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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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SMR/989 - 15

"Course on Shallow Water and Shelf Sea Dynamics"
7 - 25 April 1997

"Plume Dynamics"

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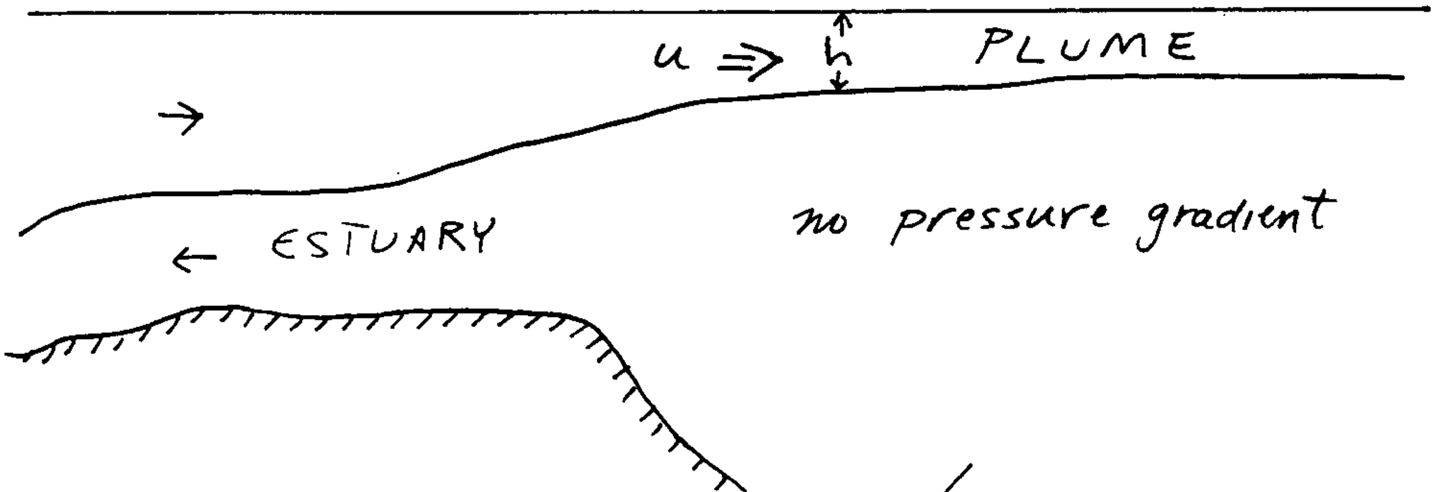
Woods Hole, MA
USA

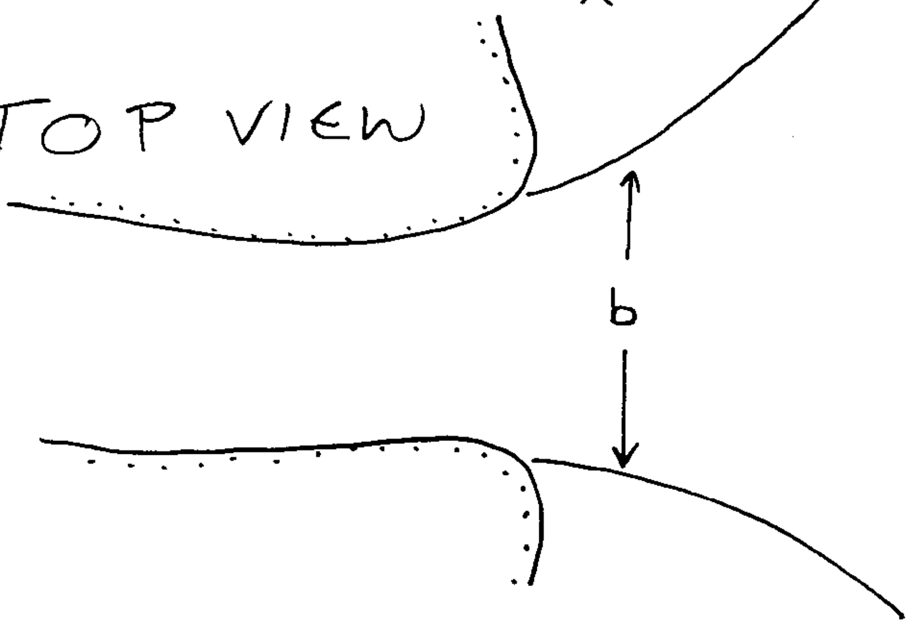
Please note: These are preliminary notes intended for internal distribution only.

PLUME DYNAMICS

SIDE VIEW



TOP VIEW



Plume mass and momentum conservation
(assume no entrainment or friction for now)

MASS: $uhb = Q = \text{constant}$ (1)

Momentum, upper layer: $u\frac{\partial u}{\partial x} + g\frac{\partial \eta}{\partial x} = 0$ (2)

lower layer: $g\frac{\partial \eta}{\partial x} = g'\frac{\partial h}{\partial x}$ (3)

where $g' = \frac{\rho_2 - \rho_1}{\rho_2}$, η is sea level anomaly

(Assuming steady-state in small region near mouth where <u>advection</u> <u>dominates</u>)

Substitute (1) and (3) in (2),

$$-\frac{Q^2}{h^2 b^2}\left(\frac{1}{h}\frac{\partial h}{\partial x} + \frac{1}{b}\frac{\partial b}{\partial x}\right) + g'\frac{\partial h}{\partial x} = 0$$

Define Froude # $F = \frac{u}{(g'h)^{1/2}} = \frac{Q}{g'^{1/2} b^{1/2} h^{3/2}}$

and re-arrange:

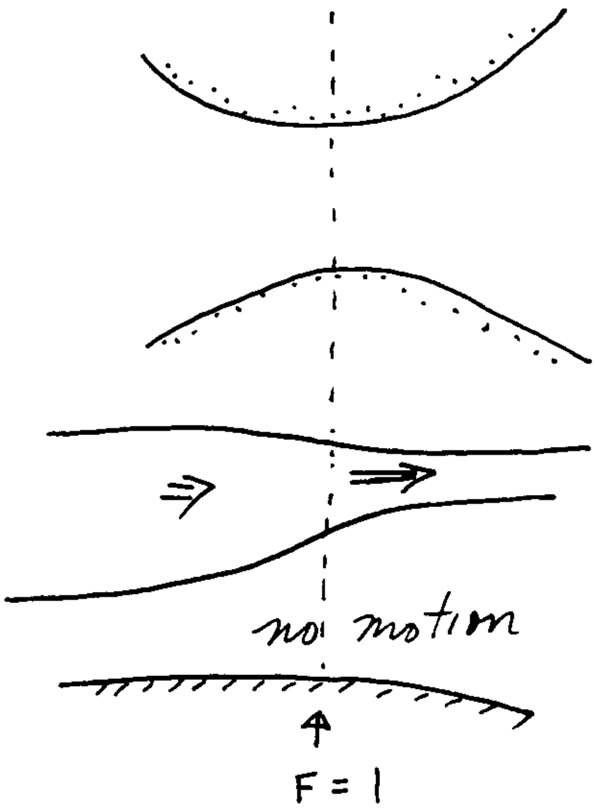
$$\left(1 - F^2\right)\frac{\partial h}{\partial x} = F^2 \frac{h}{b}\frac{\partial b}{\partial x}$$

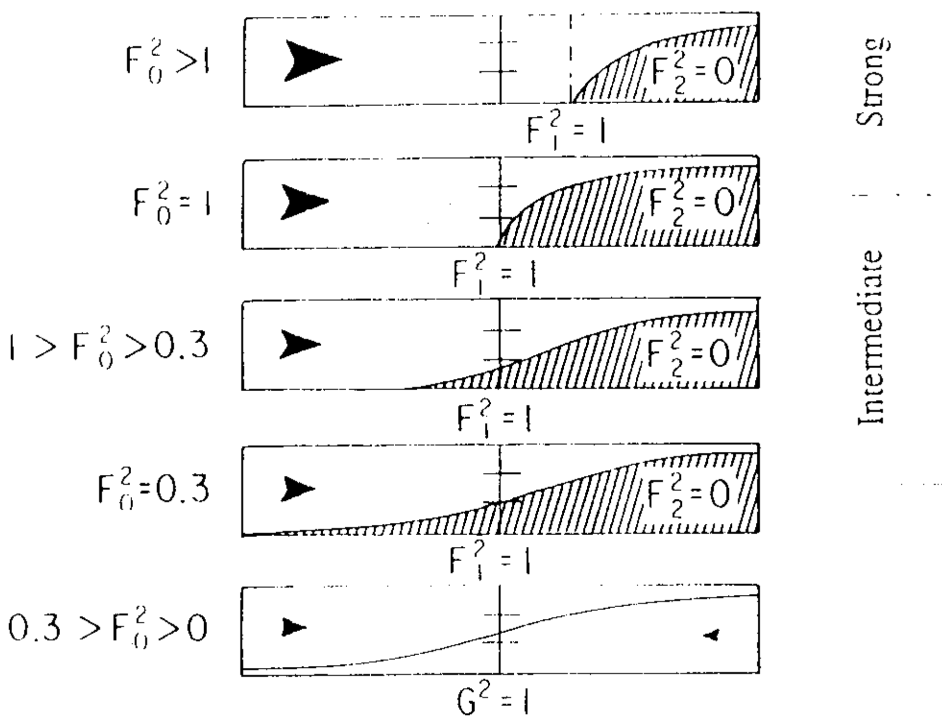
For subcritical flow $(F^2 < 1)$
an increase in width $\left(\frac{\partial b}{\partial x} > 0\right)$ cause

an increase in depth $(\partial h/\partial x > 0)$

But in <u>supercritical</u> <u>flow</u>, a lateral spreading plume $(\partial b/\partial x > 0)$ gets thinner $(\partial h/\partial x < 0)$

Hydraulic control: $F = 1$

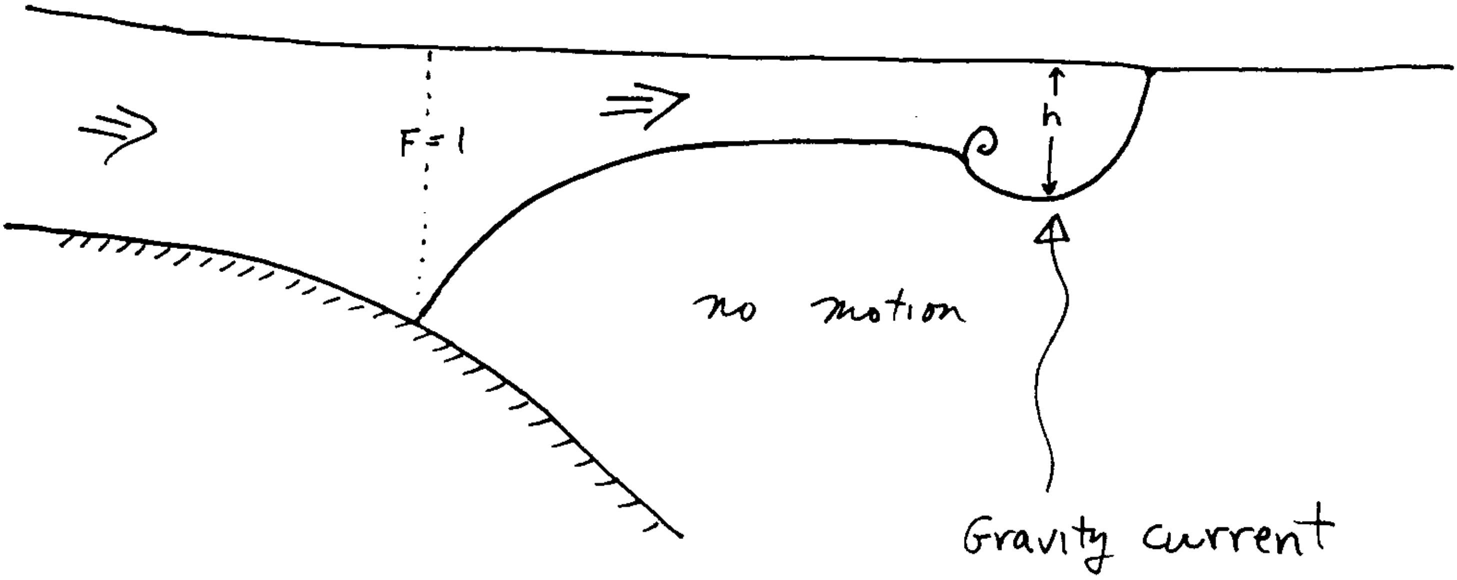




Transient Plumes, near-field (after a flood, for example)

$x \ll \frac{(g'h_0)^{1/2}}{f}$

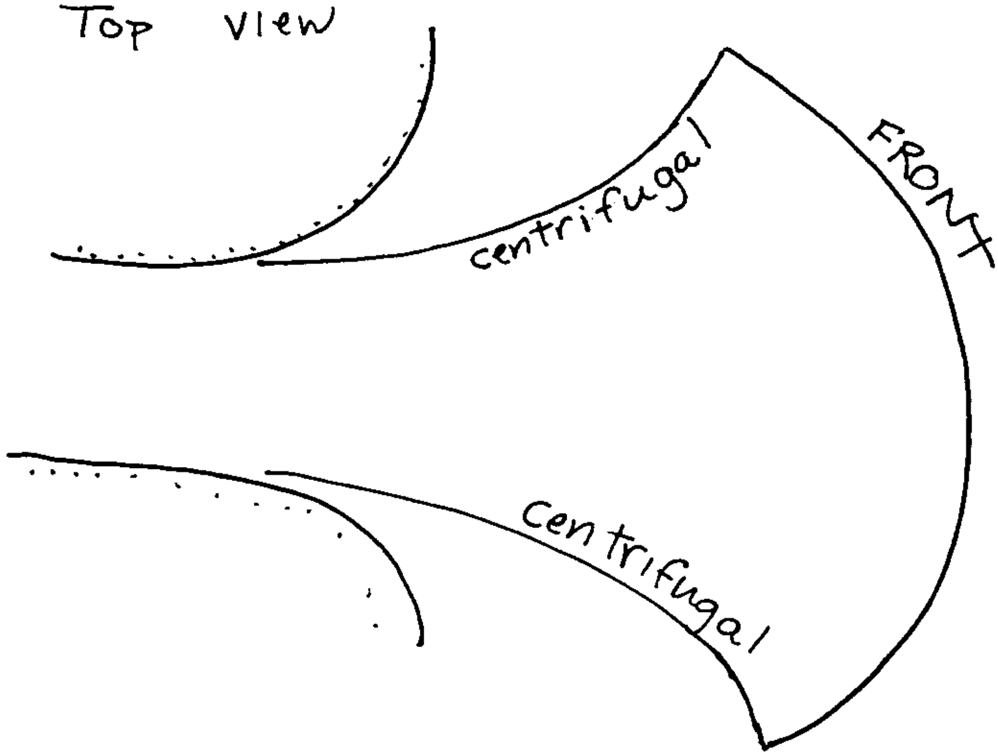
Side view



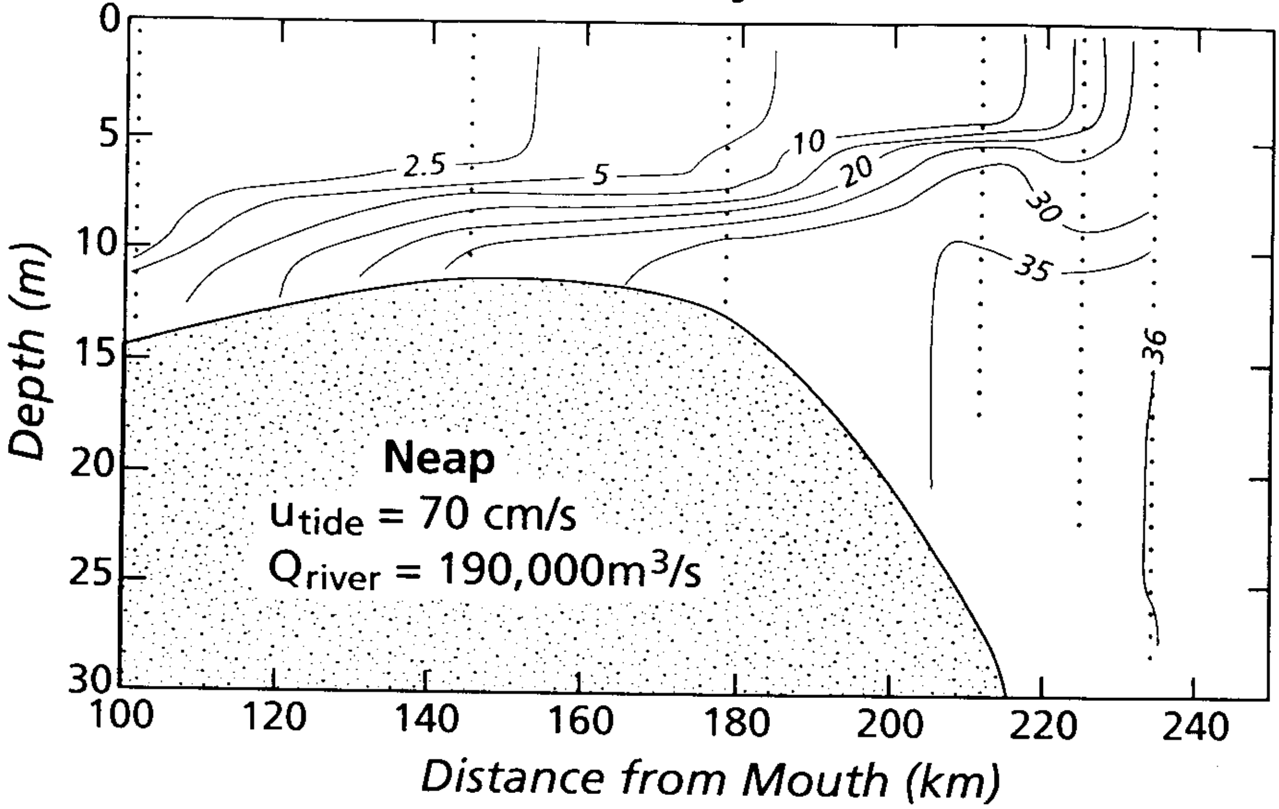
$c = \sqrt{2g'h}$

Benjamin, 1968

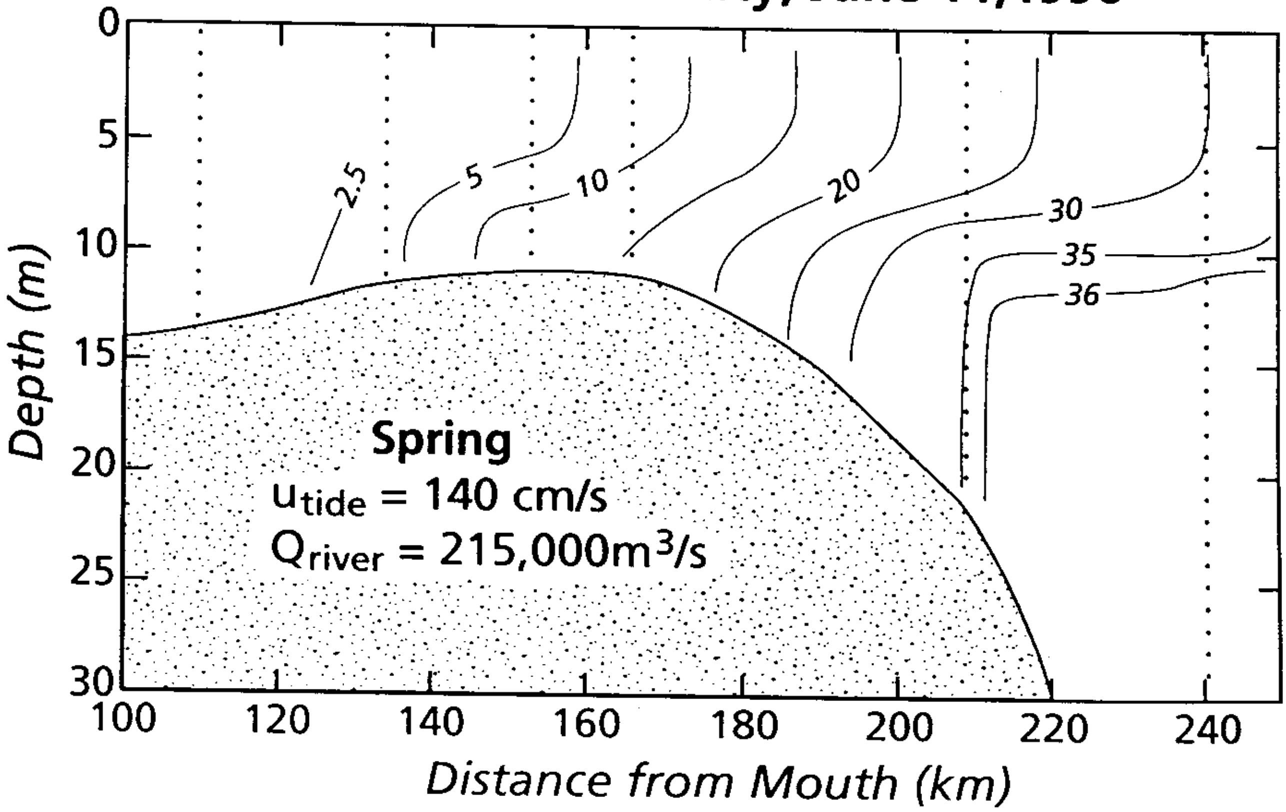
Top view



River Mouth Salinity, March 20,1990



River Mouth Salinity, June 11,1990



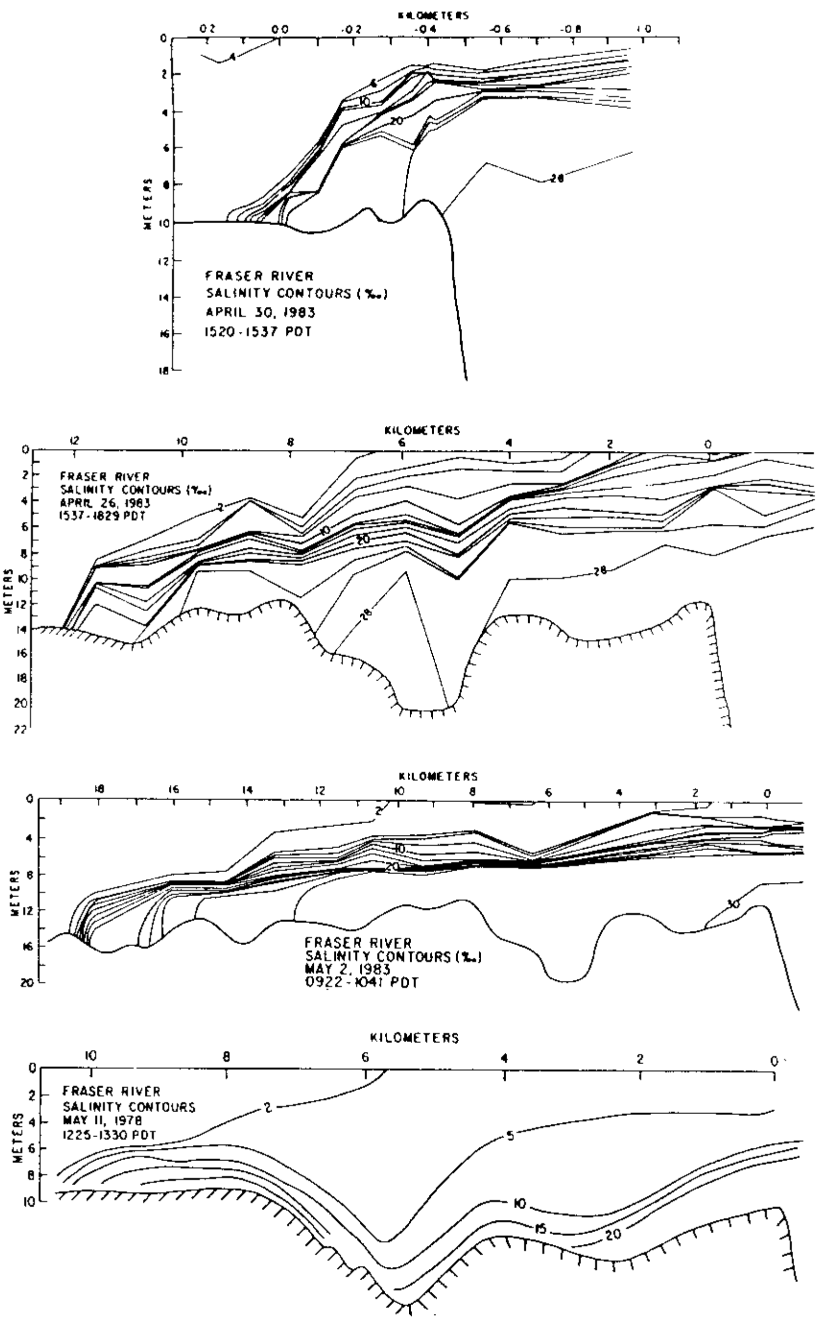
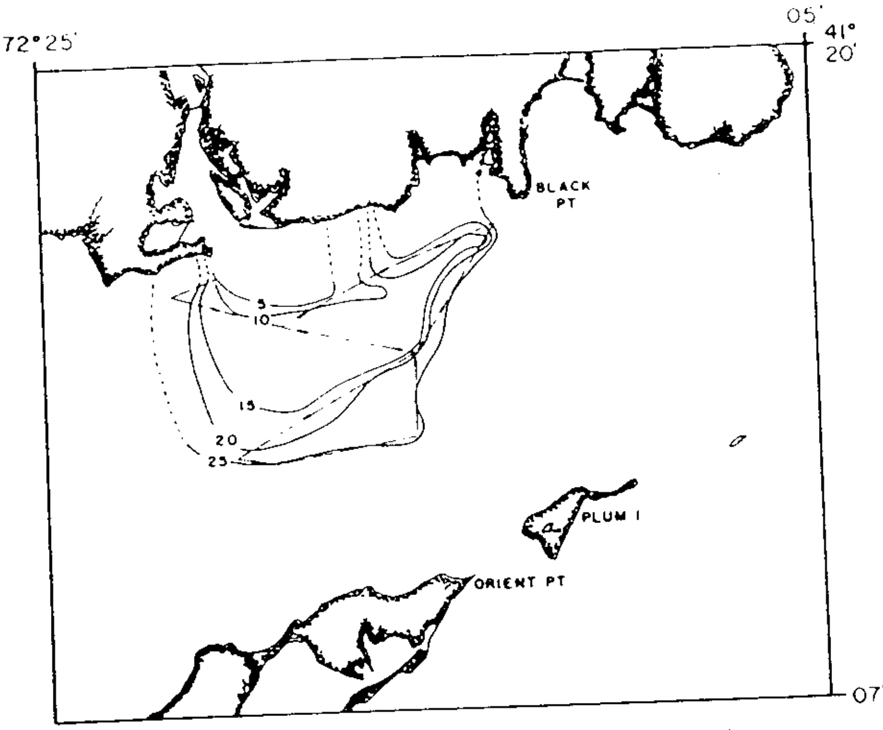


FIG. 3. Salinity sections in the Fraser estuary during different phases of the tide. (a) End of ebb. Salinity intrusion has been flushed entirely out of the estuary, with a front at the mouth. (b) Mid-flood. Salt wedge is advancing up the estuary at a speed of 0.7 m s^{-1}. (c) End of flood. Maximum intrusion distance, with a temporarily arrested condition. (d) Mid-ebb. The salt wedge has collapsed, and the high-salinity water is confined to a thin layer near the bottom.

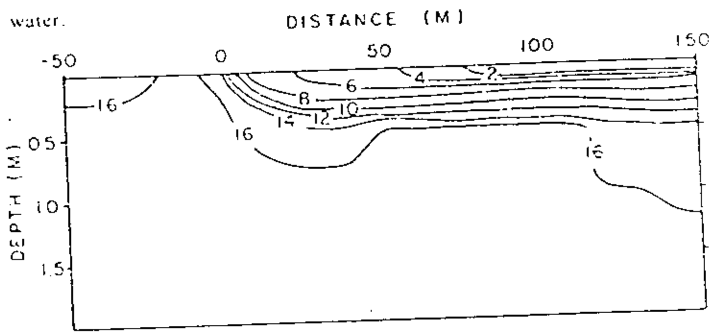
crossing the front. This speed is consistent with local velocity measurements within the intrusion, which averaged 0.7 m s^{-1} between the bottom and midpycnocline. This compares with a depth-mean inflow due to the combined influence of the tides and river runoff of roughly 30 cm s^{-1}.

A view of the spatial structure of the advancing front was obtained with the echo sounder while steaming

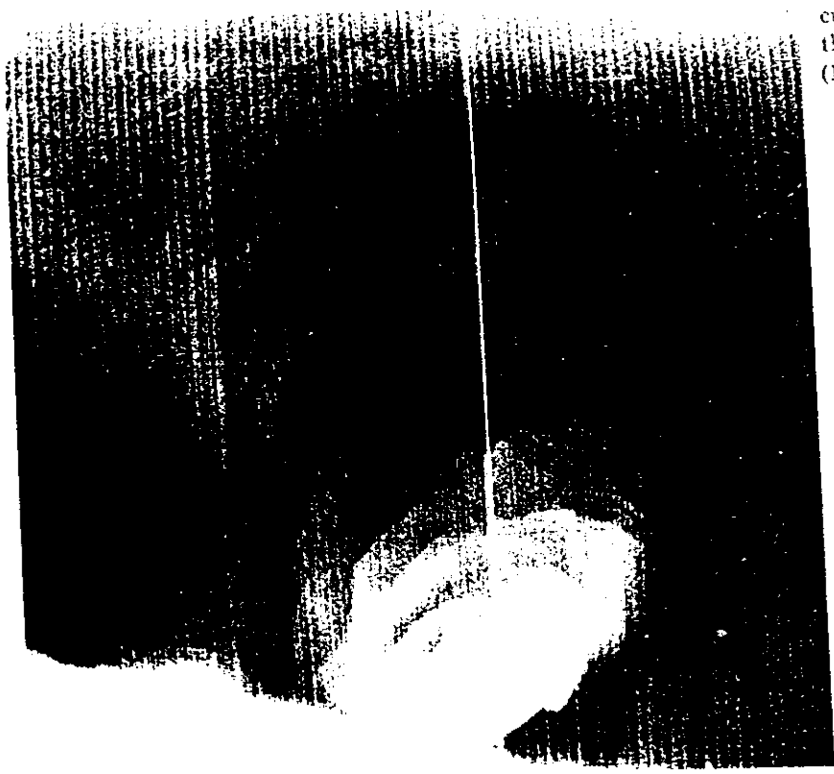
GARVINE and MONK, 1974



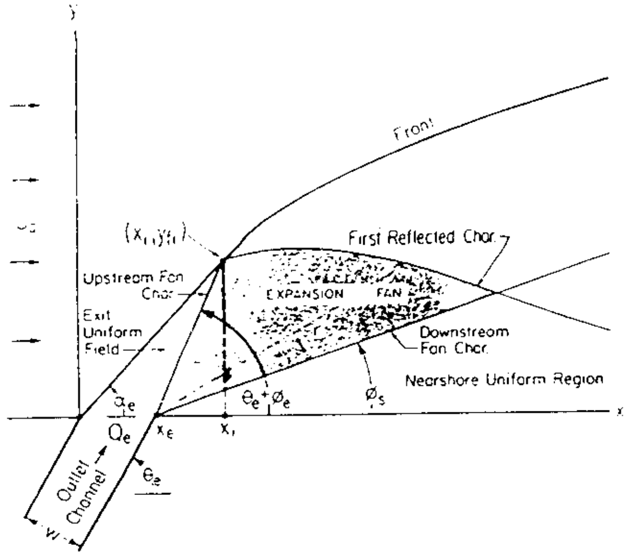
Isohalines at 0.5-m depth on March 30, 1973, at low slack water.

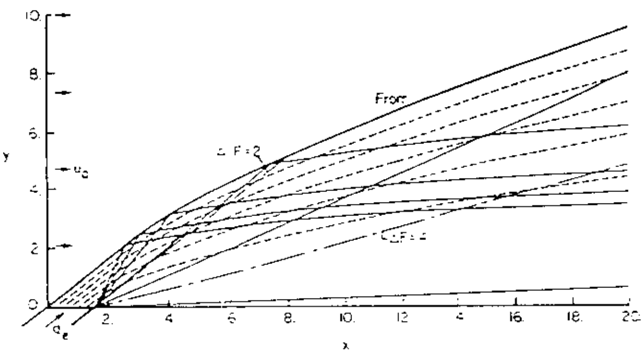


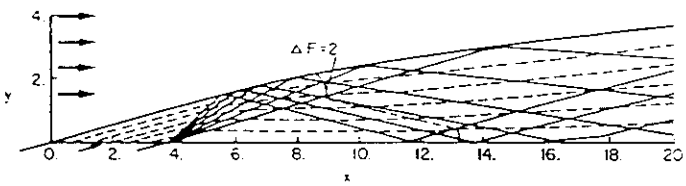
Density section normal to the front of the Connecticut River Plume. The origin of coordinates marks the location of the observed surface color boundary. From Garvine and Monk (1974).

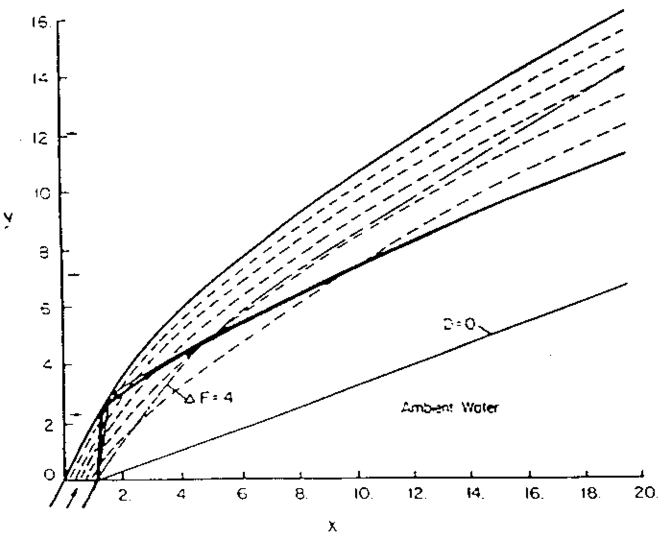


Garvine, 1982










Froude number
Bernoulli
km along coast
km from shore
30
25
20
15
10
5
0
−5
−10
0
10
5
0
2
4
2
4

What about friction?

Gradient Richardson Number

$$Ri = \frac{-g/\rho \; \partial \rho / \partial z}{(\partial u / \partial z)^2}$$

over a layer, $Ri = \dfrac{g' h}{(\Delta u)^2}$

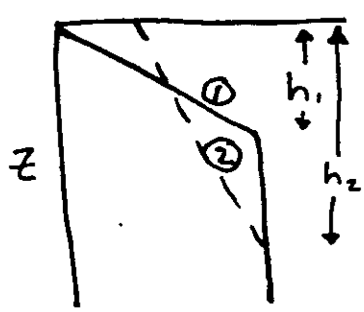
For motionless lower layer and linear shear

$$Ri \approx \frac{4}{F^2} \qquad \text{where } F = \frac{\bar{u}}{(g'h)^{1/2}}$$

Necessary condition for mixing:

$$Ri < 0.25$$

For $F \gg 1$, mixing intensifies

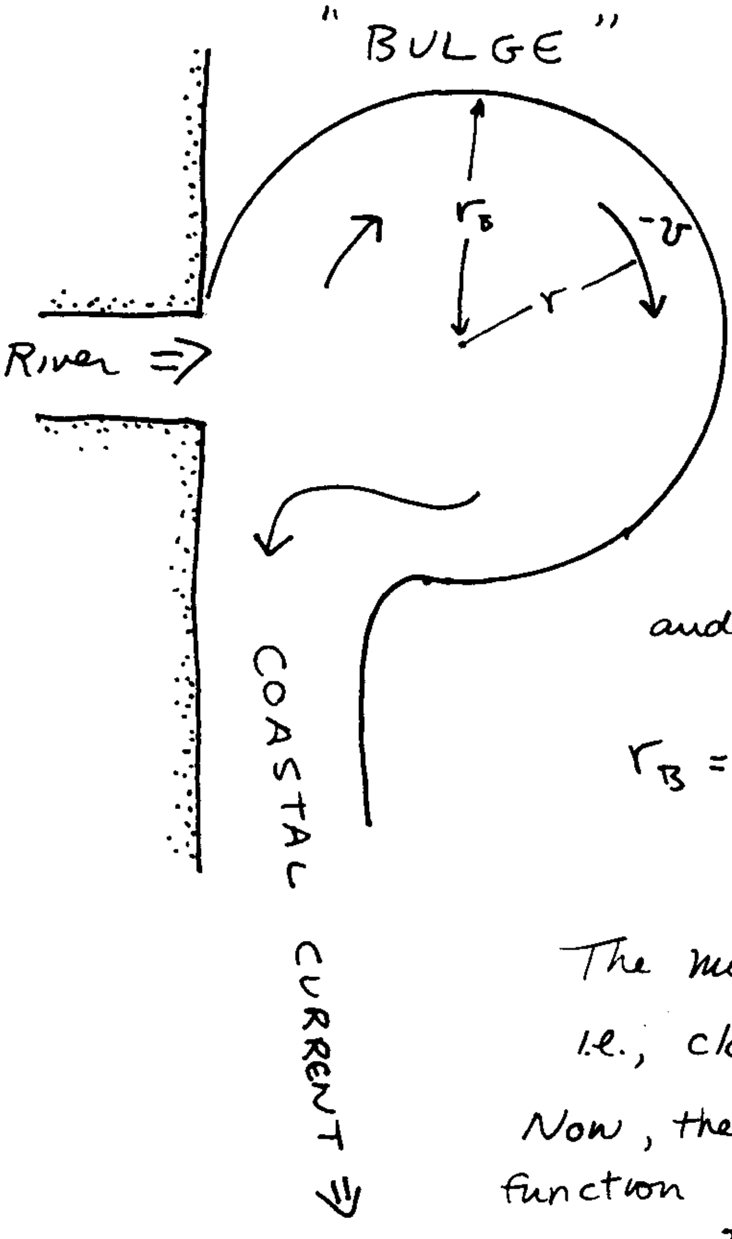


Note that $g'h = \frac{1}{\rho_0} \int \rho g \, dz$

does not change due to vertical mixing, but u decreases

Effects of Earth's rotation

Cyclostrophic balance



$$-fv - \frac{v^2}{r} = -g' \frac{\partial h}{\partial r}$$

Approximate slope in plume thickness by

$$\frac{\partial h}{\partial r} = \frac{-h_o}{r_B}$$

NOTE: I haven't worked out global solution!

and solve for r_B

$$r_B = \frac{-\left(g' h_o + v_B^2\right)}{f v_B}$$

The minus sign indicates $v_B < 0$, i.e., clockwise flow.

Now, the trick ~ use Bernoulli function to get v_B

$$B = \frac{1}{2} v_o^2 + g' h_o = \frac{1}{2} v_B^2$$

so $v_B = -\left(2g' h_o + v_o^2\right)^{1/2}$

substituting,

$$r_B = \frac{3g'h_o + v_0^2}{f(2g'h_o + v_o^2)^{1/2}}$$

$$= L_o \frac{(3 + F_o^2)}{(2 + F_o^2)^{1/2}}$$

where $L_o = \frac{(g'h_o)^{1/2}}{f}$ Deformation radius

$F_o = \frac{u_o}{(g'h_o)^{1/2}}$ Froude # at mouth

W = width of bulge = $2r_B$

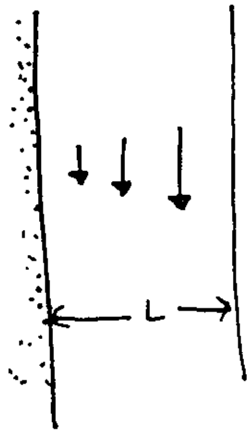
for small F_o, $W = 3\sqrt{2}\, L_o$

Note – when the radial variation of h is accounted for,

$$W = 4\sqrt{2}\, L_o$$

The coastal current

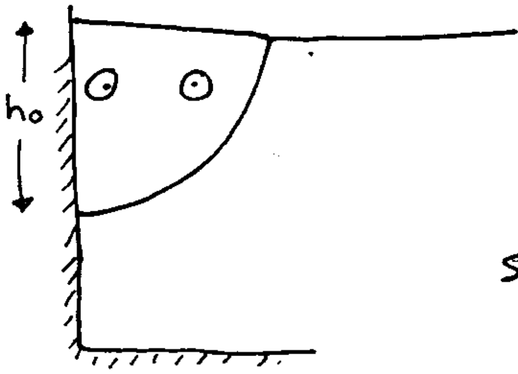
$$-fv = -g'\frac{\partial h}{\partial x}$$



Transport $Q = \int_0^L v h \, dx$

$$= \int \frac{g' h \, dh}{f}$$

$$= -\frac{1}{2}\frac{g' h_o^2}{f}$$



Zero potential vorticity solution: $\frac{\partial v}{\partial x} = -f$

so $v = -fx$

and $h = h_o\left(1 - \frac{x^2}{2L_o^2}\right)$

where $L_o = \left(\frac{g'h_o}{f}\right)^{1/2}$

Note $L = \sqrt{2}L_o$

$$\text{Plume \#} = \frac{Q_{RIVER}}{\frac{1}{2}g'h_o^2/f} = \frac{Q_{RIVER}}{Q_{\text{coastal current}}}$$

(14)

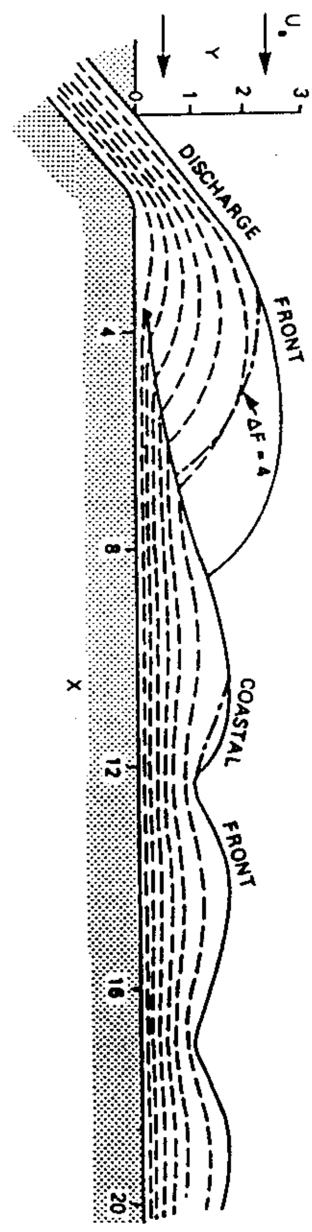


FIG. 6. Plume map for the standard case. Dash-dot line indicates where $\Delta F = 4$, solid circle where coastal front is first fitted computationally.

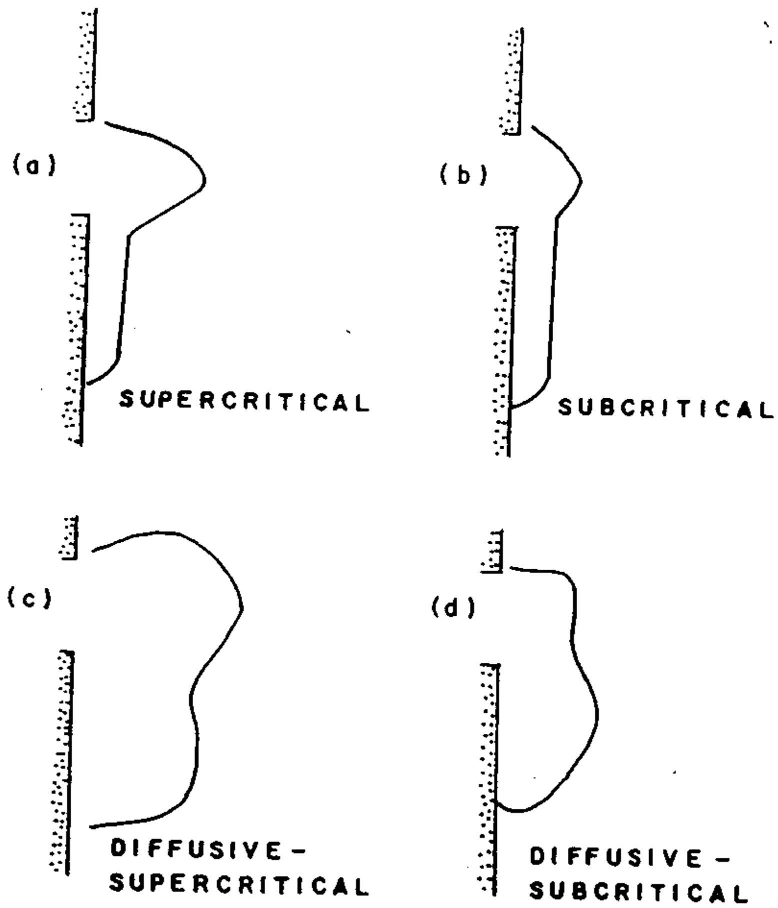
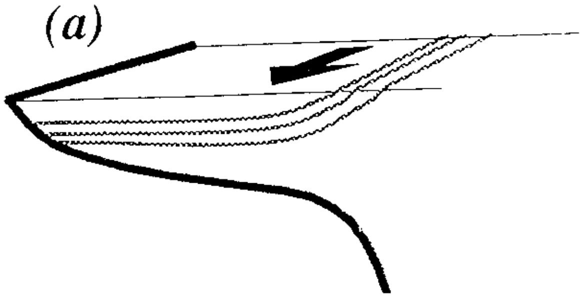


FIG. 18. Four types of midlatitude estuarine plumes, classified according to the surface salinity field.

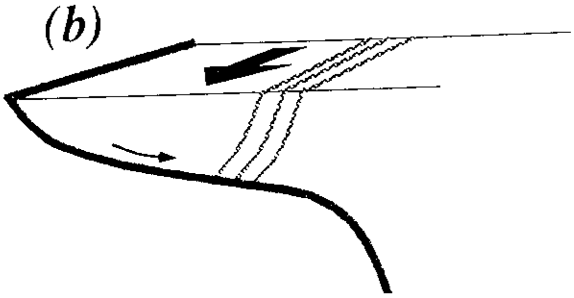
Chao '88

Bottom-attached plumes - Chapman and Lentz, 1994
Yankovsky and Chapman, 1997

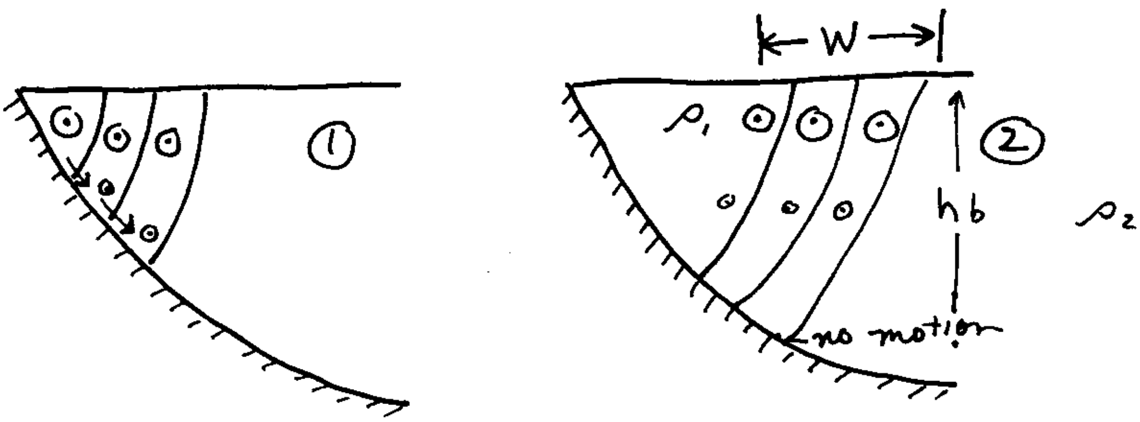
(a)



(b)

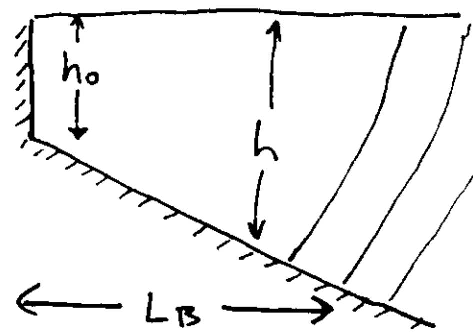


Ekman transport advects bottom front offshore until $v_{Bottom} \to 0$, then $u_E \to 0$



Thermal wind: $-f\frac{\partial v}{\partial z} = -\frac{g}{\rho}\frac{\partial \rho}{\partial x} = \frac{g'}{W}$

Transport $Q = \bar{v} h_b W = \frac{1}{2} g' \frac{h_b^2}{f}$

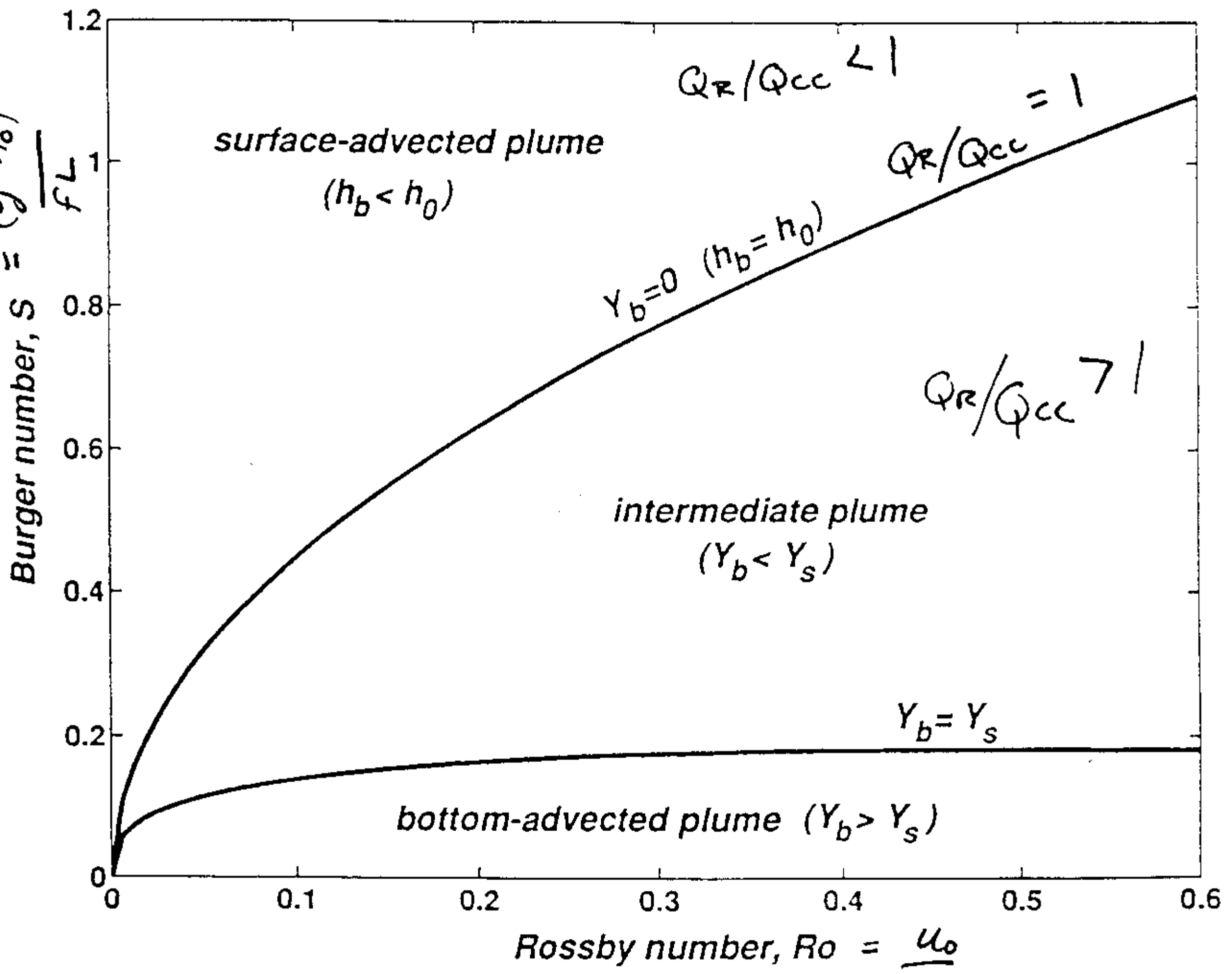


$$h = h_o + s y$$

$$L_B = \frac{h_o}{s}\left(\left(\frac{2fQ}{g' h_o^2}\right)^{1/2} - 1\right)$$

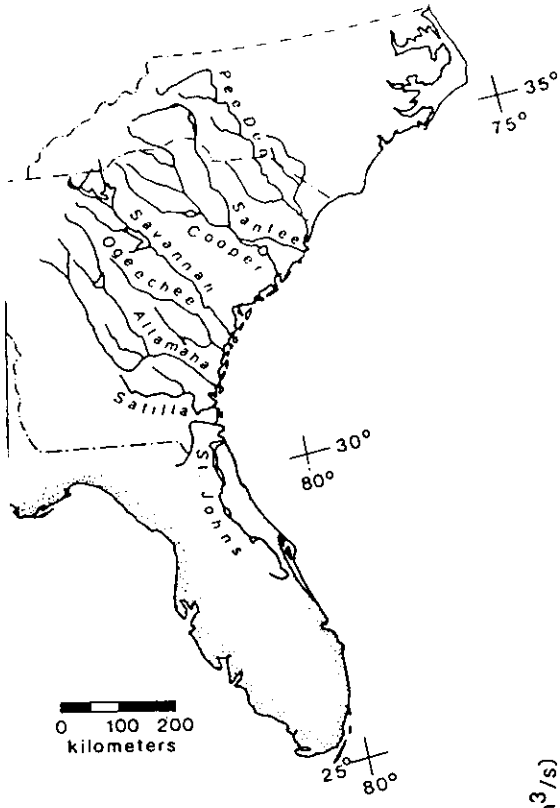
(note that $Q \neq Q_{fresh}$ if $\rho_1 \neq \rho_{river}$)

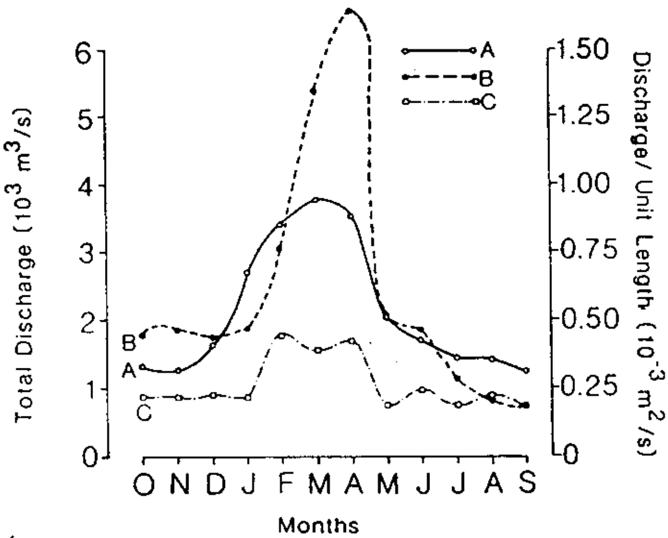
PLUME CLASSIFICATION

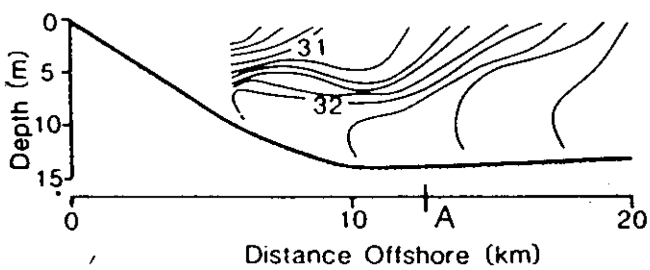


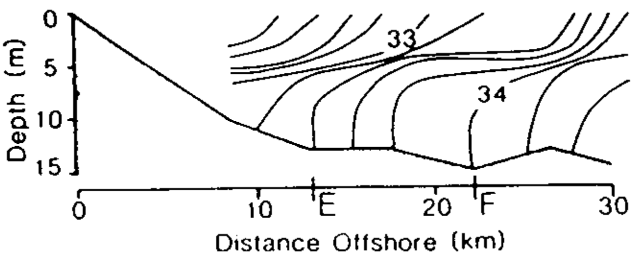
$Q_R = u_o h_o L$ ← width of river

Yankovsky and Chapman, in press

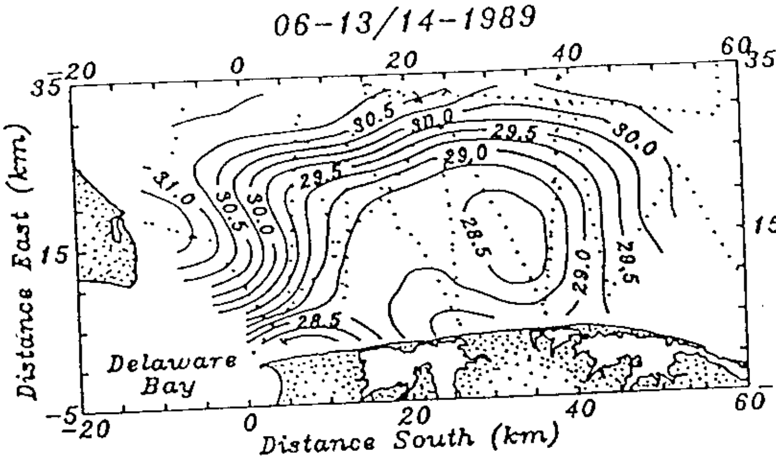

Pee Dee
Santee
Cooper
Savannah
Ogeechee
Altamaha
Satilla
St. Johns
35°
75°
30°
80°
25°
80°
0 100 200
kilometers

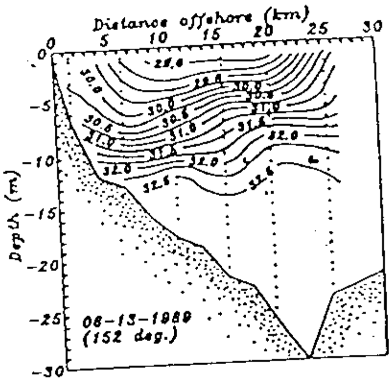

A
B
C
Total Discharge (10^3 m^3/s)
Discharge/ Unit Length (10^{-3} m^2/s)
0 1 2 3 4 5 6
0 0.25 0.50 0.75 1.00 1.25 1.50
O N D J F M A M J J A S
Months

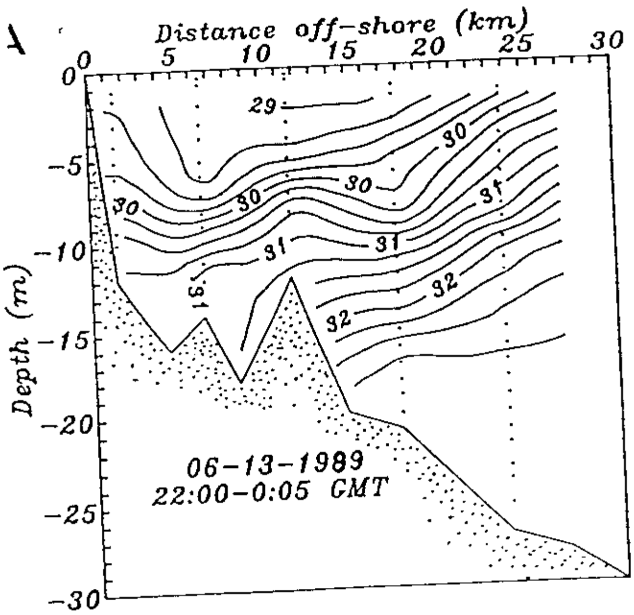

Depth (m)
0 5 10 15
31
32
0 10 A 20
Distance Offshore (km)


Depth (m)
0 5 10 15
33
34
0 10 E 20 F 30
Distance Offshore (km)

15-9
(18)

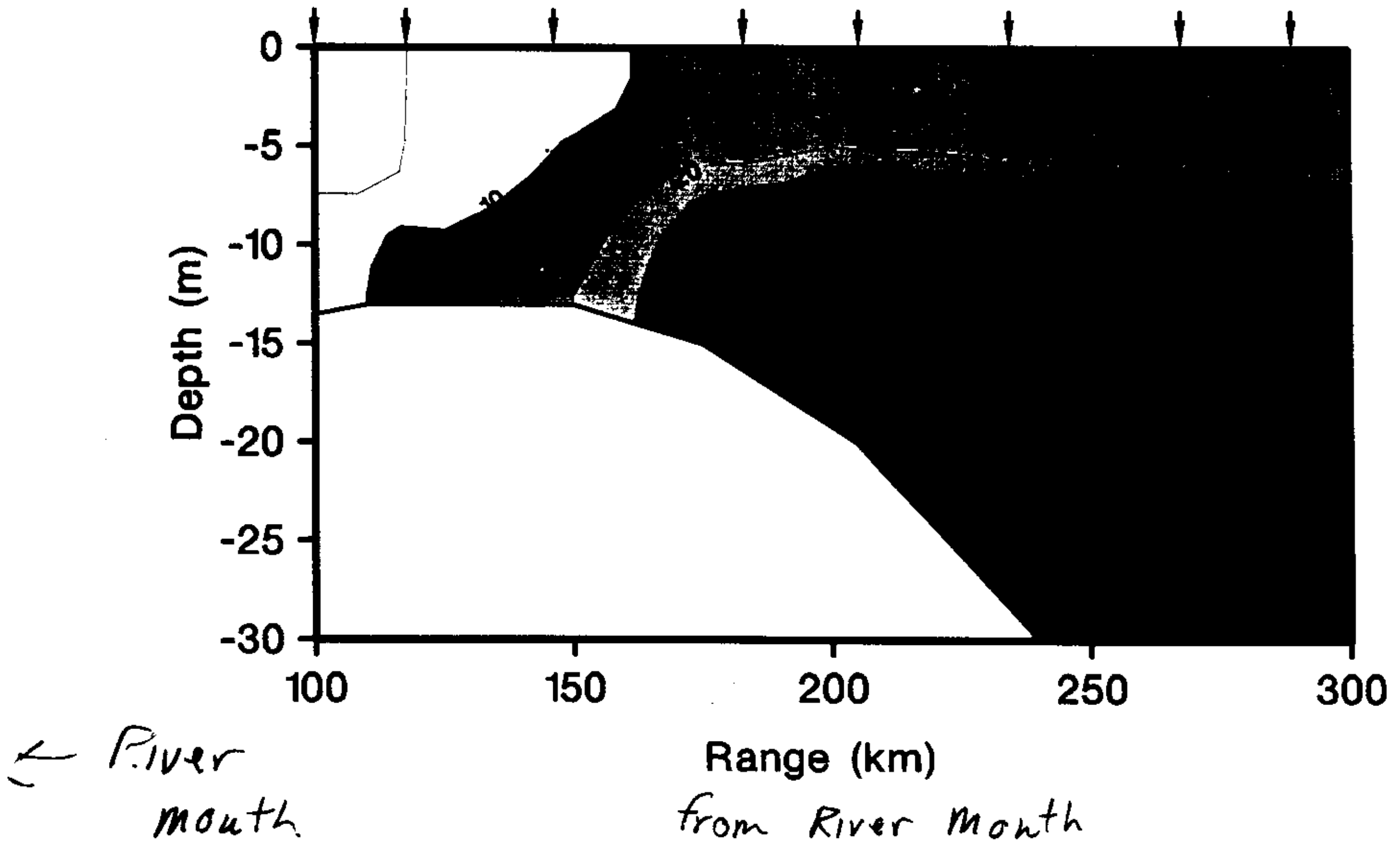






Munchow and Garvine.
Buoyancy + wind forcing
of a coastal current
J. Marine Research
1993

River Mouth Section - Mar 1 1990

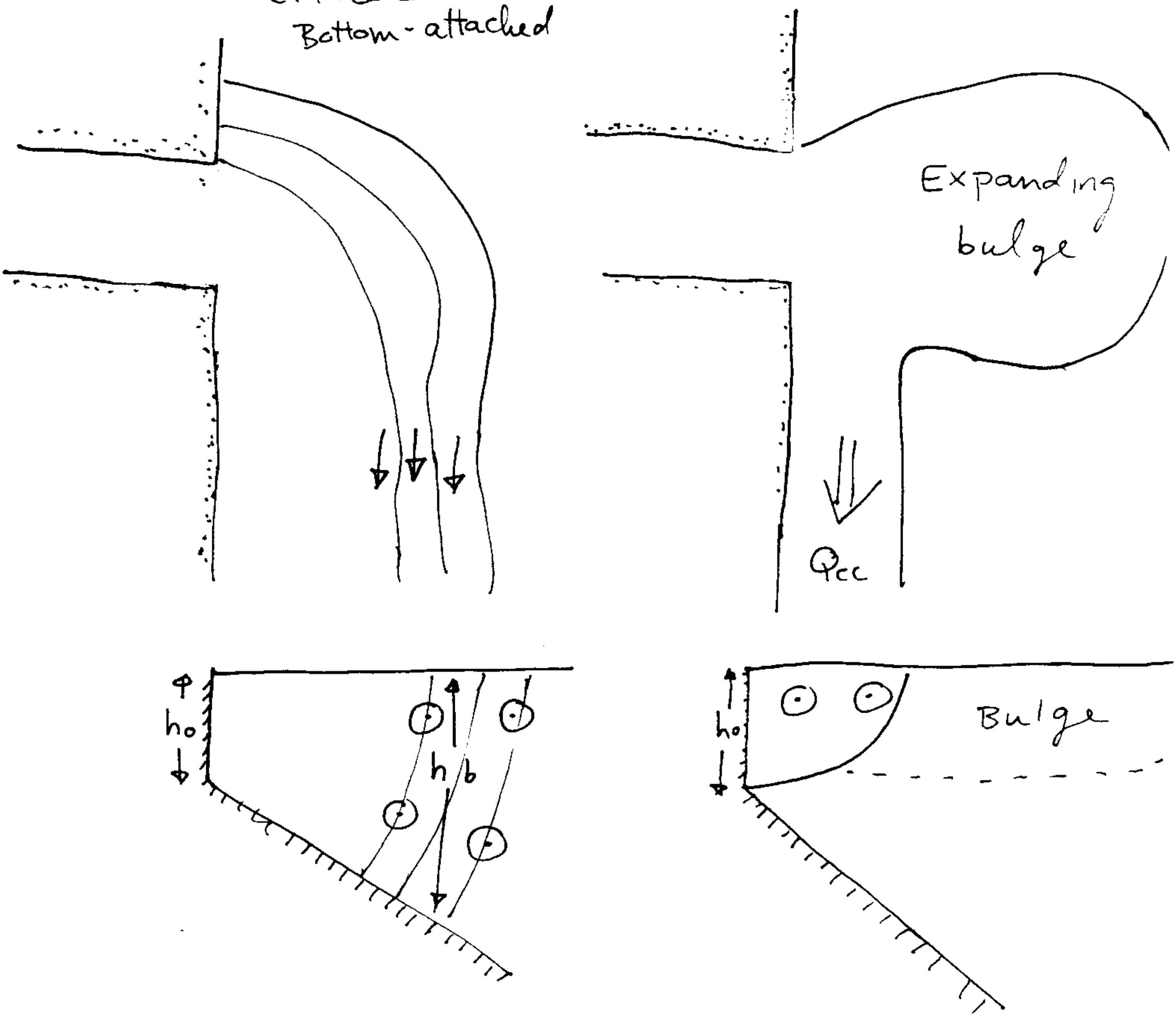


The "Bulge" Problem

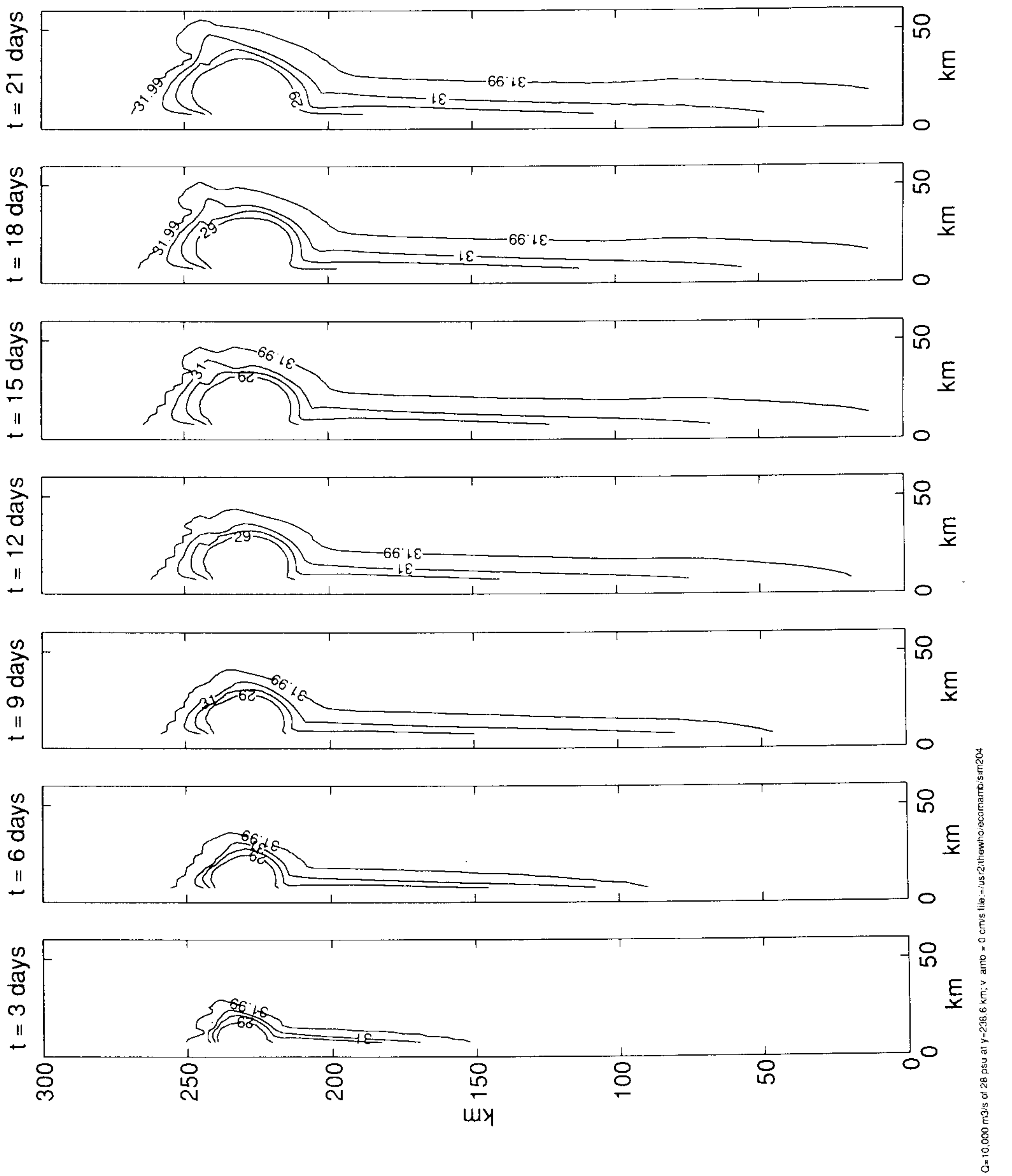
If $Q_{RIVER} > Q_{coastal\ current}$,

i.e. $\dfrac{Q_{River}}{\frac{1}{2} g' h_o^2 / f} > 1$

either Bottom-attached ... or



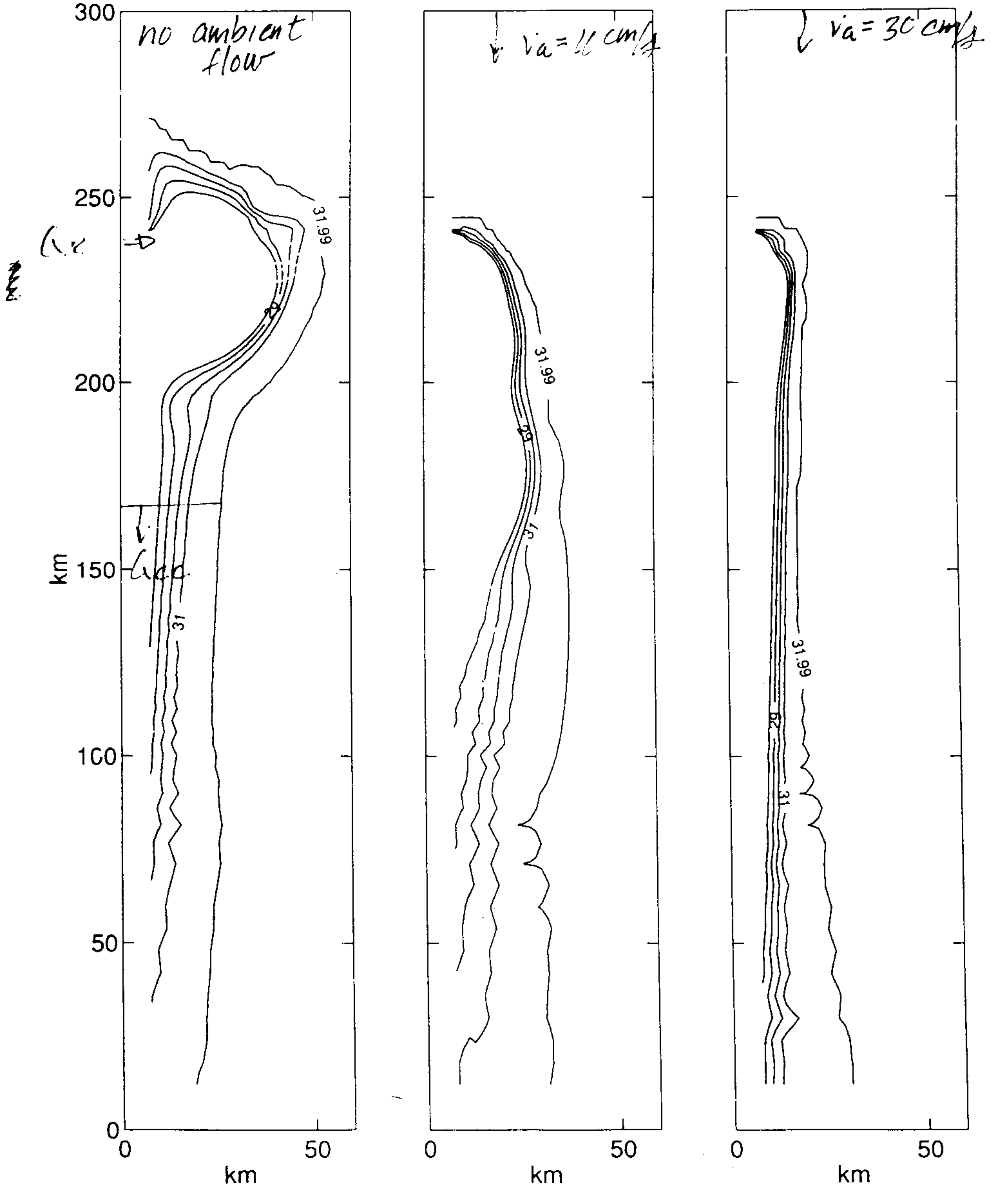
P-14a
(21)


t = 21 days
t = 18 days
t = 15 days
t = 12 days
t = 9 days
t = 6 days
t = 3 days
31.99
31
29
km
300
250
200
150
100
50
0
Q=10,000 m3/s of 28 psu at y=238.6 km; v amb = 0 cm/s file:=/usr2/thewho/ecomamb/sim204

Ambient flow (Far-field version of Garvine Problem)

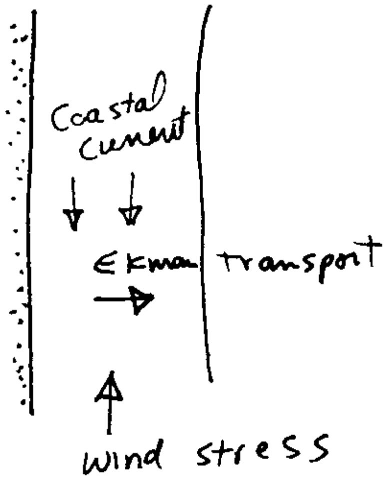
$$Q_{cc} = \frac{1}{2}\frac{g' h_o^2}{f} + L\bar{h}\, v_{ambient}$$

scaling: $v_a/(g' h_o)^{1/2} = F_a$

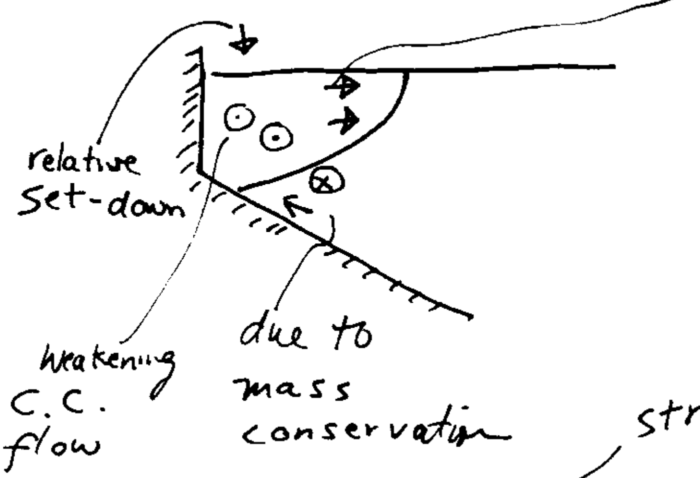


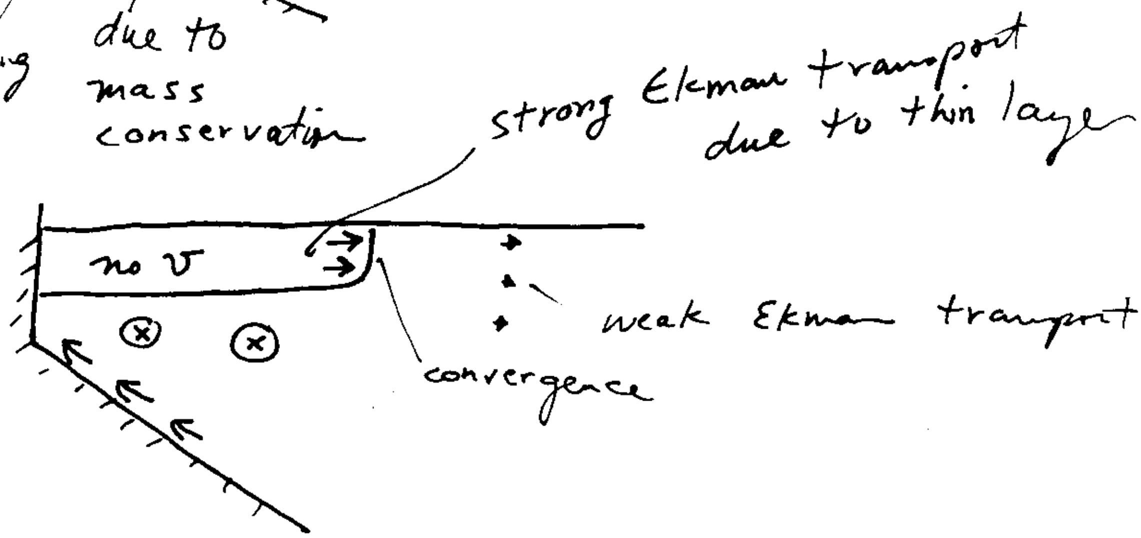
Question: Does Gcc ever balance GR?

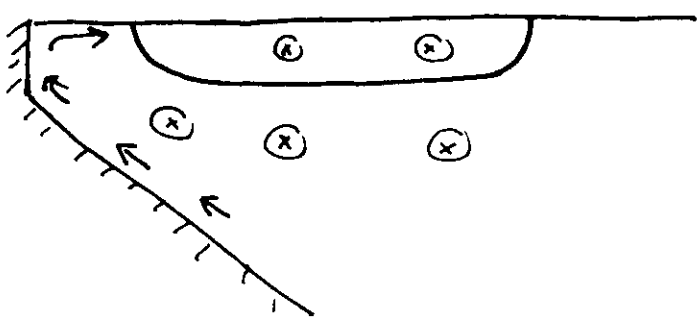
Winds - Upwelling.



$$u_E = \frac{\tau_w/\rho}{fh}$$

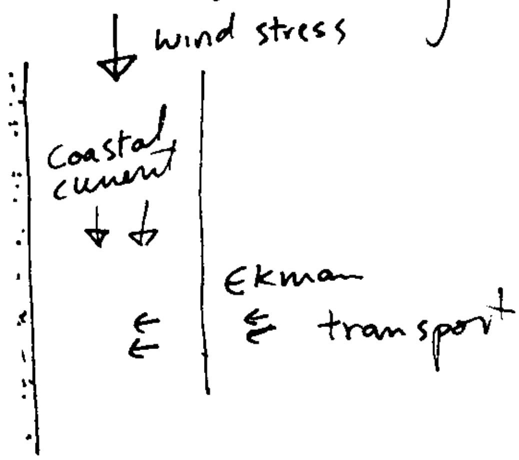


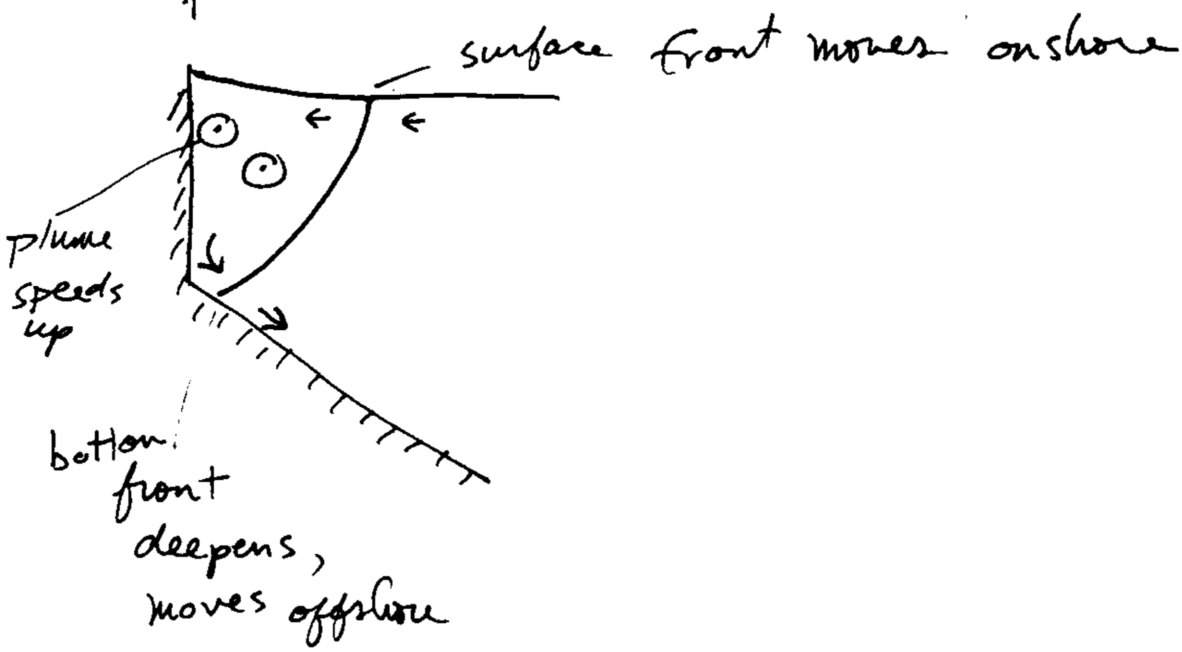


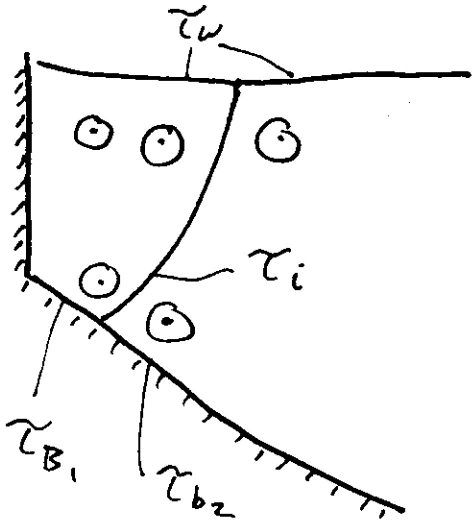


No steady solution!

Winds - downwelling







Garvine, 1996

Top and bottom boundary layers connect,

$$\tau_{wind} = \tau_{B_1} = \tau_i = \tau_{B2}$$

As $\frac{\partial \tau}{\partial z} \to 0$, $u_E \to 0$