, and group velocity, $\partial\omega/\partial \boldsymbol{K}$ follow directly from Equation (19). When $N^2 = \bar{\rho}_0^{1} g\, d\rho_0/dz > 0$, ω is always real; we restrict attention in what follows to this case of stable density stratification. (In the other case, $d\rho_0/dz < 0$, the density stratification is unstable, as evinced by the result that ω may then take imaginary values.)

Particle displacements are transverse with respect to the wave fronts, i.e. $\boldsymbol{u}_1 \cdot \boldsymbol{K} = 0$, the occurrence of nonzero values of $\boldsymbol{u}_1 \cdot \boldsymbol{K}$ being incompatible with the assumption of incompressibility (see Equation (13)). Details of the shapes of particle orbits (linear, circular or elliptical) and of concomitant disturbances of the magnetic field and of the fields of density and vorticity, are readily derived from Equations (12)–(19).

In three limiting cases, Equation (19) reduces to particularly simple forms. Thus, when $\boldsymbol{V} = \boldsymbol{N} = 0$ we have "inertial waves," for which

(20) $$\omega^2 = (2\boldsymbol{\Omega} \cdot \boldsymbol{K})^2/K^2$$

and in which rotation provides the restoring forces; when $\boldsymbol{V} = \boldsymbol{\Omega} = 0$ we have "internal waves," for which

(21) $$\omega^2 = (\boldsymbol{N} \times \boldsymbol{K})^2/K^2,$$

and in which buoyancy effects provide the restoring forces; and when $\boldsymbol{\Omega} = \boldsymbol{N} = 0$ we have "hydromagnetic (magnetohydrodynamic or Alfvén) waves," for which

(22) $$\omega^2 = (\boldsymbol{V} \cdot \boldsymbol{K})^2$$

and in which the magnetic field provides the restoring forces. Inertial waves and internal waves are highly dispersive, having group velocities that depend on $\boldsymbol{K}$. Alfvén waves, however, are nondispersive; they propagate with group velocity equal to $\pm \boldsymbol{V}$, which is independent of $\boldsymbol{K}$.

It has occurred to a number of workers (see **[23]**) that in some respects rotating fluids and stratified fluids exhibit analogous hydrodynamic behaviour (cf. Equations (20) and (21), especially when $l = 0$ and $\boldsymbol{\Omega} = (0, 0, \Omega)$, but Equation (19) shows that the analogy does not extend to magnetohydrodynamic behaviour. Thus, when $\boldsymbol{\Omega} = 0$

(23) $$\omega^2 = (\boldsymbol{V} \cdot \boldsymbol{K})^2 + \tfrac{1}{2}\{(\boldsymbol{N} \times \boldsymbol{K})^2/K^2\}[1 \pm 1]$$

(the negative sign corresponding to modes for which particle displacements are horizontal and are therefore unaffected by buoyancy forces), whereas when $N = 0$

(24) $$\omega^2 = (\boldsymbol{V} \cdot \boldsymbol{K})^2 + \frac{1}{2}\left\{\frac{(2\boldsymbol{\Omega} \cdot \boldsymbol{K})^2}{K^2}\right\}\left[1 \pm \left(1 + \frac{4(\boldsymbol{V} \cdot \boldsymbol{K})^2 K^2}{(2\boldsymbol{\Omega} \cdot \boldsymbol{K})^2}\right)^{1/2}\right].$$

The important difference between the last two equations is the presence in the latter of a term containing the (square of the) ratio of $\boldsymbol{V} \cdot \boldsymbol{K}$ to $(2\boldsymbol{\Omega} \cdot \boldsymbol{K})/K$, which has no counterpart in the former. (When $\boldsymbol{V} = 0$, $\omega^2 = (\frac{1}{2} \pm \frac{1}{2}) \times \{(\boldsymbol{N} \times \boldsymbol{K})^2 + (2\boldsymbol{\Omega} \cdot \boldsymbol{K})^2\}/K^2$.)

This "ratio term," which arises because Coriolis forces act at right angles to $\boldsymbol{u}_1$ and thus prevent the occurrence of "decoupled" modes, is of great physical importance, especially when $4(\boldsymbol{V} \cdot \boldsymbol{K})^2 \ll (2\boldsymbol{\Omega} \cdot \boldsymbol{K})^2/K^2$ (cf. Equations (20) and (22). The roots of Equation (24) are then

(25) $$\omega^2 \doteq (2\boldsymbol{\Omega} \cdot \boldsymbol{K})^2/K^2 \quad \text{and} \quad \omega^2 \doteq [(\boldsymbol{V} \cdot \boldsymbol{K})^2 K/(2\boldsymbol{\Omega} \cdot \boldsymbol{K})]^2,$$

the latter being the dispersion relationship for a hybrid type of wave discussed by Lehnert and Chandrasekhar (see **[25]**) which has no direct analogue in a stratified fluid.

The discussion of Equation (19) is less straightforward when none of the terms $(\boldsymbol{V} \cdot \boldsymbol{K})$, $(2\boldsymbol{\Omega} \cdot \boldsymbol{K})/K$ and $(\boldsymbol{N} \times \boldsymbol{K})/K$ vanishes, but when these terms are widely separated in magnitude then Equation (19) can be simplified by neglecting small quantities. There are six such limiting cases, corresponding to

(26a) $|\mathbf{V}\cdot\mathbf{K}| \gg |2\boldsymbol{\Omega}\cdot\mathbf{K}|/K \gg |\mathbf{N}\times\mathbf{K}|/K,$

(26b) $|\mathbf{V}\cdot\mathbf{K}| \gg |\mathbf{N}\times\mathbf{K}|/K \gg |2\boldsymbol{\Omega}\cdot\mathbf{K}|/K,$

(26c) $|2\boldsymbol{\Omega}\cdot\mathbf{K}|/K \gg |\mathbf{V}\cdot\mathbf{K}| \gg |\mathbf{N}\times\mathbf{K}|/K,$

(26d) $|2\boldsymbol{\Omega}\cdot\mathbf{K}|/K \gg |\mathbf{N}\times\mathbf{K}|/K \gg |\mathbf{V}\cdot\mathbf{K}|,$

(26e) $|\mathbf{N}\times\mathbf{K}|/K \gg |\mathbf{V}\cdot\mathbf{K}| \gg |2\boldsymbol{\Omega}\cdot\mathbf{K}|/K,$

(26f) $|\mathbf{N}\times\mathbf{K}|/K \gg |2\boldsymbol{\Omega}\cdot\mathbf{K}|/K \gg |\mathbf{V}\cdot\mathbf{K}|$

and the solutions are readily written down. Of particular interest in connection with the Earth's interior are the cases (26c) and (26d). When $\boldsymbol{\Omega}$ is so large that both $|\mathbf{V}\cdot\mathbf{K}|$ and $|\mathbf{N}\times\mathbf{K}|/K$ are very much less than $|2\boldsymbol{\Omega}\cdot\mathbf{K}|/K$ then, to first order of small quantities, Equation (19) reduces to

(27) $$\omega^2 \doteq \frac{(2\boldsymbol{\Omega}\cdot\mathbf{K})^2}{K^2} \quad \text{and} \quad \omega^2 \doteq \left[\frac{(\mathbf{V}\cdot\mathbf{K})^2 K}{2\boldsymbol{\Omega}\cdot\mathbf{K}}\right]^2 + \frac{(\mathbf{V}\cdot\mathbf{K})^2(\mathbf{N}\times\mathbf{K})^2}{(2\boldsymbol{\Omega}\cdot\mathbf{K})^2}$$

the dispersion relationships for the "inertial" and "magnetic" modes (see page 327). Evidently the criterion expressed by Equation (1) is probably both necessary and sufficient for inertial modes, at least for oscillations with wavelengths that are much smaller than the radius of the Earth's core, but in the case of the magnetic modes, Equation (1) must be supplemented by the more stringent condition that

(29) $$(\mathbf{N}\times\mathbf{K})^2/K^2 \ll (\mathbf{V}\cdot\mathbf{K})^2.$$

(The last point had been overlooked until, prompted by a question from the audience, I amplified the discussion given in the original version of these notes.) When $(\mathbf{N}\times\mathbf{K})^2/K^2 \gg (\mathbf{V}\cdot\mathbf{K})^2$ (but is much less than $(2\boldsymbol{\Omega}\cdot\mathbf{K})^2/K^2$), we have

(30) $$\omega^2 \doteq \frac{(2\boldsymbol{\Omega}\cdot\mathbf{K})^2}{K^2} \quad \text{and} \quad \omega^2 \doteq \frac{(\mathbf{V}\cdot\mathbf{K})^2(\mathbf{N}\times\mathbf{K})^2}{(2\boldsymbol{\Omega}\cdot\mathbf{K})^2}$$

in place of Equations (25) for the respective dispersion relationships for the inertial and magnetic modes.

References, Lecture V

1. J. M. Ade-Hall and N. D. Watkins, *Absence of correlations between opaque petrology and natural remanence polarity in Canary Island lavas*, Geophys. J. Roy. Astronom. Soc. **19** (1970), 351–360.
2. J. M. Ade-Hall and R. L. Wilson, *Opaque petrology and natural remanence in Mull (Scotland) dykes*, Geophys. J. Roy. Astronom. Soc. **18** (1969), 333–350.
3. M. J. Aitken and G. M. Weaver, *Recent archaeomagnetic results in England*, J. Geomag. Geoelect. **17** (1965), 391–394.
4. G. E. Backus, *Kinematics of geomagnetic secular variation in a perfectly conducting core*, Philos. Trans. Roy. Soc. London Ser. A **263** (1968), 239–266.
5. P. M. S. Blackett, *On distinguishing self-reversal from field reversed rocks*, J. Phys. Soc. Japan, **17** (B1) (1962), 699–710.
6. P. M. S. Blackett, E. C. Bullard and S. K. Runcorn (editors), *A symposium on continental drift*, Philos. Trans. Roy. Soc. London Ser. A **258** (1965), 1–323.
7. S. I. Braginskii, *Kinematic models of the Earth's hydromagnetic dynamo*, Geomagnetism and Aeronomy **4** (1964), 572–583.
8. G. R. R. Buchbinder, *Properties of the core-mantle boundary and observations of PcP*, J. Geophys. Res. **73** (1968), 5901–5923.
9. E. C. Bullard, *Reversals of the Earth's magnetic field*, Philos. Trans. Roy. Soc. London Ser. A **263** (1968), 481–524.
10. E. C. Bullard, C. Freedman, H. Gellman and J. Nixon, *The westward drift of the Earth's magnetic field*, Philos. Trans. Roy. Soc. London Ser. A **243** (1950), 67–92.
11. E. C. Bullard and H. Gellman, *Homogeneous dynamos and terrestrial magnetism*, Philos. Trans. Roy. Soc. London Ser. A **247** (1954), 213–278. MR **17**, 327.
12. K. E. Bullen, *An introduction to the theory of seismology*, Cambridge Univ. Press, New York, 1963. MR **29** #5626.
13. K. E. Bullen, F. Press and S. K. Runcorn (editors), *Phase transformations and the Earth's interior*, Phys. of Earth and Planetary Interiors **3** (1970), 1–518.
14. S. P. Burlatskaya, T. B. Nechayeva and G. N. Petrova, *Characteristics of secular variations of the geomagnetic field as indicated by world archaeomagnetic data*, Izv. Earth Physics **12** (1968), 62–70.
15. C. G. Collier and R. Hide, *Transient motion of a fluid in an irregular container*, (in prep).
16. A. M. Coode and S. K. Runcorn, *Satellite geoid and the structure of the Earth*, Nature **205** (1965), 891.
17. A. H. Cook, *Sources of harmonics of low order in the external gravity field of the Earth*, Nature **198** (1963), 1186.
18. A. Cox, G. B. Dalrymple and R. R. Doell, *Reversals of the Earth's magnetic field*, Sci. Amer. **216** (1967), 44–60.
19. L. Egyed, *The satellite geoid and the structure of the Earth*, Nature **203** (1964), 67–69.
20. T. F. Gaskell (editor), *The Earth's mantle*, Academic Press, New York and London, 1967.
21. P. A. Gilman, *Baroclinic, Alfvén and Rossby waves in geostrophic flow*, J. Atmospheric Sci. **26** (1969), 1003–1009.
22. P. Goldreich and S. J. Peale, *The dynamics of planetary rotations*, Ann. Rev. Astronom. Astrophys. **6** (1968), 287–320.
23. H. P. Greenspan, *The theory of rotating fluids*, Cambridge Univ. Press, New York, 1968.
24. W. H. Guier and R. R. Newton, *The Earth's gravity field as deduced from the Doppler tracking of five satellites*, J. Geophys. Res. **70** (1965), 4613–4626.

25. R. Hide, *Free hydromagnetic oscillations of the Earth's core and the theory of the geomagnetic secular variation*, Philos. Trans. Roy. Soc. London Ser. A **259** (1966), 615–647.

26. ———, *Planetary magnetic fields*, Planetary and Space Sci. **14** (1966), 579–586.

27. ———, *Motions of the Earth's core and mantle and variations of the main geomagnetic field*, Science **157** (1967), 55–56.

28. ———, *Interaction between the Earth's liquid core and solid mantle*, Nature **222** (1969), 1055–1056.

29. ———, *On hydromagnetic waves in a stratified rotating incompressible fluid*, J. Fluid Mech. **39** (1969), 283–287.

30. ———, *Dynamics of the atmospheres of the major planets*, J. Atmospheric Sci. **26** (1969), 841–853.

31. R. Hide and K.-I. Horai, *On the topography of the coremantle interface*, Physics of the Earth and Planetary Interiors **1** (1968), 305–308.

32. R. Hide and S. R. C. Malin, *Novel correlations between global features of the Earth's gravitational and magnetic fields*, Nature **225** (1970), 605–609; Nature Physical Science **230** (1971), 63.

33. W. R. Hindmarsh, F. J. Lowes, P. H. Roberts and S. K. Runcorn (editors), *Magnetism and the cosmos*, Oliver and Boyd, Edinburgh, 1965.

34. P. M. Hurley (editor), *Advances in Earth sciences*, M.I.T. Press, Cambridge, Mass., 1966.

35. E. Irving, *Paleomagnetism*, Wiley, New York, 1964.

36. ———, *Paleomagnetism of some carboniferous rocks from New South Wales and its relation to geological events*, J. Geophys. Res. **71** (1966), 6025–6051.

37. H. Jeffreys, *The Earth*, 4th ed., Cambridge Univ. Press, London, 1959.

38. H. Kanamori and F. Press, *How thick is the lithosphere?*, Nature **266** (1970), 330–331.

39. W. M. Kaula, *An introduction to planetary physics: Terrestrial planets*, Wiley, New York, 1968.

40. ———, *A tectonic classification of the main features of the Earth's gravitational field*, J. Geophys. Res. **74** (1969), 4807–4826.

41. ———, *Earth's gravity field: relation to global tectonics*, Science **169** (1970), 982–985.

42. N. Kawai, K. Hirooka and K. Tokieda, *A vibration of geomagnetic axis around the geographic north pole in historic time*, Earth and Planetary Sci. Letters **3** (1967), 48–50.

43. F. E. M. Lilley, *On kinematic dynamos*, Proc. Roy. Soc. London Ser. A **316** (1970), 153–167.

44. D. Lortz, *Exact solutions of the hydromagnetic dynamo problem*, Plasma Phys. **10** (1968), 967–972.

45. W. V. R. Malkus, *Hydromagnetic planetary waves*, J. Fluid Mech. **28** (1967), 793–802. MR **36** #4869.

46. ———, *Precession of the Earth as the cause of geomagnetism*, Science **160** (1968), 259–264.

47. B. G. Marsden and A. G. W. Cameron (editors), *The Earth-Moon system*, Plenum Press, New York, 1966.

48. H. K. Moffatt, *Turbulent dynamo action at low magnetic Reynolds numbers*, J. Fluid Mech. **41** (1970), 435–452.

49. W. H. Munk and G. J. F. MacDonald, *The rotation of the Earth*, Cambridge Univ. Press, New York, 1960.

50. M. W. McElhinny and M. E. Evans, *An investigation of the geomagnetic field in the Precambrian*, Phys. of Earth and Planetary Interiors **1** (1968), 485–497.

51. T. Nagata (editor), *Proceedings of the Benedum Earth magnetism symposium*, Univ. of Pittsburgh Press, Pittsburgh, Pa., 1962.

52. T. Namikawa and S. Matsushita, *Kinematic dynamo problem*, Geophys. J. Roy. Astronom. Soc. **19** (1970), 395–415.

53. E. Orowan, *The origin of the ocean ridges*, Sci. Amer. **221** (1969), 103–109.

54. E. N. Parker, *The occasional reversal of the geomagnetic field*, Astrophys. J. **158** (1969), 816–827.

55. T. Rikitake, *Electromagnetism and the Earth's interior*, Elsevier, Amsterdam, 1966.

56. P. H. Roberts, *An introduction to magnetohydrodynamics*, American Elsevier, New York; Longman, London, 1967.

57. M. G. Rochester, *Geomagnetic westward drift and irregularities in the Earth's rotation*, Philos. Trans. Roy. Soc. London Ser. A **252** (1960), 531–555.

58. R. B. Roden, *Electromagnetic core-mantle coupling*, Geophys. J. Roy. Astronom. Soc. **7** (1963), 361–374.

59. S. K. Runcorn (editor), *Mantles of the Earth and terrestrial planets*, Wiley, New York, 1967.

60. ———, *The application of modern physics to the Earth and planetary interiors*, Wiley, London, 1969.

61. ———, *The rotation of the planets and their interiors*, Ist. Naz. Alta Mat. Sympos. Math. **3** (1970), 193–202.

62. F. D. Stacey, *Physics of the Earth*, Wiley, London, 1969.

63. K. Stewartson, *Slow oscillations of fluid in a rotating cavity in the presence of a toroidal magnetic field*, Proc. Roy. Soc. London Ser. A **299** (1967), 173–187.

64. G. C. J. Suffolk, *Precession in a disk dynamo model of the Earth's magnetic field*, Nature **226** (1970), 628–629.

65. G. Suffolk and D. W. Allan, *Planetary magnetohydrodynamic waves as a perturbation of dynamo solutions*, in Runcorn 1969, pp. 653–656.

66. D. C. Tozer (editor), *Symposium on non-elastic processes in the mantle*, Geophys. J. Roy. Astronom. Soc. **14** (1967), 1–450.

67. R. L. Wilson, *Permanent aspects of the Earth's non-dipole magnetic field over upper Tertiary times*, Geophys. J. Roy. Astronom. Soc. **19** (1970), 417–437.

68. R. L. Wilson and N. F. Watkins, *Further correlations between the petrology and the natural remanent magnetization of basalts*, Geophys. J. Roy. Astronom. Soc. **12** (1967), 405–415.

69. T. Yukutake and H. Tachinaka, *The westward drift of the geomagnetic secular variation*, Bull. Earthquake Res. Inst. **45** (1968), 1075–1102.

70. P. Goldreich and A. Toomre, *Some remarks on polar wandering*, J. Geophys. Res. **74** (1969), 2555–2567.

71. R. Hide, *On the Earth's core-mantle interface*, Quart. J. Roy. Meteorol. Soc. **96** (1970), 579–590.

72. M. G. Rochester, "Core-mantle interactions: Geophysical and astrophysical consequences," In: *Earthquake displacement fields and the rotation of the Earth*. L. Mansinha et al. (editors), Reidel, Dordrecht, 1970, pp. 136–148.

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