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Winter College on Quantum Optics: Novel Radiation Sources

3-21 March 1997

Quantum theory of linear amplifiers
Correlated spontaneous emission laser
Quantum state measurement via emission spectrum

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Lecture 1

Quantum Theory of Linear Amplifiers

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"NOISE-FREE" AMLIFICATION

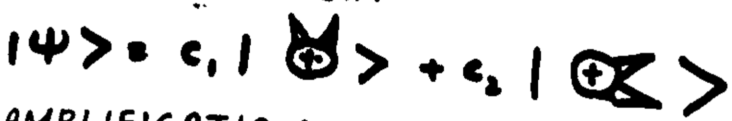
OPTICAL

LINEAR

MOTIVATION:

(i) FUNDAMENTAL

MACROSCOPIC MANIFESTATION OF MICROSCOPIC STATES, e.g., COHERENT SUPERPOSITION OF STATES
SCHRÖDINGER CAT



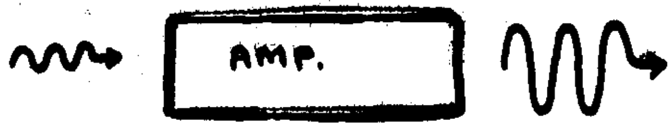
AMPLIFICATION OF NON-CLASSICAL STATES, e.g., SQUEEZED STATES

(ii) APPLIED

HIGH SPEED COMMUNICATION AND INFORMATION TRANSFER

PHOTONS — CARRIER OF INFORMATION
DETECTORS — MACROSCOPIC OBJECTS

LINEAR AMPLIFIER:



1. GAIN — STIMULATED EMISSION
2. LOSS — STIMULATED ABSORPTION
3. NOISE — SPONTANEOUS EMISSION

   (DUE TO VACUUM FLUCTUATIONS)

   FUNDAMENTAL SOURCE OF NOISE !!

NOISE-FREE AMPLIFICATION

WHEN SIGNAL IS AMPLIFIED, NOISE IN THE SIGNAL IS ALSO AMPLIFIED BUT THERE IS ALSO ADDED NOISE DUE TO SPONTANEOUS EMISSION

QUESTION: CAN WE GET RID OF ADDED NOISE IN THE OUTPUT SO THAT

$$(SNR)_{OUT} = (SNR)_{IN} \quad ?$$

VACUUM FLUCTUATIONS

(HOW DOES VACUUM LOOK LIKE ?)

$$E = \kappa\left(a e^{-i\omega t} + a^{\dagger} e^{i\omega t}\right) \; ; \quad [a, a^{\dagger}] = 1$$

HERMITIAN OPERATORS

$$a_1 = \frac{1}{2}\left(a + a^{\dagger}\right) \qquad [a_1, a_2] = \frac{i}{2}$$

$$a_2 = \frac{1}{2i}\left(a - a^{\dagger}\right) \qquad \Delta a_1 \Delta a_2 \geq \frac{1}{4}$$

$$E = \kappa\left(a_1 \cos \omega t + a_2 \sin \omega t\right)$$

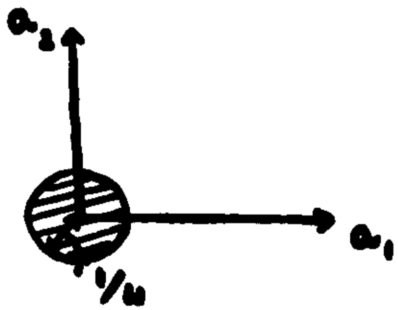
FLUCTUATIONS

$$\Delta a_1^2 = \left[\langle a_1^2 \rangle - \langle a_1 \rangle^2\right]$$

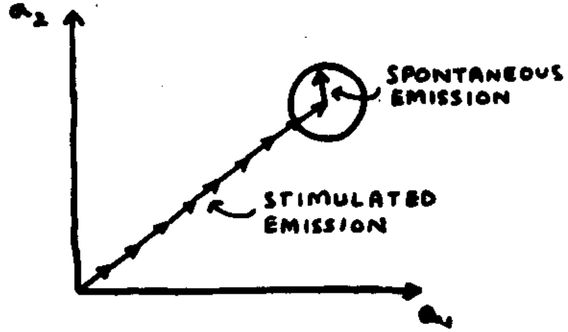
$$= \frac{1}{4}\left[\langle 0|(a^2 + a^{\dagger 2} + \underline{a a^{\dagger}} + a^{\dagger} a)|0\rangle - \langle 0|(a + a^{\dagger})|0\rangle^2\right]$$

$$= \frac{1}{4}$$

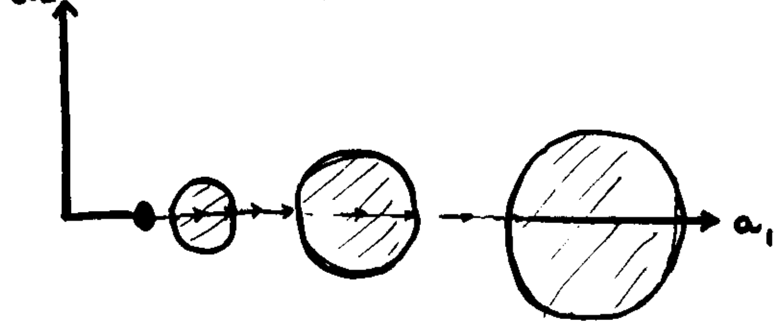
$$\Delta a_2^2 = \frac{1}{4}$$



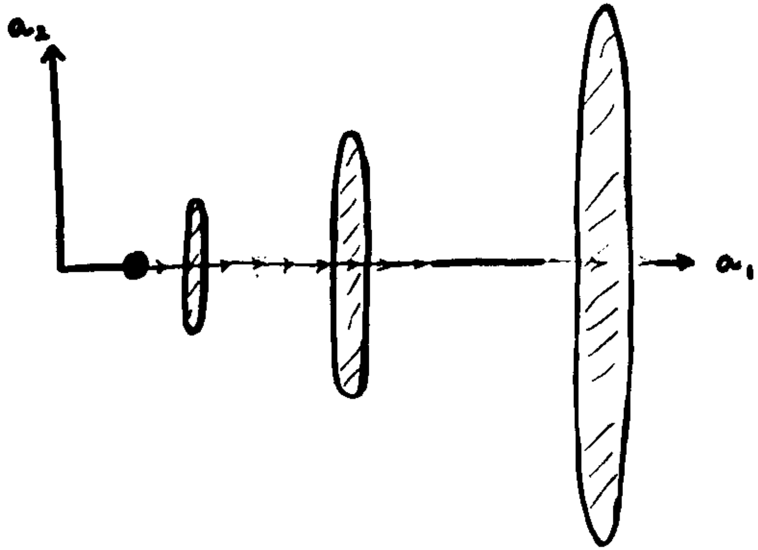
SPONTANEOUS EMISSION



VACUUM $|0\rangle$



Squeezed vacuum $|\xi\rangle$



NAIVELY

$$a_t = \sqrt{G}\, a_0 \quad ; \quad a_t^\dagger = \sqrt{G}\, a_0^\dagger$$

↳ GAIN

HERMITIAN OPERATORS

$$a_1 = \frac{1}{2}(a + a^\dagger) \quad ; \quad a_2 = \frac{1}{2i}(a - a^\dagger)$$

$\longrightarrow$

$$\langle a_1 \rangle_t = \sqrt{G}\, \langle a_1 \rangle_0$$

$$(\Delta a_1)_t = \sqrt{\langle a_1^2 \rangle_t - \langle a_1 \rangle_t^2} = \sqrt{G}\, (\Delta a_1)_0$$

$$(SNR)_{in} = \frac{\langle a_1 \rangle_0}{(\Delta a_1)_0} = \frac{\langle a_1 \rangle_t}{(\Delta a_1)_t} = (SNR)_{out}$$

WHOLE ARGUMENT IS WRONG BECAUSE

$$[a_t, a_t^\dagger] = G\,[a_0, a_0^\dagger] = G \neq 1$$

QUANTUM FLUCTUATIONS IN LINEAR AMPLIFIERS

INPUT-OUTPUT RELATIONSHIP

$$a_{OUT} = \sqrt{G}\, a_{IN} + \underbrace{\sqrt{G-1}\; F^\dagger}_{\text{ADDED NOISE}}$$

↳ GAIN

$$[F, F^\dagger] = 1$$

DEFINE

$$a_{\phi 1} = \frac{1}{2}(a e^{-i\phi} + a^\dagger e^{i\phi}) \; ; \; a_{\phi 2} = \frac{1}{2i}(a e^{-i\phi} - a^\dagger e^{i\phi})$$

$$F_{\phi 1} = \frac{1}{2}(F e^{+i\phi} + F^\dagger e^{-i\phi}) \; ; \; F_{\phi 2} = \frac{1}{2i}(F e^{i\phi} - F^\dagger e^{-i\phi})$$

$\Longrightarrow$

$$(a_{\phi 1})_{OUT} = \sqrt{G}\, (a_{\phi 1})_{in} + \sqrt{G-1}\; F_{\phi 1}$$

$$(a_{\phi 2})_{OUT} = \sqrt{G}\, (a_{\phi 2})_{in} - \sqrt{G-1}\; F_{\phi 2}$$

SIGNAL $\quad \langle a_{\phi i} \rangle_{OUT} = \sqrt{G}\, \langle a_{\phi i} \rangle_{IN}$

NOISE $\quad (\Delta a_{\phi i})^2_{OUT} = G\,(\Delta a_{\phi i})^2_{IN} + (G-1)\, \Delta F^2_{\phi i}$

$$\Delta F_{\phi 1}\, \Delta F_{\phi 2} \geq \frac{1}{4}$$

$$\Rightarrow |\Delta F|^2 = (\Delta F^2_{\phi 1} + \Delta F^2_{\phi 2}) \geq \frac{1}{2} \quad (\because a^2 + b^2 \geq 2ab)$$

$$\frac{\langle a_{\phi i}\rangle_{out}}{(\Delta a_{\phi i})_{out}} = \frac{\langle a_{\phi i}\rangle_{in}}{\left[(\Delta a_{\phi i})^2_{in} + (1-G^{-1})\,\Delta F^2_{\phi i}\right]^{1/2}}$$

$$(SNR)_{out} < (SNR)_{in}$$

IN A PHASE-<u>INSENSITIVE</u> AMPLIFIER, EQUAL NOISE IS ADDED TO BOTH QUADRATURES

IN PHASE-<u>SENSITIVE</u> AMPLIFICATION, IT SHOULD BE POSSIBLE TO ADD UNEQUAL NOISE IN THE TWO QUADRATURES WITH THE LIMITATION $|\Delta F|^2 = (\Delta F_1^2 + \Delta F_2^2) \geq \frac{1}{2}$.

NOISE-FREE AMPLIFICATION !!

PHASE-INSENSITIVE AMPLIFICATION IN TWO-LEVEL SYSTEM

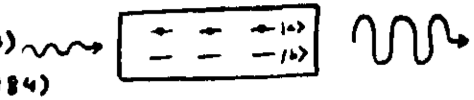
FRIBERG AND MANDEL (1983)

LOUDON AND SHEPHERED (1984)

STENHOLM (1986)

GLAUBER (1986)

⋮



$$\rho_A(t_i) = \rho_{aa}|a\rangle\langle a| + \rho_{bb}|b\rangle\langle b|$$

EQ. FOR DENSITY OPERATOR FOR THE FIELD :

$$\dot{\rho} = -\frac{A}{2}\left[\rho_{aa}\left(aa^{+}\rho - 2a^{+}\rho a + \rho a a^{+}\right)\right. \quad \text{GAIN}$$

$$\left. + \rho_{bb}\left(a^{+}a\rho - 2a\rho a^{+} + \rho a^{+}a\right)\right] \quad \text{LOSS}$$

$$\frac{d}{dt}\langle a\rangle = -\frac{A}{2}\left(\rho_{aa} - \rho_{bb}\right)\langle a\rangle$$

$$\frac{d}{dt}\langle a^{+}a\rangle = A\left(\rho_{aa} - \rho_{bb}\right)\langle a^{+}a\rangle + \underbrace{A\,\rho_{aa}}_{\text{SP. EMISSION}}$$

$$\frac{d}{dt}\langle a^{2}\rangle = A\left(\rho_{aa} - \rho_{bb}\right)\langle a^{2}\rangle$$

INPUT-OUTPUT EQS.

$$\langle a \rangle_t = \sqrt{G}\,\langle a \rangle_0$$

$$\langle a^\dagger a \rangle_t = G \langle a^\dagger a \rangle_0 + (G-1)\,\frac{\rho_{aa}}{\rho_{aa}-\rho_{bb}}$$

$$\langle a^2 \rangle_t = G \langle a^2 \rangle_0$$

$$G = e^{A(\rho_{aa}-\rho_{bb})t} \quad \text{— GAIN FACTOR}$$

$G > 1$ $(\rho_{aa} > \rho_{bb})$ — AMPLIFICATION

$G < 1$ $(\rho_{aa} < \rho_{bb})$ — ATTENUATION

DEFINE $X_\theta = \frac{1}{2}\left(a e^{-i\theta} + a^\dagger e^{i\theta}\right)$ $\quad \begin{pmatrix} X_0 = a_1 \\ X_{\pi/2} = a_2 \end{pmatrix}$

$$\langle X_\theta \rangle_t = \sqrt{G}\,\langle X_\theta \rangle_0$$

$$(\Delta X_\theta^2)_t = G(\Delta X_\theta^2)_0 + (G-1)N$$

$$N = \frac{1}{4(\rho_{aa}-\rho_{bb})} \geq \frac{1}{4}$$

PHASE-PRESERVING + PHASE-INSENSITIVE AMP.

EVEN FOR HIGHLY SQUEEZED INPUT $(\Delta X_\theta^2)_0 = 0$

SQUEEZING IS PRESERVED FOR

$$G_{max} = 2 \quad \text{(CLONING LIMIT)}$$

WHAT ABOUT SUB-POISSON STATISTICS?

$$\langle :\Delta n^2: \rangle_t = G^2 \langle :\Delta n^2: \rangle_0 + 2G(G-1)\langle a^\dagger a \rangle + (G-1)^2$$

$$\hookrightarrow \langle a^\dagger a^\dagger a a \rangle - \langle a^\dagger a \rangle^2$$

FOR INITIAL FOCK STATE $|n_0\rangle$

$$(:\Delta n^2:)_0 = -n_0 \quad \text{(SUB-POISSON)}$$

$$\Rightarrow$$

$$(:\Delta n^2:)_t = -G^2 n_0 + 2G(G-1)n_0 + (G-1)^2$$

$$= (G^2 - 2G)\,n_0 + (G-1)^2$$

$$> 0 \quad \text{FOR} \quad G \geq 2$$

GENERAL SOLUTION FOR P-REPRESENTATION

$(\rho_{aa} = 1)$

$$P(\alpha,t) = \frac{1}{\pi}\int \frac{\exp\left[-|\alpha-\beta|^2/G-1\right]}{G-1}\, P(\beta,0)\, d^2\beta$$

— AT TIME t, P FUNCTION AT POINT α IS THE AVERAGE OVER ITS NEIGHBORHOOD UNDER GAUSSIAN BELL SHAPE.

— SOURCE OF THE LOSS OF NON-CLASSICAL FEATURE

— A CLASSICAL STATE AT $t=0$ REMAINS CLASSICAL FOR ALL TIMES.

WHAT ABOUT INITIAL NON-CLASSICAL STATES? CAN A NON-CLASSICAL STATE REMAIN NON-CLASSICAL FOR ARBITRARY GAIN?

POSSIBLE ANSWERS:

1. NON-CLASSICAL FEATURES DISAPPEAR AS SOON AS $G>1$.
2. NON-CLASSICAL FEATURES DISAPPEAR AS $G>2$.
3. NON-CLASSICAL FEATURES EXIST FOR ARBITRARY GAIN.

EXAMPLES:

1. SQUEEZED STATE

$$|\Psi\rangle = |\xi, \beta\rangle$$

$\xi \to re^{0}$

P-REPRESENTATION AFTER AMPLIFICATION

$$P(\alpha,t) = \frac{1}{N}\exp\left[-\frac{2(\alpha_r - \sqrt{G}\beta_r e^{-r})^2}{G(1+e^{-2r})-2} - \frac{2(\alpha_i - \sqrt{G}\beta_i e^{+r})^2}{G(1+e^{+2r})-2}\right]$$

NORMALIZED P EXISTS IF

$$G(1+e^{-2|r|}) > 2$$

HIS CONDITION IS SATISFIED FOR ALL $|r|$

$$G > 2\ !$$

2. SCHRÖDINGER-CAT STATE

(HUANG, ZHU, ZUBAIRY, PRA 53, 1027 96)

$$|\psi\rangle = \frac{1}{\sqrt{N}}\left(|\xi_0\rangle + |-\xi_0\rangle\right)$$

↳ COHERENT STATE

$$N = 2\left(1 + e^{-2\xi_0^2}\right) \quad ; \qquad \xi_0 \text{ — REAL}$$

$$P(\alpha, t) = \frac{2}{\pi N (G-1)} \exp\left[-\frac{|\alpha|^2 + \xi_0^2 G}{G-1}\right]$$

$$\times \Bigg[\underbrace{\cosh\left(\frac{2\xi_0\sqrt{G}\,\alpha_r}{G-1}\right)}_{\text{"CLASSICAL" SUPERPOSITION OF TWO COHERENT STATES}} + \underbrace{\exp\left(\frac{2\xi_0^2}{G-1}\right)\cos\left(\frac{2\xi_0\sqrt{G}\,\alpha_i}{G-1}\right)}_{\text{COHERENT PART}}\Bigg]$$

$$\alpha_r = 0 \Rightarrow 1 \qquad \alpha_i = \frac{\pi(2n+1)(G-1)}{2\xi_0\sqrt{G}}$$

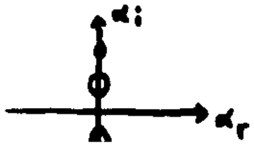
$$\Rightarrow -\exp\left(\frac{2\xi_0^2}{G-1}\right)$$

P IS NEGATIVE IF

$$\frac{2\xi_0^2}{G-1} > 0 \qquad \text{(ALWAYS TRUE)}$$

AREAS OF α IN WHICH P IS NEGATIVE ARE LOCATED PERIODICALLY ALONG IMAGINARY AXIS. SHAPE OF THESE AREAS FOR LARGE G IS GIVEN BY

$$|\alpha_r| < 1$$



PHASE-SENSITIVE AMPLIFICATION

INTERACTION OF A BOSON MODE WITH A BATH OF INVERTED HARMONIC OSCILLATORS.

BY RIGGING THE RESERVOIR — PREPARING IT IN A SQUEEZED VACUUM CENTERED ABOUT THE MODE THAT IS TO BE AMPLIFIED

DUBERTUIS, BARNETT AND STENHOLM, (1987)

MILBURN, STEYN-ROSS AND WALLS (1987)

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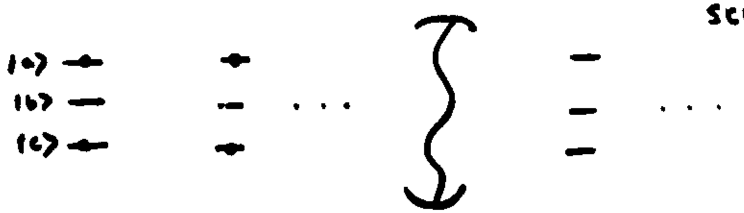
TWO-PHOTON LINEAR AMPLIFIER

SCULLY + ZUBAIRY, '88



MODEL: THREE-LEVEL ATOMS PUMPED COHERENTLY IN LEVELS $|a\rangle$ AND $|c\rangle$

$$\rho_A(t_i) = \rho_{aa}|a\rangle\langle a| + \rho_{cc}|c\rangle\langle c| + \rho_{ac}|a\rangle\langle c| + \rho_{ca}|c\rangle\langle a|$$

$$\begin{aligned}
\dot{\rho}_F = & -\frac{A}{2}\rho_{aa}\left(a a^\dagger \rho_F - 2a^\dagger \rho_F a + \rho_F a a^\dagger\right) \quad \text{GAIN} \\
& -\frac{A}{2}\rho_{cc}\left(a^\dagger a \rho_F - 2a\rho_F a^\dagger + \rho_F a^\dagger a\right) \quad \text{ABSORPTION} \\
& \left.\begin{aligned} & -\frac{A}{2}\rho_{ca}\left(a a \rho_F - 2a\rho_F a + \rho_F a a\right) \\ & -\frac{A}{2}\rho_{ac}\left(a^\dagger a^\dagger \rho_F - 2a^\dagger \rho_F a^\dagger + \rho_F a^\dagger a^\dagger\right) \end{aligned}\right\} \text{COHERENCE}
\end{aligned}$$

INPUT-OUTPUT EQUATIONS

$$\langle a\rangle_t = \sqrt{G}\,\langle a\rangle_0$$

$$\langle a^\dagger a\rangle_t = G\langle a^\dagger a\rangle_0 + \frac{\rho_{aa}}{\rho_{aa}-\rho_{cc}}(G-1)$$

$$\langle a^2\rangle_t = G\langle a^2\rangle_0 + \underbrace{\frac{\rho_{ac}}{\rho_{aa}-\rho_{cc}}(G-1)}_{\text{PHASE SENSITIVE CONTRIBUTION}}$$

$$G = \exp[A(\rho_{aa}-\rho_{cc})t]$$

DEFINE QUADRATURE COMPONENTS

$$a_{\phi 1} = \frac{1}{2}\left(a e^{-i\phi} + a^\dagger e^{i\phi}\right)$$

$$a_{\phi 2} = \frac{1}{2i}\left(a e^{-i\phi} - a^\dagger e^{i\phi}\right)$$

INPUT-OUTPUT EQUATIONS GIVE

$$\langle a_{\phi i}\rangle_t = \sqrt{G}\,\langle a_{\phi i}\rangle_0$$

$$(\Delta a_{\phi i})_t^2 = G(\Delta a_{\phi i})_0^2 + (G-1)N_{\phi i}, \quad i = 1,2$$

ADDED NOISE IN QUADRATURE COMPONENTS

$$\left.\begin{aligned} N_{\phi 1} &= \frac{1 - 2|\rho_{ac}|\cos(2\phi+\theta)}{4(\rho_{aa}-\rho_{cc})} \\ N_{\phi 2} &= \frac{1 + 2|\rho_{ac}|\cos(2\phi+\theta)}{4(\rho_{aa}-\rho_{cc})} \end{aligned}\right\} \text{UNEQUAL NOISE}$$

$$\rho_{ac} = |\rho_{ac}|\,e^{i\theta}$$

GAIN

$$G = \exp\left(A(\rho_{aa}-\rho_{cc})t\right)$$

CAN WE AMPLIFY WITHOUT ADDED NOISE?

DEFINE $\rho_{aa} = \frac{1+\eta}{2}$, $\rho_{cc} = \frac{1-\eta}{2}$, $|\rho_{ac}| = \frac{1}{2}\sqrt{1-\eta^2}$

ADDED NOISE

$$N_{\phi 1} = \frac{1 - \sqrt{1-\eta^2}\cos(2\phi+\theta)}{4\eta}$$

$$N_{\phi 2} = \frac{1 + \sqrt{1-\eta^2}\cos(2\phi+\theta)}{4\eta}$$

AMPLIFIED SIGNAL WITHOUT ADDED NOISE IS OBTAINED IN THE LIMITS:

$(2\phi+\theta) \to 0$, $\eta \to 0$, $At \to \infty$, $\eta At = C$ (FINITE)

UNDER THESE CONDITIONS

$N_{\phi 1} \to 0$ AND $N_{\phi 2} \to \infty$

ALSO

$$G = \exp((\rho_{aa} - \rho_{cc})At) = \exp(\eta At)$$
$$= \text{FINITE}$$

QUANTUM STATISTICS OF ADDED NOISE

J. ANWAR + M. S. ZUBAIRY
PRA 49, 481 (1994)

$$a_t = \sqrt{G}\, a_0 + \sqrt{G-1}\, F^{+}$$

$$[F, F^{+}] = 1$$

INITIAL STATE : VACUUM

$$\langle a_t^{+n} a_t^{m} \rangle = \langle 0 | (\sqrt{G}\, a_0^{+} + \sqrt{G-1}\, F)^n (\sqrt{G}\, a_0 + \sqrt{G-1}\, F^{+})^m | 0 \rangle$$

$$= (G-1)^{\frac{n+m}{2}} \langle F^n F^{+m} \rangle$$

NORMALLY ORDERED

ANTI NORMAL ORDERED

$$P_t(\alpha, \alpha^*) = Q_F\left(\frac{\alpha}{\sqrt{G-1}}, \frac{\alpha^*}{\sqrt{G-1}}\right)$$

COMPARISON BETWEEN Q-REPRESENTATION FOR ADDED NOISE AND Q-REPRESENTATION FOR AN IDEAL SQUEEZED VACUUM STATE SHOWS THAT BOTH ARE IDENTICAL WITH

$$\theta = \phi$$

$$\tanh r = \frac{\rho_{cc}}{\rho_{aa}}$$

NUMBER FLUCTUATIONS

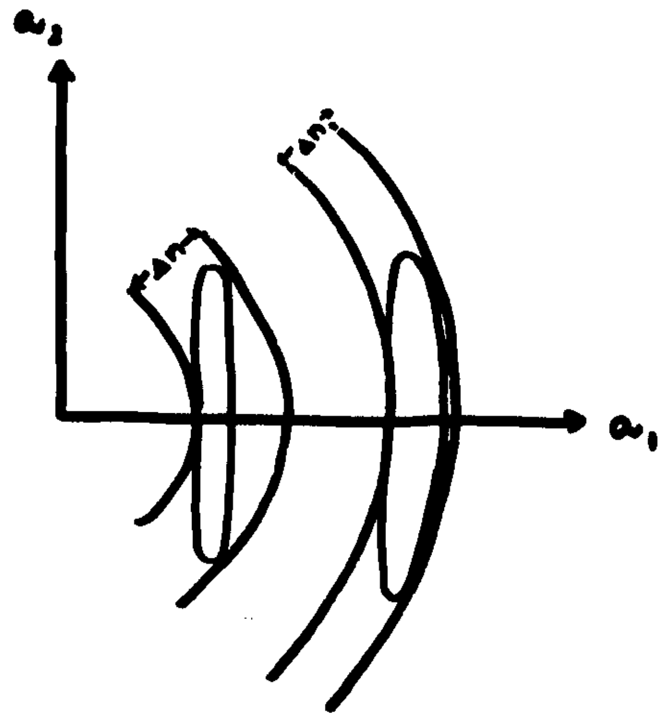
$$\langle a^\dagger a \rangle = G \langle a^\dagger a \rangle_0 + m_1$$

$$(:\Delta n^2:) = G^2 (:\Delta n^2:)_0 + \mathcal{N}$$

$$\mathcal{N} = 2G\left(m_1 \langle a^\dagger a \rangle_0 - m_2 \langle a^2 \rangle_0 - m_2^* \langle a^{\dagger 2} \rangle_0 \right) + m_1^2 + m_2^2$$

$$G = \exp\left((\rho_{aa} - \rho_{cc})At\right)$$

$$m_1 = \frac{\rho_{aa}}{\rho_{aa} - \rho_{cc}} (G-1) \quad ; \quad m_2 = \frac{\rho_{ac}}{\rho_{aa} - \rho_{cc}} (G-1)$$



CAN WE MAKE $\mathcal{N} < 0$?

YES, WHEN

(i) INPUT IS SQUEEZED

$$\langle a^{\dagger 2} \rangle_0 + \langle a^2 \rangle_0 - 2 \langle a^\dagger a \rangle_0 = x \; ; \quad 0 \leq x \leq 1$$

(ii) IF $\rho_{aa} = \frac{1+\epsilon}{2}$, $\rho_{cc} = \frac{1-\epsilon}{2}$; $\rho_{ac} = \frac{\sqrt{1-\epsilon^2}}{2} = \rho_{ca}$

THE $\mathcal{N} < 0$ IF

$$\langle a^\dagger a \rangle_0 < \frac{1}{2} \left[\frac{x\sqrt{1-\epsilon^2} - \left(1 - \frac{1}{G}\right)\left(1 + \frac{1}{\epsilon}\right)}{(1+\epsilon) - \sqrt{1-\epsilon^2}} \right]$$

NOISE-FREE AMPLIFICATION OF SQUEEZED LIGHT:

(H. HUANG, S.-Y. ZHU, M.S. ZUBAIRY, PHYS. REV. A $\underline{52}$, 4586).

$$\Delta a_i^2(t) = G\,\Delta a_i^2(0) + (G-1)\,\Delta F_i^2 \qquad (i=1,2)$$

$$\langle a_i(t)\rangle = \sqrt{G}\,\langle a_i(0)\rangle$$

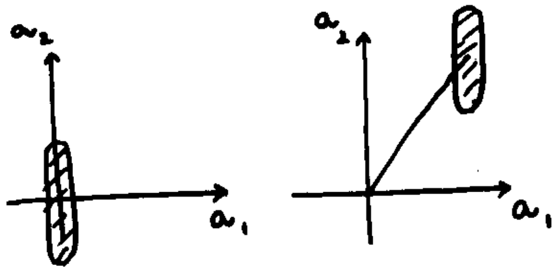
IN TWO-PHOTON LINEAR AMPLIFIER

$$\Delta F_1^2 \longrightarrow 0 \quad ; \quad \Delta F_2^2 \longrightarrow \infty$$

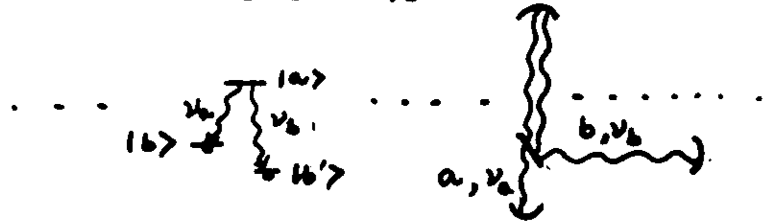
$$\Longrightarrow \Delta a_1^2(t) = G\,\Delta a_1^2(0)$$

SQUEEZING ($\Delta a_i^2 < \frac{1}{4}$) IN THE INPUT IS DEGRADED BY GAIN

CAN WE ADD COHERENT COMPONENT TO A SQUEEZED VACUUM STATE WITH NO DEGRADATION OF SQUEEZING?



TWO-MODE OPERATION:

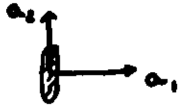


ATOMS COHERENTLY PREPARED IN LOWER LEVELS

$$|\psi_i\rangle = \frac{1}{\sqrt{2}}\left(|b\rangle + e^{-i\varphi}|b'\rangle\right)$$

"NOISE-FREE" ENERGY TRANSFER CAN TAKE PLACE FROM A STRONG COHERENT MODE TO A WEAK SQUEEZED FIELD MODE.

$t=0$: MODE a — SQUEEZED VACUUM FIELD



MODE b — STRONG COHERENT FIELD $|\alpha\rangle$

$t=t_0$: MODE a — $\langle a_1(t)\rangle = -\frac{1}{2}\left(1-e^{-2Rt_0}\right)|\alpha|\cos\theta$

$$\Delta a_1^2(t) = \frac{1}{4} + \frac{1}{4}\left(1+e^{-2Rt_0}\right)^2\left(\Delta a_1^2(0) - \frac{1}{4}\right)$$

MODE b — $\langle b_1(t)\rangle = \frac{1}{2}\left(1+e^{-2Rt_0}\right)|\alpha|\cos\theta$

$$\Delta b_1^2(t) = \frac{1}{4} + \frac{1}{4}\left(1-e^{2Rt_0}\right)^2\left(\Delta a_1^2(0) - \frac{1}{4}\right)$$

$R = g^2\,\tau^2\,r$

r → rate of injection of atoms

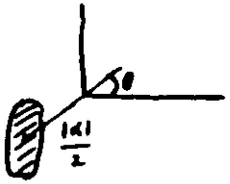
τ → transit time through the cavity

g → atom-field coupling constant

LIMITING CASES :

1. $t_0 \to \infty$

$$\langle a_1(t) \rangle \to -\frac{1}{2} |\alpha| \cos\theta$$



$$\Delta a_1^2(t) \to \frac{1}{4} + \frac{1}{4}\left(\Delta a_1^2(0) - \frac{1}{4}\right)$$

STILL SQUEEZED ; $\Delta a_1^2(t) < \frac{1}{4}$ if $\Delta a_1^2(0) < \frac{1}{4}$

COHERENT AMPLITUDE ADDED

$$\langle b_1(t) \rangle \to \frac{1}{2} |\alpha| \cos\theta$$

$$\Delta b_1^2(t) \to \frac{1}{4} + \frac{1}{4}\left(\Delta a_1^2(0) - \frac{1}{4}\right)$$

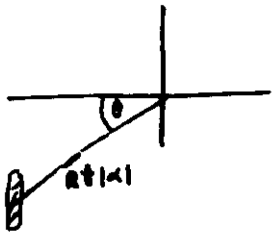
TRANSFER OF SQUEEZING FROM MODE a

COHERENT AMPLITUDE REDUCED

2. $4Rt \to 0$

$|\alpha| \to \infty$

$4Rt\,|\alpha| =$ CONSTANT



$$\langle a_1(t) \rangle \to -Rt\,|\alpha| \cos\theta \qquad \text{(LARGE)}$$

$$\Delta a_1^2(t) \simeq \Delta a_1^2(0)$$

(NOISE-FREE AMPLIFICATION)

Lecture 2

Correlated Spontaneous Emission Laser

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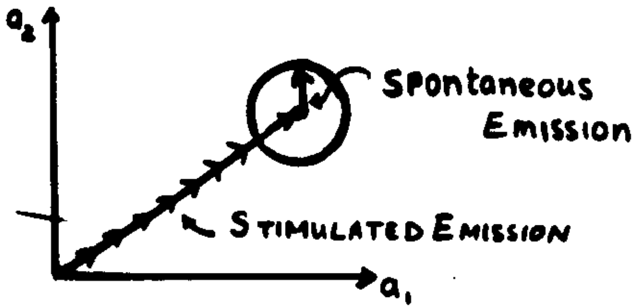
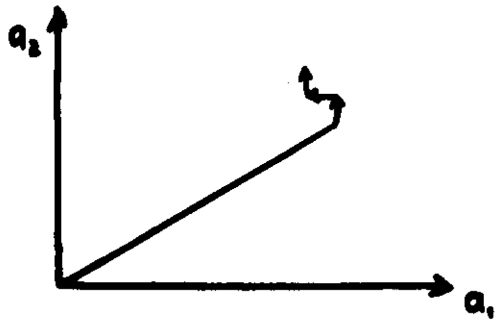
QUANTUM NOISE IN LASER.

SPONTANEOUS EMISSION



PHASE DIFFUSION MODEL



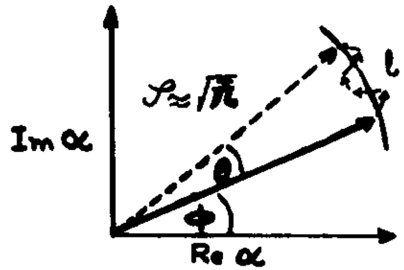
NOISE IN LASER

1. AMPLITUDE FLUCTUATIONS
   PHOTON STATISTICS
2. PHASE FLUCTUATIONS
   LASER LINEWIDTH.

LASER LINEWIDTH
PHASE DIFFUSION MODEL

SPONTANEOUS EMISSION RATE = A

FIELD AMPLITUDE $\alpha = \sqrt{\bar{n}}\, \exp(i\theta)$



PROBABILITY THAT A DISTANCE l IS TRAVELLED AFTER At STEPS

$$P(l) = \left(\frac{1}{\pi A t}\right)^{1/2} e^{-\left(\frac{l^2}{At}\right)}$$

$$P(\theta) = \left(\frac{\bar{n}}{\pi A t}\right)^{1/2} e^{-\left(\frac{\theta^2 \bar{n}}{At}\right)} \quad \text{FOR } l = \sqrt{\bar{n}}\,\theta .$$

$$\boxed{\dot{P} = D(\theta)\frac{\partial^2 P}{\partial \theta^2} \qquad D(\theta) = \frac{A}{4\bar{n}}}$$

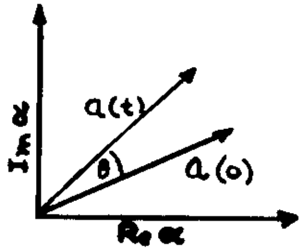
DIFFUSION EQ.

CORRELATION FUNCTION

$$\langle a^{+}(0)\, a(t)\rangle = \bar{n}\, e^{-i\omega_0 t} \langle \cos\theta \rangle$$

$$\langle \cos\theta \rangle = \int P(\theta) \cos\theta \, d\theta = e^{-Dt} \quad \Rightarrow D = \frac{A}{4\bar{n}}$$

$$\langle a^{+}(0)\, a(t)\rangle = \bar{n}\, e^{-i\omega_0 t - Dt}$$

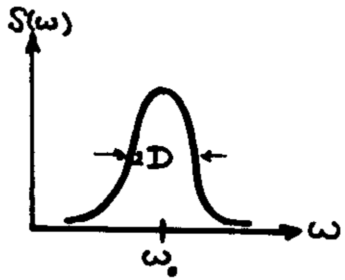


SPECTRUM:

$$S(\omega) = \mathrm{Re} \int_0^{\infty} \langle a^{+}(0)\, a(t)\rangle e^{i\omega t} dt$$

$$= \frac{D}{D^2 + (\omega - \omega_0)^2}$$

(LORENTZIAN)



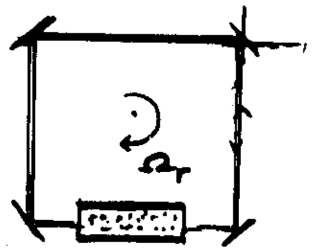
LINEWIDTH $\quad 2D = \dfrac{A}{2\bar{n}}$

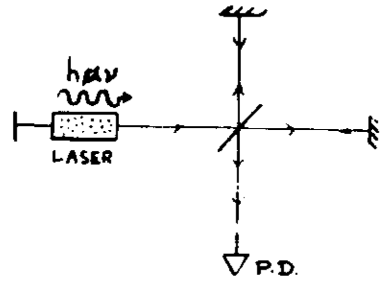
CORRELATED EMISSION LASER

MOTIVATION :

MEASUREMENTS OF SMALL DISPLACEMENTS USING LASER e.g.,

* GRAVITATIONAL WAVE DETECTION
* LASER GYROSCOPES





STANDARD QUANTUM LIMIT IS DUE TO

SHOT NOISE (IN PASSIVE SYSTEMS)
SPONTANEOUS EMISSION NOISE (IN ACTIVE SYSTEMS)

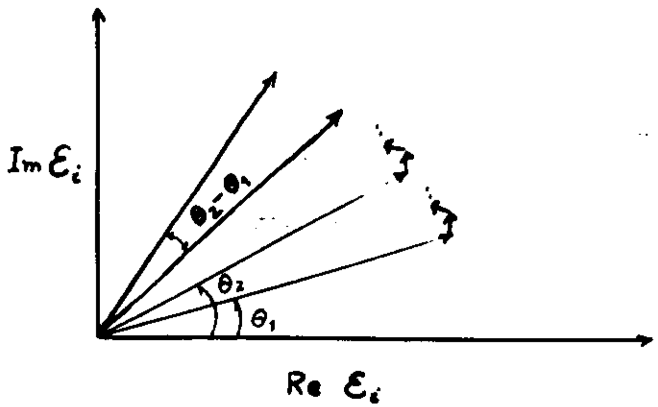
HOW TO GET ARROUND SQL ?

QUANTUM NONDEMOLITION
SQUEEZED STATES
CORELATED SPONTANEOUS EMISSION LASER (CEL)
SCULLY (1985) ; SCULLY AND ZUBAIRY (1987)

QUESTION :

CAN WE QUENCH THE SPONTANEOUS EMISSION NOISE IN THE RELATIVE PHASE ANGLE BETWEEN THE TWO-MODES ?

CORRELATED EMISSION LASER

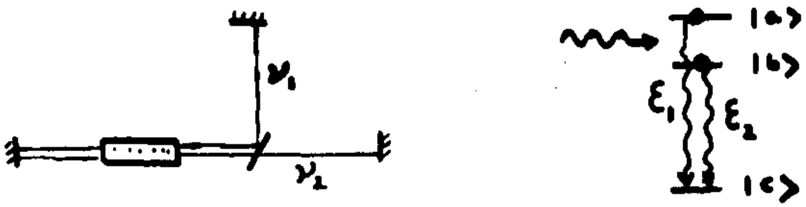


RANDOM WALK OF THE CORELATED ELECTRIC FIELD PHASORS IN THE COMPLEX $\mathcal{E}$ PLANE

CEL ACTION $\Rightarrow$ $D(\theta_1-\theta_2)=0$

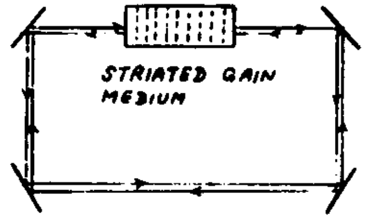
DIFFUSION COEFFICIENT

ATOMIC SYSTEM PREPARED IN A COHERENT SUPERPOSITION OF STATES

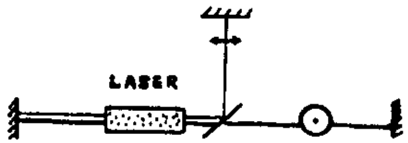


SCHEMES TO MAKE CEL

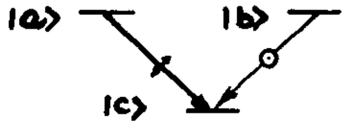
(i) "HOLOGRAPHIC" LASER



(ii) HANLE-EFFECT LASER

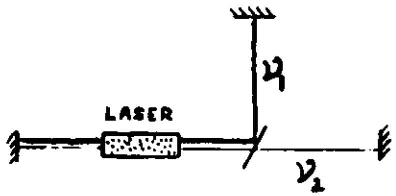


DOUBLY RESONANT CAVITY

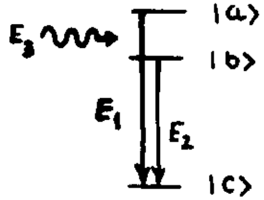


POLARIZATION INDUCED COHERENCE

(iii) QUANTUM BEAT LASER

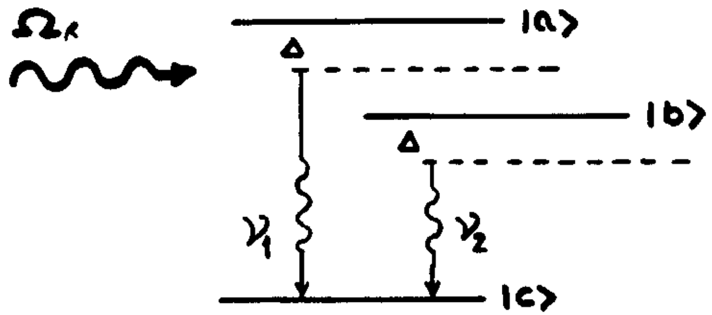


DOUBLY RESONANT CAVITY



MICROWAVE INDUCED COHERENCE

QUANTUM BEAT LASER



INTERACTION HAMILTONIAN :

$$V = \hbar g \left(a_1 e^{i\Delta t} |a\rangle\langle c| + a_2 e^{i\Delta t} |b\rangle\langle c| \right.$$
$$\left. - \frac{\hbar \Omega_R}{2} e^{-i\varphi} |a\rangle\langle b| + \text{H.C.} \right.$$

EQUATION OF MOTION FOR DENSITY OPERATOR :

$$\dot{\rho}_F = -\frac{1}{2}\alpha_{11}\left(\rho_F a_1 a_1^\dagger - a_1^\dagger \rho_F a_1\right) \quad - \text{GAIN}$$
$$\left.\begin{array}{l} -\frac{1}{2}\alpha_{12}\left(\rho_F a_2 a_1^\dagger - a_1^\dagger \rho_F a_2\right) e^{i\varphi} \\ -\frac{1}{2}\alpha_{21}\left(\rho_F a_1 a_2^\dagger - a_2^\dagger \rho_F a_1\right) e^{-i\varphi} \end{array}\right\} \text{COHERENCE TERMS}$$
$$-\frac{1}{2}\alpha_{22}\left(\rho_F a_2 a_2^\dagger - a_2^\dagger \rho_F a_2\right) \quad - \text{LOSS}$$

$$\rho = \int P(\alpha)\, |\alpha\rangle\langle\alpha|\, d^2\alpha$$

$$\frac{\partial P}{\partial t} = D \frac{\partial^2 P}{\partial \theta^2} \quad ; \quad \theta = \theta_1 - \theta_2$$

WHERE

$$D(\theta) = \frac{A}{2\langle n\rangle}\left(1 - \cos(\theta - \varphi)\right)$$

$$A = 2 r_a g^2 / \gamma^2 - \Delta^2 \quad \text{GAIN COEFFICIENT}$$

$$\psi = \theta - \varphi$$

IS THE LINEAR GAIN COEFFICIENT

$$D(\theta) \Rightarrow 0 \quad \text{when} \quad \theta = \varphi$$

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Correlated Spontaneous Emission in a Zeeman Laser

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(Received 8 June 1990)

We have observed phase-diffusion noise 40% below the Schawlow-Townes limit in the relative phase of a two-mode HeNe Zeeman laser due to the correlated-emission-laser effect.

PACS numbers: 42.50.Kb, 32.90.+a, 42.55.Wm

In recent years there has been great interest in the development of high-sensitivity laser interferometers for use in the detection of gravitational waves. Ordinarily, the sensitivity of these devices is limited by quantum fluctuations that take the form of shot noise in passive interferometers and spontaneous-emission noise in active interferometers. Proposals for increasing the sensitivity of these interferometers include squeezed states for passive devices[1] and the correlated-emission laser (CEL) for active devices.[2,3] Generation of squeezed states has now been demonstrated by several groups[4,5] and the purpose of this Letter is to describe an experimental investigation of the CEL.

The CEL utilizes a gain medium with two laser transitions sharing a common lower level (Fig. 1). A coherent superposition of the upper states is created by driving a microwave transition between these levels. This superposition results in the correlation of spontaneous emission into the two laser modes and, under appropriate conditions, leads to a reduction of phase-diffusion noise in the relative phase of the two modes.

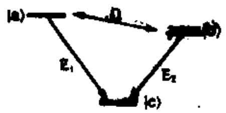


FIG. 1. CEL level diagram. Two laser transitions share a common lower level and the upper levels are coupled by an rf magnetic transition with an effective Rabi rate Ω.

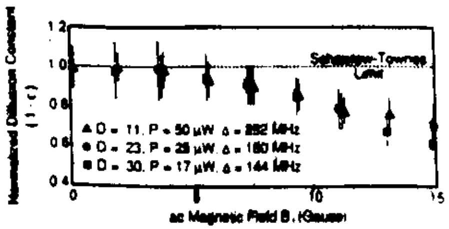


FIG. 4. Noise quenching in the CEL vs ac field strength. Data are normalized to the measured free-running diffusion constant (*D*). *P* is the laser output power per mode and Δ is the Δm = 2 Zeeman splitting.

To summarize, we have measured noise reduction 40% below the Schawlow-Townes limit in a HeNe laser due to the CEL effect. This noise reduction is in the relative phase of a two-mode laser and has potential implications for the design of optimally sensitive interferometers for gravitational wave detection. The choice may be between a passive interferometer utilizing squeezed light or an active interferometer with a CEL.

MOTIVATED BY THE ROLE OF
COHERENT SUPERPOSITION
IN QUENCHING OF SPONTANEOUS EMISSION
FLUCTUATIONS IN RELATIVE PHASE,

CAN WE GET CEL ACTION IN A SINGLE MODE LASER ??

CAN WE OBTAIN PHASE SQUEEZING IN A CEL ??

TWO-PHOTON CEL !!

- FIRST PROPOSED SOURCE OF SQUEEZED LIGHT

  TWO PHOTON LASER

  (H.P. YUEN , PHYS. REV. A <u>13</u>, 226 (1976))

- NO SQUEEZED STATES IN A TWO PHOTON LASER DUE TO SPONTANEOUS EMISSION

  (L.A. LUGIATO & G. STRINI, OPT. COMM. <u>41</u>, 447 (1982))
  (M.D. REID & D.F. WALLS, PHYS. REV. A <u>28</u>, 332 (1983))

- COMMON BELIEF THAT A LASING DEVICE [illegible] NOT PRODUCE SQUEEZED LIGHT DUE TO SPONTANEOUS EMISSION

- EMPHASIS SHIFTS TO PARAMETRIC DEVICES WHERE BELOW_ AND ABOVE _ THRESHOLD SQUEEZING IS PREDICTED AND OBSERVED EXPERIMENTALLY

- WE SHOW THAT TWO_PHOTON CORELATED_ SPONTANEOUS_ EMISSION LASER IS A SOURCE OF BRIGHT SQUEEZED LIGHT

  SCULLY, WODKIEWICZ, ZUBAIRY, BERGOU, LU AND MEYER TER VEHN, PHYS. REV. LETT. <u>60</u>, 1832 (1988).

  N. A. ANSARI, J. GEA-BANACLOCHE AND M.S. ZUBAIRY, PHYS. REV. A <u>41</u>, 5179 ([illegible]).

SQUEEZED STATE OF RADIATION FIELD

$$E(t) = \kappa \left(a e^{-i\nu t} + a^{+} e^{i\nu t} \right)$$

$$[a, a^{+}] = 1$$

HERMITIAN OPERATORS

$$a_1 = \frac{1}{2}(a + a^{+})$$

$$a_2 = \frac{1}{2i}(a - a^{+}) \quad \Rightarrow \quad [a_1, a_2] = \frac{i}{2}$$

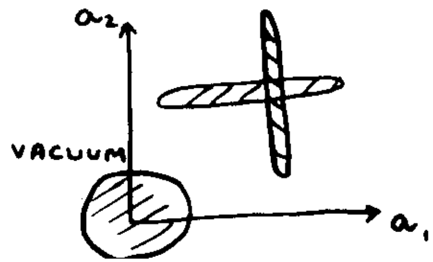
$$E(t) = \frac{\kappa}{2}\left(a_1 \cos \nu t + a_2 \sin \nu t \right)$$

UNCERTAINTY RELATION

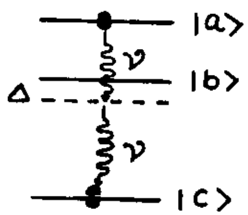
$$[a_1, a_2] = \frac{i}{2} \quad \Rightarrow \quad \Delta a_1 \Delta a_2 \geq \frac{1}{4}$$

A FIELD IS IN A SQUEEZED STATE IF

$$(\Delta a_1)^2 < \frac{1}{4} \quad \text{OR} \quad (\Delta a_2)^2 < \frac{1}{4}$$



HAMILTONIAN :



$$H = H_0 + H_1$$

$$H_0 = \hbar\Omega\, a^\dagger a + \sum_{i=a,b,c} \hbar\omega_i\, |i\rangle\langle i|$$

$$H_1 = \hbar g\, (a|a\rangle\langle b| + a|b\rangle\langle c|) + \text{H.c.}$$

ATOMS INITIALLY PREPARED IN A COHERENT SUPERPOSITION OF STATES

$$\rho = (\rho_{aa}|a\rangle\langle a| + \rho_{cc}|c\rangle\langle c| + \rho_{ac}|a\rangle\langle c| + \rho_{ca}|c\rangle\langle a|) \otimes \rho_F$$

EQUATION OF MOTION FOR THE REDUCED DENSITY MATRIX

$$\begin{aligned}
\dot\rho = \{ & -\tfrac{\alpha}{2}\rho_{aa}\mathcal{L}^*(aa^\dagger\rho - a^\dagger\rho a) && \text{GAIN}\\
& -\tfrac{\alpha}{2}\rho_{cc}\mathcal{L}(\rho a^\dagger a - a\rho a^\dagger) && \text{ABSORPTION}\\
& -\tfrac{\alpha}{2}\rho_{ca}\mathcal{L}^*(aa\rho - a\rho a) && \\
& -\tfrac{\alpha}{2}\rho_{ac}\mathcal{L}(\rho aa - a\rho a) && \Big\}\ \text{COHERENCE}\\
& -\tfrac{\gamma}{2}(a^\dagger a\rho - a\rho a^\dagger)\} + \text{H.c.} && \text{CAVITY LOSS}\\
& -i(\Omega-\nu)[a^\dagger a, \rho] && \text{FREQ. PULLING}
\end{aligned}$$

$$\alpha = \frac{2r g^2}{\Gamma^2} \quad ; \quad \mathcal{L} = \frac{1}{\Gamma + i\Delta}$$

GLAUBER-SUDARSHAN P-REPRESENTATION :

$$\rho = \int P(\alpha)\, |\alpha\rangle\langle\alpha|\, d^2\alpha \qquad (a|\alpha\rangle = \alpha|\alpha\rangle)$$

$$\alpha = r e^{i\Phi} \simeq \sqrt{\bar n}\, e^{i\Phi}$$

↳ NEGLECT AMPLITUDE FLUCTUATIONS

DENSITY MATRIX EQ. FOR ρ ⇒ FOKKER-PLANCK EQ. FOR $P(\Phi)$

$$\dot P = \underbrace{-\frac{\partial}{\partial\Phi}(d(\Phi)P)}_{\text{DRIFT}} + \underbrace{\frac{\partial^2}{\partial\Phi^2}(D(\Phi)P)}_{\text{DIFFUSSION}}$$

EQUATION OF MOTION FOR PHASE Φ :

$$\langle\dot\Phi\rangle = \langle d(\Phi)\rangle$$

$$= \Big\langle \frac{-\alpha\Delta\Gamma}{2(\Gamma^2+\Delta^2)}\Big[\underbrace{\rho_{aa} - \rho_{cc}} - \underbrace{2|\rho_{ca}|\,\text{SIN}(\theta_{ca} + 2\Phi + \tfrac{\pi}{2})}_{\text{PHASE LOCKING}}\Big] + \underbrace{\Omega - \nu}_{\text{FREQ. PULLING}} \Big\rangle$$

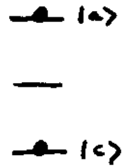
FREQUENCY-PULLING

$$\nu = \Omega + \frac{\alpha\Delta\Gamma}{2(\Gamma^2+\Delta^2)}(\rho_{aa} - \rho_{cc})$$

PHASE LOCKING

$$\theta_{ca} + 2\Phi + \frac{\pi}{2} = 0 \Rightarrow \boxed{\Phi = \frac{\theta_{ac}}{2} - \frac{\pi}{4}}$$

LINEAR GAIN OF THE TWO-PHOTON CEL



$$\langle \dot{n} \rangle = (G - \gamma)\langle n \rangle$$

$$G = \underbrace{\alpha|\mathcal{L}|^2(\rho_{aa} - \rho_{cc})}_{\text{USUAL LINEAR GAIN}} + \underbrace{2\alpha|\mathcal{L}|^2 \frac{|\rho_{ac}\Delta|}{\Gamma}}_{\text{EXTRA TWO-PHOTON CEL GAIN}}$$

$$\langle(\delta\phi)^2\rangle = \frac{1}{8\pi}\left(1 + \frac{\Gamma\rho_{aa}}{|\Delta\rho_{ac}|}\right)$$

WHEN (i) $\alpha|\mathcal{L}|^2(\rho_{aa} - \rho_{cc}) > \gamma$

(ii) $\Gamma\rho_{aa} < |\Delta\rho_{ac}|$

WE HAVE BOTH LINEAR GAIN AND SQUEEZING

FOR OPTIMUM GAIN W.R.T. DETUNING,

$$\frac{\partial G}{\partial \Delta} = 0$$

$$|\rho_{ac}\Delta| = \Gamma\rho_{cc}$$

$\Rightarrow$

$$\boxed{\begin{aligned} &G = \alpha\rho_{aa} \\ &\langle(\delta\phi)^2\rangle = \frac{1}{8\pi}\left(1 + \frac{\rho_{aa}}{1-\rho_{aa}}\right)\end{aligned}}$$

PHASE DIFFUSION COEFFICIENT

$$D(\phi) = \frac{\alpha}{4\pi}|\mathcal{L}|^2\left[\rho_{aa} + \rho_{ac}/\mathcal{L}\,\cos\left(\theta_{ac} - 2\phi + \tan^{-1}(\Delta/\Gamma)\right)\right]$$

SINCE $|\mathcal{L}| < 1$, IT SHOULD BE POSSIBLE TO ENHANCE THE CONTRIBUTION DUE TO COHERENCE ($|\rho_{ac}|$), THUS LEADING TO QUENCHING AND SQUEEZING OF PHASE NOISE.

UNDER THE PHASE LOCKING CONDITION

:— $\left(\phi_0 = \frac{\theta_{ac}}{2} - \frac{\pi}{4}\right)$

$$D(\phi) = \frac{\alpha}{4\pi}|\mathcal{L}|^2\left[\rho_{aa} - \frac{\rho_{ac}\Delta}{\Gamma}\right]$$

TOTAL PHASE NOISE

$$\langle(\delta\phi)^2\rangle = \frac{1}{4\pi} + D(\phi)\left|\frac{\partial d(\phi_0)}{\partial\phi}\right|^{-1}$$

$$= \frac{1}{8\pi}\left(1 + \frac{\Gamma\rho_{aa}}{|\Delta\rho_{ac}|}\right)$$

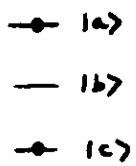
$\Gamma\rho_{aa} < |\Delta\rho_{ac}|$ $\Rightarrow$ SQUEEZING

WHEN $\rho_{aa} \to 0$, 50% SQUEEZING IS OBTAINED.

BUT IS IT COMPATIBLE WITH LASING ???

$$G = \alpha \rho_{aa}$$

$$\langle(\delta\Phi)^2\rangle = \frac{1}{8\kappa}\left(1 + \frac{\rho_{aa}}{1-\rho_{aa}}\right)$$



(a) WHEN $(\rho_{aa} - \rho_{cc}) < 0$ BUT $\rho_{aa} > 0$, LASING IS POSSIBLE (LASING WITHOUT INVERSION)

(b) WHEN $\rho_{aa} = 1/2$, $\langle(\delta\Phi)^2\rangle = \frac{1}{4\kappa}$ AND CEL ACTION IS OBTAINED

(c) FOR $0 < \rho_{aa} < 1/2$, PHASE NOISE IS SQUEEZED BELOW VACUUM LEVEL. MAXIMUM SQUEEZING (50%) IS OBTAINED AT THRESHOLD ($\rho_{aa} = 0$).

(d) SQUEEZING AND GAIN ARE COMPETING MECHANISMS

(e) NO SQUEEZING IS OBTAINED WHEN THERE IS NET POPULATION INVERSION $\rho_{aa} > \rho_{cc}$

OR $\rho_{aa} > 1/2$

Lecture 3

Quantum State Measurement via Emission Spectrum

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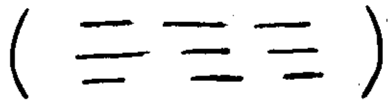
QUANTUM STATE MEASUREMENT OF LIGHT



$$|\Psi_f\rangle = \sum_n w_n |n\rangle$$

DISTRIBUTION FUNCTION : $p(n) = |w_n|^2$

QUANTUM-STATE TOMOGRAPHY

ATOMIC BEAM DEFLECTION FROM STANDING WAVES

TWO-LEVEL ATOMS INTERACTING WITH CAVITY FIELD
(QUANTUM STATE ENDOSCOPY)

MULTI-PORT HOMODYNE DETECTION

⋮

THIS TALK

QUANTUM STATE MEASUREMENT OF RADIATION FIELD FROM SPONTANEOUS EMISSION AND ABSORPTION SPECTRA OF DRIVEN SYSTEMS

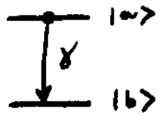
CAN WE DETERMINE THE PHOTON DISTR. FUNCTION AND STATE OF THE FIELD DIRECTLY FROM THE SPECTRA ?

AUTLER-TOWNES SPECTROSCOPY

ELECTROMAGNETICALLY INDUCED TRANSPARENCY

RESONANCE FLUORESCENCE

SPONTANEOUS EMISSION IN TWO-LEVEL ATOM



INTERACTION OF ATOM WITH VACUUM MODES:

$$H = \hbar \sum_k g_k e^{i(\omega_{ab}-\nu_k)t} |a\rangle\langle b| b_k + H.c.$$

$$|\psi(t)\rangle = c_{a,0}(t)|a,0\rangle + \sum_k c_{b,1_k}(t)|b,1_k\rangle$$

$$|\psi(0)\rangle = |a,0\rangle$$

EQS. OF MOTION FOR PROBABILITY AMPLITUDES

$$\dot{c}_{a,0} = -i \sum_k g_k e^{i\delta_k t} c_{b,1_k} \qquad \delta_k = \omega_{ab} - \nu_k$$

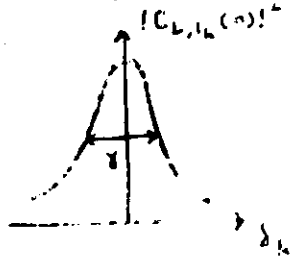
$$\dot{c}_{b,1_k} = -i g_k^* e^{-i\delta_k t} c_{a,0}$$

$$\dot{c}_{a,0} = -i \sum_k g_k e^{i\delta_k t} \left(-i g_k^* \int_0^t dt' e^{-i\delta_k t'} c_{a,0}(t')\right)$$

WEISSKOPF – WIGNER APPROX.

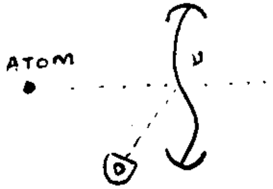
$$= -\frac{\gamma}{2} c_{a,0} \implies c_{a,0}(t) = c_{a,0}(0) e^{-\frac{\gamma}{2}t}$$

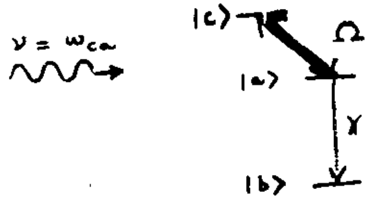
$$c_{b,1_k}(t) = \frac{i g_k^* c_{a,0}(0)}{\gamma/2 + i\delta_k}\left(e^{-(\frac{\gamma}{2}+i\delta_k)t} - 1\right)$$



$$|c_{b,1_k}(\infty)|^2 = \frac{|g_k|^2}{(\gamma/2)^2 + \delta_k^2}$$

LORENTZIAN
CENTERED AT $\omega_k = \omega_{ab}$





HAMILTONIAN

$$H = \underbrace{\frac{\hbar\Omega}{2}|c\rangle\langle a|}_{\text{DRIVING FIELD}} + \hbar\sum_k g_k e^{i\delta_k t}|a\rangle\langle b| b_k + H.c.$$

$$\delta_k = \omega_{ab} - \nu_k$$

$$|\psi(t)\rangle = c_{a,0}|a,0\rangle + c_{c,0}|c,0\rangle + \sum_k c_{b,1_k}|b,1_k\rangle$$

$$|\psi(0)\rangle = |a,0\rangle$$

EQS. FOR PROB. AMPLITUDES (WEISSKOPF-WIGNER APPROX.)

$$\dot{c}_{a,0} = -i\frac{\Omega}{2}c_{c,0} - \frac{\gamma}{2}c_{a,0}$$

$$\dot{c}_{c,0} = -i\frac{\Omega}{2}c_{a,0}$$

$$\dot{c}_{b,1_k} = -i g_k^* e^{-i\delta_k t} c_{a,0}$$

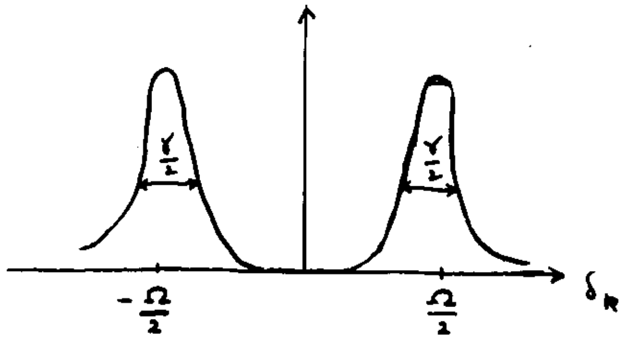
$\Longrightarrow$

$$c_{a,0}(t) = e^{-\gamma t/4}\left[\cosh(\kappa t/2) - \left(\frac{\gamma}{2\kappa}\right)\sinh(\kappa t/2)\right]$$

$$\kappa = \sqrt{\frac{\gamma^2}{4} - \Omega^2}$$

$$|c_{b,1_k}(\infty)|^2 = \frac{|g_k|^2\delta_k^2}{\left(\delta_k^2 - \frac{\Omega^2}{4}\right)^2 + \delta_k^2\gamma^2/4}$$

$$|C_{b,1_k}(\infty)|^2 = \frac{|g_{k_0}|^2 \, \delta_k^2}{\left(\delta_k^2 - \frac{\Omega^2}{4}\right)^2 + \delta_k^2 \gamma^2/4}$$

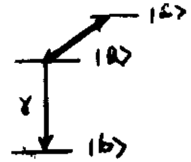


(AUTLER-TOWNES DOUBLET)

FEATURES:

1. HEIGHT IS INDEPENDENT OF Ω
2. PEAKS ARE LOCATED AT $\omega_k = \omega_{ab} \pm \Omega/2$.
3. WIDTH OF THE PEAKS IS $\frac{\gamma}{2}$

DRESSED STATE PICTURE



HAMILTONIAN

$$H = \underbrace{\left(\frac{\hbar\Omega}{2}|c\rangle\langle a| + H.c.\right)}_{H_1} + \underbrace{\left(\hbar\sum_k g_k e^{i\delta_k t}|a\rangle\langle b|\, b_k + H.c.\right)}_{H_2}$$

DEFINE

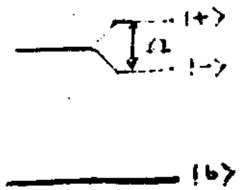
$$\underbrace{\begin{aligned}|+\rangle &= \frac{1}{\sqrt{2}}\left(|c\rangle + |a\rangle\right)\\ |-\rangle &= \frac{1}{\sqrt{2}}\left(|c\rangle - |a\rangle\right)\end{aligned}}_{\text{EIGENSTATES OF } H_1} \Rightarrow \begin{aligned}|c\rangle &= \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)\\ |a\rangle &= \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right)\end{aligned}$$

$$H_1 = \frac{\hbar\Omega}{2}\left(|+\rangle\langle +| - |-\rangle\langle -|\right)$$

$$H_{2I} = e^{-iH_1 t/\hbar}\, H_2 \, e^{iH_1 t/\hbar}$$

$$= \frac{\hbar}{\sqrt{2}}\sum_k g_k\left[e^{i(\delta_k + \frac{\Omega}{2})t}|+\rangle\langle b|\, b_k + e^{i(\delta_k - \frac{\Omega}{2})t}|-\rangle\langle b|\, b_k\right.$$

$$+ H.c.$$



ATOM INITIALLY IN

$$|a\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right)$$

(COHERENT SUPERPOSITION)

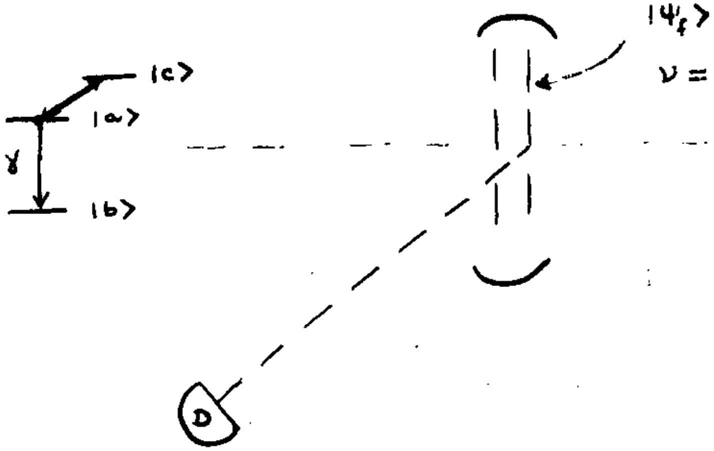
QUANTUM INTERFERENCE $\longrightarrow$ DARK LINE AT $\omega_k = \omega_{ab}$

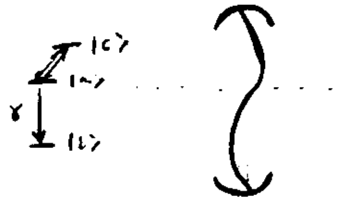
QUANTUM STATE MEASUREMENT

QUANTIZED FIELD

$|\psi_f\rangle = \sum_n W_n |n\rangle$

$\nu = \omega_{ca}$





INITIAL STATE OF THE ATOM-FIELD SYSTEM

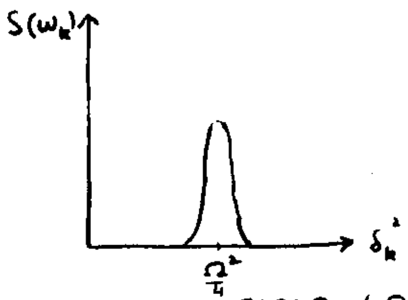
$$|\psi_i\rangle = |a\rangle \otimes \sum_n W_n |n\rangle$$

CAVITY FIELD RESONANT WITH $|c\rangle - |a\rangle$ TRANSITION

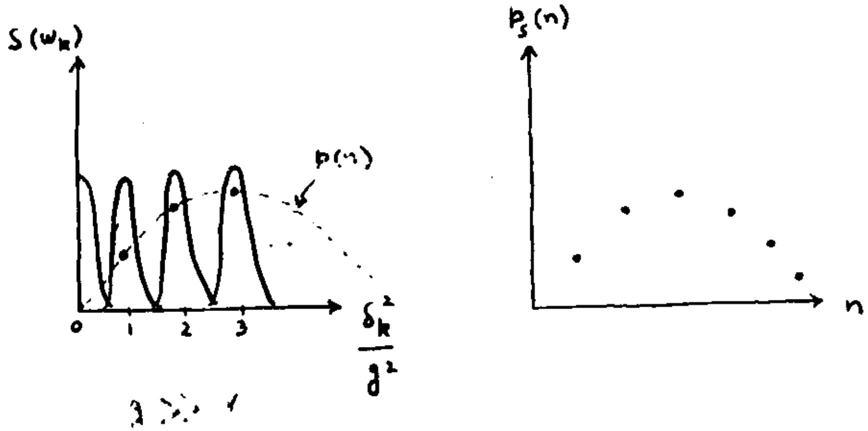
LOOK AT THE SPONTANEOUS EMISSION SPECTRUM FROM $|a\rangle - |b\rangle$

SEMICLASSICALLY

$$S(\omega_k) = I_0 \frac{\delta_k^2 \gamma/2}{(\delta_k^2 - \frac{\Omega^2}{4})^2 + \delta_k^2 \gamma^2/4}$$



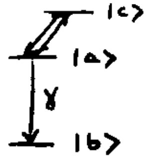
QUANTIZED FIELD $(\Omega^2 = 4g^2 n)$ WITH [illegible] DISTRIBUTED ACCORDING TO $P(n) = |W_n|^2$.



HAMILTONIAN:

$$H = \hbar g \left(|c\rangle\langle a| a + a^\dagger |a\rangle\langle c| \right) + \hbar \sum_k \left(g_k e^{i\delta_k t} |a\rangle\langle b| b_k + H.c. \right)$$

$$\delta_k = \omega_k - \omega_{ab}$$



$$H = \hbar g \left(|c\rangle\langle a| a + a^{\dagger} |a\rangle\langle c| \right) + \hbar \sum_k \left(g_k e^{i\delta_k t} |a\rangle\langle b| b_k + \text{H.c.} \right)$$

$$|\Psi(t)\rangle = \sum_n \Big[C_{a,n,0}(t) |a,n,0\rangle + C_{c,n,0}(t) |c,n,0\rangle + \sum_k C_{b,n,1_k} |b,n,1_k\rangle \Big]$$

$$|\Psi(0)\rangle = \sum_n w_n |a,n,0\rangle$$

$$\dot{C}_{a,n,0} = -\frac{\gamma}{2} C_{a,n,0} - i g \sqrt{n}\, C_{c,n-1,0}$$

$$\dot{C}_{c,n-1,0} = -i g \sqrt{n}\, C_{a,n,0}$$

$$\dot{C}_{b,n,1_k} = -i g_k^{*} e^{-i\delta_k t} C_{a,n,0}$$

$$S(\omega_k) = \sum_{n=0}^{\infty} |C_{b,n,1_k}(\infty)|^2 = \sum_{n=0}^{\infty} p(n) \left[\frac{|g_k|^2 \delta_k^2}{(\delta_k^2 - g^2 n)^2 + \delta_k^2 \gamma^2/4} \right]$$

$$S(\omega_k) = \sum_n p(n) S_n(\omega_k)$$

WHERE

$$S_n(\omega_k) = I_0 \frac{\delta_k^2 \gamma/2}{(g^2 n - \delta_k^2)^2 + \delta_k^2 \gamma^2/4}$$

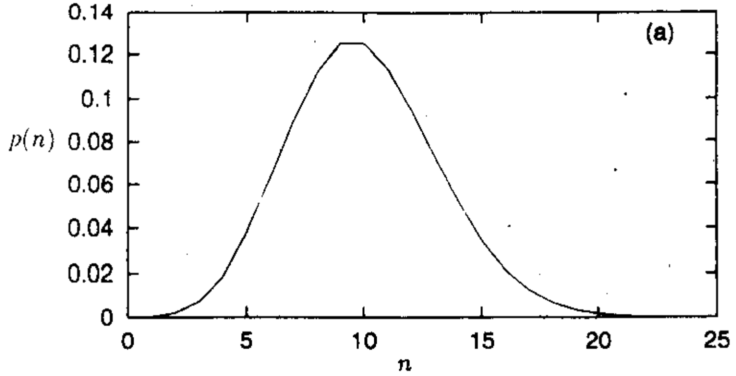
SPONTANEOUS EMISSION SPECTRUM $S_n(\omega_k)$ vs δ_k^2/g^2 HAS PEAKS OF EQUAL HEIGHTS AT $\delta_k^2/g^2 = n$ WITH FWHM OF

$$\Delta_n = \frac{1}{g^2} \sqrt{\frac{\gamma^4}{16} + \gamma^2 g^2 n}$$

$S(\omega_k)$ vs δ_k^2/g^2 WILL MIMIC $p(n)$ PROVIDED

$$g \gg \gamma \quad \text{AND} \quad \frac{n\gamma^2}{g^2} \ll 1 \quad \left(n \ll \frac{g^2}{\gamma^2} \right)$$

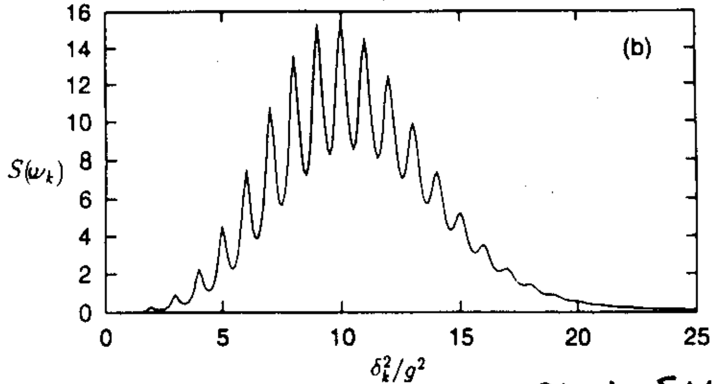
ONLY INTEGRAL VALUES OF δ_k^2/g^2 ARE COUNTED TO REPRODUCE $p(n)$.



COHERENT STATE

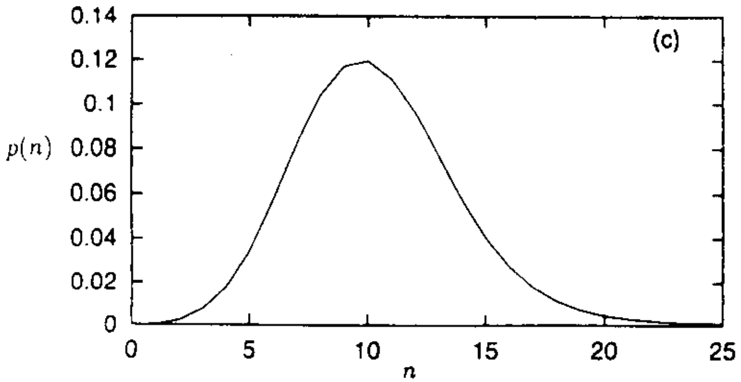
$$p(n) = \frac{e^{-\langle n \rangle} \langle n \rangle^{n}}{n!}$$

$\langle n \rangle = 10$

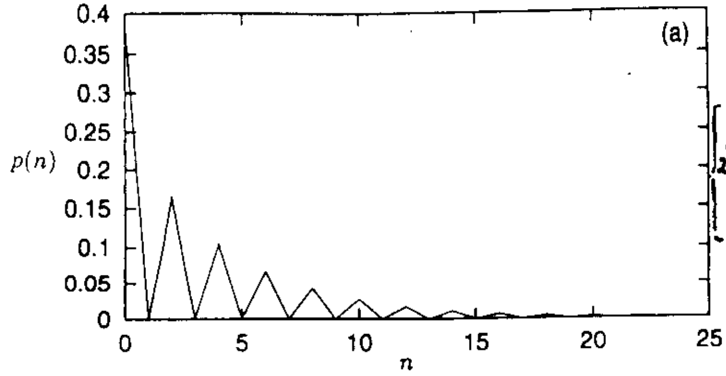


$$\frac{\gamma}{g} = 0.1$$

$$S(\omega_k) = \sum_n p(n) \frac{\delta_k^2 \gamma/2}{(g^2 n - \delta_k^2)^2 + \delta_k^2 \gamma^2/4}$$

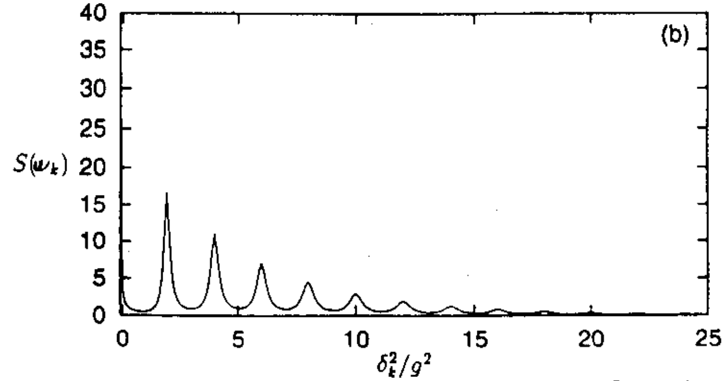


SQUEEZED VACUUM STATE



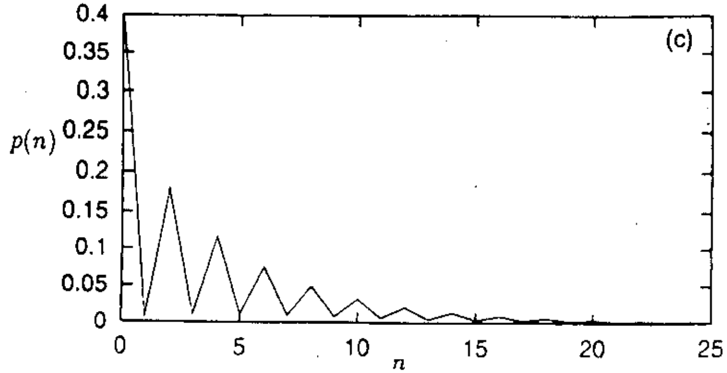
$$p(n) = \begin{cases} \frac{n!}{2^n (\frac{n}{2})!} \frac{(\tanh r)^n}{\operatorname{sech} r}, & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

$r = 1$



$$\frac{\gamma}{g} = 0.1$$

$$S(\omega_k) = \sum_n p(n) \frac{\delta_k^2 \gamma/2}{(g^2 n - \delta_k^2)^2 + \delta_k^2 \gamma^2/4}$$



CONDITIONS:

1. VACUUM RABI FREQUENCY $g \gg \gamma$. ALSO $n \ll \frac{g^2}{\gamma^2}$.

2. DETECTOR RESOLUTION $\Delta\nu \ll \gamma$

3. PASSAGE TIME THROUGH QUANTIZED FIELD:

   $\tau \gg \gamma^{-1}$

4. CAVITY FIELD DECAY TIME $\tau_c \gg \gamma^{-1}$

ADVANTAGES:

1. QUANTUM NONDEMOLITION MEASUREMENT!!!

2. INSENSITIVE TO DETECTOR EFFICIENCY

POTENTIAL PROBLEMS:

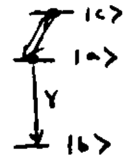
1. DECAY FROM LEVEL $|c\rangle$ TO $|a\rangle$.

QUANTIZED FIELD: $|\psi\rangle = \sum_n W_n |n\rangle$

SO FAR WE CONSIDERED $p(n) = |W_n|^2$

FOR COMPLETE STATE MEASUREMENT, WE NEED W_n

ATOM IS INITIALLY PREPARED IN A COHERENT SUPERPOSITION OF STATES: $|a\rangle$ AND $|c\rangle$



$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|a\rangle + e^{i\varphi}|c\rangle\right) \otimes \sum_n W_n |n\rangle$$

SPECTRUM:

$$S(\omega_k, \varphi) = \frac{1}{2} \sum_n \frac{\delta_k |W_n|^2 + g^2 n |W_{n-1}|^2 - 2g\sqrt{n}\,\delta_k \,\mathrm{Re}(W_n W_{n-1}^* e^{i\varphi})}{(g^2 n - \delta_k^2)^2 + \delta_k^2 \gamma^2/4}$$

WE LIKE TO KNOW $\mathrm{Re}(W_n W_{n-1}^*)$ AND $\mathrm{Im}(W_n W_{n-1}^*)$.

W_n CAN THEN BE DETERMINED (APART FROM A PROPORTIONALITY FACTOR) FROM THE RECURSION RELATION

$$W_n = \frac{a_n}{W_{n-1}^*} \qquad \text{WHERE} \quad a_n = (W_n W_{n-1}^*)$$

(PROVIDED $W_{n-1} \neq 0$)

DETERMINE $S(\omega_k, \varphi)$ FOR $\varphi = 0, \pi/2, \pi, 3\pi/4$

$$R(\omega_k) = \frac{\delta_k}{2g}\left[S(\omega_k, \pi) - S(\omega_k, 0)\right]$$

$$= \sum_n \mathrm{Re}\left(\sqrt{n}\, W_n W_{n-1}^*\right) \frac{\delta_k^2}{(g^2 n - \delta_k^2)^2 + \delta_k^2 \gamma^2/4}$$

$$I(\omega_k) = \frac{\delta_k}{2g}\left[S(\omega_k, \tfrac{3\pi}{2}) - S(\omega_k, \tfrac{\pi}{2})\right]$$

$$= \sum_n \mathrm{Im}\left(\sqrt{n}\, W_n W_{n-1}^*\right) \frac{\delta_k^2}{(g^2 n - \delta_k^2)^2 + \delta_k^2 \gamma^2/4}$$

$R(\omega_k)$, $I(\omega_k)$ YIELD $\sqrt{n}\, W_n W_{n-1}^*$ AND HENCE

$$a_n = W_n W_{n-1}^*$$

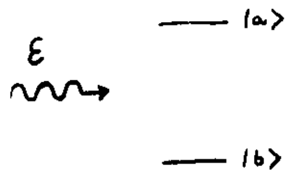
ABSORPTION SPECTRUM IN THREE-LEVEL DRIVEN SYSTEM !!

ELECTROMAGNETICALLY-INDUCED TRANSPARENCY

COMPLEX POLARIZATION

$$P = \varepsilon_0 \chi \mathcal{E} = \wp_{ba} \rho_{ab}$$

χ ↳ SUSCEPTIBILITY



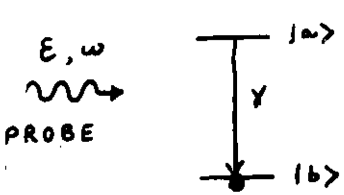
$$\chi = \frac{\wp_{ba} \rho_{ab}}{\varepsilon_0 \mathcal{E}}$$

$$\chi = \chi' + i \chi''$$

$\chi' \rightarrow$ DISPERSION

$\chi'' \rightarrow$ ABSORPTION

TWO-LEVEL SYSTEM



$$H = -\frac{\wp \mathcal{E}}{2} e^{-i\omega t} |a\rangle\langle b| + \text{H.c.}$$

$$\dot{\rho}_{ab} = -(\gamma + i\Delta)\rho_{ab} - \frac{i\wp\mathcal{E}}{2\hbar}(\rho_{aa} - \rho_{bb})$$

$\Delta = \omega_{ab} - \omega$

$$\Longrightarrow \quad \rho_{ab}^{(1)} = -\frac{i\wp\mathcal{E}}{2\hbar}\frac{(\rho_{aa}^{(0)} - \rho_{bb}^{(0)})}{\gamma + i\Delta}$$

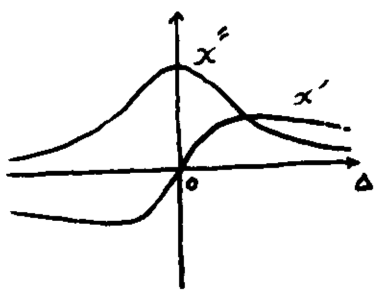
$$= \frac{i\wp\mathcal{E}}{2\hbar}\frac{1}{\gamma + i\Delta} \quad \text{FOR } \rho_{aa}^{(0)} = 0, \ \rho_{bb}^{(0)} = 1$$

LINEAR SUSCEPTIBILITY

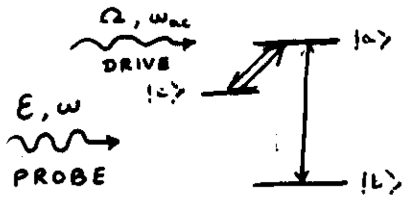
$$\chi = \chi' + i\chi'' = \frac{\wp\rho_{ab}^{(1)}}{\epsilon_0 \mathcal{E}}$$

$$\chi' = \frac{\wp^2}{\epsilon_0\hbar}\frac{\Delta}{\gamma^2 + \Delta^2}$$

$$\chi'' = \frac{\wp^2}{\epsilon_0\hbar}\frac{\gamma}{\gamma^2 + \Delta^2}$$



ABSORPTION SPECTRUM IN DRIVEN THREE-LEVEL SYSTEM:



$$H = -\frac{\wp\mathcal{E}}{2} e^{-i\omega t}|a\rangle\langle b| - \frac{\hbar\Omega}{2} e^{-i\omega_{ac}t}|a\rangle\langle c| + \text{H.c.}$$

$$\dot{\rho}_{ab} = -(\gamma_{ab} + i\Delta)\rho_{ab} - \frac{i\wp\mathcal{E}}{2\hbar}(\rho_{aa}^{(0)} - \rho_{bb}^{(0)}) + \frac{i}{2}\Omega\rho_{cb}$$

$$\dot{\rho}_{cb} = -(\gamma_{cb} + i\Delta)\rho_{cb} - \frac{i\wp\mathcal{E}}{2\hbar}\rho_{ca}^{(0)} + \frac{i}{2}\Omega\rho_{ab}$$

$\Delta = \omega_{ab} - \omega$

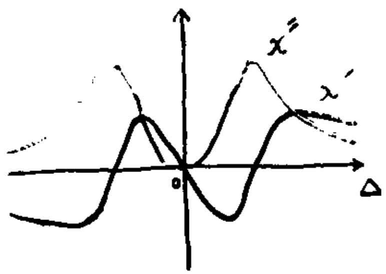
$$\rho_{aa}^{(0)} = \rho_{ca}^{(0)} = 0, \quad \rho_{bb}^{(0)} = 1$$

$$\rho_{ab} = \frac{i\wp\mathcal{E}(\gamma_{cb} + i\Delta)}{2\hbar[(\gamma_{cb} + i\Delta)(\gamma_{ab} + i\Delta) + \Omega^2/4]}$$

$$\gamma_{cb} \simeq 0$$

$$\chi' = \frac{\wp^2}{\epsilon_0\hbar}\frac{\Delta(\Delta^2 - \Omega^2/4)}{(\Delta^2 - \Omega^2/4)^2 + \Delta^2\gamma_{ab}}$$

$$\chi'' = \frac{\wp^2}{\epsilon_0\hbar}\frac{\Delta^2\gamma_{ab}}{(\Delta^2 - \Omega^2/4)^2 + \Delta^2\gamma_{ab}}$$



AT $\Delta = 0$, $\chi' = \chi'' = 0$!!

ELECTROMAGNETICALLY-INDUCED TRANSPARENCY

QUANTIZED DRIVING FIELD

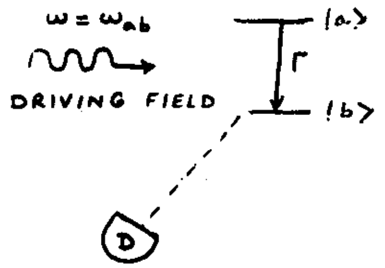
$$\chi' = \frac{\wp^2}{\epsilon_0 \hbar} \sum_n \frac{\Delta\,(\Delta^2 - g^2(n+1))}{\left[(\Delta^2 - g^2(n+1))^2 + \Delta^2 \gamma_{ab}\right]}\; p(n)$$

$$\chi'' = \frac{\wp^2}{\epsilon_0 \hbar} \sum_n \frac{\Delta^2 \gamma_{ab}}{\left[(\Delta^2 - g^2(n+1))^2 + \Delta^2 \gamma_{ab}\right]}\; p(n)$$

ABSORPTION SPECTRUM IS THE SAME AS BEFORE

THUS χ'' VS Δ^2 YIELDS $p(n)$ AS BEFORE !!

RESONANCE FLUORESCENCE FROM TWO-LEVEL ATOM



INTERACTION PICTURE HAMILTONIAN

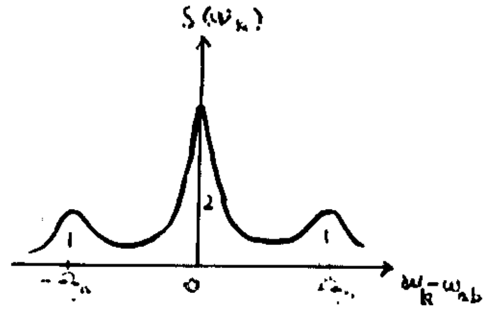
$$H_I = \hbar g \left(|a\rangle\langle b|\, a + a^\dagger\, |b\rangle\langle a| \right)$$

EIGENSTATES

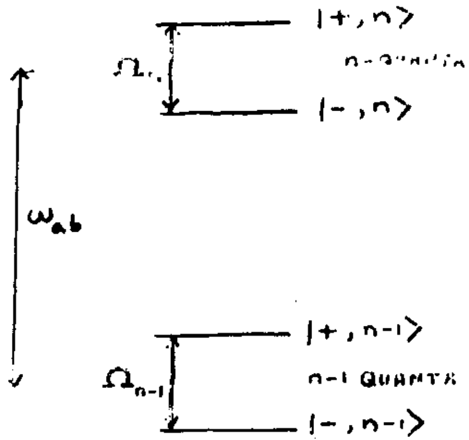
$$|\pm, n\rangle = \frac{1}{\sqrt{2}} \left(|a, n\rangle \pm |b, n+1\rangle \right)$$

EIGENVALUE

$$\lambda_\pm = \pm \hbar \frac{\Omega_n}{2} = \pm \hbar g \sqrt{n+1}$$

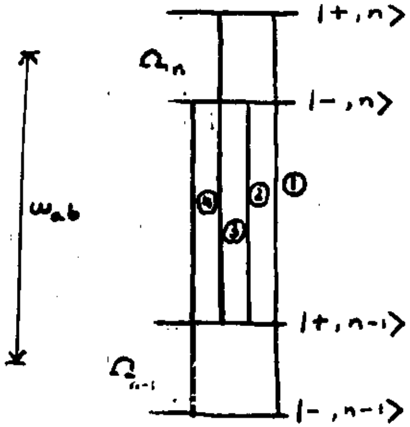


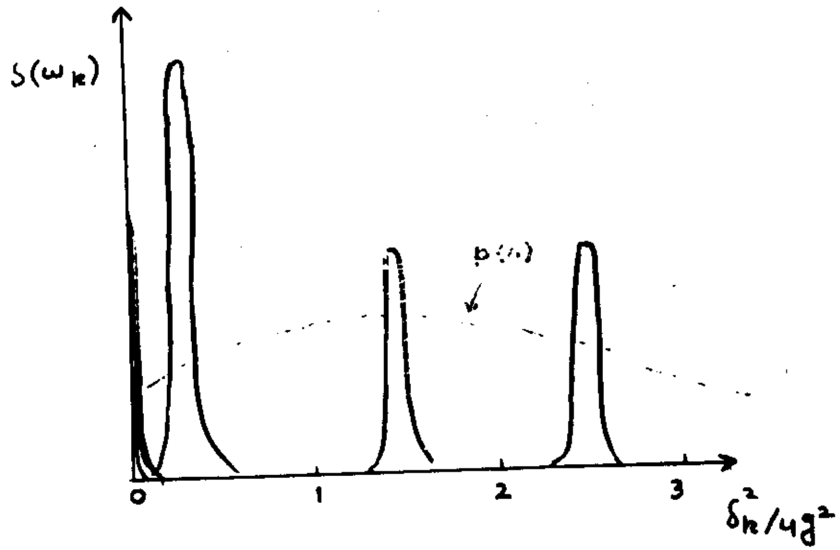
$(\Omega_n \gg \Gamma)$



$$|\pm, n\rangle = \frac{1}{\sqrt{2}}\left\{|a,n\rangle \pm |b,n+1\rangle\right\}$$

$$\Omega_n = 2g\sqrt{n+1}$$





TRANSITION FREQUENCIES:

1. $\nu_k = \omega_{ab} + \left(\frac{\Omega_n + \Omega_{n-1}}{2}\right) = \omega_{ab} + g(\sqrt{n+1} + \sqrt{n})$

2. $\nu_k = \omega_{ab} - \left(\frac{\Omega_n + \Omega_{n-1}}{2}\right) = \omega_{ab} - g(\sqrt{n+1} + \sqrt{n})$

3. $\nu_k = \omega_{ab} + \left(\frac{\Omega_n - \Omega_{n-1}}{2}\right) = \omega_{ab} + g(\sqrt{n+1} - \sqrt{n})$

4. $\nu_k = \omega_{ab} - \left(\frac{\Omega_n - \Omega_{n-1}}{2}\right) = \omega_{ab} - g(\sqrt{n+1} - \sqrt{n})$

PLOT OF $S(\omega_k)$ vs $(\omega_k - \omega_{ab})^2/4g^2$ SHOULD HAVE PEAKS AT

$\frac{1}{4}(\sqrt{n+1} + \sqrt{n})^2$ AND $\frac{1}{4}(\sqrt{n+1} - \sqrt{n})^2$

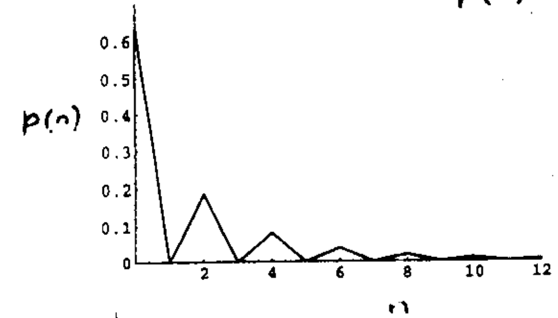
1. PEAK HEIGHTS INDEPENDENT OF n FOR $n > 0$.
2. FOR $n = 0$, BOTH PEAKS ARE AT $1/4$
   (TWICE AS HIGH AS AT OTHER VALUES OF n??)
3. FOR $n > 1$, PEAKS LOCATED AT $n + \frac{1}{2}$ AND 0.

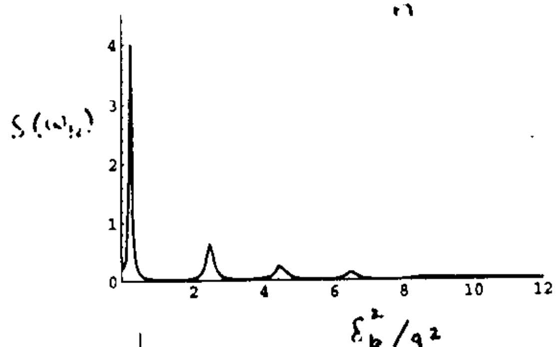
	$n = 0$	$n = 1$	$n = 2$	$n = 3$
$\frac{(\sqrt{n+1} + \sqrt{n})^2}{4}$	0.25	1.46	2.47	3.48
$\frac{(\sqrt{n+1} - \sqrt{n})^2}{4}$	0.25	0.043	0.025	0.018

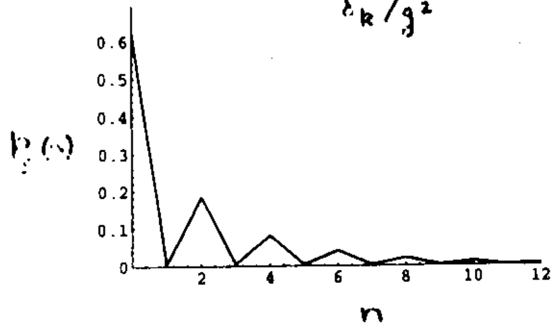
$$b_S(n) = \begin{cases} \dfrac{S\left(\frac{\delta_k^2}{4g^2} = (\sqrt{n+1} + \sqrt{n})^2/4\right)}{\frac{1}{2} S\left(\frac{\delta_k^2}{4g^2} = 0.25\right) + \sum_{n=1}^{\infty} S\left(\frac{\delta_k^2}{4g^2} = (\sqrt{n+1} + \sqrt{n})^2/4\right)} & n \geq 1 \\[2ex] \dfrac{\frac{1}{2} S\left(\frac{\delta_k^2}{4g^2} = 0.25\right)}{\frac{1}{2} S\left(\frac{\delta_k^2}{4g^2} = 0.25\right) + \sum_{n=1}^{\infty} S\left(\frac{\delta_k^2}{4g^2} = (\sqrt{n+1} + \sqrt{n})^2/4\right)} & n = 0 \end{cases}$$

Squeezed Vacuum State

$$p(n) = \begin{cases} \dfrac{n!}{2^n \left(\frac{n}{2}!\right)^2} \dfrac{(\tanh r)^n}{\operatorname{sech} r} & ; n \text{ even} \\ 0 & ; n \text{ odd} \end{cases}$$







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Quantum state measurement via Autler–Townes spectroscopy

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Abstract

We consider the spontaneous emission from a three-level atom in which the two upper levels are driven by a quantized radiation field and the intermediate level decays to the lower level via spontaneous emission. It is shown that the spontaneous emission spectrum has the same characteristics as the photon statistics of the driving field. The proposed scheme can also be used to find the full state vector of the field for pure states.

In recent years, a large class of the states of the radiation field have been studied. Several of them display nonclassical features, such as the squeezed state (for a review, see Ref. [1]) and the so-called Schrödinger cat state (which is a coherent superposition of two coherent states) [2]. These states exhibit interesting features in their quantum statistical properties. For example, they may have oscillatory photon distributions. A number of schemes have been proposed that may be used in the measurement of the field distribution functions. These include methods based on dispersive atom–field coupling in a Ramsey method of separated oscillatory fields [3], atomic beam deflection [4], the quantum state tomography [5], and the conditional measurements on the atoms in a micromaser setup [6]. In this paper, we present a new scheme in which the quantum state of the field can be directly measured from the Autler–Townes spectrum.

In the Autler–Townes scheme, the spontaneous emission spectrum is split in a doublet when the upper or lower level is coupled to another level via a driving field [7]. The appearance of the doublet can be easily understood in the dressed-state picture. Interesting quantum interference effects have been studied in such a system where a dark line is predicted for the upper-level coupling case with no such dark line for the lower-level coupling [8,9]. All studies of the Autler–Townes splitting appear to assume the driving field to be classical.

In this paper we assume the driving field to be quantum statistical which is described by a photon distribution function $p(n)$. We show that, under appropriate conditions, the Autler–Townes spectrum yields the function $p(n)$. We also show that the complete state vector of the field can be determined from this method for pure states of the field.

We consider a system of three-level atoms in which the transition $|c\rangle-|a\rangle$ is driven by a quantum field with photon statistics $p(n)$ and the transition $|a\rangle-|b\rangle$ is coupled to the vacuum modes (see Fig. 1). The interaction Hamiltonian in the dipole and the rotating-wave approximation is given by

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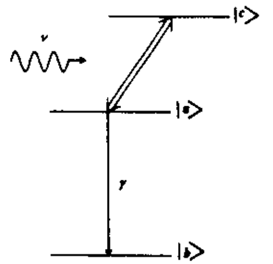

Fig. 1. Level scheme of the atom.

$$H = \hbar g(|c\rangle\langle a|a + |a\rangle\langle c|a^\dagger) + \hbar\sum_k \left(g_k e^{i(\omega_{ab}-\nu_k)t}|a\rangle\langle b|b_k + g_k^* e^{-i(\omega_{ab}-\nu_k)t} b_k^\dagger |b\rangle\langle a|\right), \tag{1}$$

where a and $a^\dagger$ are the annihilation and creation operators of the driving field, g is the corresponding vacuum Rabi frequency, b_k and $b_k^\dagger$ are the annihilation and creation operators of the reservoir modes with wave vector k and frequency $\nu_k = ck$, and g_k is the corresponding coupling constant. Here ω_{ab} is the transition frequency between the levels $|a\rangle$ and $|b\rangle$ and we have assumed the driving field to be at resonance with the $|c\rangle$–$|a\rangle$ transition.

Next we derive the equations of motion for the probability amplitudes. At any time the atom-field wavevector $|\psi(t)\rangle$ can be written as

$$|\psi(t)\rangle = \sum_n \left(C_{a,n,0}(t)|a,n\rangle|0\rangle + C_{c,n,0}(t)|c,n\rangle|0\rangle + \sum_k C_{b,n,1_k}(t)|b,n\rangle|1_k\rangle \right), \tag{2}$$

where $C_{a,n,0}(t)$ and $C_{c,n,0}(t)$ represent the probability amplitudes for the atom to be in levels $|a\rangle$ and $|c\rangle$, respectively, with n photons in the driving field of frequency ν and no photons in the reservoir modes, whereas $C_{b,n,1_k}(t)$ is the probability amplitude for the atom in the level $|b\rangle$ with n photons in the driving field and one photon in mode k. The equations of motion of these amplitudes are given by

$$\dot C_{a,n,0} = -ig\sqrt{n}C_{c,n-1,0} - i\sum_k g_k e^{i\delta_k t} C_{b,n,1_k}, \tag{3}$$

$$\dot C_{c,n-1,0} = -ig\sqrt{n} C_{a,n,0}, \tag{4}$$

$$\dot C_{b,n,1_k} = -ig_k^* e^{-i\delta_k t} C_{a,n,0}, \tag{5}$$

where $\delta_k = \omega_{ab} - \nu_k$. It follows, on formally integrating Eq. (5) and substituting the resulting expression for $C_{b,n,1_k}$ into Eq. (3), that, in the Weisskopf–Wigner approximation, Eq. (3) reduces to

$$\dot C_{a,n,0} = -ig\sqrt{n}C_{c,n-1,0} - \gamma C_{a,n,0}, \tag{6}$$

where 2γ is the spontaneous emission rate from the level $|a\rangle$ to the level $|b\rangle$. Eqs. (3) and (6) form a closed set and can be solved easily.

If we assume the atom to be initially in level $|a\rangle$ and the driving field to be in the state $\sum_n w_n|n\rangle$, the solution for the amplitude $C_{a,n,0}$ is given by

$$C_{a,n,0}(t) = e^{-\gamma t/2}[\cosh(\kappa_n t/2) - (\gamma/\kappa_n)\sinh(\kappa_n t/2)]w_n, \tag{7}$$

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where $\kappa_n = (\gamma^2 - 4g^2 n)^{1/2}$. This expression for $C_{a,n,0}(t)$ can in turn be substituted in Eq. (5), which can then be integrated to yield the desired expression of $C_{b,n,1_k}(t)$. A steady-state value is obtained for $t \gg \gamma^{-1}$. The steady-state expression for the spontaneous emission spectrum is given by (apart from a proportionality factor)

$$S(\nu_k) = \sum_{n=0}^{\infty} |C_{b,n,1_k}(\infty)|^2 = \sum_{n=0}^{\infty} p(n)\frac{|g_k|^2\delta_k^2}{(g^2 n - \delta_k^2)^2 + \delta_k^2\gamma^2}, \tag{8}$$

where $p(n) = |w_n|^2$ is the photon statistics of the driving field. Thus we see that the spontaneous emission spectrum from level $|a\rangle$ to level $|b\rangle$ depends on the photon statistics of the driving field between the levels $|c\rangle$ and $|a\rangle$. This result is valid for arbitrary states of the field and not just for pure states.

We now address the question whether it is possible to gain information about the photon statistics of the driving field from a knowledge of the spectrum. In order to do so, we rewrite Eq. (8) in the form

$$S(\nu_k) = \sum_{n=0}^{\infty} p(n) S_n(\nu_k), \tag{9}$$

where

$$S_n(\nu_k) = \frac{|g_{k_0}|^2\delta_k^2}{(g^2 n - \delta_k^2)^2 + \delta_k^2\gamma^2}. \tag{10}$$

Here we have replaced g_k by its value at $k_0 = \omega_{ab}$ which is a reasonable approximation in the region of interest. Now $S_n(\nu_k)$ is the spontaneous emission spectrum associated with the excitation n which is zero at $\nu_k = \omega_{ab}$ unless $n = 0$ when it is a Lorentzian of width 2γ. The function $S_n(\nu_k)$ has double peaks (the well-known Autler–Townes doublet) located at $\delta_k = \pm g\sqrt{n}$. The height of these peaks is the same and is given by $|g_{k_0}|^2/\gamma^2$. An important and interesting fact is that the height of the peaks is independent of the excitation number n. For a plot of $S_n(\nu_k)$ versus δ_k^2, there is only one peak located at $g^2 n$ whose full width at half maximum is

$$\Delta_n = \gamma^2 + \sqrt{\gamma^4 + 4\gamma^2 g^2 n}. \tag{11}$$

The complete spontaneous emission spectrum consists of contributions from all the photon excitations in the photon distribution function $p(n)$ in Eq. (9). It is clear that, for the decay rate γ much less than the vacuum Rabi frequency g of the driving field, the spontaneous emission spectrum ($S(\nu_k)$ versus δ_k^2) will mimic the photon distribution function $p(n)$. In the following, we consider some examples of the photon statistics and see how they can be recovered from the corresponding spontaneous emission spectra.

In Fig. 2a, we present the photon distribution function for a coherent state, namely

$$p(n) = \frac{e^{-\bar n}\bar n^n}{n!}, \tag{12}$$

with the mean number of photons $\bar n = 10$. It may be seen from Fig. 2b that the spontaneous emission spectrum $S(\nu_k)$ has an oscillatory behavior with peaks occurring at $\delta_k^2 = ng^2$. These oscillations are manifestations of the spectra $S_n(\nu_k)$ corresponding to the photon excitations n. If we want to recover the photon distribution function $p(n)$ from the spectrum given in Fig. 2b, the only meaningful values of δ_k^2/g^2 are the integral values as is evident from Eq. (8). On retaining the contributions only from these values and properly normalizing the resulting distribution, we obtain Fig. 2c, which reproduces the Poissonian photon distribution of Fig. 2a to a good degree.

As another example, we consider the driving field to be a squeezed vacuum state whose photon distribution function is given by [10]

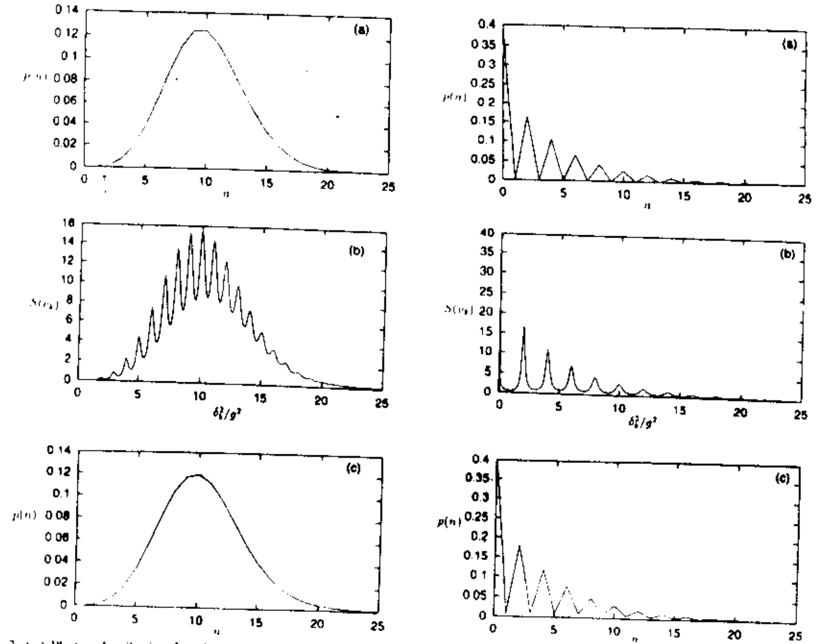

Fig. 2. (a) Photon distribution function $p(n)$ for a coherent state with $\bar{n} = 10$. (b) The spontaneous emission spectrum $S(\nu_k)$ (in arbitrary units) versus δ_k^2/g^2 for $\gamma/g = 0.1$. (c) The photon distribution function recovered from (b).

Fig. 3. (a) Photon distribution function $p(n)$ for a squeezed coherent state with $r = 1.6$. (b), (c) As in Fig. 2.

$$p(n) = (\cosh r)^{-1} \frac{n!}{[(\frac{1}{2}n)!]^2} (\tfrac{1}{2}\tanh r)^n, \qquad \text{when } n \text{ is even},$$
$$= 0, \qquad \text{when } n \text{ is odd}, \tag{13}$$

where r is the squeezing parameter. This photon distribution is highly oscillatory as shown in Fig. 3a. The corresponding spontaneous spectrum and the recovered photon distribution are shown in Figs. 3b and 3c, respectively. The nonzero values at odd n are due simply to the contributions from the neighboring values of n.

We also note that the usual results for the Autler-Townes spectrum are obtained for an initial Fock state, for which there are no oscillations in the spectrum. This is surprising in view of the fact that the Fock state is a highly nonclassical state.

So far we have shown how the diagonal elements of the field density matrix can be determined from the spontaneous emission spectrum when the atoms in the level $|a\rangle$ interact with the field. However, a complete determination of the field state requires, in addition to the diagonal density matrix elements, the off-diagonal

elements as well. In particular, if the state of the field is given by $\sum_n w_n|n\rangle$ then we need to find the amplitudes w_n for all values of n. We now show that this can also be done if the atoms are prepared in a coherent superposition of states $|a\rangle$ and $|c\rangle$. We follow a procedure similar to the one adopted in Ref. [6].

For the atom initially in a coherent superposition of states

$$|\psi_A(0)\rangle = \frac{1}{\sqrt{2}}(|a\rangle + e^{i\phi}|c\rangle), \tag{14}$$

where ϕ is a relative phase, Eqs. (4) and (6) yield

$$C_{a,n,0}(t) = \frac{e^{-\gamma t/2}}{\sqrt{2}}\{[\cosh(\kappa_n t/2) + (\gamma/\kappa_n)\sinh(\kappa_n t/2)]w_n + (ig\sqrt{n}/\kappa_n)\sinh(\kappa_n t/2)w_{n-1}\}. \tag{15}$$

The corresponding expression for the spectrum $S(\nu_k,\phi)$ for the choice of the coherence angle ϕ is

$$S(\nu_k,\phi) = \tfrac{1}{2}\sum_n |g_{k_0}|^2 \frac{\delta_k^2|w_n|^2 + g^2 n|w_{n-1}|^2 - 2g\sqrt{n}\delta_k \operatorname{Re}(w_n w_{n-1}^* e^{-i\phi})}{(g^2 n - \delta_k^2)^2 + \delta_k^2\gamma^2}. \tag{16}$$

The spectrum now depends not only on the diagonal elements of the density matrix for the field but also on the off-diagonal elements $w_n w_{n-1}^*$ as well. The real and imaginary values of $w_n w_{n-1}^*$ can then be obtained by determining the spectrum for four choices of the phase ϕ, namely $\phi = 0, \pi/2, \pi$, and $3\pi/4$ and forming the combinations

$$R(\nu_k) = \frac{\delta_k}{2g}[S(\nu_k,\pi) - S(\nu_k,0)] = \sum_n \operatorname{Re}(\sqrt{n}w_n w_{n-1}^*)\frac{|g_{k_0}|^2\delta_k^2}{(g^2 n - \delta_k^2)^2 + \delta_k^2\gamma^2}, \tag{17}$$

$$I(\nu_k) = \frac{\delta_k}{2g}[S(\nu_k,3\pi/2) - S(\nu_k,\pi/2)] = \sum_n \operatorname{Im}(\sqrt{n}w_n w_{n-1}^*)\frac{|g_{k_0}|^2\delta_k^2}{(g^2 n - \delta_k^2)^2 + \delta_k^2\gamma^2}. \tag{18}$$

Thus a determination of $R(\nu_k)$ and $I(\nu_k)$ yields the values of $\sqrt{n}w_n w_{n-1}^*$, and hence $a_n = w_n w_{n-1}^*$, in the same way as before.

The probability amplitudes w_n can now be determined (apart from an arbitrary and uninteresting phase factor) from the recursion relation

$$w_n = \frac{a_n}{w_{n-1}^*}. \tag{19}$$

The advantage of the present approach over the method presented in Ref. [6] is that the amplitudes w_n can be directly determined from the spectral information and no numerical inversion of relations (17) and (18) is required. This is particularly useful when the field has a large mean number of photons.

In summary, we have shown that the Autler-Townes spectrum can be used to recover the photon distribution function of the driving field. It may also be used to determine the complete reduced density matrix of the field when the field is in a pure state. The proposed scheme can be used in a configuration where the driving field, the atomic beam, and the detector are mutually perpendicular. This scheme can also be used to probe the field inside a cavity. This would, however, require a number of preparations due to the quantum demolition character of the measurements.

There are a number of potential noise sources in this scheme. In our model we have neglected the spontaneous emission from the level $|c\rangle$ to the level $|a\rangle$ which can substantially alter the results. However, it can be shown that the results presented in this paper remain essentially unaffected if the decay rate from level $|c\rangle$ to level $|a\rangle$ is much smaller than the vacuum Rabi frequency g between these levels. This condition is experimentally achievable as has been shown in recent experiments on the realization of a "one-dimensional" atom [11]

Another important consideration is the finite detector bandwidth $\Delta\nu$ which should be much less than the atomic decay rate γ.

The scheme presented in this paper is one of a large class of systems where the emission or probe absorption characteristics may carry interesting quantum statistical properties of the pump or driving fields. These system are experimentally accessible and a detailed study is required to identify the optimal systems and their potential to extract information regarding not only the diagonal but also the off-diagonal elements of the density matrix of the field.

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