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Booker's Mode Theory of Guidance of Electromagnetic Waves in the Magnetosphere.

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8. BOOKER'S MODE THEORY OF GUIDANCE OF ELECTROMAGNETIC WAVES IN THE MAGNETOSPHERE

8.1 Introduction

In Sect. 6 I have presented the ray theory of Smith et al. (1960) for whistler trapping in field aligned ducts of density ionization. This theory, which gives the critical fractional density gradients for ray trapping as a function of initial wave normal angle with respect to the magnetic field and as a function of frequency, is based on the whistler dispersion equation (6.1) and hence refers to the FMS branch in the frequency region

$$\omega_{Bi} << \omega < |\omega_{Be}|$$

A quite different theory of guidance in ducts was developed in 1962 by H. Booker and is the object of the present section. Booker's theory is different from the work of Smith et al. (1960) in several respects. First of all it is a mode theory (and not a ray theory) of wave guidance. The duct is modelled by Booker as a thin field aligned density discontinuity which is contrary to the ray theory of Smith et al. where the thickness of the duct must exceed the wavelength of the waves whose propagation is considered. We will come back later to the physical meaning of the discontinuity model of Booker. More important, Booker's theory is done using the dispersion relation for purely longitudinal propagation and thus, in fact, ignores the anisotropic properties of the medium. The point of view here is that of deciding under which conditions a ray which, at a given point in the magnetosphere is field aligned, continues to stay so during propagation. The use of the longitudinal dispersion relation (eqs. (1.21) (1.22))

which is exact at all frequencies, implies that Booker's theory of guidance covers all possible electromagnetic waves from hydromagnetic to radio frequencies. Finally, curvature of the Earth's magnetic field (which was neglected in the whistler theory of Smith et al. (1960) but has been later included by Walker (1966b) in a study also directed to whistler waves), plays, as we will see, a crucial role in Booker's theory.

8.2. Magnetospheric model

The magnetic field of the Earth is, in the simplest approximation, assumed to be that of a central dipole (see Fig. 36) and a particular line of flux is conveniently identified by the latitude λ at which it intersects the surface of the Earth. With s we denote the distance, measured along a line of flux of a given point P on that line from the point A where the line intersects the equatorial plane; r will denote the distance of P from the center of the Earth and the angle AOP will be indicated by θ. Then, if a is the radius of the Earth, the equation of the line of flux of latitude Λ is

$$r = a\frac{\cos^2\theta}{\cos^2\Lambda} \qquad (8.1)$$

The variation of the radial distance r with θ for specified lines of flux is illustrated in Fig. 37. For most purposes, it is convenient to identify position on a line of flux of latitude Λ by means of the angular distance θ from the equatorial plane. In some cases, however, it may be necessary to refer to the actual distance s along the line. It is easy to see that the relation between the two is given by the equation

$$\frac{1}{s}\frac{ds}{d\theta} = \frac{\cos\theta}{\cos^2\Lambda}\left(4-3\cos^2\theta\right)^{1/2} \qquad (8.2)$$

Let us indicate with f_{MO} the value of the electron gyrofrequency at the equator of the Earth. In terms of this, the gyrofrequency at the point r, θ of the ionosphere or magnetosphere will be given, for the assumed dipole field of the Earth, by

$$f_M = f_{MO}(a/r)^3\ (4-3\cos^2\theta)^{1/2} \qquad (8.3)$$

For the line of latitude Λ, we obtain the variation with θ of f_M by substituting (8.1) into (8.3). The result is

$$f_M = f_{MO}\frac{\cos^6\Lambda}{\cos^6\theta}\left(4-3\cos^2\theta\right)^{1/2} \qquad (8.4)$$

This variation (and the analogous for the proton gyrofrequency) is illustrated in Fig. 38 for a number of lines of flux (and for f_{MO}=0.87 M_c/s).

Going now to the density variations, Booker assumes the following model for the smooth density variations of the magnetospheric plasma

$$N = N_o(a/r)^P \cos^q\theta \qquad (8.5)$$

with p and q positive. Correspondingly, one obtains for the electron plasma frequency

$$f_{pe} = f_{Po}(a/r)^{1/2P}\cos^{1/2q}\theta \qquad (8.6)$$

Using eq. (8.1) for the line of flux corresponding to latitude Λ, we obtain, for the variation of the electron plasma frequency along a given line of flux

$$f_{pe} = f_{po}\frac{\cos^p\Lambda}{\cos^{p-\frac{1}{2}q}\theta} \qquad (8.7)$$

The variation of this frequency with θ along several lines of flux is given in Fig. 39 corresponding to the following parameter values

$p = 4$, $q = 0$, $f_{po} = 1$ Mc/s

Notice that the plasma distribution assumed by Booker refers really to the magnetosphere and would have to be modified at levels below ∿1000 km.

8.3 Inner and outer zones of longitudinal propagation

As already mentioned in Sect. 8.1, Booker's theory of guidance refers to electromagnetic waves for longitudinal propagation. The two refractive indices are then given (see Sect. 1) by

$$n_\pm^2 = 1 - \frac{\omega_{pi}^2}{\omega(\omega \mp \omega_{Bi})} - \frac{\omega_{pe}^2}{\omega(\omega \pm \omega_{Be})} \qquad (8.8)$$

Here, as we saw in Sect. 1.3, the upper sign refers to L waves (singularity at $\omega=\omega_{Bi}$) and the lower to R waves (singularity at $\omega=\omega_{Be}$). We recall that the two waves are also commonly denoted, especially at radio frequencies, as ordinary (L) and extraordinary (R) waves. In the range of frequencies $\omega \lesssim \omega_{Be}$ (and for the ionospheric and magnetospheric plasmas for which $\omega_{Be} \ll \omega_{pe}$), the extraordinary (R) wave, which is the FMS branch using Shafranov's notation (see Sect. 1), becomes the parallel electron whistler for $\omega>\omega_{Bi}$ and the hydromagnetic magnetoacoustic mode for $\omega<\omega_{Bi}$. The ordinary (L) wave, on the other hand, is the Alfvén branch for $\omega<\omega_{Bi}$ going over to the ion cyclotron wave at frequencies ω close to ω_{Bi} and then essentially it propagates (SE branch of Fig. 1) only at frequencies above the cut-off frequency $\omega_o^{(3)} \sim (\omega_{pe}^2 + 1/4\omega_{Be}^2)^{1/2} - \frac{1}{2}|\omega_{Be}|$ (which, for the ionosphere magnetospheric plasma, are in general above ω_{Be}).

It is important now to point out that, given the refractive index (8.8), for any fixed wave frequency, the magnetosphere can be divided into zones where longitudinal propagation is possible and zones where it is forbidden. The transition between the two regions occurs, as we know, when n^2 passes through either of the values 0 or ∞.

Referring to the R wave, a resonance occurs when

$$\omega = \omega_{Be} \tag{8.9}$$

Thus, for this wave, at a given frequency, one boundary of the forbidden zone for longitudinal propagation occurs at the level in the magnetosphere where the local electron gyrofrequency is equal to the frequency under consideration. For a frequency of 100 kc, this gives the inner surface of the stapled area of Fig. 40a (corresponding to the magnetospheric model of Sect. 8.2). The outer surface corresponds to the condition $n^2=0$. This is the frequency $\omega_o^{(3)}$ of Shafranov given by

$$\omega_o^{(3)} = (\omega_{pe}^2 + \frac{1}{4}\omega_{Be}^2)^{1/2} - \frac{1}{2}|\omega_{Be}| \tag{8.10}$$

above which we have propagation of the SE branch (see Fig. 1). Thus, in the stapled region in Fig. 40a propagation is not possible and the two conditions $n_+^2=\infty$, $n_+^2=0$ separate the magnetosphere into an inner region and an outer region of propagation. In the outer zone a wave supposed to be guided along the lines of flux of the Earth's magnetic field is returned when it encounters the outer surface of the forbidden

zone (where $n_+^2=0$). On the other hand, a wave travelling in the inner zone (and also supposed to be guided), when it intersects the $n_+^2=\infty$ surface is not reflected but rather it dissipates there causing some heating of the magnetosphere.

Fig. 40a refers to the R wave at 100 Kc. The forbidden magnetospheric zone will of course vary with frequency. The variation of its location in the equatorial plane, as a function of frequency, is indicated by the horizontal shading in Fig. 41. From this we see clearly that, at frequencies below $\sim$1 Kc the inner zone for propagation for the R wave occupies virtually the entire magnetosphere. These are frequencies typical of whistlers which can in general propagate from one to the other hemisphere without encountering zones of forbidden propagation. On the other hand, we see from the same figure that high frequencies do essentially propagate only in the outer zone.

The same type of reasoning can be applied to the n_- wave (L wave). There, it is $n_-^2=\infty$ when

$$\omega = \omega_{Bi} \tag{8.11}$$

and $n_-^2=0$ at the frequency

$$\omega_o^{(2)} \sim \omega_{pe} \tag{8.12}$$

which give the boundaries of the inner and outer zones of longitudinal propagation for this wave. In the inner region we have therefore propagation of the A branch and in the outer region of the FE branch. At the same frequency ω=100 Kc (considered before for the R wave), we see now for the L wave, from Fig. 40b, that there is an outer zone of propagation for the L wave but no inner zone (except below the ionosphere). On the other hand, at the very low frequency of 10 c/s, Fig.

40c, always referring to the L wave (which is now the Alfvén wave) indicates that there is essentially no outer zone of propagation but instead there is a substantial inner zone of longitudinal propagation. It is in this inner zone that Alfvén waves possibly propagate between conjugate hemispheres of the Earth.

The location of the forbidden zone of propagation for the L wave is shown, at the equator and as a function of frequency, in Fig. 41, denoted with a vertical shading. As the external boundary of the inner region of propagation for the L wave occurs at $\omega=\omega_{Bi}$, whereas it occurs at $\omega=\omega_{Be}$ for the R wave, we see that the forbidden zone of propagation of the L wave is enormously thicker than that of the R wave. At audio frequencies (the whistler's frequencies) this forbidden zone occupies virtually the all magnetosphere (and it is only the R mode whistlers that propagate). It is only for the very low frequencies that we have an almost entirely inner zone of propagation, as seen in Fig. 40c.

Fig. 40a-c, giving the inner and outer zone of magnetospheric propagation, indicate also that, for a given wave frequency, there is a maximum latitude of line of flux for which longitudinal propagation entirely within the inner zone is possible between conjugate hemispheres. Conversely, for a given latitude of a flux line, there is a maximum frequency ~~above~~ below which longitudinal propagation entirely within the inner zone is possible between conjugate hemispheres. These important features of magnetospheric propagation are presented in Fig. 42 which gives in fact the maximum frequencies for interemispheric propagation (within the inner zone) as a function of latitude of the line of flux, both for the R (dashed line) and the L waves (solid line). In this same figure one finds indicated both the whistler and the Alfvén wave regimes.

8.4 Field curvature

An important quantity which must be computed is the curvature of the Earth's magnetic field. The condition for guiding of electromagnetic waves that will be later imposed will be that the curvature of a given ray follows the curvature of magnetic field lines.

With reference to Fig. 43, let MN and M'N' be two adjacent magnetic lines of force of any curvature. Consider then two equipotential lines AB and A'B' of length dr separated by a small distance and inclined by an angle α. If OA=OA'=r and the magnetic fields along AA' and BB' and H + dH respectively, we can write

$$(H+dH)(r+dr)d\alpha = Hr\, d\alpha$$

from which

$$H\, dr + r\, dH = 0 \qquad (8.13)$$

Then the local curvature 1/r of Fig. 43 will be given by

$$K_F \equiv \frac{1}{r} = -\frac{1}{H}\frac{dH}{dr} = -\frac{1}{f_M}\frac{df_M}{dr} \qquad (8.14)$$

where, in the last expression, f_M is the cyclotron frequency. More generally it is advisable to write

$$K_F = -\frac{1}{H}\nabla_\perp H = -\frac{1}{f_M}\nabla_\perp f_M \qquad (8.15)$$

to recall that the gradient which enters is that along the local normal to the magnetic field line. Eq. (8.15) is general and can be applied to a variety of field patterns. Let us see, in particular, the results it gives for the magnetic field of the Earth. We want the field curvature at the point θ on the line of latitude Λ. An easy calculation, using for example (8.4), leads to

$$\frac{K_F}{K_E} = 3\cos^2\Lambda \frac{2-\cos^2\theta}{\cos\theta\left(4-3\cos^2\theta\right)^{1/2}} \qquad (8.16)$$

where, as it is convenient, the field curvature has been normalized to the curvature of the Earth

$$K_E = 1.57 \times 10^{-7} \text{ rad/meter} \qquad (8.17)$$

The θ dependence of the ratio K_F/K_E is reported in Fig. 44 for various values of Λ. For low latitude lines, the field curvature exceeds the Earth's curvature while the opposite is true for high latitude lines. Note, however, that, apart from special cases, the field curvature is not of a different order of magnitude with respect to the Earth's curvature.

8.5 Ray curvature and excess field curvature

Consider a curved magnetic field and an electromagnetic wave, considered as a ray, bent into a curve owing to refractive effects and inclined at a small angle ϕ at P and $\phi-d\phi$ at Q, as shown in Fig. 45, PQ being ds, an element of the ray path. For simplicity, let the field line and the ray be in the same plane. Let the normal distance from Q to the field line through P be dr. If n is the refractive index for the ray, by Snell's law, the curvature of the ray can be expressed as

$$K_R = -\frac{d\phi}{ds} = -\frac{1}{n}\frac{dn}{dr}\cos\phi \qquad (8.18)$$

For small values of ϕ, we have

$$K_R = -\frac{1}{n}\nabla_\perp n \qquad (8.19)$$

i.e. the ray curvature is given by the fractional downward gradient of refractive index in the outward principal normal direction to the magnetic field line.

Using now the expression (8.8) for longitudinal refractive index, we obtain

$$K_R = \frac{1-n^2}{n^2}\frac{1}{N}\nabla_\perp N + \frac{K_F}{2n^2}\left[\frac{1}{\omega\,\omega_{Bi}}\,\frac{\mp\,\omega_{pi}^2}{(1 \mp \omega/\omega_{Bi})} \pm \frac{\omega_{pe}^2}{\omega\,\omega_{Be}(1 \pm \omega/\omega_{Be})}\right] \qquad (8.20)$$

with K_F given by (8.15). There are therefore two separate contribution to the ray curvature, one arising from the fractional gradient of the magnetic field and the other from the fractional gradient of the ionization density.

The condition for guiding of electromagnetic rays along the curved magnetic field line is clearly

$$\frac{K_R}{K_F} = 1 \qquad (8.21)$$

In relation to guiding it is important to introduce the quantity

$$K = K_F - K_R \qquad (8.22)$$

which is the excess of field over ray curvature, (or briefly excess field curvature). It is this quantity, as we will see, which enters the notion of modified refractive index (see Sect. 8.7) and it is the consequences of this curvature differential that have to be overcome by means of field aligned irregularities of ionization, if guiding along the lines of flux of the Earth's magnetic field is to take place. Using the model of magnetic field and density described in Sect. 8.2, Booker derives the variation of the differential curvature along a number of lines of flux and for a number of wave frequencies. His results are shown in Figs. 46a-d where the excess curvature on the ordinate axes is normalized to Earth's curvature. We see from these results that the curvature differential between the rays and the lines of flux is, broadly speaking, of the order of magnitude of the Earth's curvatures. Furthermore K is always positive. This means that, for the magnetic field and density models assumed, additional downward curvature is required at all places and all frequencies to produce guiding along the lines of flux.

8.6 Model for field aligned irregularities of ionization

The model assumed by Booker to study guided propagation in field aligned ducts is the following. Take a particular line of flux of the Earth's magnetic field and rotate it around the magnetic axis, thereby forming a geometrical surface in the magnetosphere. Booker assumes that, upon crossing this surface from inside to outside there is a discontinuous increase in electron density ΔN, either positive or negative. Inside the surface the ionization density is assumed to be given by eq. (8.5). Outside, the density variation is still represented by (8.5) but with a sligtly different value of the constant $(N_o+\Delta N)$. Thus, with respect to the ray theory of Smith et al. (1960) for guiding of whistlers in ducts which was requiring density variation across the field containing many wavelengths, Booker's approach is quite different in that it refers to a thin field aligned discontinuity.

As will be seen, [illegible] Booker's treatment such a surface of discontinuity is capable [illegible] acting as a guide for transmission between conjugate points in the northern and southern hemispheres. A ray on the inside of the surface of discontinuity can, under suitable conditions, suffer successive internal reflections from the interface and thus convey energy along a line of flux.

8.7 Modified refractive index

The method of modified refractive index to study ray propagation with account taken of the curvature of the Earth's magnetic field was first introduced by Booker and Walkinslaw (1946) in a study of tropospheric propagation.

In the free space between the Earth and the ionosphere a ray travels in a straight line. An observer who thought that the Earth was flat would say that this line is not straight but has an upward curvature which be might ascribe to a variation of refractive index with height. If the actual refractive index is n, let us call $n'(z)$ the fictitious refractive index (a function of the vertical) which one would have to imagine with a flat earth and let us derive what is the form of $n'(z)$. Suppose then that, in this fictitious medium, the ray is inclined at an angle ψ to the horizontal and let s be the distance measured along the ray. From Snell's law we have that $n'\cos\psi=\text{const}$ so that

$$\frac{dn'}{d\psi} = n' \operatorname{tg}\psi$$

As a consequence, the curvature of the ray is

$$\frac{d\psi}{ds} = \frac{dn'}{ds} \Big/ \frac{dn'}{d\psi} = \frac{dn'}{dz}\sin\psi \Big/ n' \operatorname{tg}\psi = \frac{1}{n'}\frac{dn'}{dz}\cos\psi \qquad (8.23)$$

Now, again referring to free space, an observer who thinks that the earth is flat will think that the ray has an upper curvature $\cos\psi/R_E$, with R_E the Earth's radius. Hence, equating this to (8.23) we obtain

$$\frac{1}{n'}\frac{dn'}{dz} = \frac{1}{R_E}$$

$$n' = \exp\left(\frac{z-h}{R_E}\right) \qquad (8.24)$$

where h is a constant. For example h can be the level of the ionosphere so that, just below, n'=1. Now, as in practice z-h is always much smaller than R_E we can expand (8.24) and obtain

$$n'^2 = 1 + \frac{2(z-h)}{R_E} \qquad (8.25)$$

This formula, as already mentioned, applies to the free space below the ionosphere. However the trick of substituting the actual refractive index with one containing a linear variation in z to simulate curvature ~~of the Earth's line~~ can be applied more generally and, in fact, was applied by Booker to study the guiding of rays along a density discontinuity. Then, in studying nearly longitudinal propagation near a particular point of a particular line of magnetic flux, the line of flux and the associated field aligned density irregularity are considered to be straight but, instead of the actual refractive index n, a modified index n' is employed to study propagation, given by

$$n' = n(1 + Kz) \qquad (8.26)$$

Here z is a coordinate measured transverse to the field in the outward direction and K is the differential curvature defined in (8.22). The modified refractive index (8.26) has therefore an additional transverse

gradient across the magnetic field not possessed by the actual refractive index and actually proportional to the differential curvature K. As a consequence, the curvature of wave rays with respect to the straight line of flux in this model is the same as the curvature of the actual rays relative to the bent lines of force in the real world. This technique requires that the radius of curvature of the rays and of the lines of magnetic flux to be large compared with the local wavelength in the medium. After the line of flux has been straightened out in accordance with the concept of modified refractive index, a ray which is successively reflected at the interface has the form shown in Fig. 47. Each segment of the ray has now the differential curvature K calculated from (8.22) and plotted in Fig. 47. As we move along the guide, the curvature K of the ray segments in Fig. 47 changes slowly in accordance with the variations shown in Figs. 46.
In conclusion it should be pointed out that, as clear from Fig. 47, the linear z variation of the modified refractive index (8.26) refers to the width of the wave guide.

8.8 Phase integral condition in Booker's theory

Let α be the elevation angle of a wave at the plasma interface, as shown in Fig. 47. We shall assume this angle to be small so that use can be made of the longitudinal expression (8.8) for the refractive index n. We will investigate a posteriori to which extent this is justified. Corresponding to the discontinuity in density (from N_o + to N_o+ΔN) at the interface of Fig. 47 we will have a discontinuity in refractive index. Reflection at the interface is described by Fresnel's formulae. The critical value α_c of α below which total internal reflection occurs is found (see Appendix B) to be given by

$$\alpha_c^2 = \frac{1-n^2}{n^2} \frac{\Delta N}{N} \tag{8.27}$$

The phase change ϕ_1 of the waves at this reflection, corresponding to the value α of the inclination angle is given approximately (see Appendix B) by

$$\Phi_1 = 2 \operatorname{arctg} \left(\frac{\alpha_c^2}{\alpha^2} - 1 \right) \tag{8.28}$$

From this formula we see that, when $\alpha > \alpha_c$, ϕ_1 is complex corresponding to imperfect reflection.

We wi l now write the phase integral condition of mode theory (see Sect. 7) for the wave guide of Fig. 47. There, one of the boundaries of the wave guide is the interface across which there is the density jump. The other edge is the caustic surface indicated in Fig. 47 which corresponds to reflection at a zero of q (the z projection of the refractive index vector). We have already calculated in Sect. 7 the change in phase of a wave while crossing twice the wave guide in the case of a linear variation of refractive index (see eq. 7.10). For the Booker's case ($\alpha \ll 1$) this becomes

$$2 \frac{2\pi\alpha^3}{3K\lambda} - \frac{\pi}{2} - \Phi_1 = 2\pi (p-1) \tag{8.29}$$

where p=1 is now the lowest order mode and the term $-\pi/2$ is the phase change at the caustic. Substituting eq. (8.28) for ϕ_1, we rewrite (8.29) as

$$\frac{2\pi\alpha^3}{3K\lambda} - \operatorname{arctg} \left(\frac{\alpha_c^2}{\alpha^2} - 1 \right)^{1/2} = \left(p - \frac{3}{4} \right) \pi \tag{8.30}$$

At a given point of a line of flux of the Earth's field, and for a given frequency, the differential curvature K of eq. (8.22) can be

obtained from Figs. 46. Also, for a given fractional density change $\Delta N/N$ across the interface, the critical angle α_c is obtained from (8.27). When all this is used, one determines, from (8.30), the angle of elevation α of the p-th mode.

8.9 <u>Minimum fractional density gradient for trapping and track width</u>

It is important to derive the minimum fractional density change $\Delta N/N$ that produces reasonable good guiding of one mode. This occurs when the mode changes from leaky to trapped behaviour and, therefore, when the mode angle α is close to α_c. In these conditions, the second term on the left hand side of eq. (8.30) (the phase change at the interface) can be neglected and, referring to the lowest order mode p=1, the equation reduces to

$$\frac{2\pi\alpha^3}{3K\lambda} = \frac{1}{4}\pi \tag{8.31}$$

of which we must impose, for minimum guiding, the solution to be

$$\alpha = \alpha_c \tag{8.32}$$

Thus the condition for the lowest mode to be guided becomes

$$\alpha_c = (3/8\ K\lambda)^{1/3} \tag{8.33}$$

Using now eq. (8.27) for α_c, it follows that <u>the minimum fractional change in ionization density necessary to guide the lowest order mode</u> is given by

$$\frac{\Delta N}{N} = \frac{n^2}{1-n^2} \left(\frac{3}{8} K\lambda \right)^{2/3} \tag{8.34}$$

necessary fo guiding as a function of position (θ) along different magnetic flux lines.

Fig. 48a refers to the frequency f=1Kc/s. At this frequency, the lines of magnetic flux for latitudes Λ=15°, 30°, 45°, 60°, 75° which are reported in the figure, lie entirely in the inner zone of longitudinal propagation for the FMS (R) wave and in the forbidden zone for the L wave (see Sect. 8.3). Thus all the curves in Fig. 48a refer to R waves (whistlers). It is seen that, at 1Kc/s, guidance can be produced by fractional variations of ionization density of the order of 1%. The track width of the lowest order mode, for the same frequency, is plotted as a function of θ, for different latitudes Λ, in Fig. 49a and, as it can be seen, varies from ∿10 to ∿100 km.

Going now to a low frequency, of 10 c/s, Fig. 48b gives the minimum $\Delta N/N$ required for guiding. In this case inner zone longitudinal propagation is possible not only for the R but also for the L wave (correspondingly different curves, dashed and not dashed respectively are given in the Figure). Here we see that guidance of such waves round the lines of flux of the Earth requires higher values of $\Delta N/N$, from 10% to 25%, than in the case of the 1Kc/s wave, the smaller values being adequate for the higher latitude lines. The track width at this frequency, given in Fig. 49b is also much higher than for the 1Kc/s wave and ranges roughly from 100 to 1000 km.

On the other hand, moving to much higher frequencies and, for example, for f=10Mc/s, we see from Fig. 48c that we need $\Delta N/N$∿10% to ensure guiding for low latitude lines (and guiding takes place now in the outer zone). For high latitudes and near the equatorial plane, guiding becomes impossible. The track width, given in Fig. 49c, is, for this frequency, much smaller than at the lower frequencies and, in general, below 1 km.

Thus, the overall impression that we get from these (and other)

numerical results, is that guidance disappears as we go to high frequencies, of the order of 10 Mc/s, and also going to low frequencies, of the order of 10 c/s or smaller. On the other hand, guidance is most easily obtained for whistler waves, in the inner zone of propagation, at intermediate frequencies (Kc/s). For a given mode, at a given frequency and on a given flux line, the variations of $\Delta N/N$ with position (θ) along the line are not very remarkable but, in any case, one gets somewhat larger values of $\Delta N/N$ when the line crosses the equatorial plane.

A very convenient way of looking globally at these results is that of fixing θ, at the position of crossing of the equatorial plane for any given line, and there plot the values of $\Delta N/N$ required for guiding and the corresponding track width as a function of frequency. This is done in Figs. 50a-d where the left hand ordinate scale refers to $\Delta N/N$ and the right hand scale to the width w and the successive figures (a-d) refers to lines of different latitude. The curves for $\Delta N/N$ show strikingly the disappearance of guiding both as we go to high radio frequencies and as we go to low hydromagnetic frequencies. The low frequency branches refer, as it is explicitly indicated, to inner zone of propagation and the high frequency branches to outer zone of propagation. These curves show that, for the R waves, there are about 5 decades in frequency where guidance is possible for relatively small values of $\Delta N/N$. From the sequence a-d of the figures, it is also seen that this frequency range slides down by about 3 decades as we move from low latitude to high latitude lines of flux. In other words, guidance of radio waves in the outer zone is easier at low latitudes than at high latitudes, while the opposite is true for guidance of hydromagnetic waves in the inner zone (easier at high than at low latitudes).

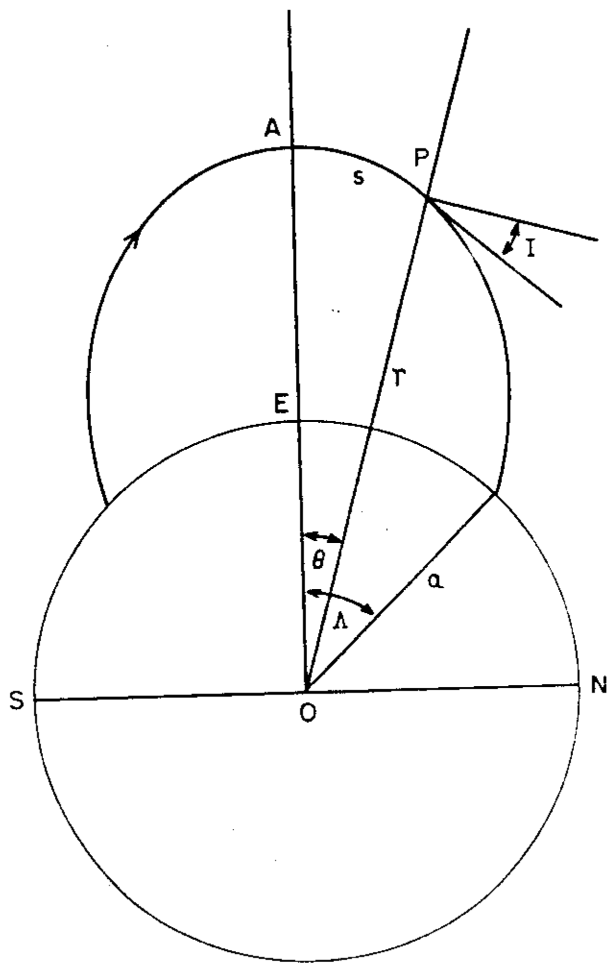
To explain in physical terms, at least in a qualitative way, these numerical results we must give some comments on the meaning of the track width of the modes. As commented by H. Booker, the model taken of a discontinuity in density at an interface parallel to a flux line, must be interpreted in the sense that the results obtained are valid for any continuous density variation across the magnetic flux line of scale somewhat less than the track width w obtained by the calculations(*). It follows then that the track width obtained may be also interpreted as the (maximum) lateral extension of the field aligned density irregularity. Let us look now from this point of view at the results shown in Figs. 50a-d. The values of w reported in these figures can be interpreted as the maximum transverse scale of the density irregularity for which (under the calculated value of ΔN/N) trapped propagation (of the lowest order mode) is obtained. For the intermediate frequencies (Kc's), where guiding is obtained for relatively small values of ΔN/N (down to 1%), we see that the transverse scale runs from ∿2 km to ∿100 km for the 15° line of flux and from 50 km to ∿1000 km for the 75° line of flux.

For f=10 Mc/s the transverse scale is less than 1 km. Most likely inhomogeneities on this scale are present in the magnetosphere. However, what inhibits guidance at these radio frequencies, are the higher values of ΔN/N required.

(*) On the other hand, being the calculation based on the use of the phane integral condition, such track width must large in comparison with the wavelength of the mode considered. Thus we see that, in the end, the Booker's discontinuous model is not very different from the duct models used in the ray theory of guided whistlers by Smith et al. (1960).

On the other hand, at hydromagnetic frequencies, the transverse scale w becomes exceedingly large (as it is also seen in Fig. 50) and, in fact, it approaches, at the low frequency limit, the radius of the Earth. Such large scale inhomogeneities are at least unlikely. It is seen therefore that guidance at very low hydromagnetic frequencies in the magnetosphere may be impossible because such large scale inhomogeneities are simply not accomodated.

FIG. 36



$$z = a \frac{\cos^2\theta}{\cos^2\Lambda}$$

FIG 37

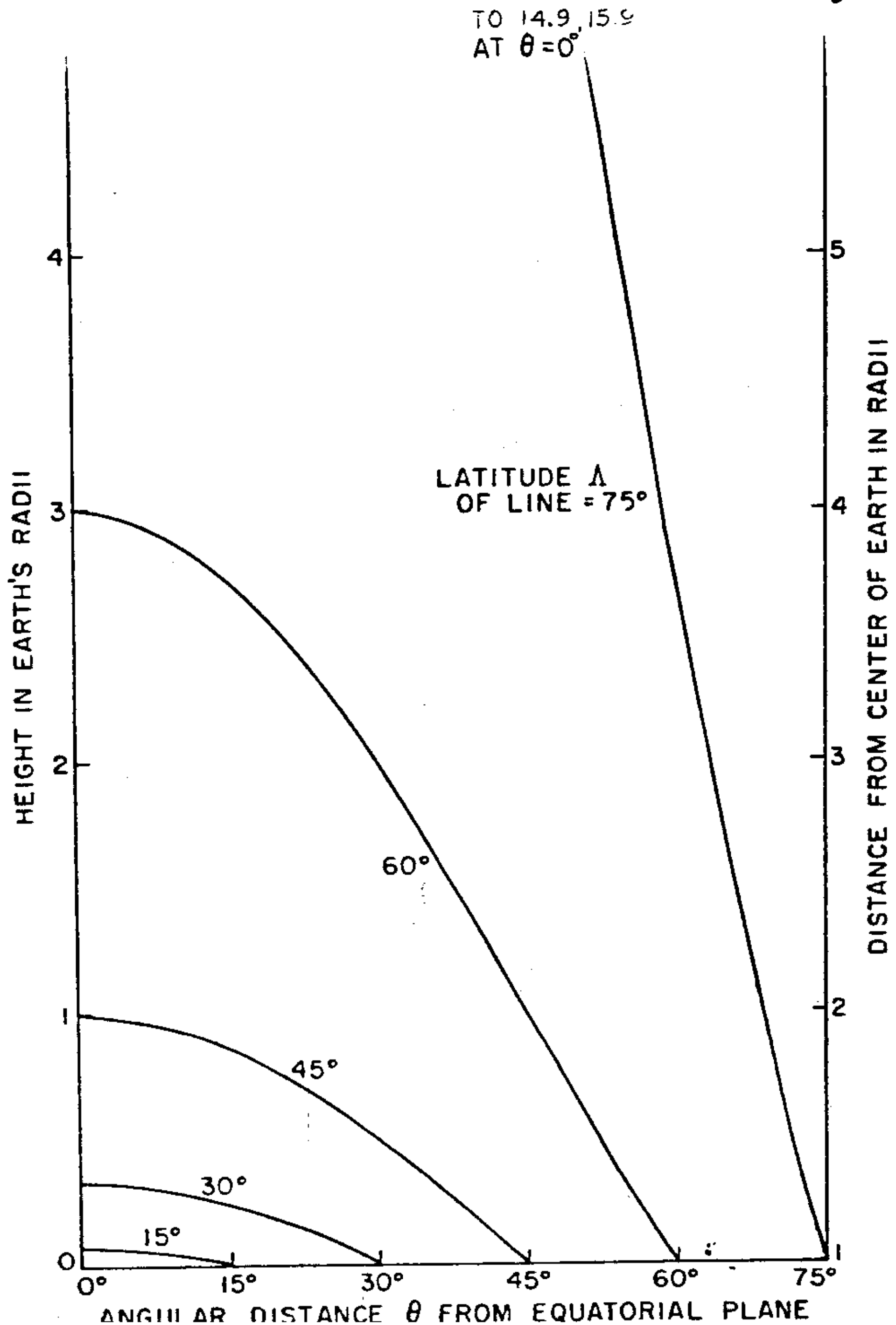


FIG 38

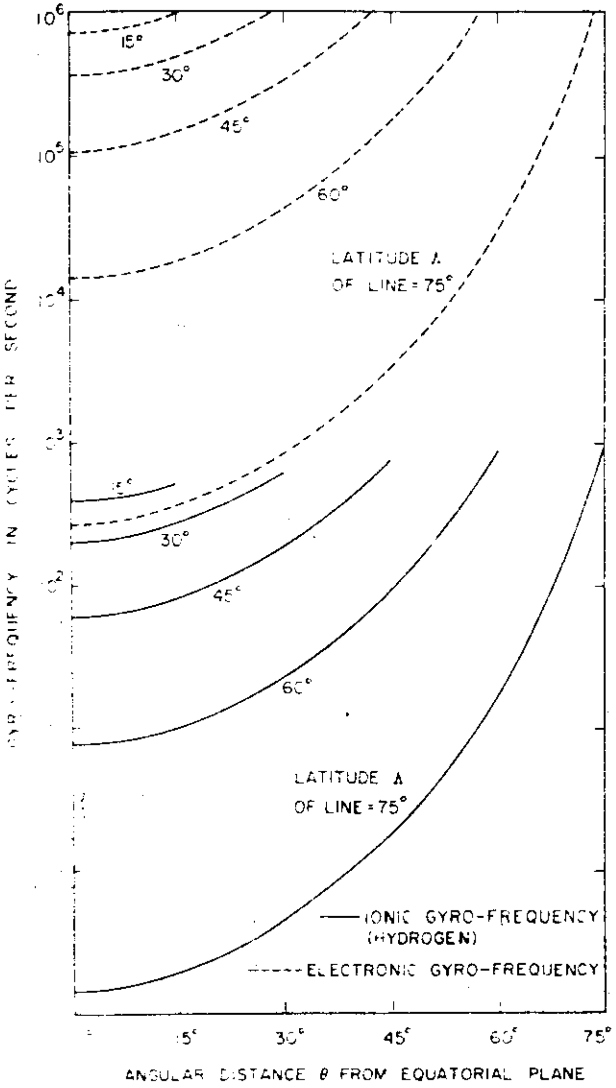
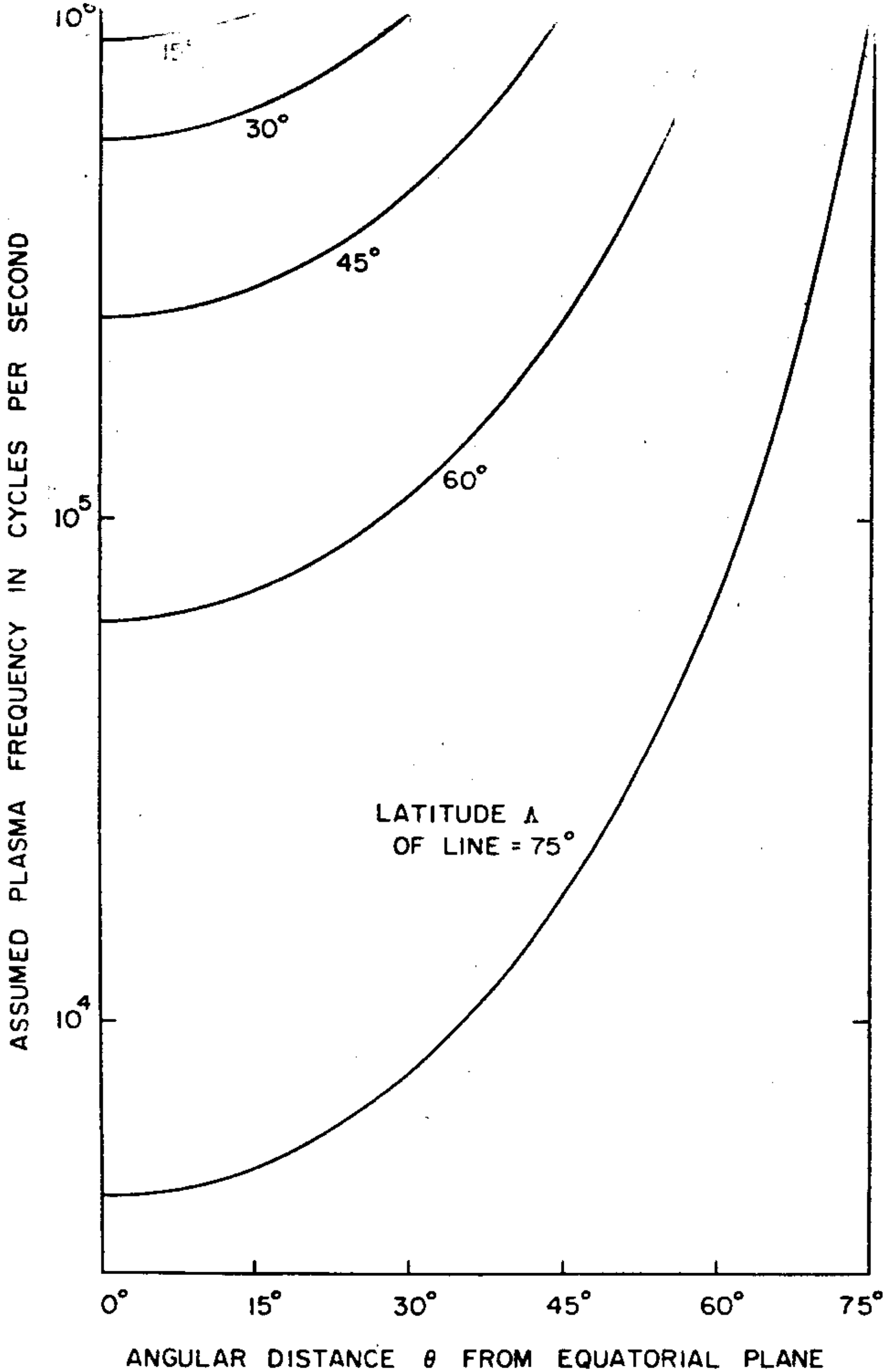

15°
30°
45°
60°
LATITUDE Λ
OF LINE = 75°
15°
30°
45°
60°
LATITUDE Λ
OF LINE = 75°
IONIC GYRO-FREQUENCY
(HYDROGEN)
ELECTRONIC GYRO-FREQUENCY
15°
30°
45°
60°
75°
ANGULAR DISTANCE θ FROM EQUATORIAL PLANE

FIG. 39


ASSUMED PLASMA FREQUENCY IN CYCLES PER SECOND
10^5
10^4
15°
30°
45°
60°
LATITUDE Λ
OF LINE = 75°
0°
15°
30°
45°
60°
75°
ANGULAR DISTANCE θ FROM EQUATORIAL PLANE

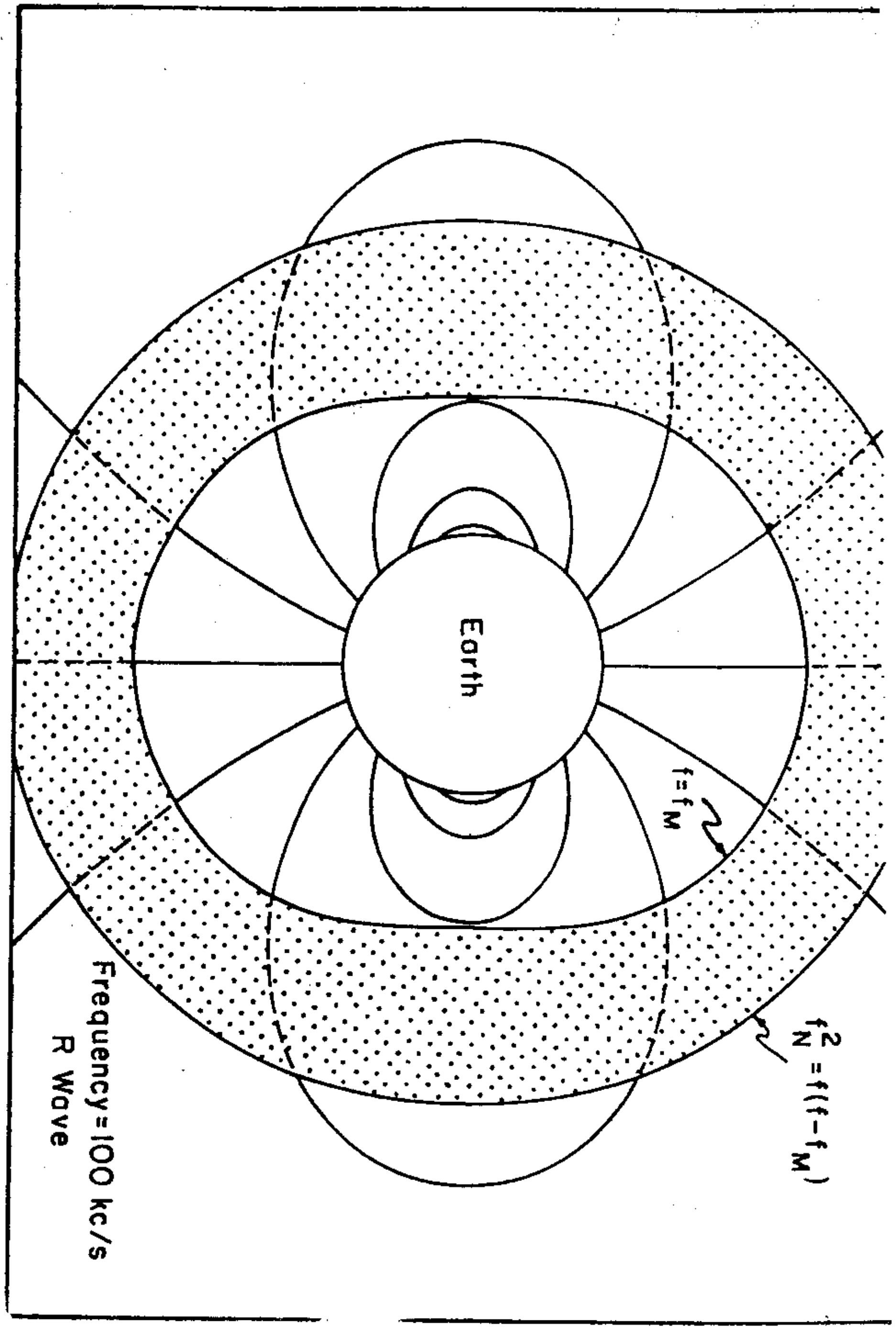


FIG 10a

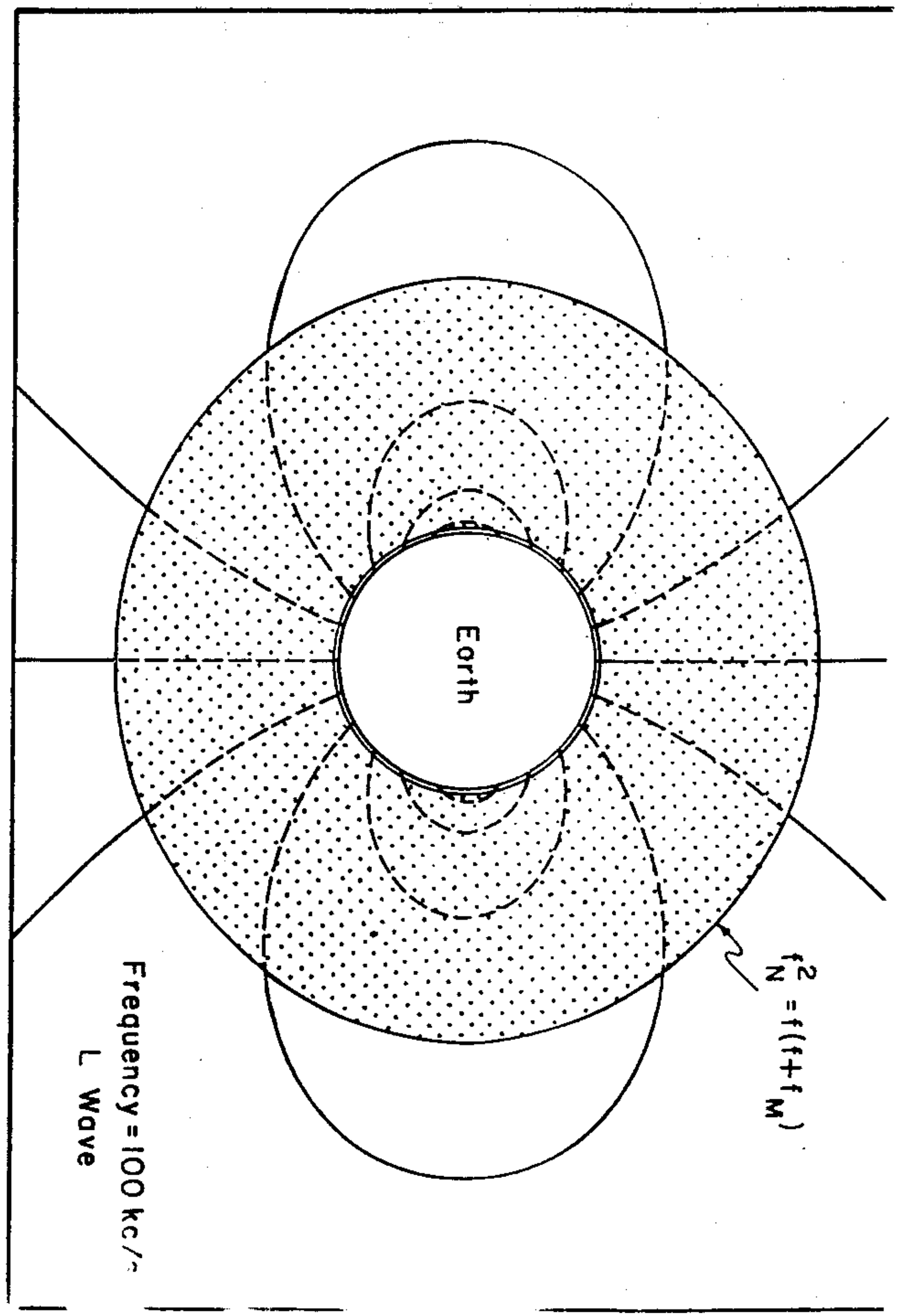


FIG 10b

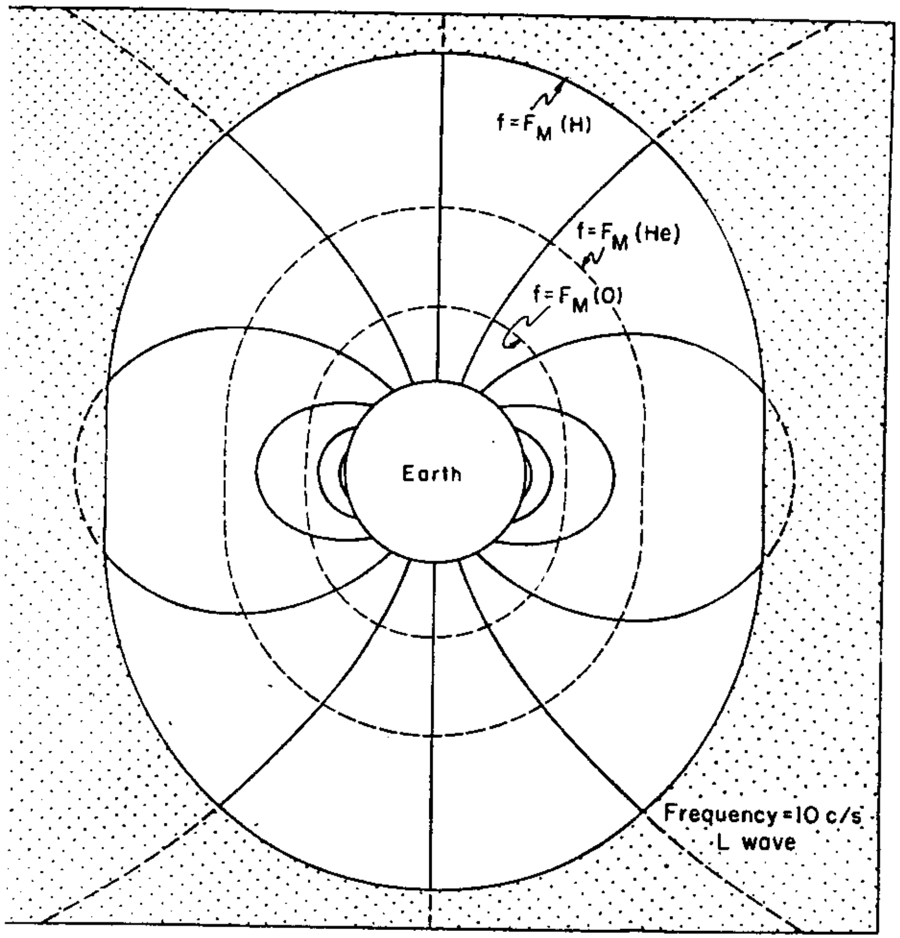


FIG.40c

FIG.41

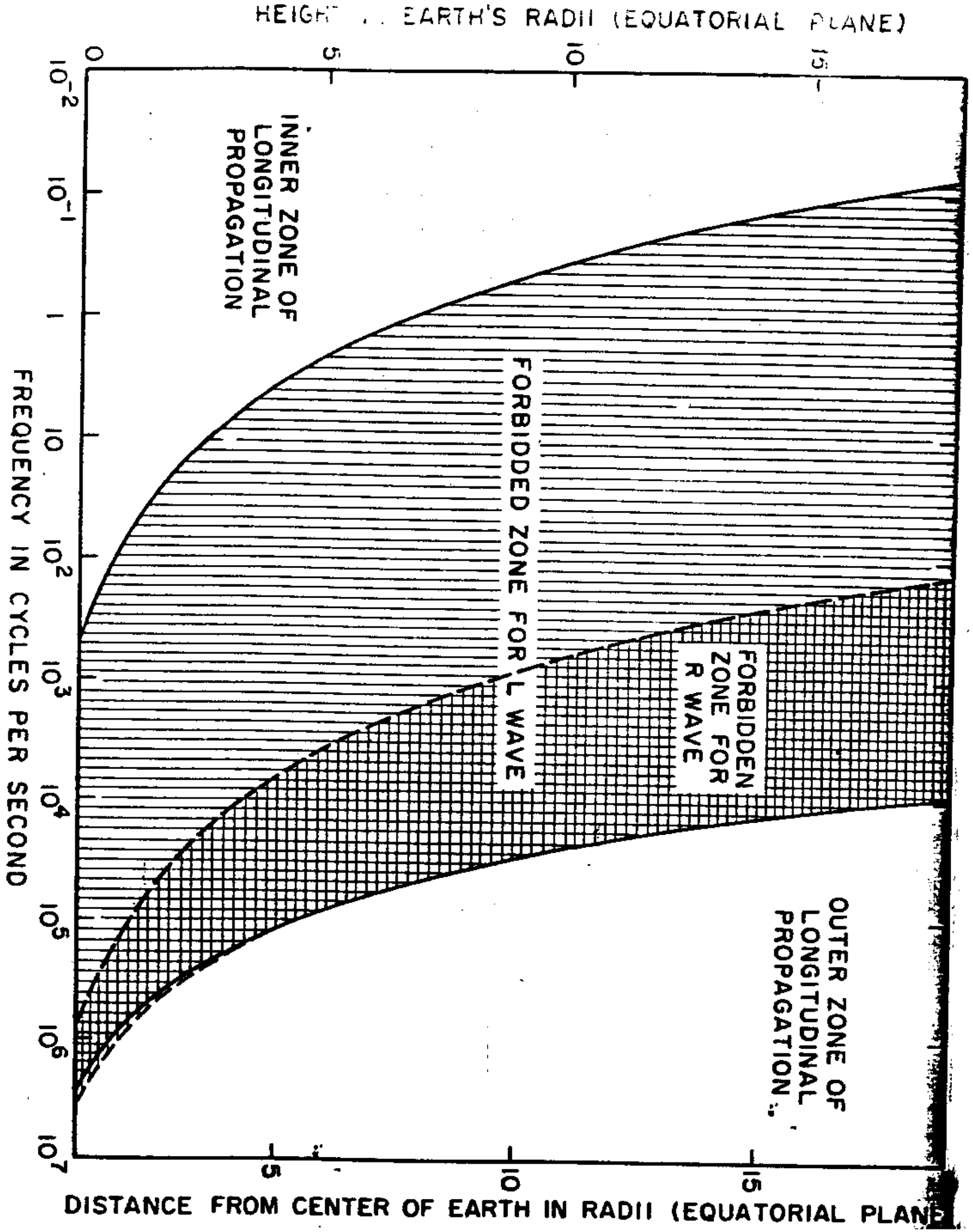


FIG. 42

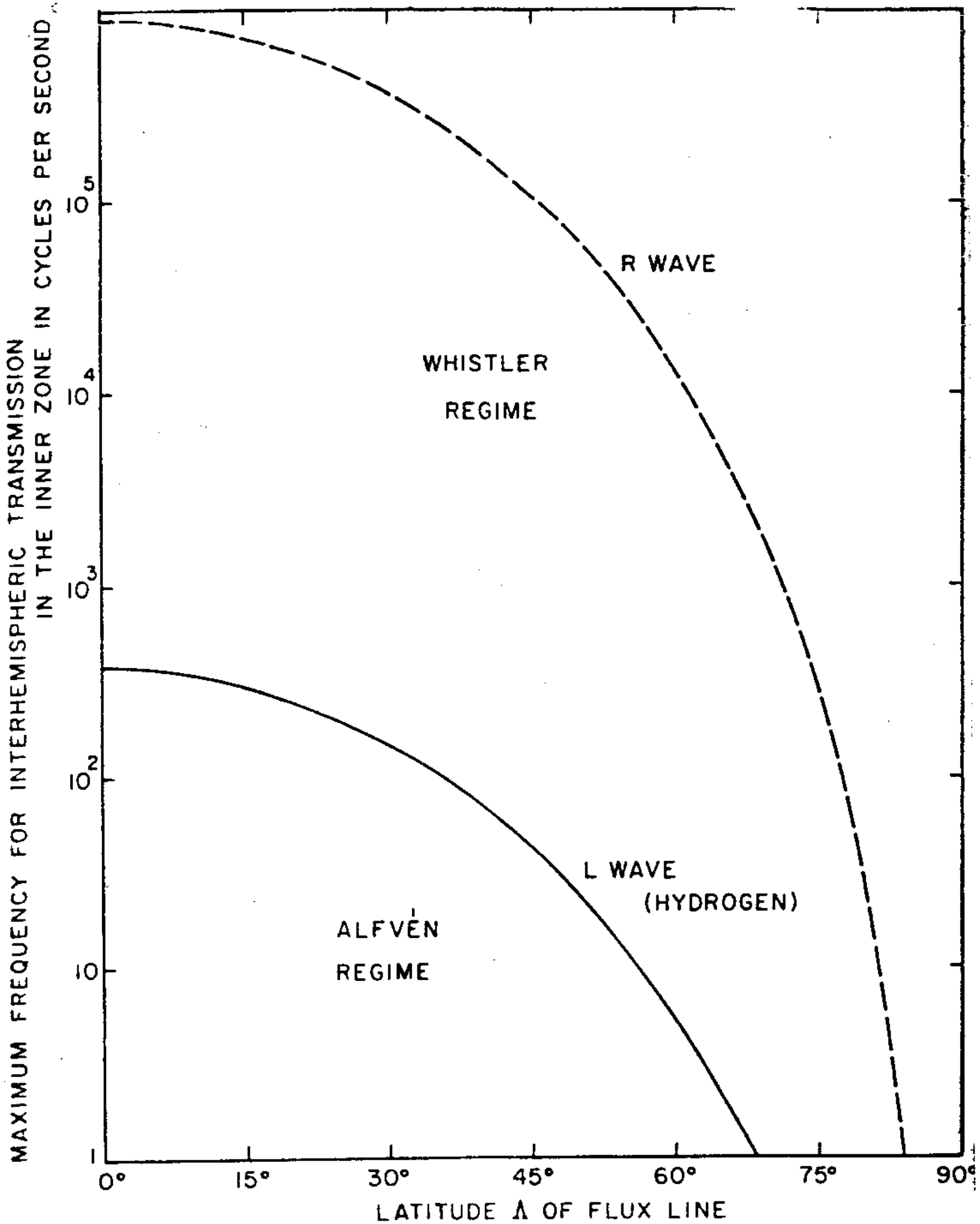

MAXIMUM FREQUENCY FOR INTERHEMISPHERIC TRANSMISSION IN THE INNER ZONE IN CYCLES PER SECOND
10^5
10^4
10^3
10^2
10
1
R WAVE
WHISTLER REGIME
L WAVE (HYDROGEN)
ALFVÉN REGIME
0°
15°
30°
45°
60°
75°
90°
LATITUDE Λ OF FLUX LINE

FIG. 43

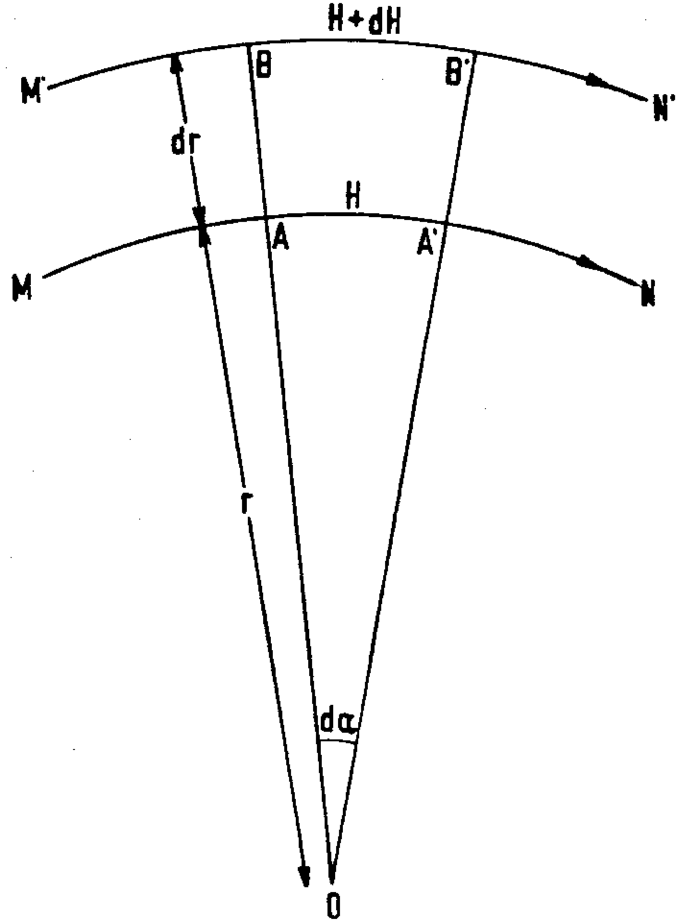
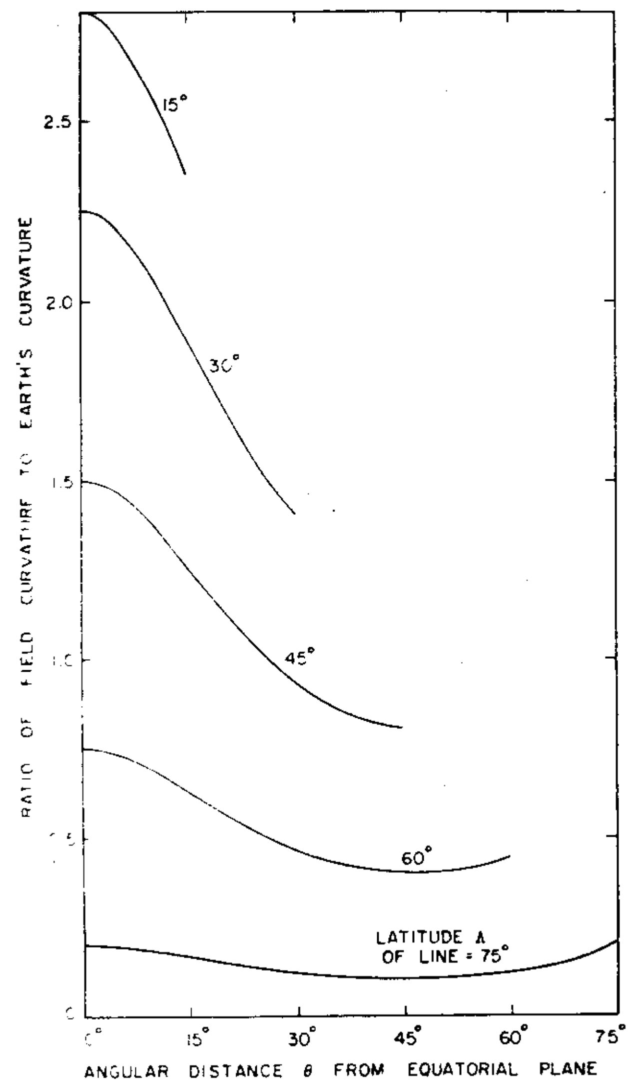

H+dH
M'
B
B'
N'
dr
H
A
A'
M
N
r
dα
O

FIG. 44


RATIO OF FIELD CURVATURE TO EARTH'S CURVATURE
2.5
2.0
1.5
1.0
0.5
0
15°
30°
45°
60°
LATITUDE λ
OF LINE = 75°
0°
15°
30°
45°
60°
75°
ANGULAR DISTANCE θ FROM EQUATORIAL PLANE

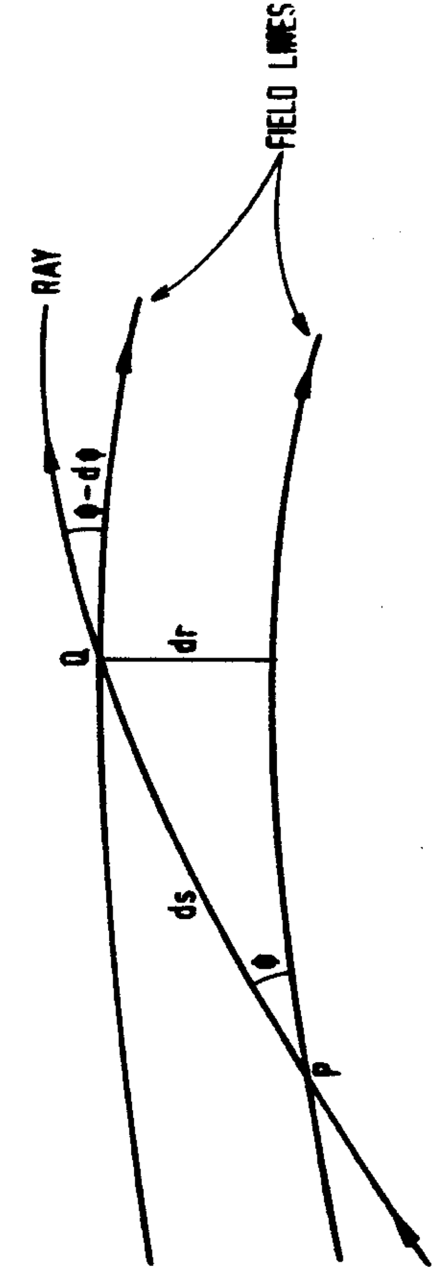

FIELD LINES
RAY
φ-dφ
dr
Q
ds
φ
P

FIG. 45

FIG. 46a

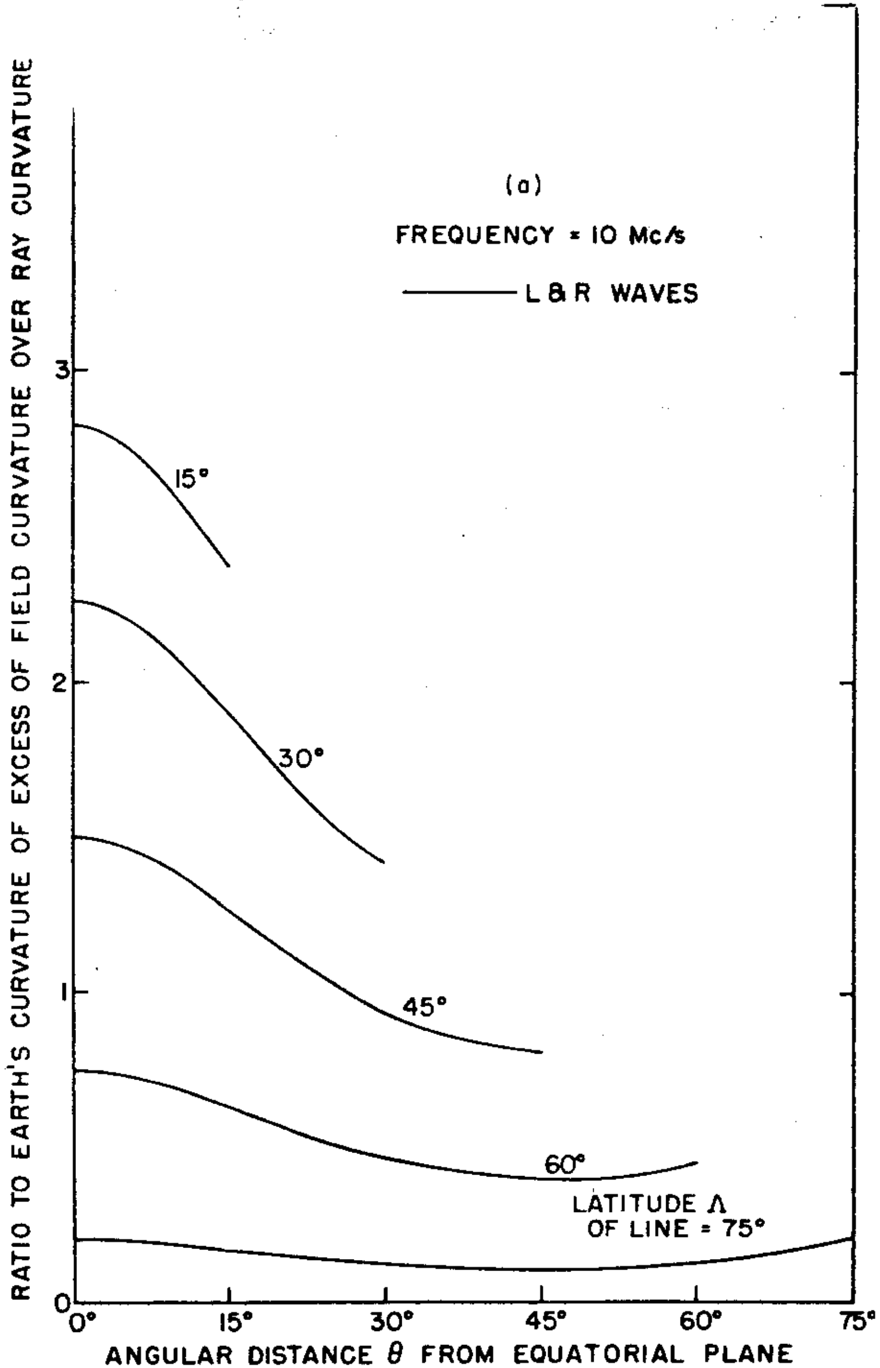


FIG 46b

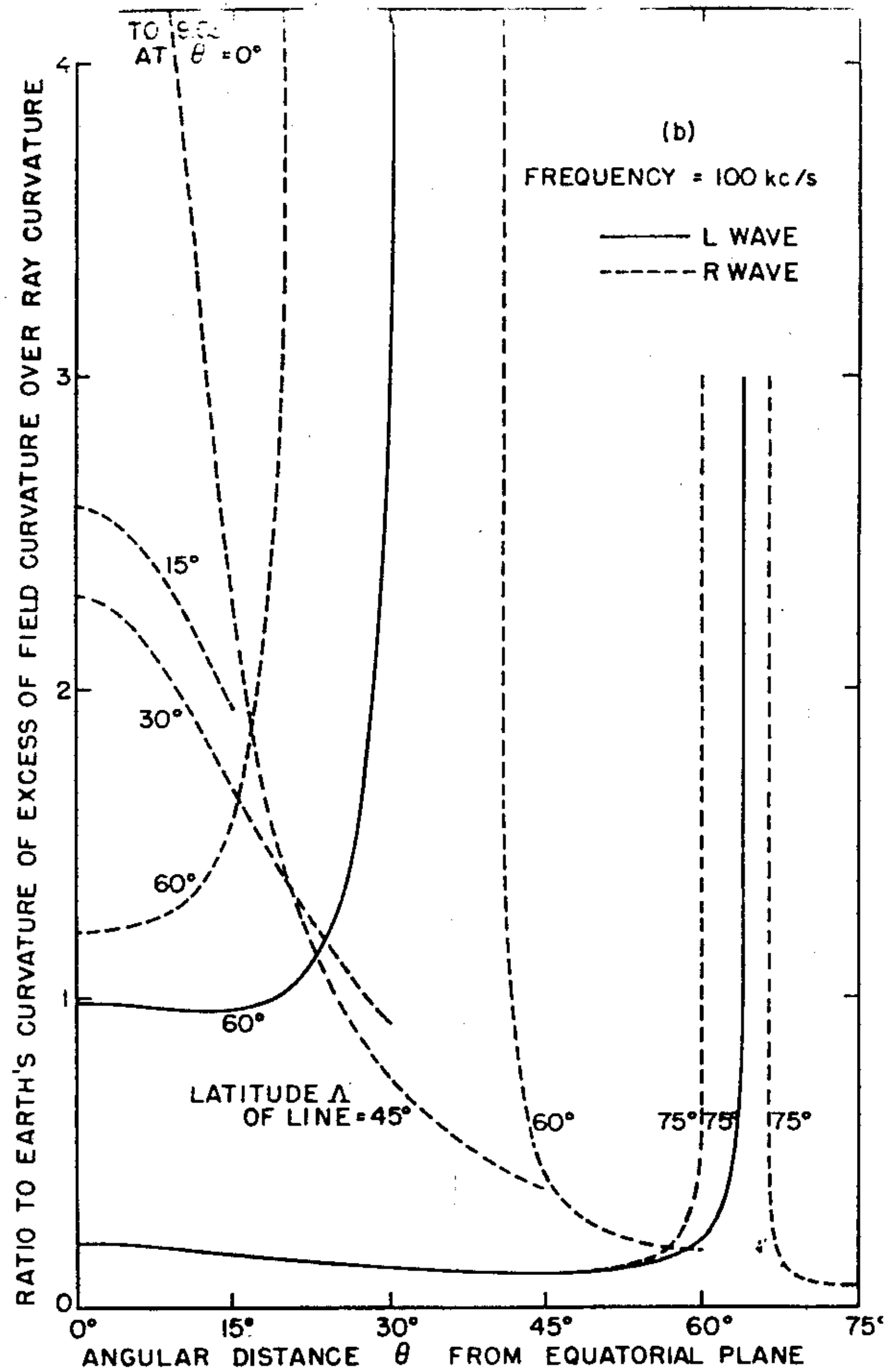


FIG 46c

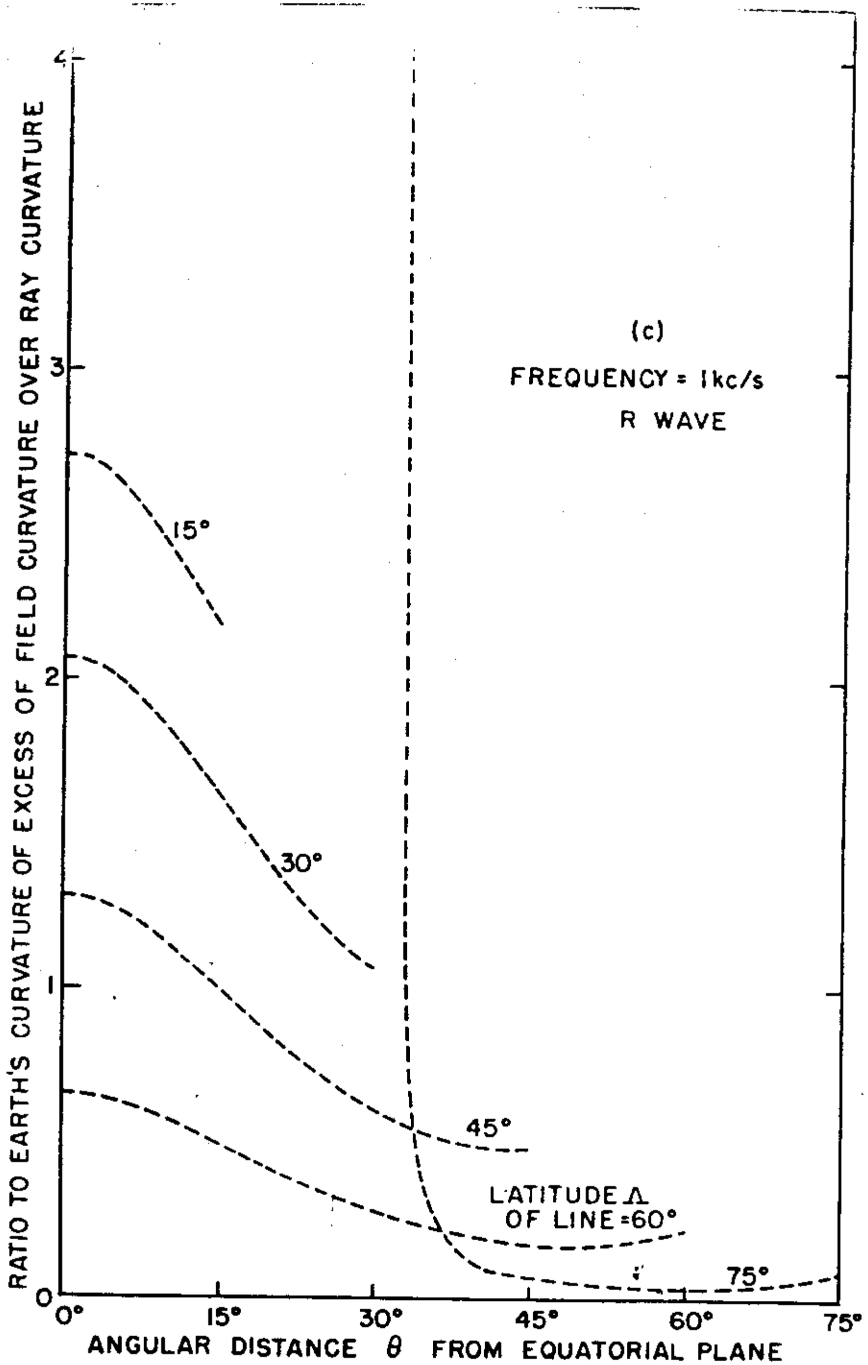
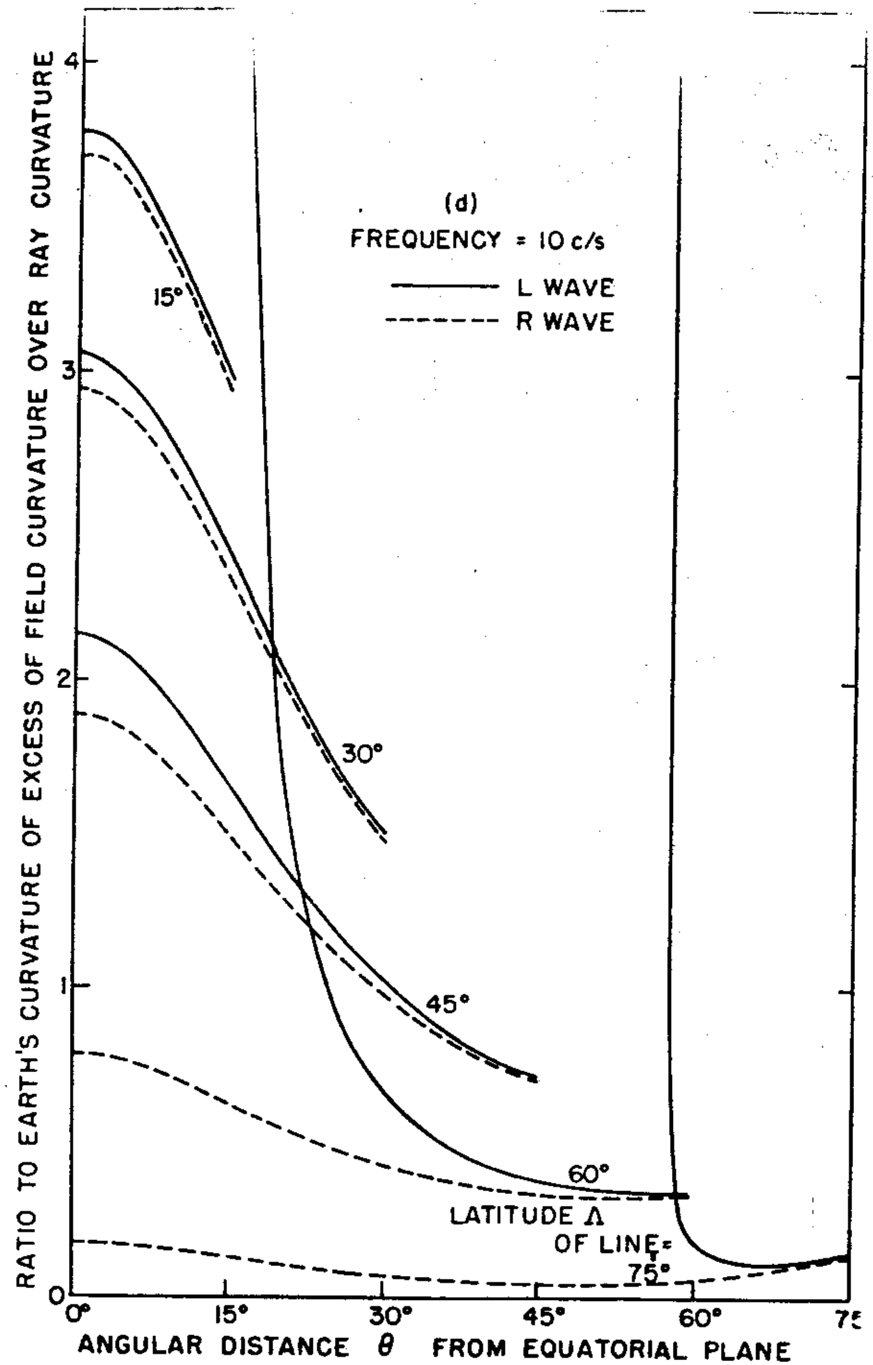


FIG 46d



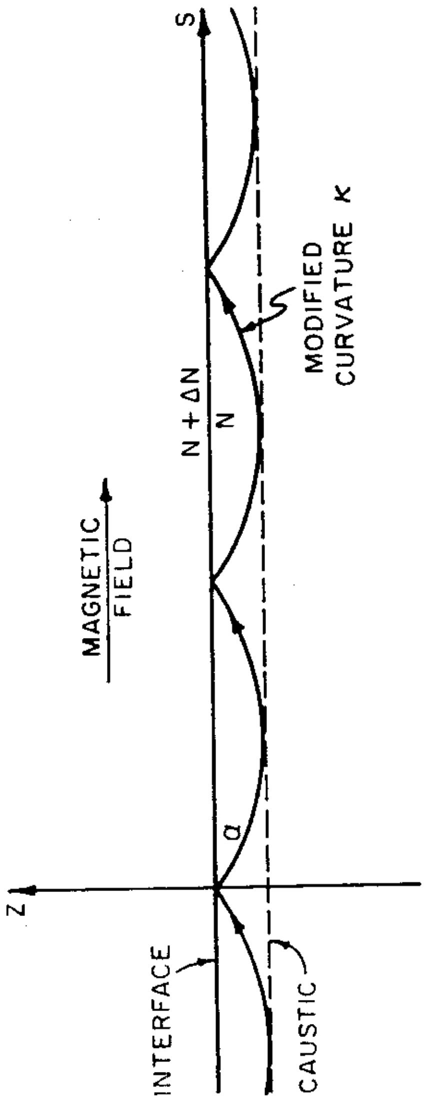


FIG 47

FIG 48a

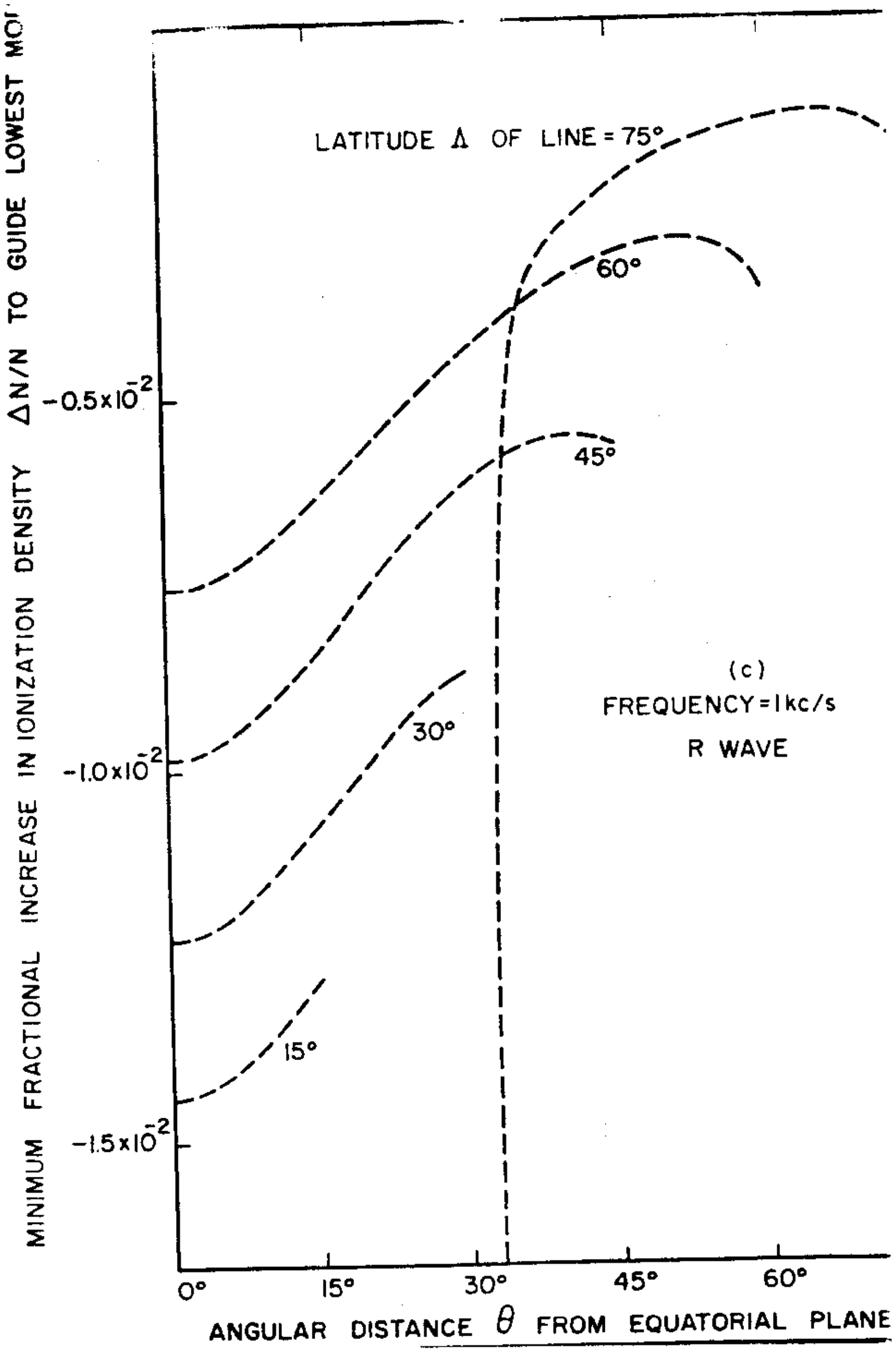


FIG 48b

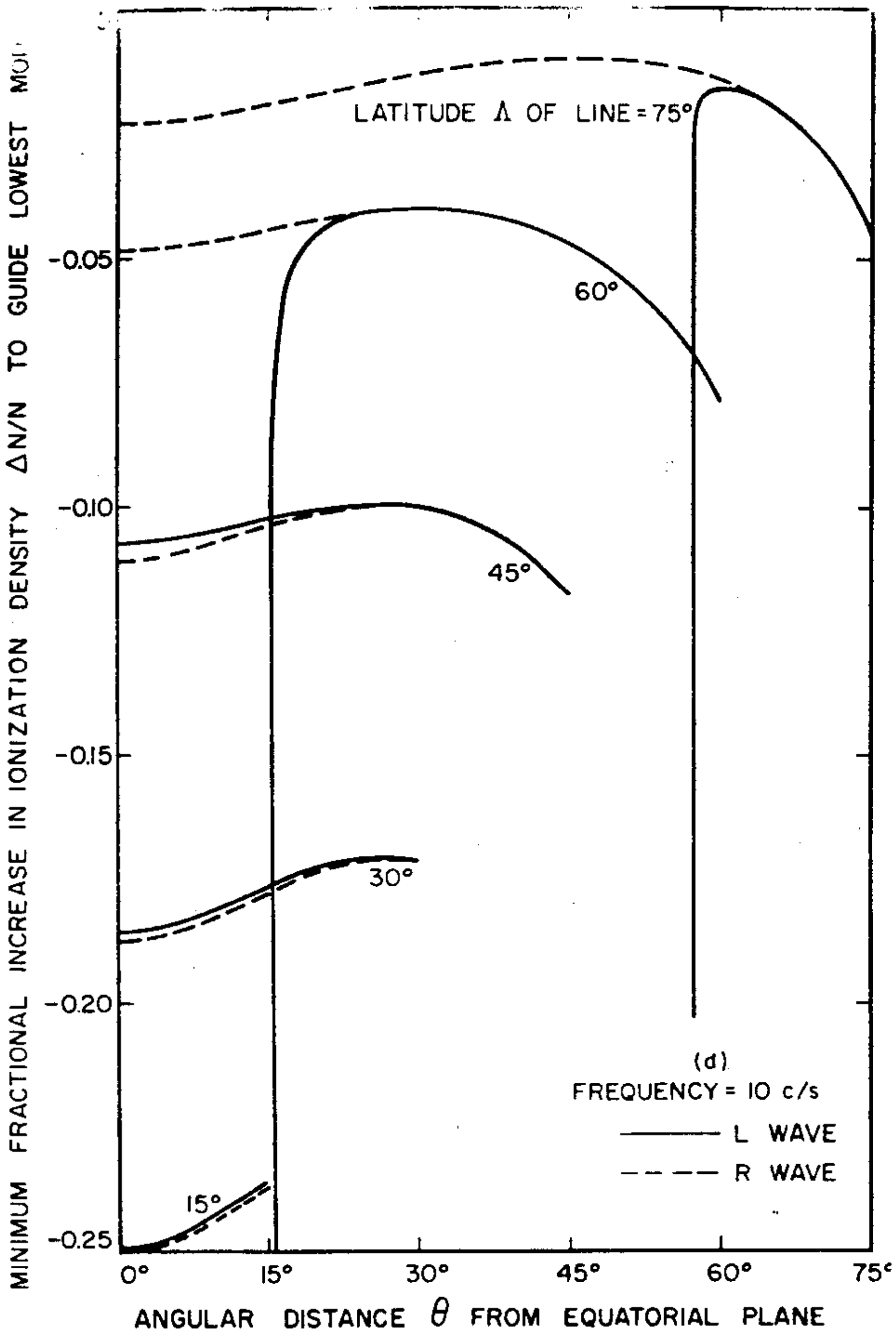


FIG 48c

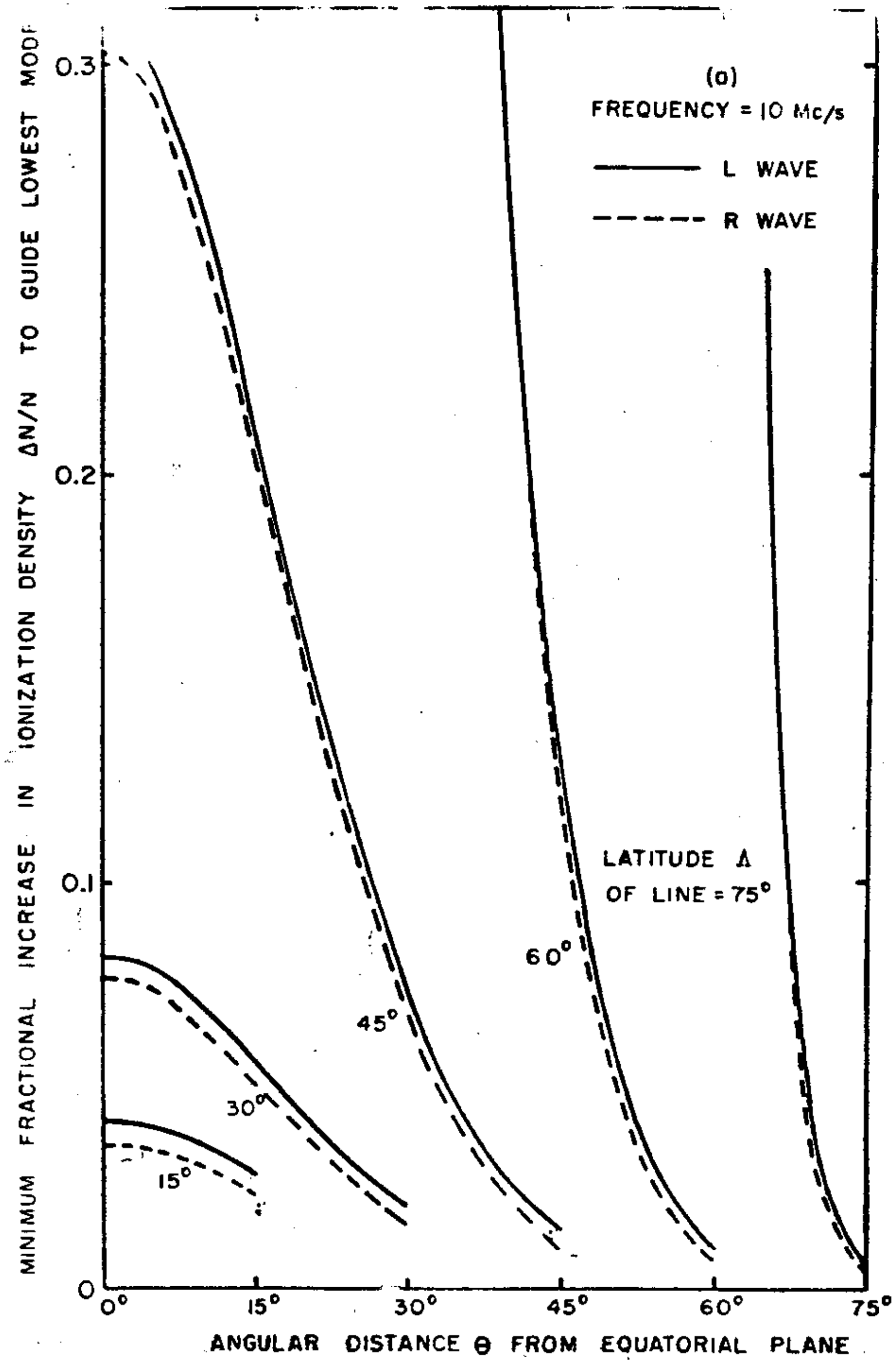


FIG 49a

(41)

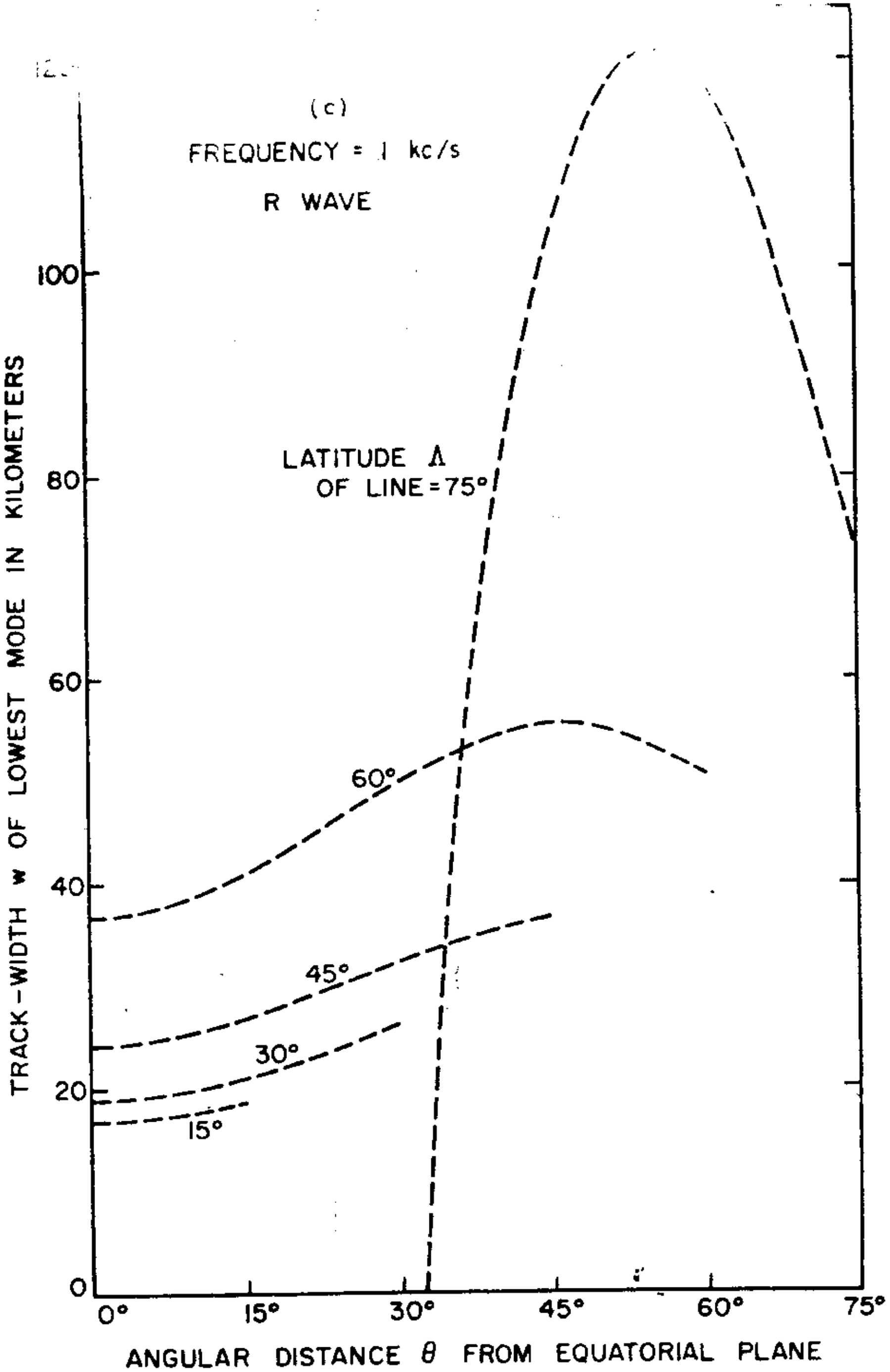


FIG 49b

(42)

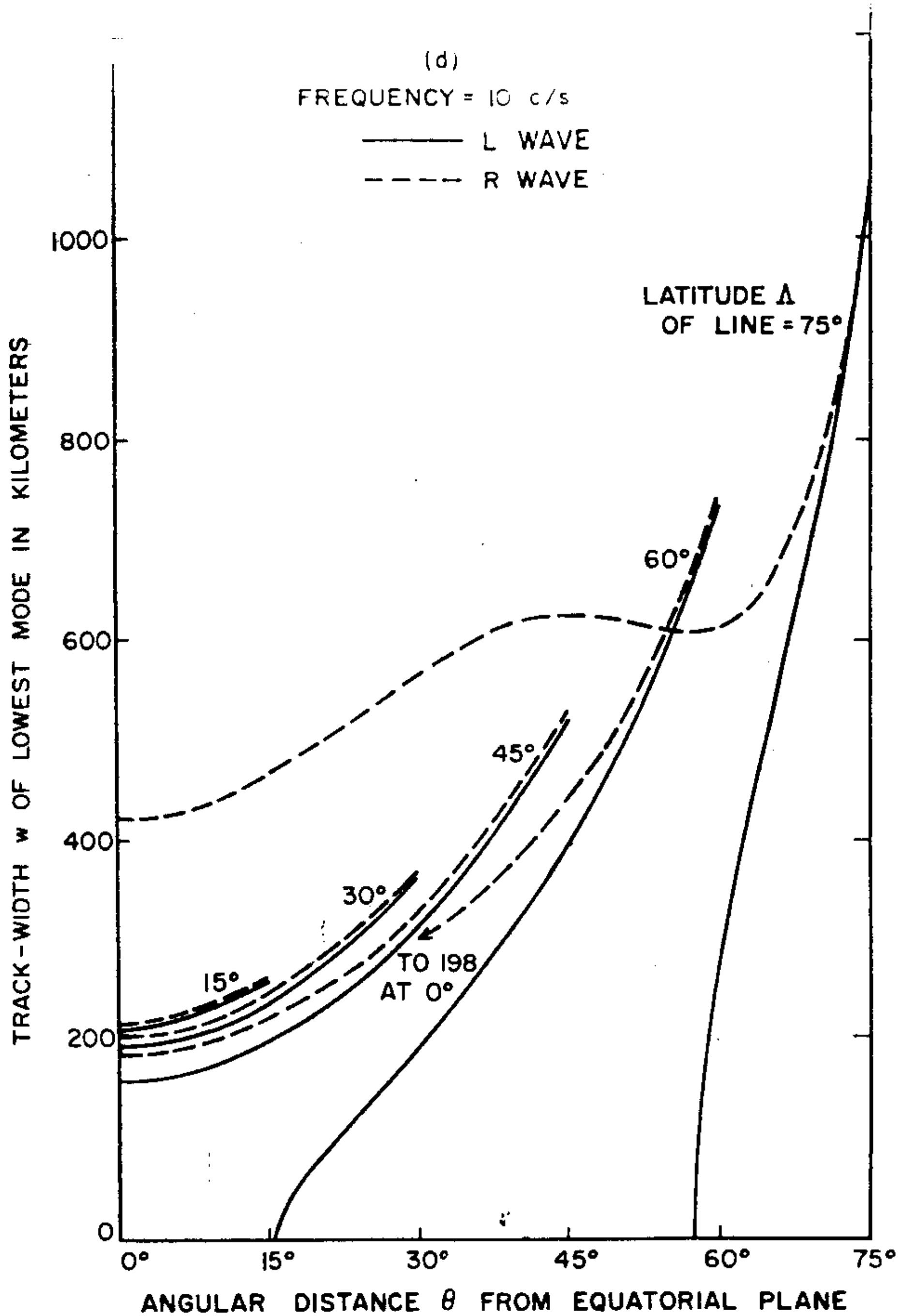


FIG 49c

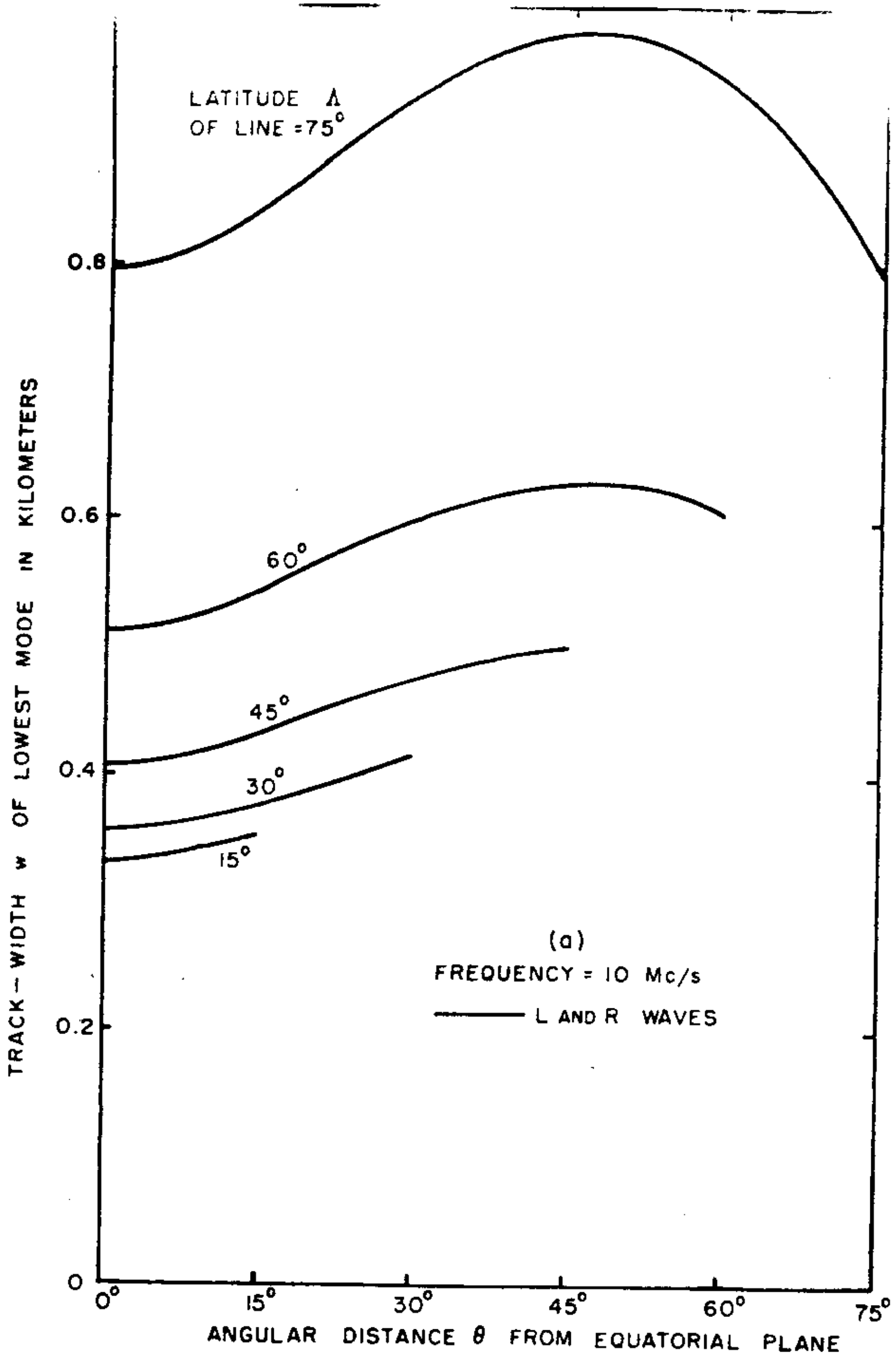


FIG 50a

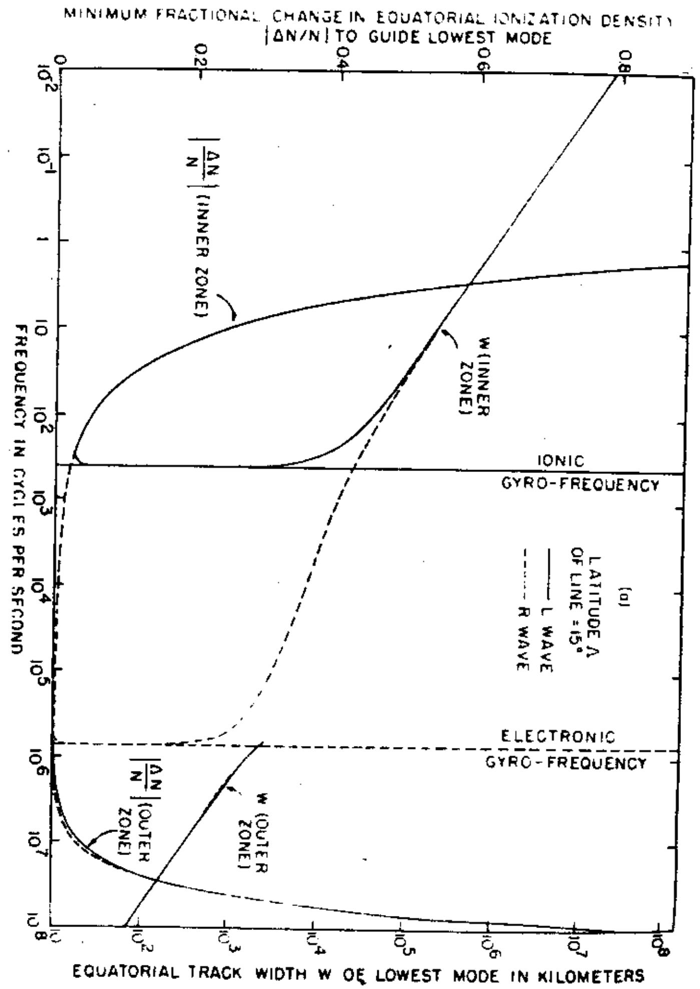


FIG 50b

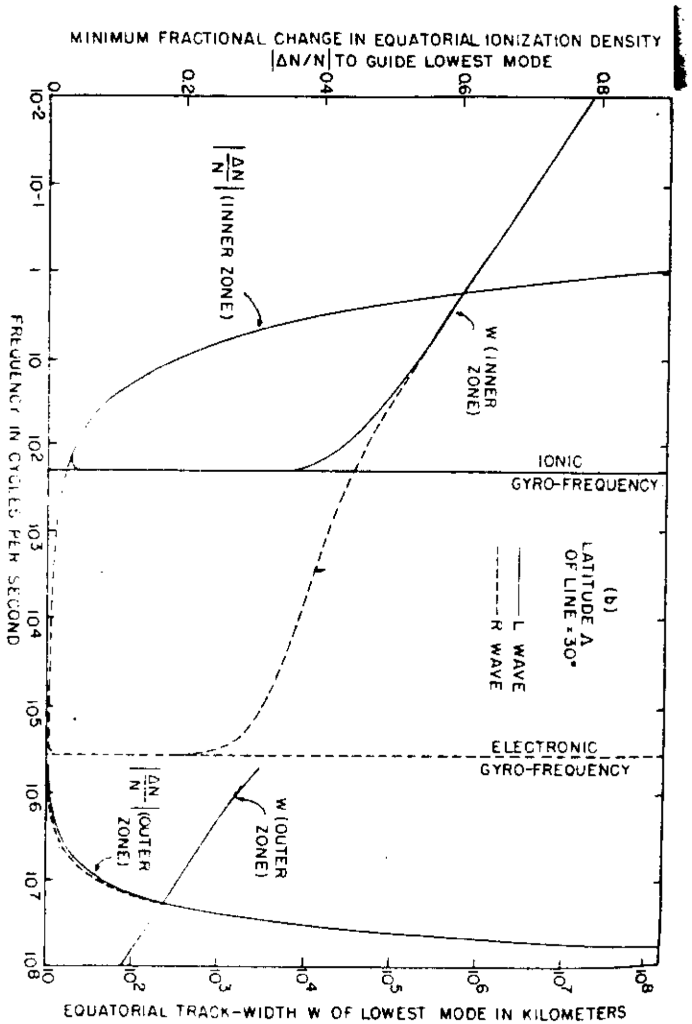


FIG. 50c

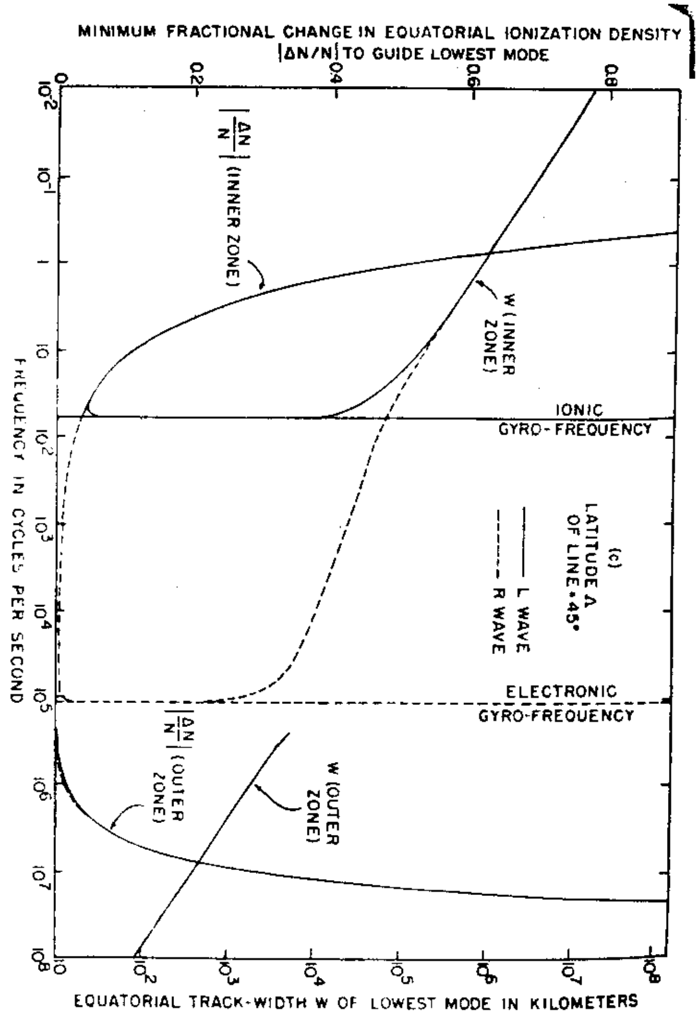
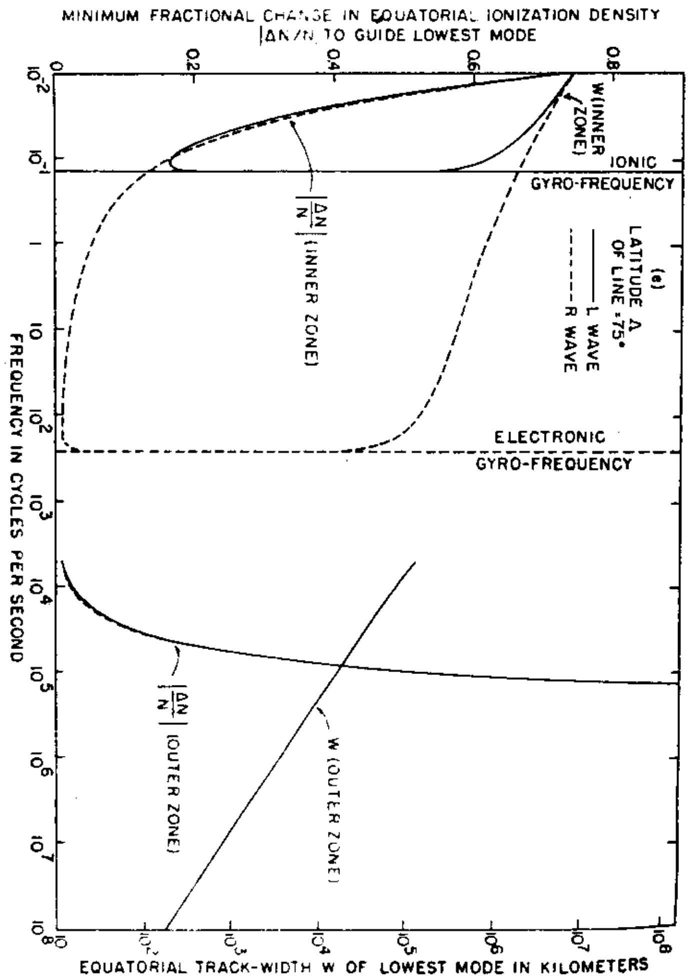


FIG. 50d


MINIMUM FRACTIONAL CHANGE IN EQUATORIAL IONIZATION DENSITY
|ΔN/N| TO GUIDE LOWEST MODE
0
0.2
0.4
0.6
0.8
W(INNER ZONE)
IONIC
GYRO-FREQUENCY
|ΔN/N| (INNER ZONE)
(e)
LATITUDE Λ OF LINE = 75°
L WAVE
R WAVE
ELECTRONIC
GYRO-FREQUENCY
|ΔN/N| (OUTER ZONE)
W (OUTER ZONE)
FREQUENCY IN CYCLES PER SECOND
EQUATORIAL TRACK-WIDTH W OF LOWEST MODE IN KILOMETERS