INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
34100 TRIESTE (ITALY) - P.O.B. 586 - MIRAMARE - STRADA COSTIERA 11 - TELEPHONES: 224281/2/3/4/5/6
CABLE: CENTRATOM - TELEX 460392-I

SMR/98 - 52

AUTUMN COURSE ON GEOMAGNETISM, THE IONOSPHERE AND MAGNETOSPHERE

(21 September - 12 November 1982)

TROPOSPHERIC PROPAGATION - RAIN ATTENUATION.

N.J. McEWAN

School of Electrical and Electronic Engineering
University of Bradford
Bradford
West Yorks, BD7 1DP
U.K.

TROPOSPHERIC PROPAGATION II – RAIN ATTENUATION

N. J. McEwan

<u>Introduction</u>. Fig 1 shows the attenuation (per km) expected for microwaves of a given frequency, at a number of fixed rainfall rates (mm hr^{-1})

Noticeable features are (1) A rises very rapidly with frequency (roughly as f^2) for frequencies $\lesssim$ 10 GHz.

(2) A levels out at very high frequencies, eventually falling slightly with frequency

It would be helpful if this could be explained physically.

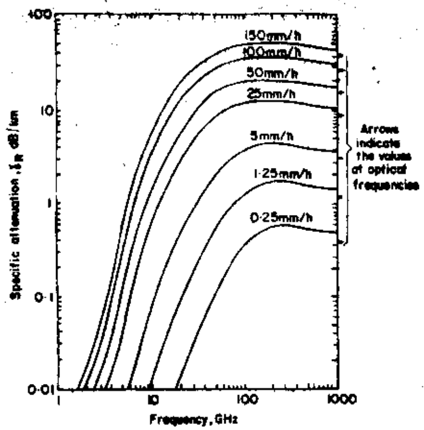
A drop of radius a and refractive index m would be expected to enter resonant scattering conditions as $|mk_0a|$ approaches 1. Taking eg. 10 GHz, $|m| \approx 7$:– $|mk_0a| = 1$ gives $a = 0.7$ mm

Since many raindrops are bigger than this, there is no simplified theory of scatter in the frequency range 10–100 GHz which is of most interest.

For $|mk_0a| \ll 1$, Rayleigh scattering predicts that a sphere of radius a will behave as a dipole radiator of total dipole moment proportional to $a^3\left(\frac{\varepsilon-1}{\varepsilon+2}\right)$ (independently of f, excluding any dependence of ε_r on f) ($\varepsilon_r = m^2$)

The absorption cross section is readily shown to be proportional to $a^3 f \; Im\left(\frac{\varepsilon_r-1}{\varepsilon_r+2}\right)$

Fig 1

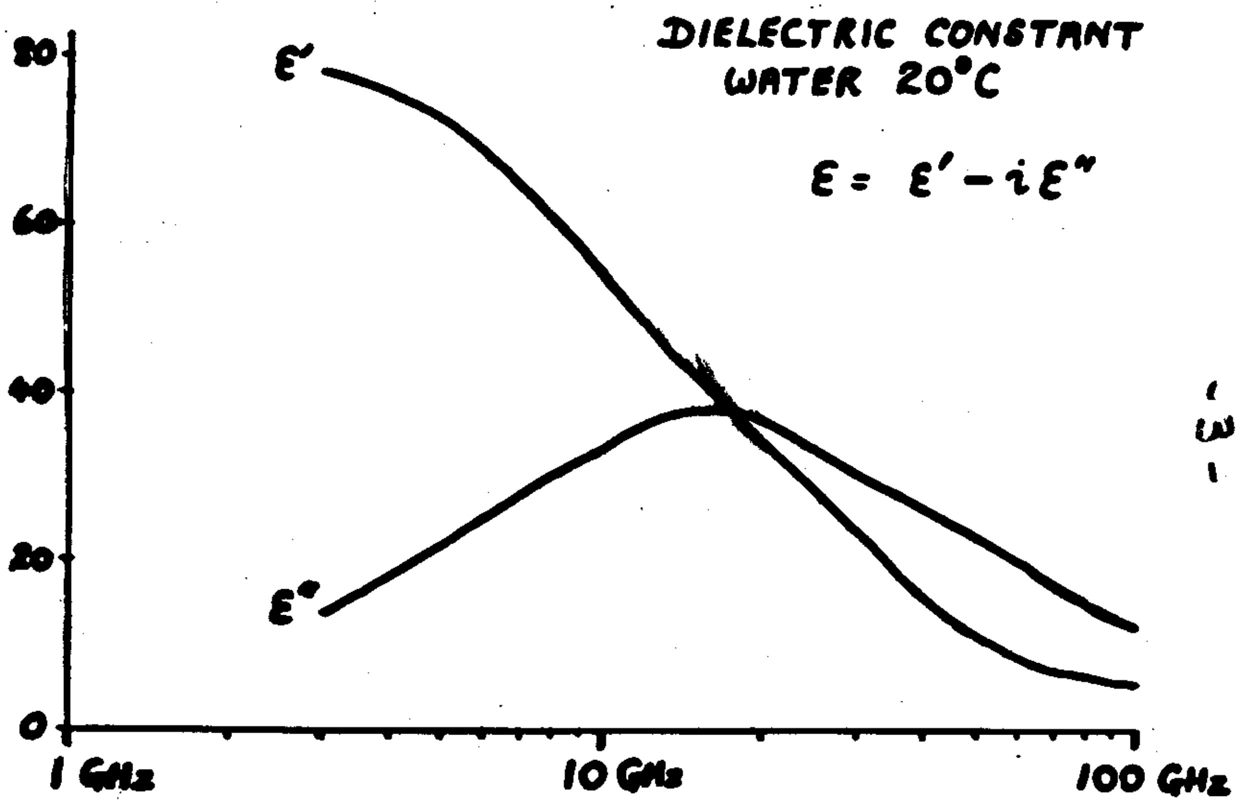


Specific attenuation due to rain

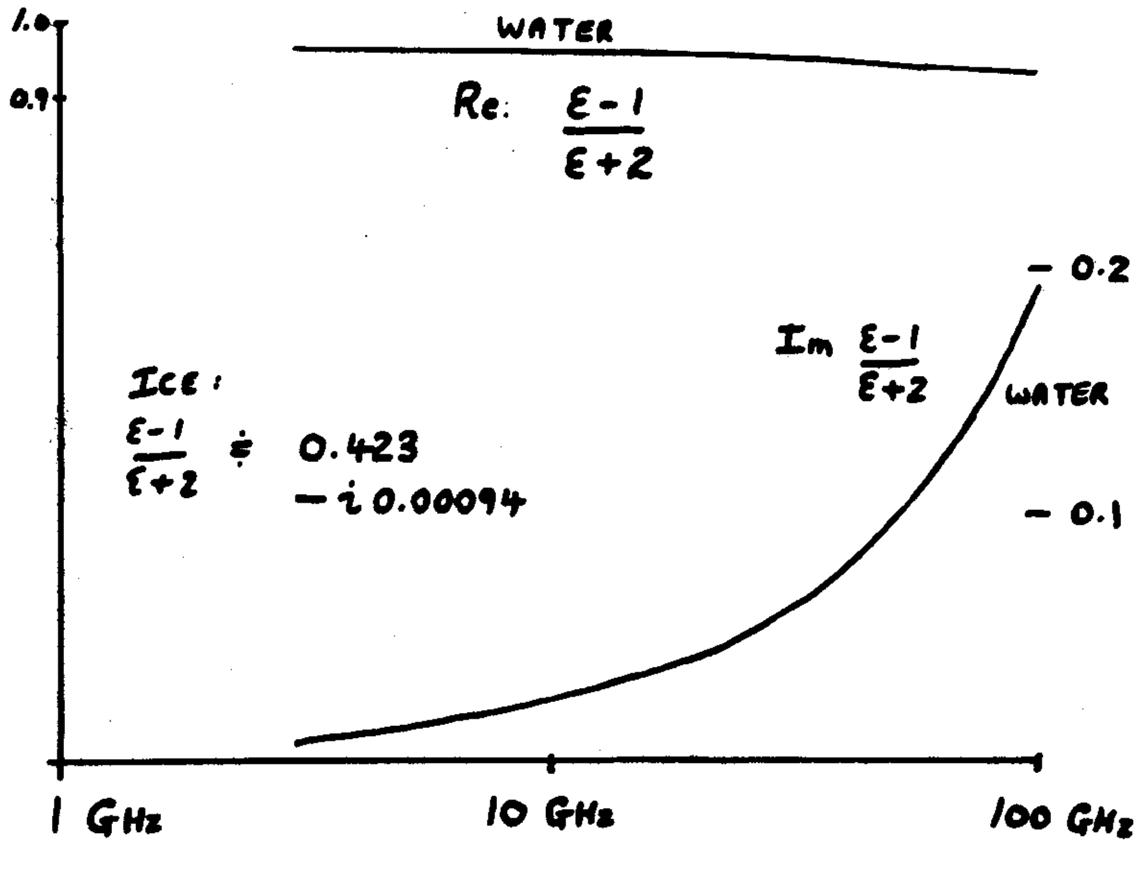
This behaves as a constant energy loss per cycle, ie $A \propto f$, if ε_r is constant.

Fig 2 shows a plot of ε_r versus frequency. The peak in $Im(\varepsilon_r)$ at ≈ 20 GHz can be regarded as the 22 GHz resonance water line, greatly broadened in the liquid state. The plot of $\left(\frac{\varepsilon_r-1}{\varepsilon_r+2}\right)$ (Fig 3) shows an imaginary part increasing ~~monotonically~~ with f. We thus expect an absorption term increasing faster than f.

The <u>scattered</u> power of the radiating dipole should be proportional to $a^6 f^4 \left|\frac{\varepsilon_r-1}{\varepsilon_r+2}\right|^2$. Fig 3 clearly shows that the ε_r dependent term will ~~not~~ vary very much. This also predicts ~~a loss~~ an attenuation term rising much faster than f in the Rayleigh region. In fact



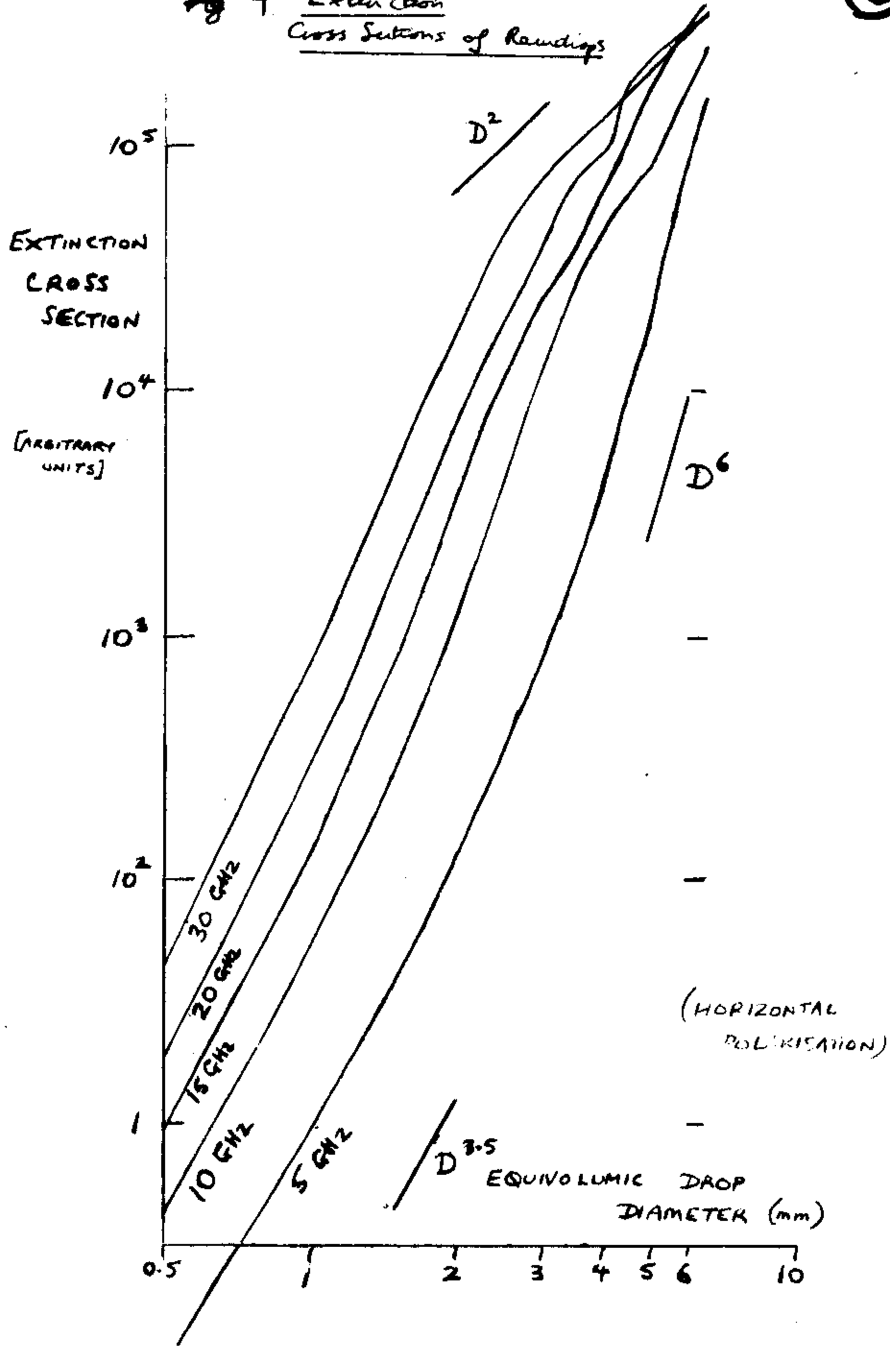
[ICE: $\varepsilon \approx 3.2 - i\,0.0085$]



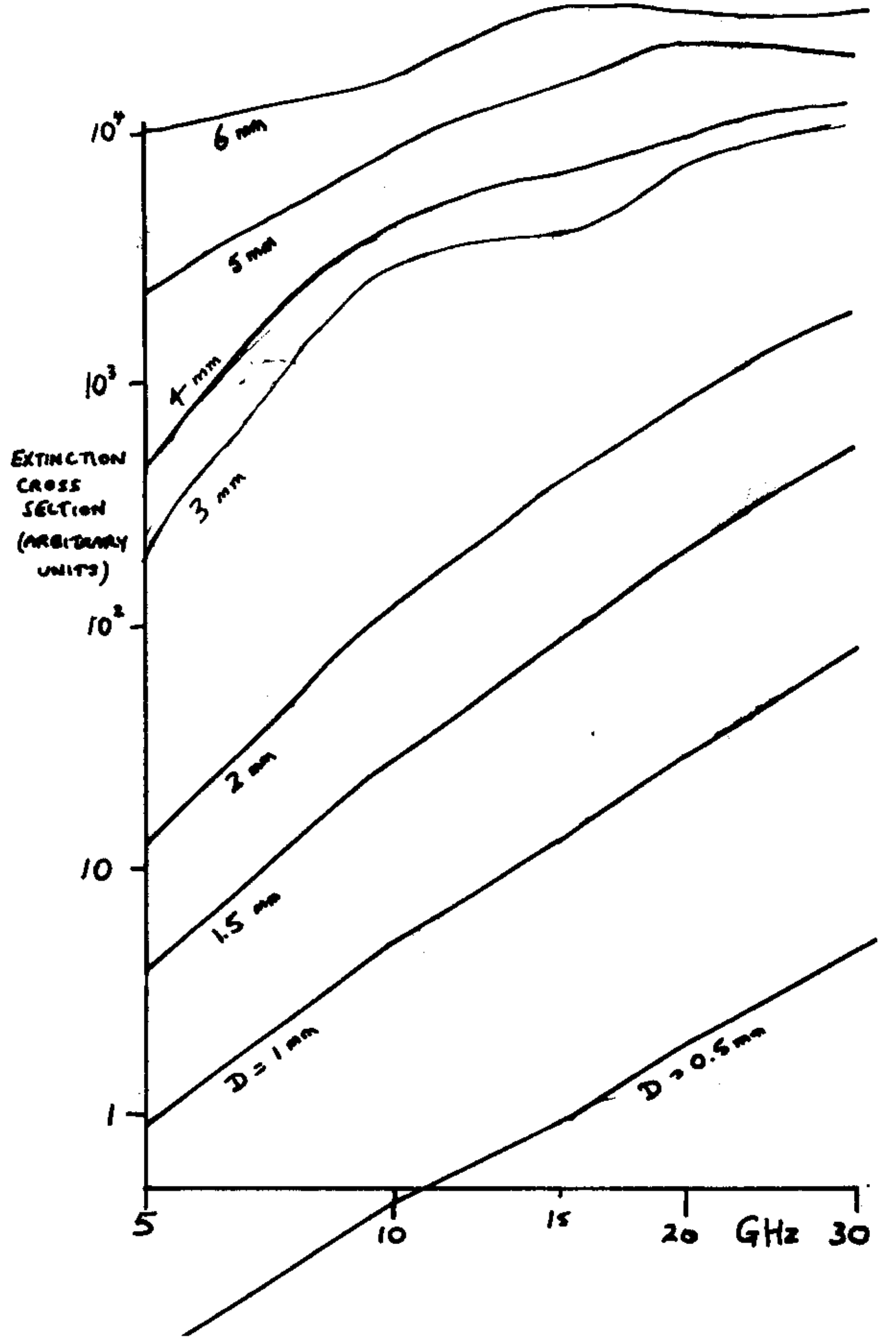
4

(5)

Fig 7 Extinction Cross Sections of Raindrops

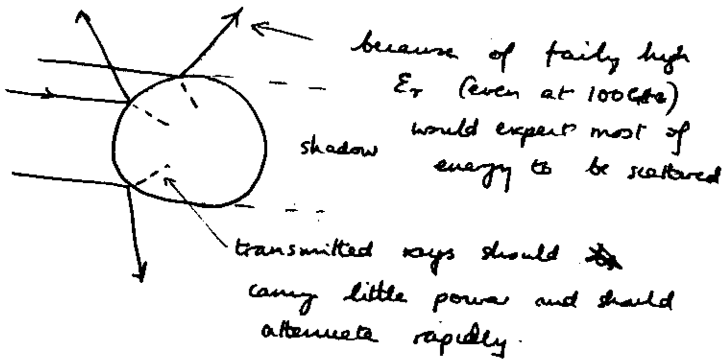


(6)



the scattering cross section dominates the attenuation at $f \gtrsim 30$ GHz. These two effects give a very rough explanation of the rapid rise in extinction with respect to frequency, at the lower frequencies.

At very high frequencies, we should expect a quasi-optical approximation to hold – we can think of individual incident rays with a reflected and transmitted ray as for incidence on a plane surface:



Would therefore expect extinction cross section $\approx 2 \times$ geometrical cross section at very high f.

Exact Cross Sections: See Figs 4 and 5.

(N.B. 1 – 6 mm equivolumic diameter is most important range of drop sizes normally. Very few drops above 6 mm, 9 mm is an [almost] absolute limit due to drop stability – break-up occurs)

Fig 4 shows (as might be expected) cross sections initially rising as * $D^{3.5}$ and tending to steepen at larger sizes as ~~cross~~ scattering increasingly dominates over absorption – but at large sizes slope falls rapidly as resonance region is entered – this happens at a smaller size as the frequency is raised.

(would expect D^3 for pure absorption – no scattering.)

Fig 5 shows that in certain size ranges, extinction is increasing like f^3 or f^4. (Some fine structure in Figs 4, 5 is evidence of resonance effects.)

Drop Size Distributions

Basic Question: How many parameters does it take to describe a drop size distribution well enough for practical radiometeorology??

In practise a 2-parameter family of functions works well for most purposes relating to microwaves – except possibly at the higher frequencies, say $\gtrsim 30$ GHz:

$$N(D) = N_0 \exp(-\Lambda D)$$

(N_0 and Λ: 2 free parameters)

[D always denotes equivolume drop diameter]

$N(D)\,dD$ = no. of drops per unit volume having ~~diameter~~ in range $D, D+dD$

(Assume cut off above 9 mm.)

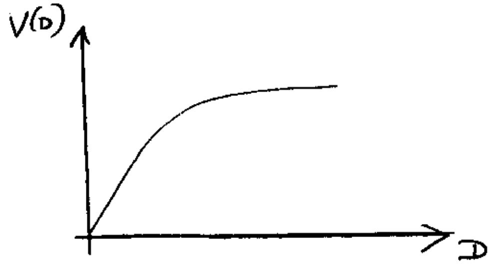
[This distribution (of the so called Marshall Palmer form) is most uncertain at the small drop end - there are suggestions of a cut off. Obviously small drops density is seriously affected by updrafts, and smallest drops merge into heavy cloud or fog — unrelated to any ground rainfall.]

Rainrate Integral

$$R = \int_0^\infty N(D)\,V(D)\,D^3\,dD$$

$V(D)$ ↓ terminal velocity — rather a weak function of size, possibly approximate by D^n with n in range 0.5 - 1.

D^3 ↓ [dominant term]



Attenuation Integral

$$A\ (\text{dB/km}) = \int_0^\infty N(D)\,Q(D)\,dD$$

$Q(D)$ ↓ Extinction cross section

N.B. If $Q(D)$ happens to be the same function as $V(D)D^3$, we should get a constant ratio A/R, independent of $N(D)$! This is never quite true, but clearly isn't too far from the truth in the 10-30 GHz range. (Fig 4)

One Parameter Models. For many purposes, eg. estimating A from R, or R from radar reflectivity, we are obliged to make the crude approximation that the drop size distribution functions form a one-parameter family.

This can be done by assuming that N_0 and Λ are functionally related to R. The "standard" Marshall-Palmer uses:

$$N_0 = 0.0662\,R^{0.021}$$
$$\Lambda = 41\,R^{-0.21} \qquad (\Lambda \text{ in cm}^{-1},\ R \text{ in mm hr}^{-1})$$

↓ N.B. Heavy rain ⇒ more large drops in distribution.

Useful estimates of the errors in the one-parameter approximation can be obtained by taking two particular one-parameter families as bounds, for extreme rain types:

$$\left.\begin{aligned} N_0 &= 0.338\,R^{-0.030} \\ \Lambda &= 57\,R^{-0.21} \end{aligned}\right\}\ \text{"Joss drizzle"}$$

$$\left.\begin{aligned} N_0 &= 0.0131\,R^{0.084} \\ \Lambda &= 30\,R^{-0.21} \end{aligned}\right\}\ \text{"Joss thunderstorm"}$$

Since the functions $Q(D)$ and $D^3V(D)$ are not too far from power-laws, we should expect the one-parameter model to yield a relation of the form $A = aR^b$ — which is a good fit to the numerically derived relation.

If $Q(D)$ increases faster than $D^3V(D)$, we expect $b > 1$ — Fig 4 suggests that $b > 1$ at 5 GHz, and will fall at higher

In fact Fig 4 shows that for $f \gtrsim 10$ GHz, $Q(D)$ will be rising about as D^n with

* $5 \gtrsim n \gtrsim 3$, suggesting that the relation between A and R of the form aR^b will be fairly "tight". Following examples confirm some of these predictions:

(a) Values of b in expression aR^b for 'standard' Marshall - Palmer:

f =	5	10	15	20	30	60 GHz
b =	1.10	1.17	1.14	1.12	1.06	0.87

(b) Values of specific attenuation A possible for various rain rates (A in dB/km)

R =		1	10	25	100	150 mm hr⁻¹
10 GHz	I	0.02	0.23	0.67	3.32	5.32
	II	0.01	0.17	0.49	2.49	3.96
	III	0.013	0.11	0.26	(0.97)	(1.42)
20 GHz	I	0.09	0.99	2.54	10.6	16.1
	II	0.06	0.78	2.17	10.2	16.1
	III	0.06	0.59	1.45	(5.7)	(8.5)
30 GHz	I	0.26	2.07	4.8	16.7	24.2
	II	0.15	1.78	4.73	20.7	31.8
	III	0.15	1.50	3.79	(15.5)	(23.3)
60 GHz	I	0.75	4.40	8.88	(25.7)	(35.0)
	II	0.66	4.94	11.0	(36.8)	(52.4)
	III	0.63	6.38	16.0	(64.6)	(97.1)

I = thunderstorm II = standard III = drizzle.

Bracketed values are ignorable because either R is too large to be caused by drizzle, or A is so large as to make a practical system inoperative — such large values are of no interest to the system designer as the system could not be designed to cope with them.

The spread of values at a given R indicates the likely maximum error incurred in the one parameter approximation. The Table suggests that rainfall ~~measurements~~ rate measurements are likely to give a reliable measure of attenuation provided the spatial variations of R are adequately known. (Also, for purposes of ~~stats~~ long term statistical predictions, the ~~errors~~ errors will tend to average out.) (R may be a less satisfactory estimate above 30 GHz since the smallest drops give an increasingly important contribution to A and ~~this~~ the drop size spectrum is badly-behaved at the small-diameter end.)

The Prediction Problem.

The most common engineering problem is to predict statistics of total attenuation (not dB/km) for a given terrestrial path, starting only from statistics of R measured at a single point near that path.

Note: In ~~some~~ most cases special statistical techniques are needed to convert ~~to~~ some forms of rain statistic (eg. 10 minute time average statistics, or extreme monthly totals) into statistics of 'instantaneous' R. (Effects of raingauge integration time must be removed if significant.) [These techniques are already in a usable state.]

The problem reduces to finding $\langle R^b \rangle$ statistics (average taken over a given path) given R statistics.

There are 3 basic methods, all of which can be made to work with useful accuracy.

(1). Use a structural model of a rain cell, eg. Gaussian, cylindrical etc, to calculate relation between two statistics.

(2) Use a purely empirical relation (equiprobability) between R and $\langle R^b \rangle$.

(3) Convert a time record of R from a single gauge into an inferred spatial ~~stats~~ structure. (assume 'frozen' structure moving with wind velocity as measured at 700 mb height.)

N.B. The ~~$R/\langle R^b \rangle$ conv~~ $R \leftrightarrow \langle R^b \rangle$ conversion in methods (1) and (2) will normally have been obtained from a multi-raingauge experiment at a quite different location — an assumption of statistical similarity of rain structure ~~is made~~ must be made, even if the simple R statistics are quite different at the site for which prediction is made. (This may also be true for method 3)

Satellite (Earth-Space Paths) — prediction problem:

The above methods can be extended to satellite paths (elevations > 20°) if there

is sufficient knowledge of the vertical structure of rain.

For temperate climates, usable statistics can be obtained by assuming that R is uniform with height up to the 0°C level and that attenuation above that height is negligible.

In temperate ~~[illegible]~~ climates, rain frequently has a stratiform structure in which this picture is quite accurate. Embedded in the stratiform structure may be weak convective cells in which the freezing level is only slightly elevated, but the rain intensifies below that level. (ie., does not consist purely of water that was previously above the 0°C level ~~and~~ in a frozen state)

Those cells, and even stronger isolated convective cells eg. thunderstorms, do not apparently invalidate the simple picture as a basis for calculating attenuation statistics.

A Problem for Tropical Climates:

Rain forming well below the 0°C level appears to be the normal situation in these climates — a satisfactory vertical structure model for attenuation prediction remains to be found!