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AUTUMN COURSE ON MATHEMATICAL ECOLOGY

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DYNAMICS OF INTERACTING AGE STRUCTURED POPULATIONS

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Dynamics of Interacting Age Structured Populations

I. Biological Examples
II. Model formulation- brute force
    A. Leslie Matrix approach
    B. von Foerster approach
III. Results from detailed models
    A. Simple examples
        1. effect on equilibria and isoclines
        2. effect on stability
        3. effect on dynamics, periodic solutions
    B. Simulation of complex models
        1. summary of results
        2. relationship to analytic treatments
IV. Simple approaches incorporating time delays
    A. Various simplification approaches
        1. Assumptions about birth
        2. Assumptions about death
        3. Assumptions about maturity
        4. Stochastic nature of models
    B. Methods of anlysis
    C. Results
        1. effect on equilibria
        2. effect on stability
        3. effect on dynamics
V. Conclusions
    A. Are there common themes ?
    B. How much complexity is needed in models ?
    C. Unanswered questions

The Problem:

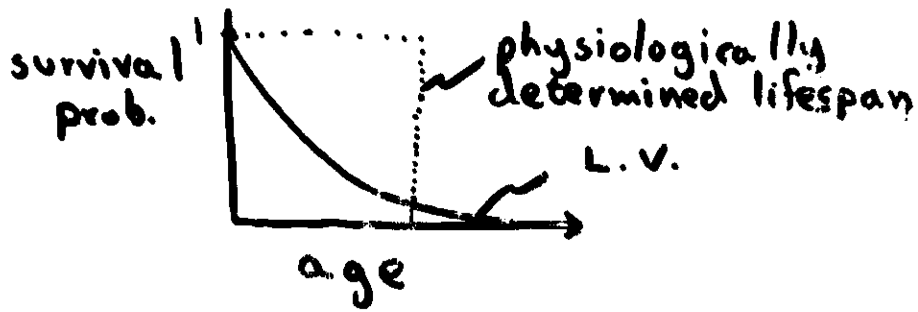
Role of age or size in interactions between populations

I. Biological Examples

A. Amphibians - young compete with young, adults with adults

B. Competition among sunfish of different size (Werner, 1977)

C. Predators whose age structure deviates from the Lotka-Volterra ideal, and those with significant delays in 'conversion' of prey killed.



D. Predators which eat prey discrimately.

Moose eaten by wolves on Isle Royale

E. Cannabilism

F. Competition between age classes.

II. Model Formulation - Brute Force.

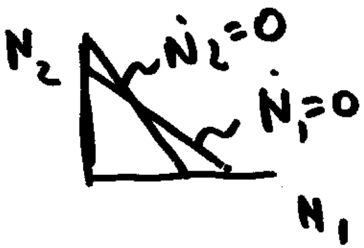
Use Leslie matrices or von Foerster (PDE) models for one or both species.

Large number of parameters
Very Detailed
Deterministic

First - a simple example

(Hassell & Comins, 1976)

Discrete time competition with 2 age classes.



$$X_{t+1} = x_t \exp[-a(x_t + \alpha y_t)]$$

$$Y_{t+1} = y_t \exp[-c(y_t + \beta x_t)]$$

$$x_{t+1} = X_t \exp[r - a'(X_t + \underline{\alpha}' Y_t)]$$

$$y_{t+1} = Y_t \exp[s - c'(Y_t + \underline{\beta}' X_t)]$$

x, y = # in larval stage
X, Y = # in Adult stage

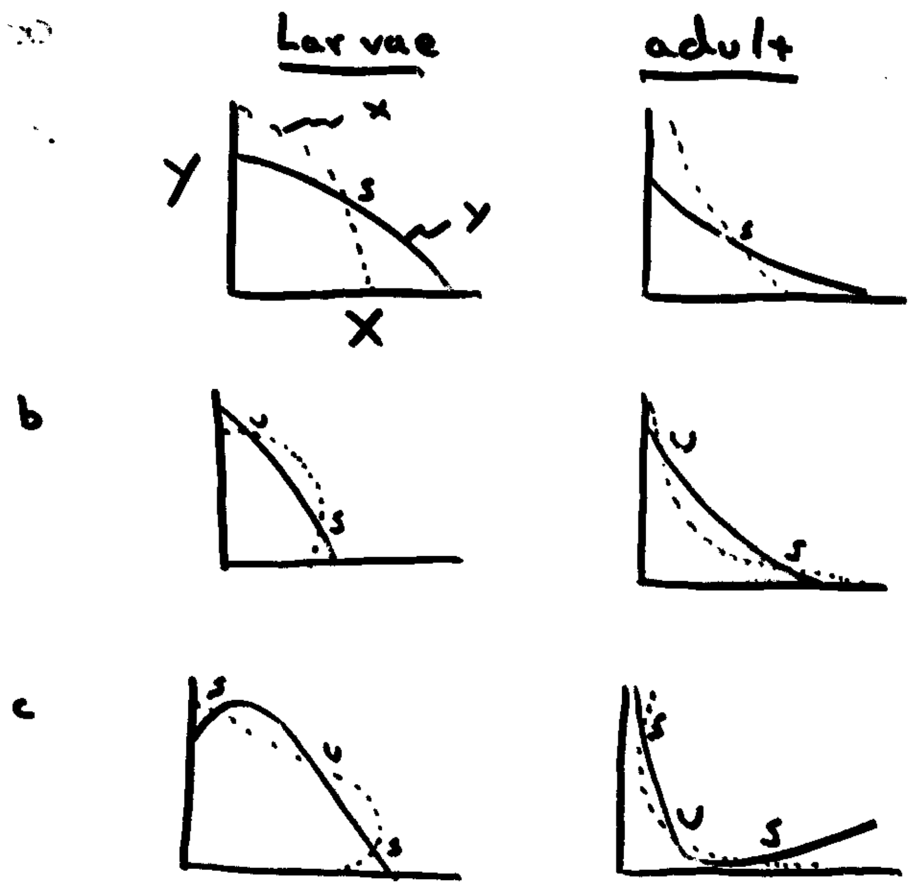
Possible zero growth curves for the case where $\alpha' = \beta' = 0$.

$$X_{t+1} = X_t [\qquad\qquad]$$

$\tau = 1$

$$y_{t+1} = y_t [\qquad\qquad]$$

census time



Multiple equilibria

Non linear isoclines · Gilpin & Ayala

A von Foerster type approach

(Auslander, Oster, & Huffaker, 1974)

Parasite-host interaction

Adult parasites lay eggs in early instars of host.

Let $p(a,t)$ be a density function on age a for number of parasites.

Let $h(a,t)$ be a density function for number of hosts.

$$p_t + p_a = -\mu^p(t,a)\,p \qquad 0 \le a \le \gamma$$

$$h_t + h_a = -\mu^h(t, a, H_t, \tilde{H}_0, \tilde{P})\,h$$

$H = \int_0^\infty h(a,t)\,da$ # adult hosts

$$\left[\tilde{H}_0 \equiv \int_\beta^{\beta+\delta} h(a,t)\,da,\right]$$

hosts with $\beta < a < \beta+\delta$ are parasitized

$$\tilde{P} = \int_\alpha^{\alpha+\gamma} p(a,t)\,da$$

Parasites with $\alpha < a < \alpha + \gamma$ are adults

$p(0,t)$; $h(0,t) =$

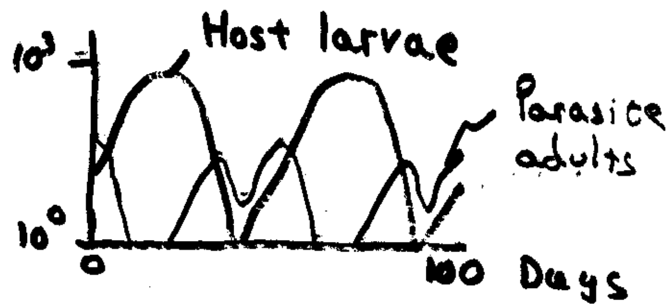
SIMPLIFY BY CHOOSING FUNCTIONAL FORMS - SIMULATE

Results of simulation:

Depending on parameters, 2 types of solutions

1) Equilibrium, all ages of parasite & host represented

2) "Discrete generation model"



one or the other of 1 or 2.

An increase or a decrease in searching efficiency of parasite (habitat heterogeneity)

increase – Discrete generation into equilibrium — extinction

decrease – Parasite population size decreases, persistence unaffected.

Justifies use of discrete time models.

Can we understand this behavior in a simpler model?

A simple, analytically tractable model (Beddington & Free)

2 age classes of predator & one of prey

$$N_{t+1} = N_t\left(1 + r(1 - N_t/k) - a_1 P_t^1 - a_2 P_t^2\right)$$

$$P^1_{t+1} = \alpha a_2 N_t P_t^2$$

$$P^2_{t+1} = s P_t^1$$

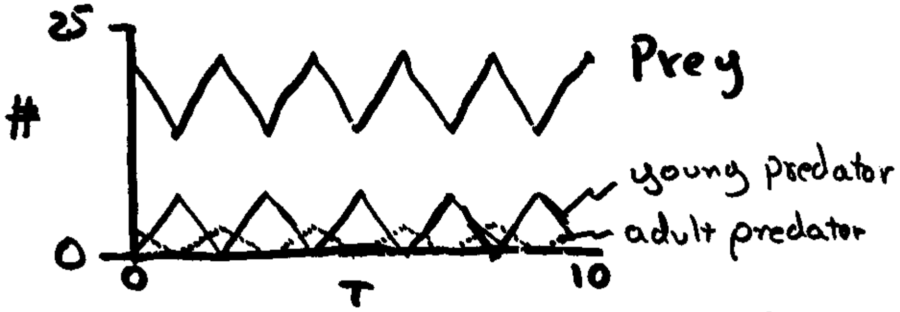
can find eq., and stability conditions:

2 conditions - a complicated one, &

$$1 > \theta \equiv a_2 s / a_1$$

As in previous model, increase a_2, equilibrium solution unstable.

Can show there exists a stable 2 point cycle.



$r=4$, $q=0.55$, $\theta=1.08$ $K=40$

Also, solutions where P^1, P^2 do not dip to zero are possible.

Other ways in which age structure affects predator-prey systems.

1) Maturity rate (or time) for predator in cont. time models - Volterra 1930's (Cushing & Saleem, 1982)

2) Departure from Lotka-Volterra survivorship curve (Hastings & Wollkind)

$$\frac{dP}{da} = -kP$$

$$\frac{dH}{dt} = \underline{Hg(H)} - \underline{Pf(H)}$$

$$\frac{\partial p}{\partial t} + \frac{\partial p}{\partial a} = -\mu(a,t)p$$

$$p(0,t) = cPf(H)$$

$$P = \int_0^\infty p(a,t)\,da$$

assume $\mu = 0$ $\quad a < T$
and all pred. die at T

$$\begin{cases} \frac{\partial p}{\partial t} + \frac{\partial p}{\partial a} = 0 & 0 < a < T \\ P = \int_0^T p(a,t)\,da \\ p(0,t) = cPf(H) \end{cases}$$

solve:

$$P(t) = \int_{t-T}^{t} \underline{cP(s)f(H(s))}\,ds$$

$$\frac{dH}{dt} = Hg(H) - Pf(H)$$

examine stability: $\frac{dP}{dt} = \quad -$

markedly different
less stable

3) Age dependent predation

Smith & Mead
Hastings
Gurtin & Levine
Important effects possible

4)
$$\left.\begin{aligned} \frac{dH}{dt} &= rH(t-T_1) - aPH \\ \frac{dP}{dt} &= cPH(t-T_2) - dP \end{aligned}\right]$$

Conclusions:

Both analytic & simulation studies necessary –

goal find appropriate simple models

theme age structure different from other forms of structure (space, genetic) because of role of time dependence, leading to periodic solutions, and stability depending on 'competing' time delays

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