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"Meteorologically Forced Subinertial Flows Through the Strait of Gibraltar"

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Meteorologically Forced Subinertial Flows Through the Strait of Gibraltar

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The observed transport due to subinertial flows, with periods ranging from days to a few months, through the Strait of Gibraltar reaches values up to 10^6 m^3 s^{-1}, and is well correlated with atmospheric pressure fluctuations over the Mediterranean Sea. This transport is barotropic and accounts for 84% of the variance observed at these frequencies. A second mode accounts for 12% of the variance and is characterized by a node (zero amplitude) located at the mean depth of the interface between Atlantic and Mediterranean waters. This baroclinic mode is modulated by the spring-neap tidal cycle, corresponding to an increased exchange of waters between Atlantic and Mediterranean at neap tides. A simple model of the Mediterranean, which includes two basins and two straits, indicates that the system has two resonant frequencies at periods of 1.2 and 5.6 days. The 1.2-day resonance corresponds to the eastern and western basins oscillating through the Strait of Sicily, while that at 5.6 days is the resonance of the Mediterranean Sea with the North Atlantic Ocean. A good approximation to the observed flows is obtained when this model is forced with the most energetic mode of the atmospheric pressure over the eastern and the western Mediterranean and a friction term is included to limit the flow through the Strait of Gibraltar. Restricting the flow through the Strait of Sicily does not improve the agreement between the model and the observations for the flows at Gibraltar.

1. INTRODUCTION

The Strait of Gibraltar connects the Mediterranean Sea with the North Atlantic Ocean. It has a length of about 60 km, and at its narrowest section (Point Cires section) the width is 12 km (Figure 1). A principal sill with a maximum depth of 300 m is located between Point Paloma (Spain) and Point Altares (Morocco). Toward the east the channel deepens to about 600 m south of Tarifa and to 900 m at the Gibraltar-Ceuta section. To the west the depth reaches about 450 m north of Tangier, but shallows again to 350 m at a secondary sill to the north of Cape Spartel and then slopes gently down to the deep North Atlantic.

The mean exchange of water through the Strait of Gibraltar has long been recognized to be density driven in response to an excess of evaporation over precipitation in the Mediterranean Sea [*Nielsen*, 1912]. Mass and salt conservation imply an inflow slightly larger than the outflow by about 5%. *Lacombe and Richez* [1982] have suggested that the flow fluctuations through the strait are of three distinct types: tidal, subinertial, and long term. This general classification has been confirmed in our observations, which show that fluctuations in each of these regimes have similar magnitude, from 0.5 to 1 m s^{-1}. Tidal flows are principally forced by the north Atlantic tide, although the locally generated tide inside the Mediterranean must be considered in order to understand the details of the tidal behavior in the strait, in particular in relation to the tidal energy flow through the strait [*Stock and Filloux*, 1975]. Subinertial flows, with periods from a few days to a few months, are mainly meteorologically forced. *Crépon* [1965] suggested that they were related to atmospheric pressure fluctuations over the western Mediterranean, and *Garrett* [1983] extended Crépon's work to include the effect of pressure changes over the whole Mediterranean. Both tidal and subinertial flows are mainly barotropic (depth independent), although a baroclinic contribution can be identified for these two types of flow. At subinertial frequencies, part of the baroclinicity observed might be due to the effects of local winds over the strait, which are characterized by the presence of frequent strong events, reaching speeds up to 25 m s^{-1} [*Air Ministry*, 1962]. It has been observed that the along-strait winds are highly coherent with the atmospheric pressure fluctuations over the western Mediterranean and also to the surface currents, but their effect is mainly felt within the upper few tens of meters [*Lacombe and Richez*, 1982]. The seasonal and longer-term flow fluctuations are baroclinic. Several recent studies [*Bryden and Stommel*, 1984; *Bormans et al.*, 1986; *Farmer and Armi*, 1986] speculate on their possible forcing and control mechanisms, but the true dynamical role of the strait in limiting these flows remains a subject of debate.

Crépon [1965] noted a high degree of correlation between the negative of the atmospheric pressure ($-Pa$) and sea level fluctuations at several locations in the western Mediterranean, as might be expected if the sea surface responded to atmospheric pressure as an inverted barometer. Assuming that the across-strait sea level difference ($\Delta\eta$) between Ceuta and Gibraltar was a measure of the flow through the strait by a geostrophic balance argument, he found that $\Delta\eta$ related directly to $-Pa$ rather than to the time derivative of Pa, as would suggest conservation of volume. Crépon concluded that the dynamics of the strait, particularly the effects of friction and nonlinearities, might explain this result.

Garrett [1983] suggested that the exchange through the Strait of Sicily and differing sea levels in the eastern and western Mediterranean basins should be included in an analysis of the response. Using a two-basin and two-strait model in which the flow through the straits is limited by geostrophic control, and considering eastward propagating atmospheric pressure systems, he showed that both sea level in the western Mediterranean and flows through Gibraltar become more in phase with minus the atmospheric pres-

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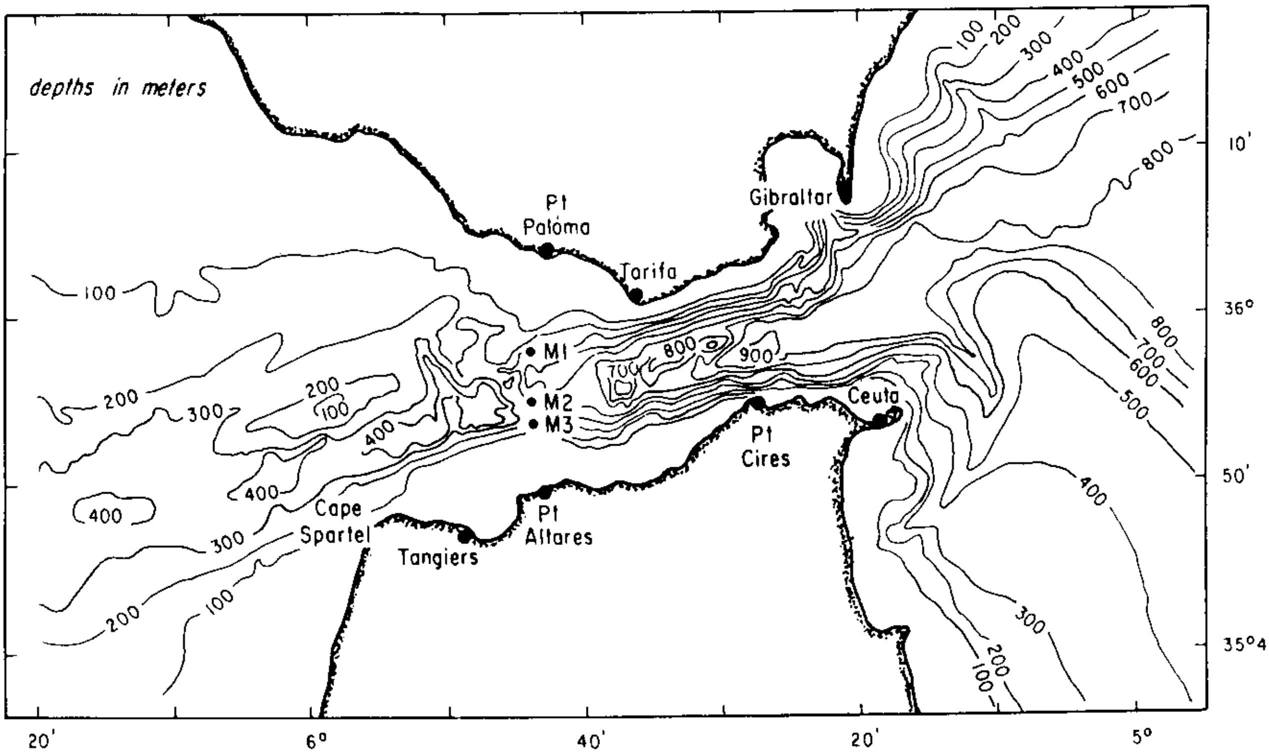

Fig. 1. Map of the Strait of Gibraltar showing places referred to in the text. Also shown are the locations of the three sill moorings used for the subinertial current estimate: M1, M2, and M3.

sure, as observations indicate. As a consequence, the model predicted a significant noninverse barometer response of sea level in the eastern Mediterranean at periods of several days. Also because of traveling pressure systems the atmospheric pressure fluctuations over the eastern and western Mediterranean were expected to become uncorrelated at subinertial frequencies. These two aspects were partially confirmed by *Garrett and Majaess* [1984], although their analysis was based on only 5 months of data from one sea level and three atmospheric pressure stations.

The objective of the present paper is to show the relation of the observed subinertial flow in the Strait of Gibraltar to the atmospheric pressure field over the Mediterranean Sea and to explain the mechanism by which this comes about by developing a simple analytical model.

2. Subinertial Flow Observations

Observations of currents, temperature, conductivity, and pressure are available for the period between October 1985 and October 1986 at several locations in the strait [*Pillsbury et al.*, 1987]. The measurements from moorings M1, M2, and M3 from the sill section between Point Paloma and Point Altares during October 1985 to May 1986 (Figure 1) are considered in detail here. The meters on moorings M1 and M3 have a common measuring time interval of about 6 months (October 22, 1985 to April 21, 1986). Mooring M1 had four instruments, and mooring M3 had three instruments. Mooring M2 broke off after a month but was recovered, with data available from October 22 to November 23, 1985 at five depths. Instrument depths along with basic statistics of the observations are presented in Table 1. The location of the interface between Atlantic and Mediterranean waters, taken as the 37.5‰ isohaline, can be estimated from the pressure, temperature, and conductivity observations. The mean depth of the interface is found to be 119 m at M1, 169 m at M2, and 156 m at M3 during their respective full recording period, thus only the top instruments on M2 and M3 were in the upper layer. High-frequency fluctuations, with periods shorter than 1 day, are eliminated from the raw observations using the PL64 low-pass filter [*Flagg et al.*, 1976]. This filter has a half power point at 38 hours. The principal axis components of each low-passed time series is computed and listed in Table 1. The principal axis corresponds to the orientation at which two orthogonal components of a horizontal vector time series become uncorrelated and the variance of the component along this axis is a maximum with respect to any other orientation chosen.

Covarying modes of fluctuation for the subinertial currents at the sill are defined using empirical orthogonal functions [e.g., *Kundu et al.*, 1975] (EOFs). The covariation matrix between principal axis components of the current from each instrument on moorings M1 and M3 (or M1, M2, and M3) is formed, and the eigenvalues and modes of that matrix are sought.

Due to the different measuring intervals that the meters in M2 have with respect to those in M1 and M3, and to the desirability of having both a long time series and a good spatial coverage, two EOFs analyses were performed. The first consisted in using the 12 time series from all the meters in the three moorings, which have 1 month of simultaneous data. The first mode explains 84.0% of the variance, and its spatial weights, which are the standard deviation of the currents due to this mode at each location, are listed in Table 1 and shown in Figure 2. This figure also illustrates the cross section of the sill and indicates the location of the current meters on each mooring. In this mode, currents fluctuate everywhere in phase, with some amplitude variation. The time coefficients of this first mode are shown in Figure 3*a* by the dashed line that covers the period October 25 to November 20, 1985.

TABLE 1. Statistics of Current Meter Measurements From Moorings M1, M2, and M3

Mooring	Depth of Instrument, m	Mean		Subinertial Currents Principal Axis Component			EOF Modes	
		Magnitude, cm/s	Orientation, deg	Orientation, deg	Semimajor Axis cm/s	Semiminor Axis cm/s	Mode 1, cm/s	Mode 2, cm/s
M1	143	15.7	-143.6°	-5.6°	15.3	3.3	11.2(14.4)	7.6(6.5)
M1	156	18.8	-164.3°*	18.6°	13.5	3.6	9.1(12.3)	7.5(6.6)
M1	167	20.4	-150.0°	-8.6°	12.9	3.6	9.3(11.9)	7.2(6.0)
M1	215	11.5	167.9°	-20.8°	6.8	2.4	4.1(5.6)	4.0(3.3)
M2	123	11.1	-116.8°	13.7°	23.0	3.3	24.4	-3.3
M2	143	17.2	-150.3°	5.7°	16.4	2.5	18.2	0.6
M2	153	27.9	-146.7°	10.2°	17.4	2.7	19.5	2.3
M2	254	53.2	-149.7°	22.9°	8.2	4.6	4.6	7.9
M2	306	38.3	-133.8°	24.4°	7.0	5.9	4.1	5.5
M3	110	13.4	50.5°	-8.8°	20.3	5.6	19.4(17.3)	-8.4(-10.3)
M3	140	6.2	137.6°	-2.0°	17.9	6.5	20.3(17.4)	-5.5(-4.6)
M3	180	24.8	-157.0°	19.0°	13.0	3.9	12.4(12.0)	0.3(-0.5)

Main statistics of the current meter measurements used to estimate the subinertial flows through the Strait of Gibraltar. The north sill mooring M1 (35°58.26'N, 05°44.62'W) was at a depth of 222 m, its four instruments registered for the period October 22, 1985 to May 4, 1986. The center sill mooring M2 (35°54.79'N, 05°44.41'W) was at a depth of 321 m, its five instruments registered for the period October 22 to November 23, 1985. The south sill mooring M3 (35°53.42'N, 05°44.20'W) was at a water depth of 190 m and its three instruments registered for the period October 26, 1985 to April 21, 1986. The orientation of the mean and principal axis subinertial currents is measured counterclockwise from the east. The semimajor and semiminor axes of fluctuation are denoted by the standard deviation of the subinertial current along that axis. Two estimates of the spatial weights for the subinertial current modes are shown. One corresponds to the EOF analysis with all 12 instruments in moorings M1, M2, and M3, and the other to the analysis with only the seven instruments from moorings M1 and M3 (given in parentheses). The variance represented by each mode is 84.0%(79.8%) for Mode 1 and 12.7%(16.3%) for Mode 2.

*The orientation on this instrument is suspect. There seems to be a constant offset on the compass. This does not affect any of our analysis results which only consider the principal axis component.

The second EOF analysis used only the meters on moorings M1 and M3, for which observations are available for a period of about 6 months. The first mode of this second EOF analyses explains 79.8% of the variance, its spatial weights are also listed in Table 1 (in parentheses), and its time coefficients are shown in Figure 3*a* by the solid line. The inclusion of the M2 observations does not significantly alter the time evolution of the principal mode of motion obtained by either of the EOFs. This is an indication of the high degree of coherence between observed subinertial currents across the section. The values of the spatial weights for mode 1 at M1 and M3 given in Table 1 are not appreciably affected by considering the measurements from M2, although its inclusion gives a more detailed description of the spatial structure of this mode across the section. It is important to note that the

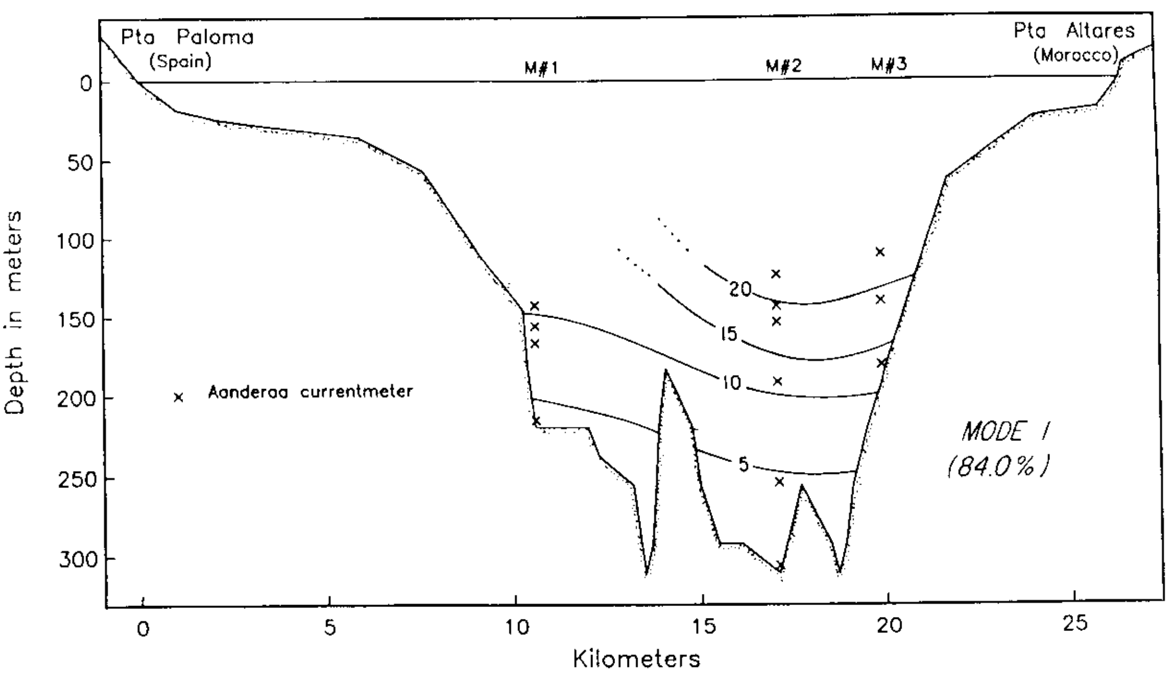

Fig. 2. Spatial distribution of the first empirical orthogonal mode of the subinertial principal axis currents, which explains 84% of the variance at these frequencies (< 0.5 cpd). The location of the current meters from the three sill moorings is shown on the cross section at Gibraltar's main sill from Point Paloma to Point Altares. This empirical orthogonal function analysis used data from all 12 instruments on moorings M1, M2, and M3 from October to November 1985.

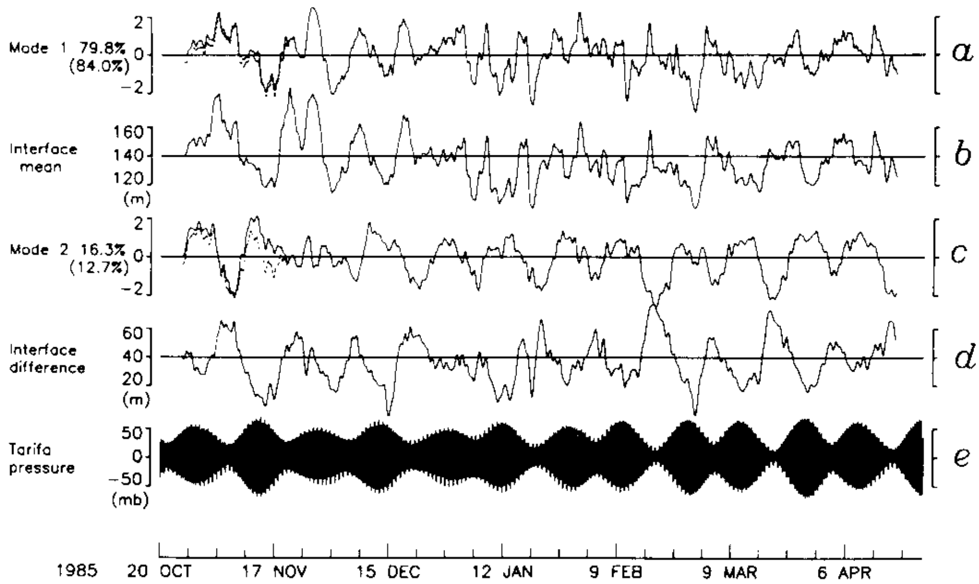

Fig. 3. (*a*) Time series of time coefficients of mode 1 of the subinertial principal axis currents. Results from two different EOF analysis are shown. The dashed line, covering only the first month, is obtained by considering all 12 measurements available from the three moorings at the sill section. The solid line, covering a period of about 6 months, gives the time evolution of mode 1 when only the seven instruments from moorings M1 and M3 are considered. Both series are normalized to have a variance of 1. (*b*) Time series of mean depth of the interface across the strait in meters, taken as the 37.5‰ isohaline. This series was obtained by averaging the observed low-pass time series of interface depth at moorings M1 and M3. (*c*) Time series of time evolution of mode 2 of the subinertial currents. Results from the two different EOF analysis are shown as in Figure 3*a*. (*d*) Time series of interface depth difference across the strait between moorings M3 (south) and M1 (north) in meters. Positive values indicate deeper interface depths at the south mooring. (*e*) Time series of tide prediction of the bottom pressure at the Port of Tarifa in millibars, shown here as a reference to identify periods of spring and neap tides.

spatial average of the weights for mode 1 is about the same for the two EOFs. A weighted integration of the subinertial currents across the section is not justified with the 12 available measuring points; instead, a straight average of the spatial weights of mode 1 was calculated. This gives 13.05 cm s^{-1}, which, when multiplied by the time coefficients of mode 1 (Figure 3*a*), and by the cross-sectional area at the sill (2.951 10^6 m^2) gives an estimate of the subinertial transports through Gibraltar during this period. This is illustrated by the thick line in the bottom plot of Figure 6 (the units in which the transport is given are Sverdrups, 1 Sv = 10^6 m^3 s^{-1}). This estimate is nearly the same as that obtained by a direct average of the subinertial current measurements available across the section. Also multiplying the average of the spatial weights of mode 1 times the cross-sectional area at the sill, gives a root mean square value of 0.4 Sv for the subinertial transports through the strait.

The mode illustrated in Figure 2 is taken to represent the subinertial currents through the strait. While the amplitude of this mode might be somewhat uncertain due to the lack of sampling in the upper layer, the time evolution of this function (Figure 3*a*) is felt to correctly represent the subinertial current variability, since high coherence and relatively little phase lag is found between nearly all pairs of current observations at the sill. Although direct velocity observations are not available near the surface, shallow (< 10 m) pressure sensors were deployed on either side of the strait at Point Paloma, Spain, and Point Altares, Morocco, from April 26 to August 4, 1986. The difference in pressure across the strait can be taken as representing surface layer currents averaged across the width of the strait, if the cross-strait momentum balance is geostrophic. Geostrophic currents thus derived are found to be significantly correlated to the few direct current measurements in the deep layer during the same period (not shown). This supports the idea that subinertial currents in the surface layer are coherent with those at depth at the sill section.

Averaging the low-passed time series of the depth of the interface (taken as the 37.5‰ isohaline) at M1 in the north and M3 in the south, it is possible to obtain the time evolution of the mean depth of the interface across the strait at the sill (Figure 3*b*). There is a high degree of correlation between the subinertial transports through Gibraltar, represented here by mode 1 (Figure 3*a*), and the depth fluctuations of the interface between Mediterranean and Atlantic waters. This correlation implies that the ratio of Atlantic to Mediterranean waters which are being exchanged increases when the subinertial flows are toward the Mediterranean.

The second mode of the EOF analyses accounts for 12.6 or 16.3% of the variance, depending on the number of measurements used. Its time coefficients are shown in Figure 3*c*. They are characterized by marked fortnightly tidal modulation, which can be verified by comparing them with the bottom trace (Figure 3*e*), which shows the tidal predictions for the bottom pressure at the Port of Tarifa. The spatial weights of the second mode are also listed in Table 1 and shown in Figure 4. This fortnightly modulation of the flows through the strait seems to be a persistent signal and not an artifact of the EOF decomposition or related to mooring motion. In the sections that follow, a meteorologically forced signal will be identified in the flows present in the strait, and when this is subtracted from any of the individual current records obtained at the sill, the remaining current always has an identifiable fortnightly modulation, in accordance with the structure depicted in Figure 4 by mode 2. With respect to mooring motion, the fortnightly modulation is present in all the instruments

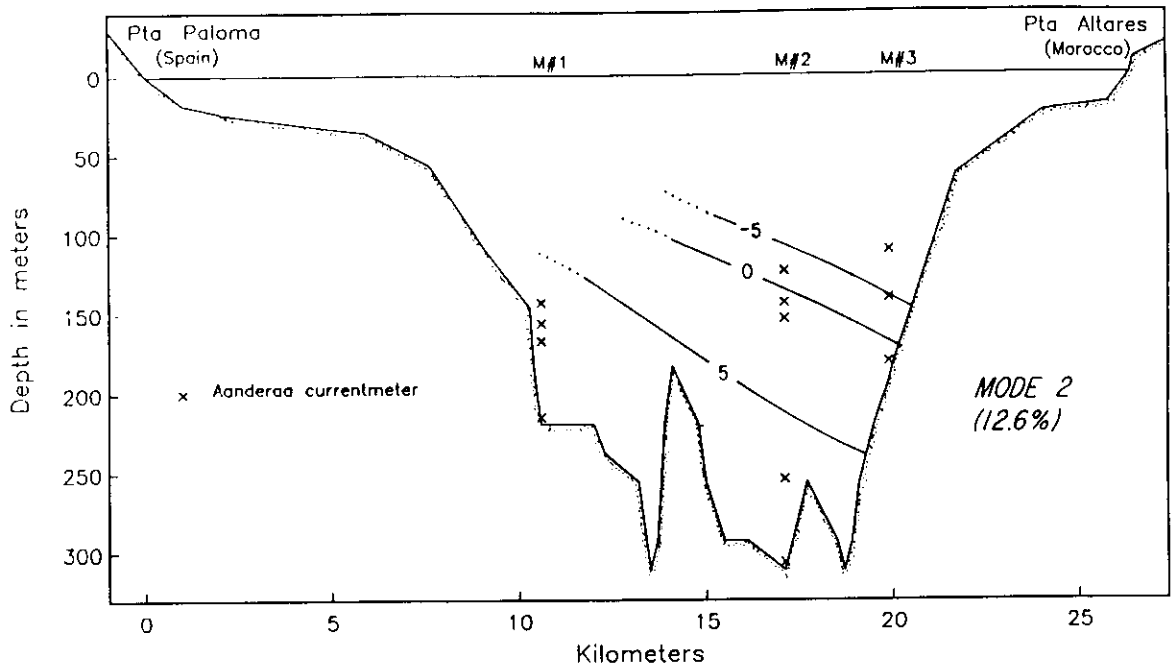

Fig. 4. Spatial distribution of the mode 2 of the subinertial principal axis current in cm s^{-1}, which explains 12.6% of the variance at these frequencies (< 0.5 cpd). This EOF analysis used data from all 12 instruments in moorings M1, M2, and M3, from October-November 1985.

on mooring M1 and the lower meters on moorings M2 and M3, all of which had negligible depth excursions during their recording periods.

The spatial structure of mode 2 depicted in Figure 4 represents a first baroclinic mode of motion with a zero contour coinciding approximately with the mean interface location between the inflowing Atlantic and outflowing Mediterranean layers. Considering the superposition of mode 2 with the persistent long-term baroclinic exchange between Mediterranean and Atlantic waters, it is observed that the mean shear between the two layers is enhanced during periods of neap tides and reduced during spring tides. An independent evidence for this behavior of the shear at the sill is provided by calculating the difference of the depth of the interface between the south mooring (M3) and the north mooring (M1), which is shown in Figure 3*d*. It is evident from this plot that the slope of the interface across the strait, which is related to the shear through the thermal wind balance, is persistently more pronounced during periods of neap tides in relation to spring tides. A dynamical explanation for the existence and behavior of this second mode of motion, clearly related to the tidal dynamics in the strait, is beyond the scope of the present paper and will not be pursued here. The baroclinic nature of mode 2 is such that no appreciable net transport through the strait is associated with it at any given time. Its apparent tidal origin suggests that it is not related to meteorological forcing.

3. ATMOSPHERIC PRESSURE OVER THE MEDITERRANEAN

The Mediterranean Sea extends zonally for approximately 4600 km, while on the average the north-south extent is only about 1000 km, it has a mean depth of 1500 m with a maximum of about 4000 m in the Ionian Sea (Figure 5). For oceanographic purposes it is usually regarded as being divided into a western and an eastern basin connected by the Strait of Sicily. Because of the high orographic relief characteristic of the Italian Peninsula, Sicily and Tunisia, the two-basin division is also practical from a meteorological point of view. Observations of atmospheric pressure sampled every 6 hours from 64 meteorological stations bordering the sea (Figure 5) were obtained from the European Center for Medium-Range Weather Forecasts (Reading, England). The measurements covered the period between October 1985 and October 1986. Obviously false values were removed, missing observations were interpolated using splines, and the resulting time series were filtered using the PL64 filter described earlier.

Covarying patterns of atmospheric pressure fluctuations are obtained using the same EOF method described earlier. The spatial structure of the first mode, which accounts for 65% of the variance, is illustrated in Figure 5. This mode represents a homogeneous pattern without sign change. It has larger amplitudes over the western Mediterranean with maximums in the Ligurian and North Adriatic Seas; smaller amplitudes are found toward the eastern Mediterranean and minimum values occur near Cyprus. This mode represents a standing pattern unrelated to traveling weather fronts. An average of the weights shown in Figure 5 gives 4.31 mbar. Considering the values separately for the two basins, an average of 5.17 mbar for the western Mediterranean and of 3.56 mbar for the eastern Mediterranean result. The forcing over the sea represented by this principal mode can be thought of as consisting of two atmospheric pressure functions, one for each basin, which have the same time evolution but differing magnitudes. Multiplying the spatial average of 4.31 mbar by the time coefficients of the mode gives the series shown on the top plot of Figure 6. This is nearly the same time series that we would have obtained by averaging all the available stations at each sampled time, which is shown later by the top plot of Figure 12. EOFs, however, extract the spatial structure of this mean pattern as well as an estimate of the variance it represents. Modes 2 and 3 of the EOF of atmospheric pressure represent 15 and 10% of the variance, respectively. Since both are related to smaller spatial structures than the mode considered above, their integrated

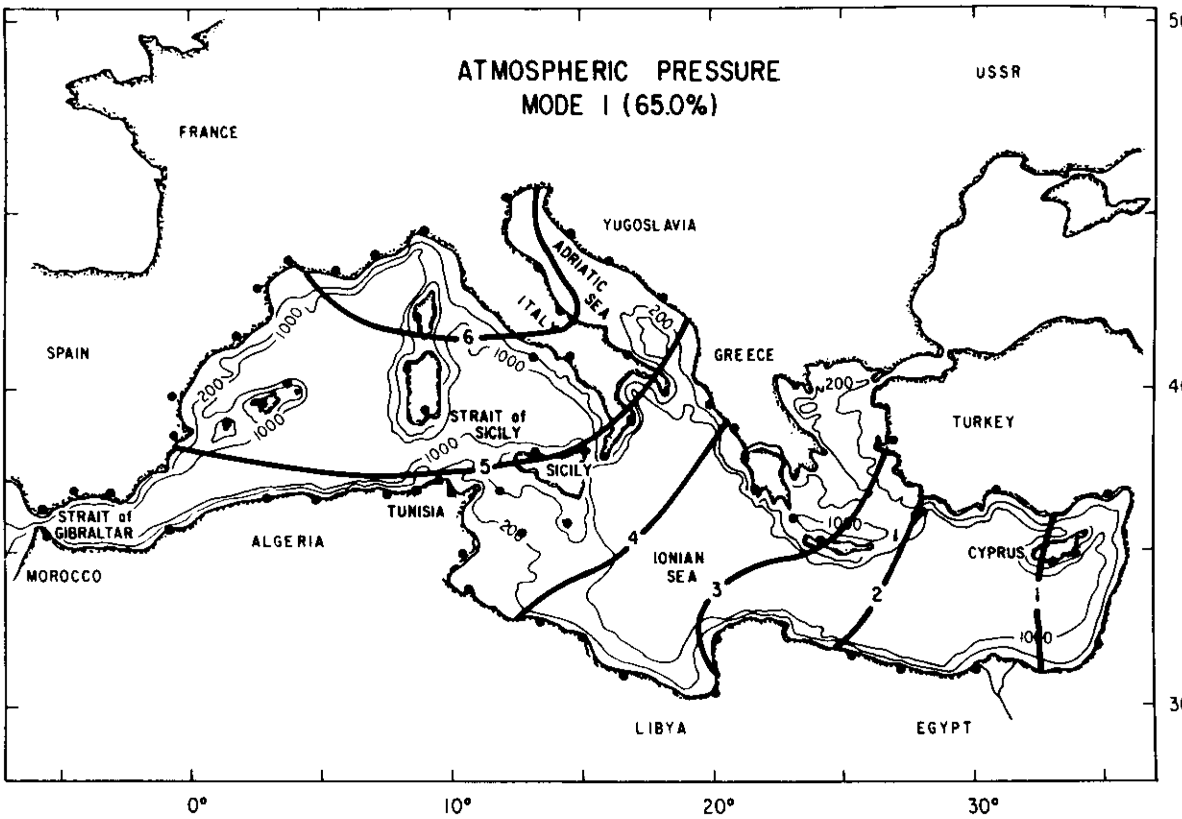

Fig. 5. Map of the Mediterranean Sea with depth contours in meters and the location of places referred to in the text. The large dots show the location of the 64 meteorological stations used to obtain the atmospheric pressure patterns over the sea. Contour lines correspond to the spatial weights (in mb) for the principal EOF mode of the atmospheric pressure field over the Mediterranean, which explains 65% of the variance at subinertial frequencies.

contributions over the whole sea to the forcing of flows at Gibraltar should not be appreciable. Mode 2, however, has a spatial structure that separates the Mediterranean into two parts with a zero crossing meridionally at approximately the same longitude as Sicily and partially accounts for the observed eastward traveling pressure disturbance characteristic of the region [*Air Ministry*, 1962]. The possible contribution of this mode to the flows through the Strait of Sicily needs to be considered.

4. Relation Between Observed Flows and Atmospheric Forcing

Figure 6 illustrates the time series for the forcing mode of atmospheric pressure over the whole Mediterranean and the subinertial transports through Gibraltar. The atmospheric mode only accounts for 65.0% of the total variance of the atmospheric pressure field at subinertial frequencies and is clearly correlated

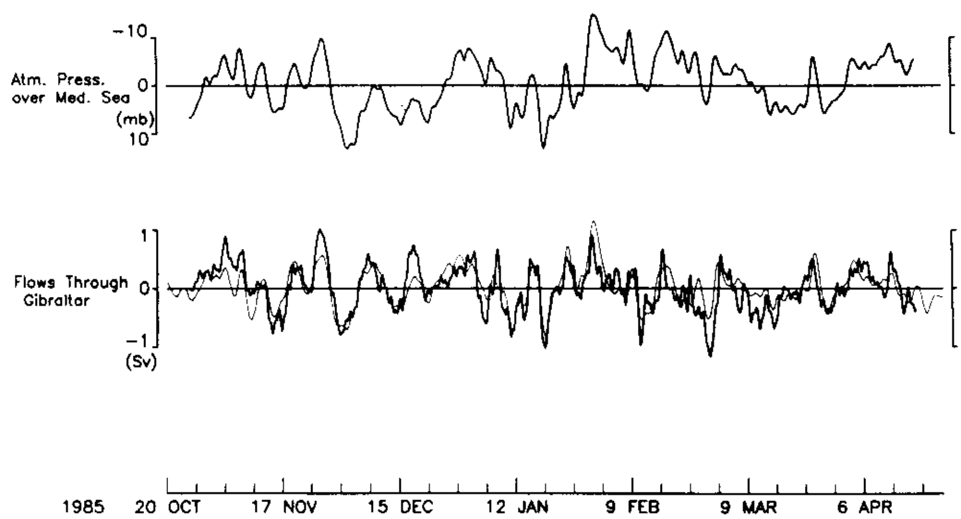

Fig. 6. Time series of the uniform atmospheric pressure fluctuations over the Mediterranean Sea (upper plot), and of measured subinertial transports through Gibraltar (lower plot, thick line), in Sverdrups, i.e., 1 Sv = $10^6 m^3 s^{-1}$. The thin line superposed on the lower plot corresponds to the model "prediction" for Q_1 that is discussed in Section 6.

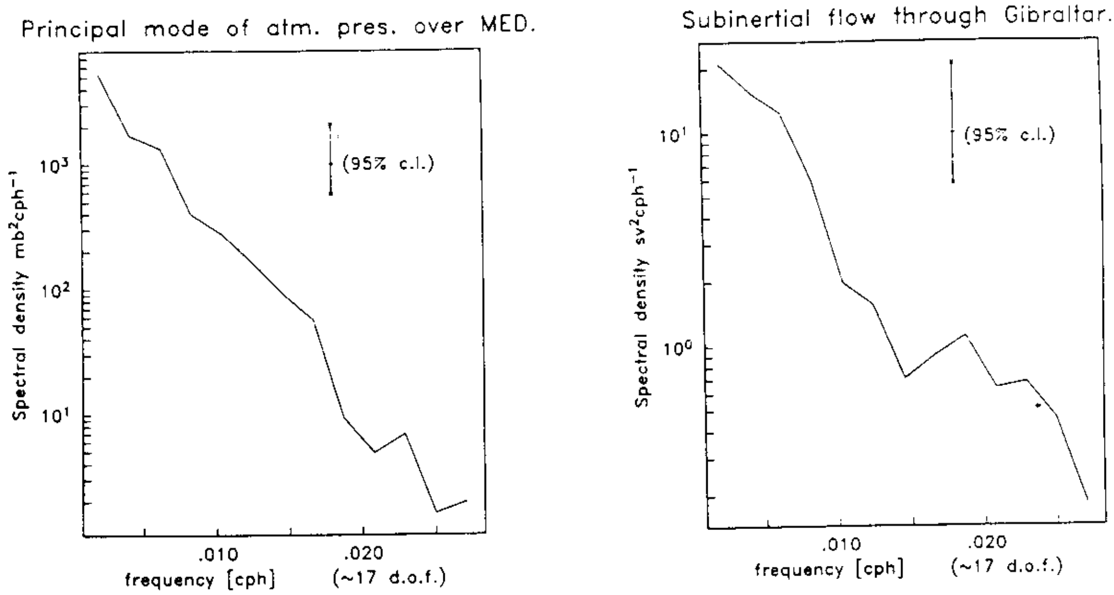

Fig. 7. Power spectra of the time series shown in Figure 6, the principal mode of the atmospheric pressure fluctuations over the Mediterranean Sea and the observed subinertial transports through Gibraltar. A band width of 0.00208 cph was specified, which gave approximately 17 degrees of freedom for the estimates.

with the mode representing transport through the strait, although the latter appears to have relatively more energy at higher frequencies. Power spectra of the two series calculated with a bandwidth of 0.00208 cph are illustrated in Figure 7. The transports at Gibraltar have a more even energy distribution than the atmospheric pressure for the range of frequencies considered.

Results of cross-spectra calculations between the two series are illustrated in Figure 8. The coherence between the two series is high and significant at the 95% confidence level up to the 0.01042-cph frequency band corresponding to a period of 4 days, then drops below the significance level and is just significant again at periods of 2.8 days. The gain has a value of 0.03 Sv/mbar at the lowest resolvable frequency corresponding to an 80-days periodicity and then rises steeply to a maximum of 0.09 Sv/mbar at a frequency band centered at 0.00625 cph (6.67 days periodicity), then falls from this maximum, somewhat more gently, toward higher frequencies until the coherence is lost. The maximum in gain may be related to the resonant frequency of the Mediterranean Sea oscillating with the north Atlantic as discussed in the next section. The phase relation shows a value near 270° at the lowest frequency band and then drops rapidly to values around 180°, with a decreasing slope toward higher frequencies. Such a phase relation might be expected since at very low frequencies there ought to be enough time for the water to flow in or out through the strait, allowing the sea surface in the sea to respond uniformly as an inverted barometer. Conservation of volume would thus imply a 270° phase difference between the atmospheric pressure forcing and the transport, i.e., high atmospheric pressure leading outflow (which is negative inflow) by 90°. At higher frequencies the strait may not have time to accommodate all the required volume exchange, and a different response is observed.

5. A SIMPLE MODEL

A simple model is presented here to explain some of the features of the observations. The work of *Garrett* [1983] suggests that the model should allow different mean atmospheric pressures over the eastern and western basins. This requires the consideration of a two basin–two strait model as sketched in Figure 9. Within each basin it is assumed that long gravity waves smooth the sea level and subsurface pressure fields as to render them spatially uniform at subinertial time scales. For the first basin, conservation of volume is given by

$$A_1 \frac{\partial \eta_1}{\partial t} = Q_1 - Q_2 \tag{1}$$

where A_1 is the surface area of the basin, η_1 is the uniform sea level elevation, Q_1 is the volume flow through the first strait (i.e., $Q_1 = u_1 A_G$, A_G being the cross-sectional area of the strait and u_1 the average velocity over this section) and Q_2 that through the second strait ($Q_2 = u_2 A_S$). In the second basin we have

$$A_2 \frac{\partial \eta_2}{\partial t} = Q_2 \tag{2}$$

In each basin the subsurface, spatially uniform, pressure is taken to be given by the mean atmospheric pressure over the basin plus the hydrostatic contribution, so we have

$$P_1 = Pa_1 + \rho g \eta_1 \tag{3}$$

$$P_2 = Pa_2 + \rho g \eta_2 \tag{4}$$

for the first and second basins respectively. Pa_1 is the spatially average atmospheric pressure over basin 1 and Pa_2 that over basin 2, ρ is the water density, and g is the acceleration due to gravity.

At first, a balance between the pressure gradient force along the strait and the acceleration of the flow is taken to be the main dynamical balance,

$$\frac{\partial Q_1}{\partial t} = -\frac{A_G}{\rho} \frac{\partial P}{\partial x} \tag{5}$$

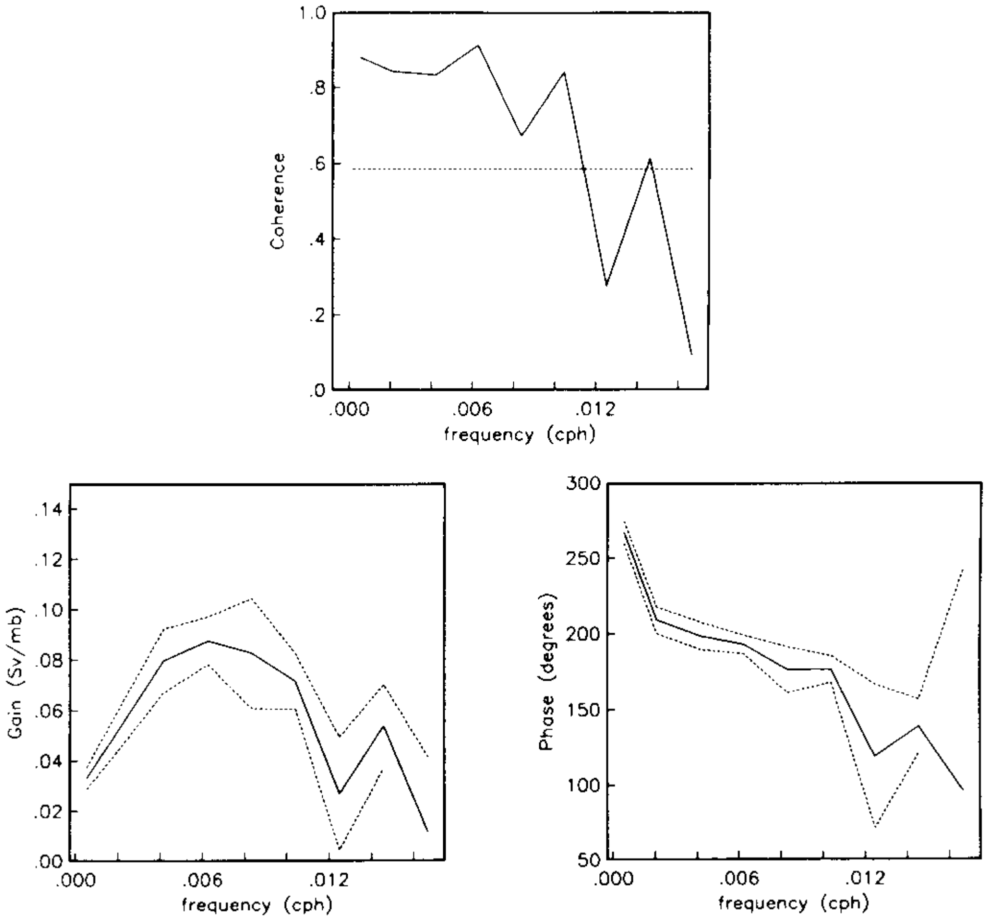

Fig. 8. Coherence, gain and phase relations (continuous line in each plot) between the subinertial transports through Gibraltar and the atmospheric forcing over the Mediterranean Sea, obtained from the cross-spectra calculation between the two series shown in Figure 6. The dashed line in the coherence plot indicates the 95% significance level. The gain and phase normalized standard error [*Bendat and Piersol*, 1986, p. 317] are indicated by dashed lines in each respective plot.

for the first strait and

$$\frac{\partial Q_2}{\partial t} = -\frac{A_S}{\rho}\frac{\partial P}{\partial x} \qquad (6)$$

for the second strait, x being the along-strait coordinate.

To have a closed system of equations, a relation for the subsurface pressure (P_0) in the outside ocean, which implicitly enters in (5), needs to be specified. Here it is assumed that the sea surface outside the Strait of Gibraltar responds isostatically to the subinertial atmospheric pressure forcing, so as to cancel any variation of P_0 due to the atmosphere, i.e., P_0 is equal to a constant taken here to be zero for convenience. With this last assumption we have six equations with six unknowns (i.e., $Q_1, Q_2, \eta_1, \eta_2, P_1$, and P_2) and two forcing functions (Pa_1 and Pa_2). Taking all of them to behave as exp $(i\omega t)$ and approximating spatial derivatives by

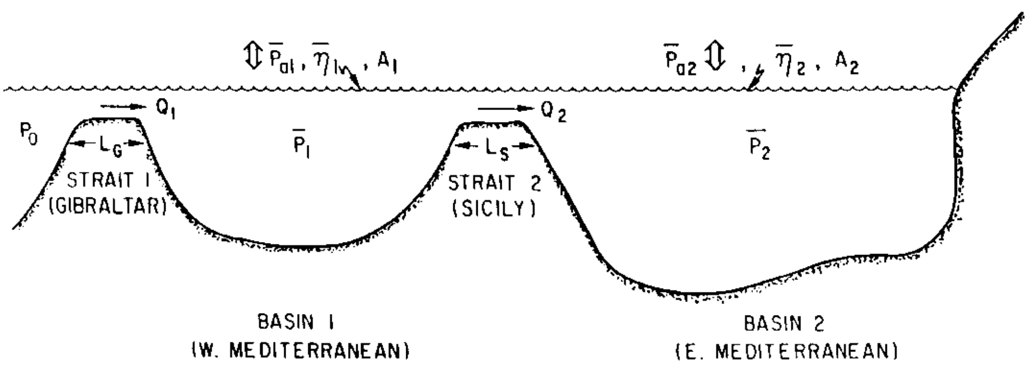

Fig. 9. Sketch of the two basin-two strait model. The variables considered, η_i, P_i, and Pa_i, correspond to uniform sea level, subsurface pressure and atmospheric pressure forcing for basin i. Q_i is the transport through strait i and P_0 corresponds to the subsurface pressure, exterior to the system, in the open ocean side of the first strait. A_i is the surface area of basin i. L_G is the characteristic length of the first strait (Gibraltar) and L_S that of the second strait (Sicily).

TABLE 2. Numerical Values of the Constants Used to Evaluate the Model Results

Value of Constant	Description
$A_1 = 8.646 \times 10^{11}$ m^2	surface area of the western Mediterranean, between the Straits of Gibraltar and Sicily
$A_2 = 1.6703 \times 10^{12}$ m^2	surface area of the eastern Mediterranean without the Black Sea
$A_G = 2.951 \times 10^6$ m^2	minimum cross-sectional area of the Strait of Gibraltar
$A_S = 1.8926 \times 10^7$ m^2	minimum cross-sectional area of the Strait of Sicily
$g = 9.81$ m s^{-2}	acceleration due to gravity
$\rho = 1027$ kg m^{-3}	mean density of seawater
L_G* $= 6 \times 10^4$ m	length of the Strait of Gibraltar
L_S* $= 10^5$ m	length of the Strait of Sicily

*These lengths are not well-defined quantities, but the values given are the ones used for the evaluation of the model results.

finite differences, we can, after some simple algebra, obtain a relation for Q_1 as a function of the forcings given by

$$Q_1 = i\left[\left(\frac{\omega A_1}{\rho g D_0} - \frac{\omega^3 L_S A_2 A_1}{\rho A_S g^2 D_0}\right) Pa_1 + \frac{\omega A_2}{\rho g D_0} Pa_2\right] \quad (7)$$

where the denominator D_0 is given by

$$D_0 = -1 + \left[\frac{L_G A_2}{A_G g} + \frac{L_S A_2}{A_S g} + \frac{L_G A_1}{A_G g}\right]\omega^2 - \frac{L_S L_G A_1 A_2}{A_G A_S g^2}\omega^4 \quad (8)$$

and L_G and L_S are the characteristic length of the first and second straits respectively. Numerical estimates are obtained using the values listed in Table 2.

The model exhibits two resonant frequencies defined by the roots of equation (8). The first occurs at a period near 5.6 days and can be identified with the Helmholtz-type resonance of the Mediterranean communicating with the Atlantic through Gibraltar. The second resonance occurs at a period of 1.2 days, close to the diurnal frequency, and is related to the cooscillation of the western and eastern basins connected through the Strait of Sicily. This second resonant frequency must have important implications on the tidal behavior of the Mediterranean.

Equation (7) indicates that the forcing leads the flow by 90°, as expected for this case in which there is no flow restriction at the straits. If we evaluate the order of magnitude of the flow given by (7) at frequencies away from the two resonant ones, say that related to a 10-day periodicity, we get 1.1 10^6 m^3 s^{-1} (considering that the spatial average for Pa_1 is 5.2 mbar and that for Pa_2 is 3.6 mbar) which is of the same order of magnitude as that of the observed flows.

6. EFFECT OF FRICTION IN THE STRAITS

Intuitively, it is expected that straits limit the amount of flow between adjacent basins. Friction is one obvious mechanism which might constrain the flow. The simplest way to include friction in the dynamical equations is through a linearized parameterization, including a term λQ in equations (5) and (6), where λ is taken as the product of a drag coefficient times a root mean square velocity divided by the depth [*Csanady*, 1976]. *Garrett* [1983] proposed that a different mechanism term "geostrophic control" limits the flow through the strait. This hypothesis [*Toulany and Garrett*, 1984] implies that in straits where $(fW/L) >> \omega, \lambda$ (f is the local Coriolis parameter, W the width of the strait, and L its length) the flow is limited by geostrophy, in the sense that the sea level difference across the strait cannot be greater than the sea level difference between the two ocean basins it connects [*Garrett*, 1983]. The statement of geostrophic control is equivalent to using a linearized friction, but with $\lambda = fW/L$. In any case, the friction term used in equation (9) below can be thought of as being the addition of a true friction plus geostrohic control, when the latter is proved to be dynamically present. The addition of a restraint term λQ in equation (5) is viewed here as a generic form of constraint. The observations described earlier and the results of this simple model are not sufficient to allow for the clear identification of the constraining mechanism.

A further consideration is that, with the values of L_G and L_S used the presence of geostrophic control implies that in Sicily there would be a constraint term at least an order of magnitude larger than at Gibraltar, which does not seem likely. Sicily has a cross-sectional area about 6 times larger than Gibraltar, and it is reasonable to expect that water velocities at all frequencies (except maybe near the diurnal frequency) and frictional effects are smaller there. Also, a recent study by *Wright* [1987] shows that for a strait connecting two basins, both of which are small compared to the external Rossby radius of deformation, which is the case here, geostrophic control can be proven not to hold, if dissipation of cyclonically propagating Kelvin waves in each basin can be neglected.

Maintaining our previous assumption that $P_0 = 0$, the introduction of friction at Gibraltar implies using instead of equation (5) the equation

$$\frac{\partial Q_1}{\partial t} = -\frac{A_G}{\rho}\frac{\partial P}{\partial x} - \lambda Q_1 \quad (9)$$

With this and the rest of our previous equations (1) to (4) and (6), we arrive at a solution which is like (7) but with a different D equal to the sum of the old D_0, given by (8), plus an imaginary part, i.e.,

$$D = D_0 + i\left[-\frac{\lambda L_G(A_1 + A_2)}{A_G g}\omega + \frac{\lambda L_G L_S A_1 A_2}{A_G A_S g^2}\omega^3\right] \quad (10)$$

The results of this model, which does not consider friction at Sicily, are shown in Figure 10. The response of the flows through Gibraltar (Q_1), predicted by the model for three different values of λ are indicated. For comparison, the solid circles indicate the actual measurement results given previously in Figure 8. Although there are some discrepancies, the model seems to reproduce the observed gain and phase well, considering its simplicity.

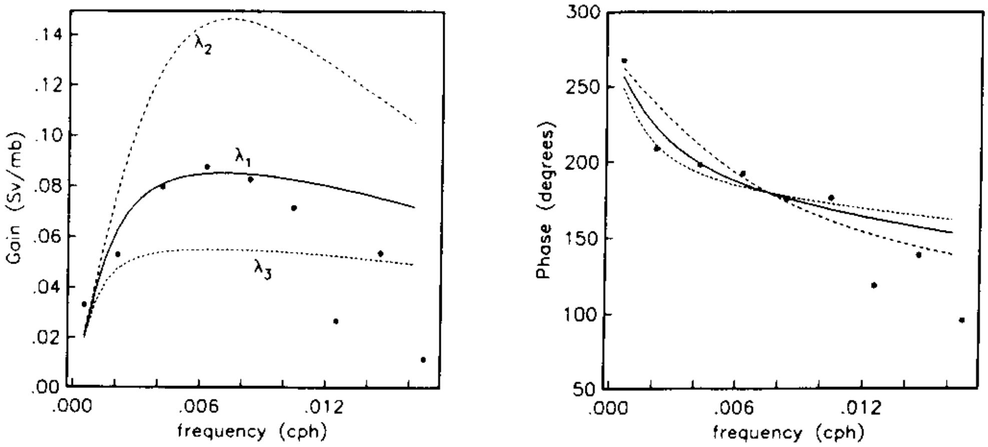

Fig. 10. The response of the flows through the Strait of Gibraltar, obtained from the two basin-two strait model with linearized friction at Gibraltar and forced by the principal atmospheric pressure mode. The results of the model for three different values of λ are shown: $\lambda_1 = 5.17\ 10^{-5}s^{-1}$, $\lambda_2 = 3.0\ 10^{-5}s^{-1}$ and $\lambda_3 = 8.0 \times 10^{-5}s^{-1}$. The dots reproduced, as reference, the observed response at Gibraltar given also in Figure 8.

It is interesting to note that at very low frequencies the model results are quite insensitive to the value of λ used. But the overall best results were obtained using the value of $\lambda_1 = 5.17 \times 10^{-5}\ s^{-1}$. A reasonable value can be argued to be given by $\lambda = C_0U_0/H$; where C_0 is a dimensionless drag coefficient, U_0 a characteristic current velocity and H the hydraulic depth, i.e., the cross-sectional area over the width of the strait. Using $C_0 = 3 \times 10^{-3}$, $U_0 = 0.5$ m s^{-1} and H ~ 120 m, we get λ = 1.27 ×

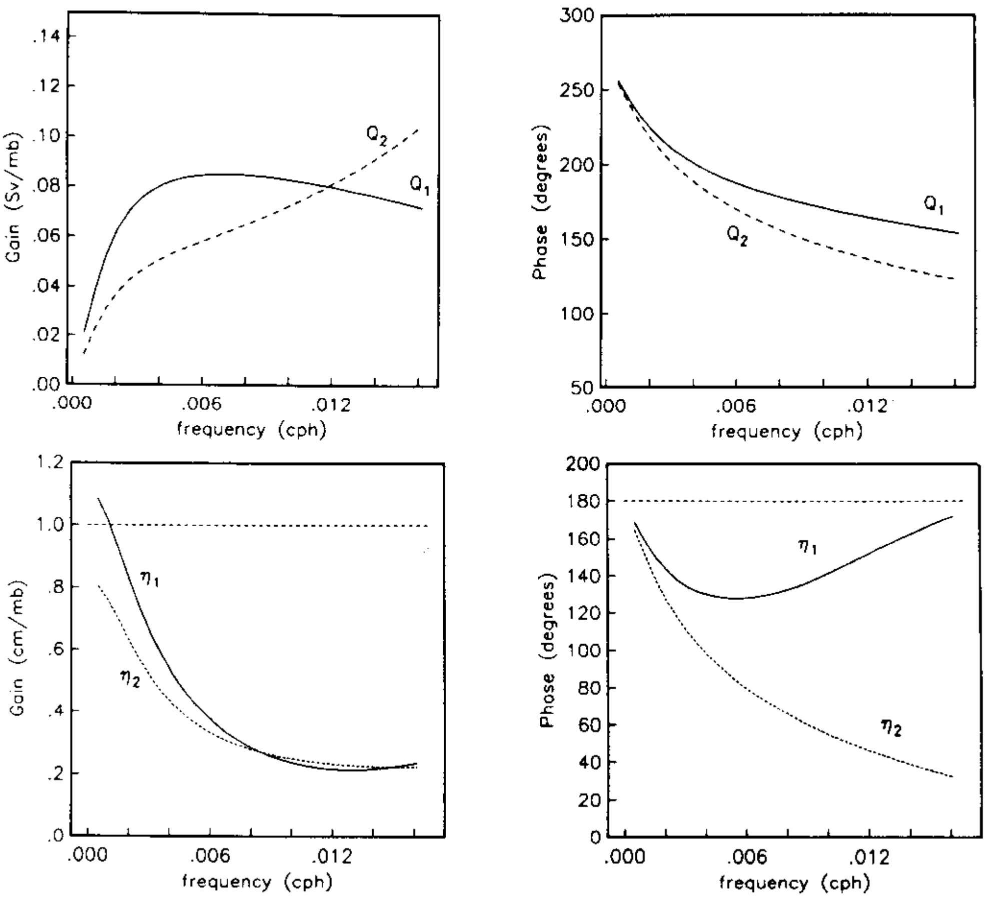

Fig. 11. Model results for the behavior of the flows through the straits (Q_1 and Q_2) and the sea level response in each basin (η_1 and η_2), when forced by the principal atmospheric pressure mode and subject to linearized friction at only the first strait ($\lambda = 5.17\ 10^{-5}s^{-1}$). Q_1 is the response of the flow through Gibraltar (continuous line) also given in the previous figure and reproduced here for reference. Q_2 is the flow response at Sicily (dashed line). η_1 corresponds to the sea level response of the Western Mediterranean (continuous line) and η_2 to that of the Eastern Mediterranean (dashed line).

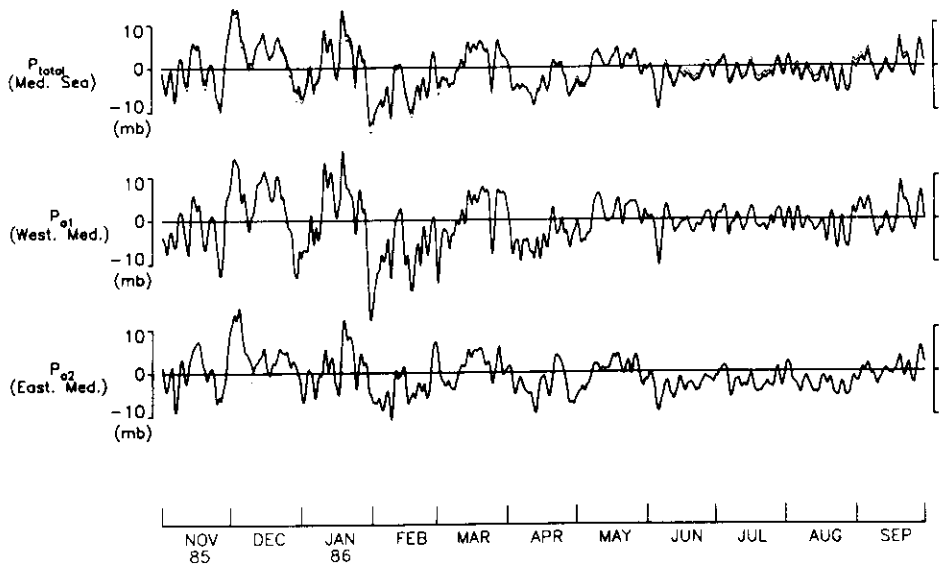

Fig. 12. Time series of average atmospheric pressure in mb over the whole Mediterranean Sea (top plot thick line). The superposed thin line reproduces the time coefficients of mode 1 of atmospheric pressure multiplied by the mean of its spatial weights shown also in Figure 5. The middle plot is the average atmospheric pressure in mb over the Western Mediterranean (Pa_1) and the lower plot that of the Eastern Mediterranean (Pa_2).

10^{-5} s^{-1}. So the value of λ_1 is 4 times larger. One consideration is that if geostrophic control is present at Gibraltar, it would account for a part of λ_1 given by $\frac{1}{2} f\ W/L$ [*Wright*, 1987], though the appropriate value of L is uncertain. Using the model response shown in Figure 10 for λ_1 and taking as input the mode 1 of atmospheric pressure given in the top plot of Figure 6, it is possible to obtain the time domain "predictions" of the transport through Gibraltar which is plotted by the thin line on the bottom plot of Figure 6, superposed to the observed transports (thick line). The agreement is quite good. The correlation coefficient between the observed and model "predicted" transports is 0.76 to the 99% significance level. The value of λ_1 implies strong damping, as indicated by calculating an estimate of the "Q" of the system given by the ratio ω_0/λ_1 (ω_0 being the resonant frequency), which gives a value of ~ ¼.

In Figure 11 the model results for the response of the flows through Sicily (Q_2, dashed line) and for the sea level response of the two basins is given (η_1 for the western and η_2 for the eastern basins, respectively), all for the friction parameter $\lambda_1 = 5.17 \times 10^{-5}$ s^{-1} applied only at Gibraltar. These results for η_1 compare well with observations from Naples reported by *Palumbo and Mazzarella* [1982, Figure 5], but the resemblance can be fortuitous for various reasons. One is that the model results relate to the uniform sea level fluctuations being forced by only part of the atmospheric pressure field. Local wind, atmospheric pressure, and oceanographic conditions can imply differing local sea level response for distinct places within each basin. These can be argued to be the case in the reported sea level response at Katakolon (Greece) by *Garrett and Majaess* [1984, Figure 6], where the results of our model (η_2) do not match their observations. In any event the results given in Figure 11 should constitute an important part of the flows through the straits and sea level response observed at any locality within each basin. Unfortunately there are no direct observations of the flows through Sicily to compare with the curves for Q_2 shown.

7. EFFECTS OF INCOHERENCE IN ATMOSPHERIC PRESSURE OVER THE TWO BASINS

As mentioned earlier in section 3, it is reasonable to expect that the flows through Gibraltar are quite insensitive to small-scale perturbations and incoherences of atmospheric pressure or sea level that occur inside the Mediterranean Sea, since the transports at Gibraltar represent an integrated effect of whatever happens inside the sea. Nevertheless, this is obviously not the case for the sea level response in each of the basins or for the flows through the Strait of Sicily, which should be sensitive to incoherences of atmospheric pressure between the two basins. Strictly speaking, the model proposed in the previous sections uses as forcing functions the mean atmospheric pressures over each basin separately and not only the part that is coherent over the whole sea, which was the forcing used to get the results shown in Figures 10 and 11. This was done partly for simplicity and also because the main interest was on the flows at Gibraltar. In this section the behavior of the model variables is investigated when the actual observed average atmospheric pressures over each basin (Pa_1 and Pa_2) are used as the forcing functions in the model.

Figure 12 shows time series of average atmospheric pressure over the whole Mediterranean Sea (top plot, thick line), over the western Mediterranean only (Pa_1, middle plot), and over the eastern Mediterranean (Pa_2, bottom plot), for a period of 11 months (November 1985 to September 1986). It is clear by comparing the plots for Pa_1 and Pa_2 that there are appreciable discrepancies between the average pressure in each basin. To quantify the degree of coherency, the cross spectra between the two series is calculated and shown in Figure 13. The two series are significantly coherent from the lowest resolvable frequency band centered at periods near 80 days up to periods near 6 days. The results indicated for the lowest resolvable frequency band should be treated with caution, since with the 13 months of data available it is not possible to properly resolve the behavior of the motions in this band. The gain is usually larger than 1.0 for the

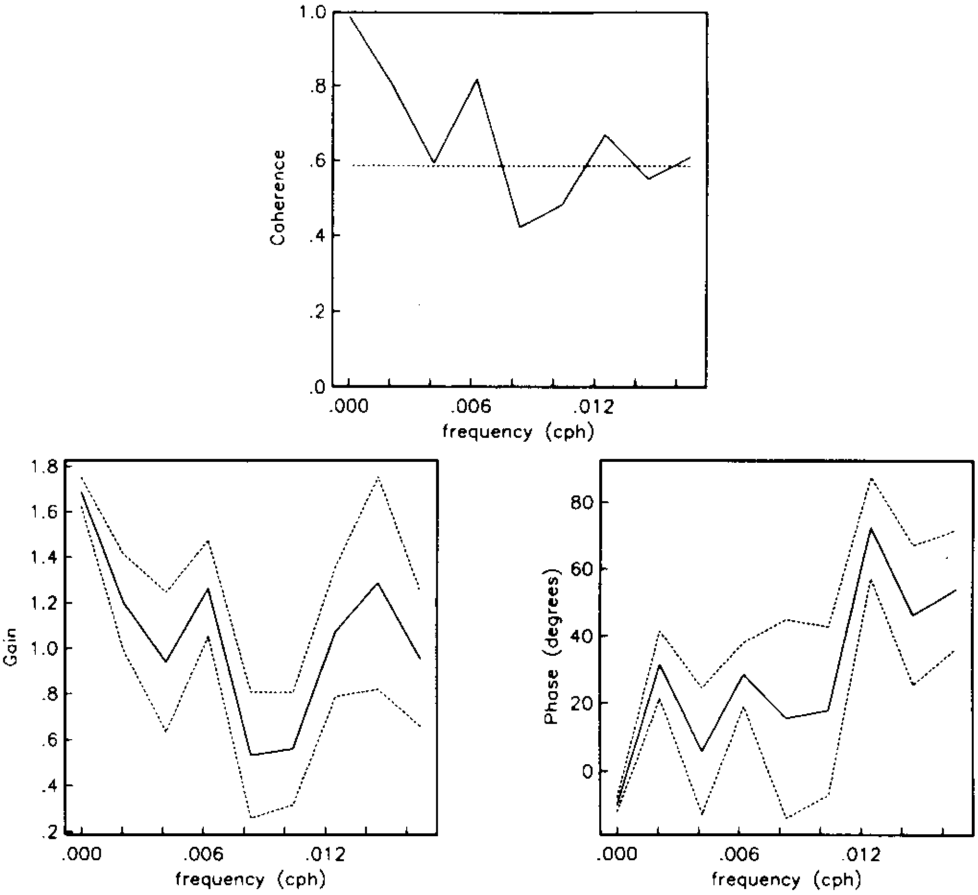

Fig. 13. Coherence, gain, and phase relation (continuous line in each plot) between the mean atmospheric pressure over the Western Mediterranean (Pa_1) and that over the Eastern Mediterranean (Pa_2), obtained from the cross-spectra calculations between middle and lower series shown in Figure 12. The dashed line in the coherence plot indicates the 95% significance level. The gain and phase normalized standard error [*Bendat and Piersol*, 1986, p. 317] are indicated by dashed lines in each respective plot.

bands where the coherence is significant (to the 95% confidence level), indicating more energetic fluctuations over the western basin as is observed. Positive phases are indicative of Pa_2 lagging Pa_1.

A way to compare the observations with the model results would be to perform a multiple regression analysis in the frequency domain, taking Pa_1 and Pa_2 as input series and the observed transports at Gibraltar as output and then to compare the

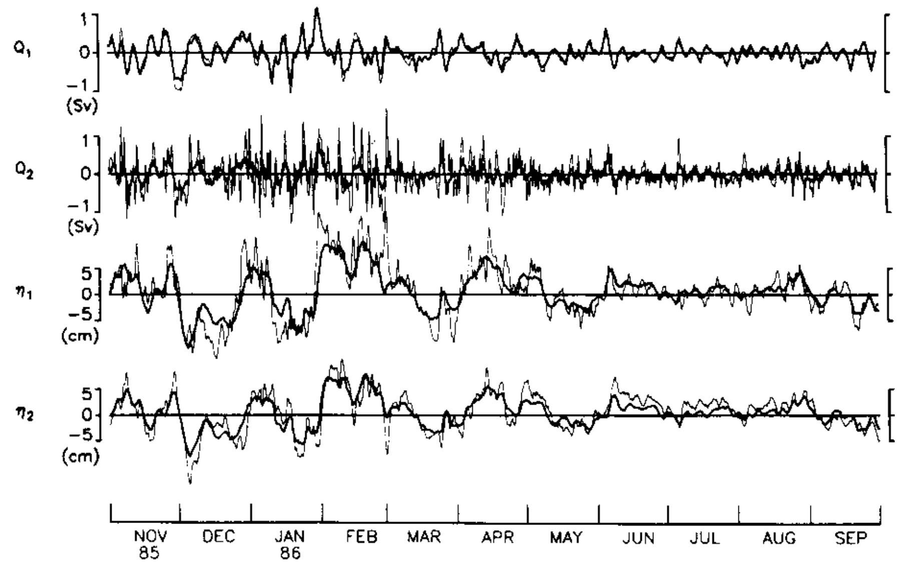

Fig. 14. Model time "predictions" for the variables Q_1, Q_2, η_1, and η_2; when forced by only the coherent mode of atmospheric pressure over the whole Mediterranean Sea (thick line in each respective plot) and when forced by the individual mean atmospheric pressure over each basin (Pa_1 and Pa_2), indicated by the thin lines in each plot. For both cases only friction at Gibraltar is considered, with $\lambda = 5.17\ 10^{-5} s^{-1}$.

results of this analysis with the model coefficients for Pa_1 and Pa_2 on equation (7) using D given by (10) instead of D_0. In practice, however, this procedure is not possible due to the fact that the two input series (Pa_1 and Pa_2) are too well correlated to represent appropriate input series on a multiple regression analysis. Nevertheless, using equation (7) with the appropriate D given by (10) and the Fourier transforms of Pa_1 and Pa_2, it is possible to perform a Fourier synthesis and obtain "predictions" for the four relevant variables of the model, i.e., Q_1, Q_2, η_1, and η_2. These are plotted in Figure 14 (thin line) superposed to the model "predictions" when only the coherent part of the pressure field over the Mediterranean is used, i.e., responses from Figure 11 and first mode of atmospheric pressure given in the top plot of Figure 12 used as a forcing function. It is observed that the flows at Gibraltar (Q_1) are only slightly modified when Pa_1 and Pa_2 are used instead of only the first mode of atmospheric pressure or the average atmospheric pressure over the whole sea. But is is clear that the other variables are appreciably modified, and their "predicted" series when forced by Pa_1 and Pa_2 should follow more closely their actual behavior.

A further effort was made to develop a model that also considered friction at Sicily, but its inclusion did not improve the resemblance with the observations of the flows at Gibraltar. This result suggests that the Strait of Sicily does not impose any important restriction to the water exchange between eastern and western Mediterranean basins. However, including friction at Sicily does reduce some of the high frequency fluctuations present in the Q_2 plots of Figure 14, which might be more realistic.

8. CONCLUSIONS

The observations analyzed in this paper confirm earlier work [*Crépon*, 1965; *Garrett*, 1983] supporting the idea that the principal forcing for the subinertial flows through the Strait of Gibraltar is the uniform atmospheric pressure field over the Mediterranean. The simple analytical model proposed indicates that as a first-order approximation the system behaves like a forced simple oscillator subject to some friction. However, further work needs to be done in clarifying the physics behind the constraint term used in the model.

With a practical predictive objective in mind, a possible next step is attempting a parameterization of the model, for example, making λ a function of frequency. Due to the uncertainties in the available data, principally in the estimate of the flow through Gibraltar, it may be more appropriate to seek additional independent measurements and then do a more complete data inversion. For example, sea level observations from one or both basins and flow measurements at Sicily could be obtained simultaneously to determine the values of appropriately chosen relevant parameters.

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