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Lecture I

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

2 June - 4 July 1997

NEUTRINO PHYSICS

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ICTP Summer School
June '97

S. Pakvasa — Neutrinos.

Tentative Schedule

I. Neutrino Mass in SM & beyond
Neutrino Mixing & Oscillation
Neutrino Mass Experiments

II. Neutrino Interactions & x-sections
Atmospheric Neutrinos

III. Solar Neutrinos

IV. LSND & Accel. ν's
SuperNova Neutrinos
High Energy ν-Astronomy
Neutrinos & Cosmology.

Useful References

- M. Fukugita & T. Yanagida
  in Neutrino Astrophysics
  Springer 1994
  (ed. A. Suzuki & M.F.)

- C. W. Kim & A. Pevsner,
  Neutrino Physics (1995).

- F. Boehm & P. Vogel.
  Physics of Massive
  Neutrinos (1991?)

- Ed. K. Winter
  (Collection of original papers)
  → Conf. Proceedings.
  1972 — 1996.

Neutrino Masses & Mixings

- who cares? Why?

  → fundamental parameters

- $m_\nu = 0$?
  or $m_\nu \neq 0$?

  → if $\sim O(eV)$ profound impact on cosmology & large scale structure

  → only differen bet. families u, c, t; d, s, b, e, μ, τ is <u>MASS</u>. why ν_i should be diff.

Why ν properties are interesting?

"SM" particle content:

Leptons $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad e_R, \mu_R, \tau_R$

Quarks $\begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \begin{pmatrix} c_i \\ s_i \end{pmatrix}_L \begin{pmatrix} t_i \\ b_i \end{pmatrix}_L \quad u_{iR}, d_{iR}$ etc.

weak currents coupling to $W_\mu^\pm$:

Quarks: $\overline{(u\ c\ t)}_L \gamma_\mu U_{KM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$

Leptons: $\overline{(\nu_e \nu_\mu \nu_\tau)}_L \gamma_\mu \underline{1} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L$

Any evidence for $m_{\nu_i} \neq 0$, $\theta_i \neq 0$

$\mu_{\nu_i} \neq 0$

points to new physics (beyond SM).

Theoretical Prejudice: • $m_i \neq 0$ • $\theta_i \neq 0$ • Lepton # not conserved $\Rightarrow$ Majorana ν's

Neutrino Mass. Why interesting?

1) Fundamental Parameter.
Is it 0? few eV? 10^{-7} eV? ....

2) $\hookrightarrow$ 0?

3) If few eV. $\Rightarrow$ useful for "closing" the universe; account for the "missing mass" in Galactic Clusters etc.

4) Now we know only difference between "flavors" e.g. d, s, b or u, c, t ; e, μ, τ is masses. Hence expect ν_e, ν_μ, ν_τ to have different masses.

We are here primarily interested in ν_e. (Not Really!)

What about $m_\nu \equiv 0$?

(1) There ~~is no~~ does not seem to be any fundamental reason for m_ν to be 0 as for m_γ or m_{gluon} or graviton (General Covariance) (Gauge invariance).

(2) Only other massless (besides gauge quanta) particles are Nambu-Goldstone particles. due to spontaneous breaking of Global symmetries. If ν were such a particle then it would obey soft-ν theorems - analogues of soft-π theorems. Hence

$$\lim_{q \to 0} A(\nu) \equiv 0$$

all ν couplings vanish as

$q_\nu \to 0$. Phenomenologically this implies

(a) In β-decay mimics $\kappa \propto (Q-T)^2$ a

~~non~~-zero m_ν !! But expect a bigger effect than few eV. NOT SEEN.

Kurie Plot NOT ST. LINE!!

(b) In neutral currents reactions

$\nu A \longrightarrow \nu X$.

expect (1) matrix element $\propto J^{e.m.}_{I=1}$

(2) $\sigma \propto \alpha G_F^2$

$\Big\}$ as $q_\nu \to 0$

NOT SEEN.

Hence ν is not massless by being a Nambu-Goldstone particle. [What about Chiral Symmetry?

Yes, what about it !

Laboratory:

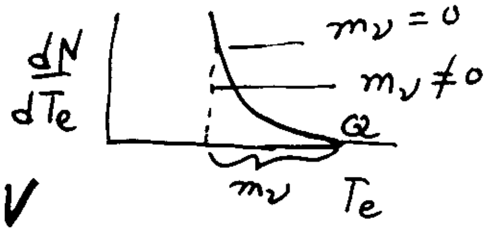
Troitszk

$m_{\bar{\nu}_e} < 4.5$ eV 95% C.L.

β-decay end point : $n \to p e^- \bar{\nu}_e$

$dN/dT_e \sim (Q-T)^2$ $m_\nu = 0$

$\sim \sqrt{(Q-T)^2 - m_\nu^2}$



160 KeV

$m_{\nu_\mu} <$ 95% C.L.

{energy-mmtm cons. in $\pi \to \mu + \nu_\mu$}

$m_{\nu_\tau} < 27$ MeV (BEPC)

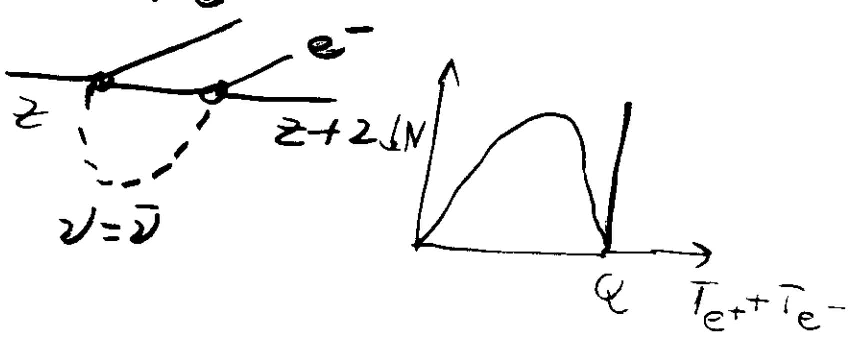
{end point in $\tau \to 5\pi + \nu_\tau$}

0.6 eV

$\langle m_{\nu_e}^{eff} \rangle <$ 90% C.L.

(Double Beta Decay)

of e^- ^{76}Ge



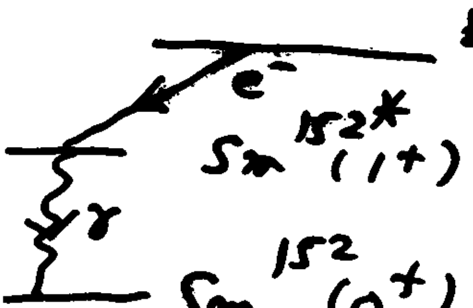
$\mu_\nu^e < 4 \cdot 10^{-10} \mu_B$

[$\bar{\nu} e \to \bar{\nu} e$]

(Proposal)

How do we know ν_e is Left-Handed?

Goldhaber, Grodzins, Sunyar (1957).
[Beautiful Experiment]
K-capture of Eu^{152} (G.T.) $\Delta J = 1$.



$$e^- + Eu^{152} \longrightarrow Sm^{*152} + \nu$$
$$Sm^* \longrightarrow Sm^{152} + \gamma$$

Finally: $\underset{(0^+)}{e^- + Eu^{152}} \longrightarrow \nu_e + \underset{(0^+)}{Sm^{152}} + \gamma$

- Using Resonant Scattering detect only antill to ν i.e. γ has the extra recoil energy to give $Sm \longrightarrow Sm^*$. Kinematics :

$\underbrace{e^- + Eu}_{\text{Both nearly at rest}} \longrightarrow$ $\overset{\nu}{\longleftarrow} \bullet \overset{\gamma\text{-Ray}}{\longrightarrow}$; $Sm^{152} \longleftarrow$

$$p_\nu + p_{Sm} = p_\gamma$$

Quantizing along p ~~as the z-axis~~:

$$J_z = L_z + S_z \quad \text{but } L_z = 0.$$
$$= S_z.$$

Initially $S_z = S_z^e = \pm\frac{1}{2}$

Finally $S_z = S_z^\nu + S_z^\gamma$

To get $\pm\frac{1}{2}$: when $S_z^\nu = +\frac{1}{2}$, $S_\gamma = -1$

& $S_z^\nu = -\frac{1}{2}$, $S_\gamma = +1$

Both possible. But by measuring circ. pol. of γ, they found that on RHP i.e. $S_\gamma = +1$

Hence $S_\nu = -\frac{1}{2}$ ERGO.

To produce resonance scattering, the γ-ray energy must slightly exceed the 960 keV to allow for the nuclear recoil. It is precisely the "forward" γ-rays, carrying with them a part of the neutrino-recoil momentum, which are able to do this, and which are therefore automatically selected by the resonance scattering.

(iv) The last step is to determine the polarization sense of the γ-rays. To do this, they were made to pass through magnetized iron before impinging on the ^{152}Sm absorber. An electron in the iron with spin σ_e opposite to that of the photon can absorb the unit of angular momentum by spin-flip; if the spin is parallel it cannot. This is indicated in Fig. 6.8(d). If the γ-ray beam is in the same direction as the field **B**, the transmission of the iron is greater for left-handed γ-rays then for right-handed.

A schematic diagram of the apparatus is shown in Fig. 6.9. By reversing **B** the sense of polarization could be determined from the change in counting rate.

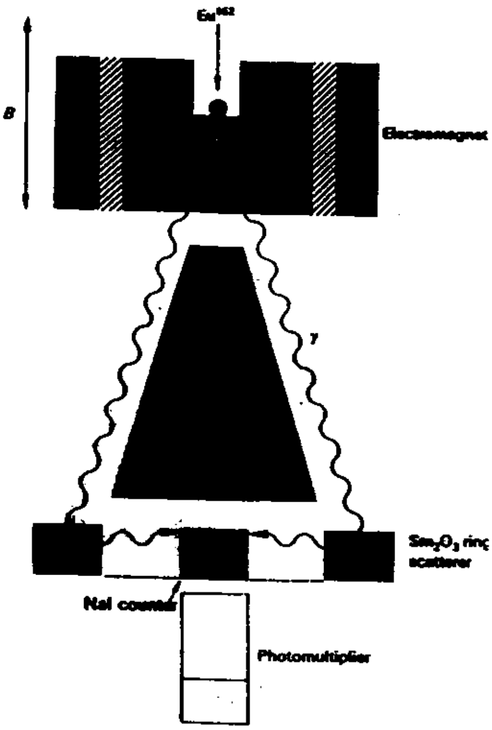


Fig. 6.9 Schematic diagram of apparatus used by Goldhaber *et al.* in which γ-rays from the decay of $^{152}Sm^*$, produced following K-capture in ^{152}Eu, undergo resonance scattering in Sm_2O_3, and are recorded by a sodium iodide scintillator and photomultiplier. The transmission of photons through the iron surrounding the ^{152}Eu source depends on their helicity and the direction of the magnetic field **B**.

Coherence Conditions for Oscillations

- We assumed plane waves for ν_1 & ν_2 with $\underline{p}_1 = \underline{p}_2$ but $E_1 \neq E_2$

  Is this justified?

- If one uses a more rigorous, wave packet treatment, same final result. {Kayser (1980)}

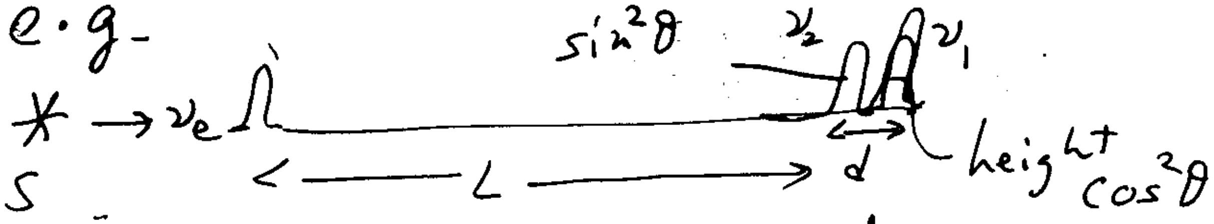
  [But unlike usual beam preparation with slits etc. e.g. in Stern-Gerlach or light, not clear what packet is produced in e.g. $n \rightarrow p e \bar{\nu}_e$ or $\pi \rightarrow \mu \nu_\mu$?]

Okun et al. 1981 Sov JNP

- If as the ν travels distance L from source to detector, the ~~wave packet~~ two mass eigenstates separate [due to mass difference they travel at different speeds]

  If this separation is too big no

e.g.



The separation d is given by

$$d = \frac{1}{2}\frac{\cancel{(\delta m c^2)^2}}{E_\nu^2} L \quad (m_1^2 - m_2^2)c^4$$

For oscillations to occur d must be less than width of the wave packet $\Delta X(L)$.

$$d < \Delta X(L) \simeq \Delta X_0$$

(if spreading negligible).

If $d > \Delta X$ then two pulses arrive with relative intensities $\cos^2\theta$ & $\sin^2\theta$ separated in time by d/c.

$\Gamma_i(m\nu_i)$

- Kinematics
- e, μ, τ oscillations? (No! Okun et al. 1997)

Kinds of ~~Neutrino~~ ~~Masses~~

- Weyl Fields (massless).

2-comp

$$\psi_L \xrightarrow[\text{anti}]{} \psi_L^c \equiv \chi_R$$

$$\phi_R \xrightarrow[\text{anti}]{} \phi_R^c \equiv \xi_L$$

(mass term $L \longleftrightarrow R$).

- ~~Da~~ Majorana Mass (self conjugate)

2-comp.

$$m_L \, \overline{\psi_L} \chi_R + h.c.$$

$$m_R \, \overline{\phi_R} \xi_L + h.c.$$

violate lepton # by 2.

- Dirac Mass.

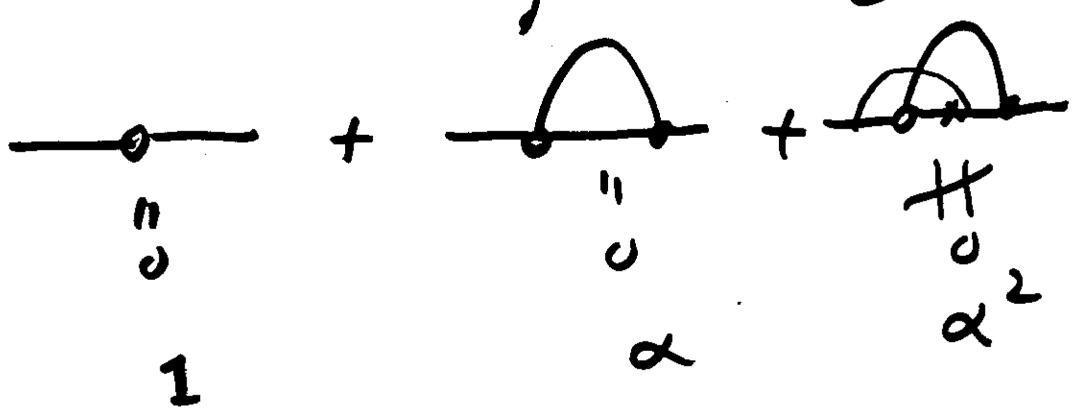
4-comp.

$$m_D \left[\overline{\psi_L} \phi_R + \overline{\phi_R} \psi_L \right]$$

conserves lepton #.

(i) Arrange the theory so that at tree level $m_{\nu_e} = 0$. Also at one loop level $m_{\nu_e} = 0$. But maybe at 2-loops $m_{\nu_e} \neq 0$

$m_{\nu_e} \sim$



Then expect $m_{\nu_e} \sim \alpha^2 (\,) m_e$

$\approx eV$!!

It can be either Dirac or Majorana.

(Ma)

(ii) "See-Saw" [redacted]

In general case when both Dirac & Majorana mass terms present: 2×2 mass matrix:

$$\overline{(\psi_L \ \ \xi_L)} \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \chi_R \\ \phi_R \end{pmatrix}$$

$\longrightarrow$ e.v. m_1, m_2

e. states. χ_1, χ_2 $\longrightarrow$ Majorana

if $m_R \gg m_D \gg m_L$

e. values are

$$m_1 \approx m_D^2/m_R$$

$$m_2 \approx m_R.$$

[redacted]

When $m_1 = m_2 = m$
$\psi = \chi_1 + \chi_2$
Dirac Case

Standard Electro-weak The[ory]

$SU(2) \times U(1)$. Fermions assigned

$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad e_R$.

Electron mass : $m_e \underbrace{(\bar{e}_L e_R + \bar{e}_R e_L)}_{I = 1/2}$

breaks $SU(2) \times U(1)$.

$\underline{\underline{\nu_e}}$

(1) If no ν_{eR} then no Dirac mass. $(m_{\nu_e} \bar{\nu}_{e_L} \nu_{eR} - \cdots)$. [exactly]

(2) If no $I = 1$ mass terms (i.e. $m \underbrace{\overline{\nu_{e_L}^c} \nu_{e_L}}_{I=1} + \cdots$) then no Majorana mass terms.

(1) & (2) $\Longrightarrow$ $m_{\nu_e} = 0$.

However, in general, $I = 1$ mass possible, then expect $m_{\nu_e} \sim m_e^2 / M$

and ↳ Majorana

and $\underline{m_1 \ll}$ [redacted]

So m_ν can be much smaller than $m_D \sim m_e, m_{q_i}$ etc.

In this case

(i) ν's are Majorana not Dirac.

(ii) Expect

$$m_{\nu_e} \sim \frac{m_e^2}{m_R} \text{ or } \frac{m_u^2}{m_R} \text{ etc}$$

$$m_{\nu_\mu} \sim m_\mu^2/m_R, \text{ etc.}$$

$$m_{\nu_\tau} \sim m_\tau^2/m_R$$

& $m_{\nu_e}/m_{\nu_\mu}/m_{\nu_\tau} \sim m_e^2/m_\mu^2/m_\tau^2 \ldots$

Standard GUTS.. $SU(5)$

Fermions assigned in $\underline{5} \oplus \underline{10}$

$$\begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ \nu_e \end{pmatrix}_L \oplus \begin{pmatrix} (u,d)^i_L \\ u^c_{i_L} \\ e^+_L \end{pmatrix}$$

(no ν_{e_R})

Hence no "room" for ν_{eR}. and again ① $m_{\nu_e}^{Dirac} \equiv 0$

② With standard way of breaking $SU(5)$ no $I=1$ masses so $m_{\nu_e}^{Majorana} = 0$.

Again, as in $SU(2) \times U(1)$, possible to induce $I=1$ mass term.

$$m_{\nu_e}^{Majorana} \sim m_e^2/M.$$

ut now $M \sim m_x \sim 10^{14}$ GeV. $\longrightarrow m_{\nu_e} \lesssim 10^{-11}$ eV!!

Different Bigger GUTS: $SO(10)$

Fermions assigned in $\underline{16}$

under $SU(5)$ $\quad \underline{16} \longrightarrow \underline{\bar{10}} \oplus \underline{5} \oplus \underline{1}$

ν_{eL} ν_{eR}

1) So $SO(10)$ has both ν_{eL} & ν_{eR} !

2) Dirac mass of ν_e related to m_u

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L e_R \ \nu_{eR} \longleftrightarrow \begin{pmatrix} u \\ d \end{pmatrix}_L d_R, u_R .$$

$$\text{So } \quad m_{\nu_e}^{\text{Dirac}} = m_u \sim \text{few MeV.}$$

3) No $I=1$ Majorana mass for ν_{eL}.

4) But large $\underline{I=0}$ Majorana mass for ν_{eR}. $\quad M \overline{\nu^c_{eR}} \nu_{eR}$

Precise value of M depends on details of symmetry breaking, can be as large ...

ν-mass matrix in SO(10) is:

$$(\bar{\chi}_R \;\; \bar{\nu}_{eR}) \begin{pmatrix} 0 & m_u \\ m_u & M \end{pmatrix} \begin{pmatrix} \nu_{eL} \\ \chi_L \end{pmatrix}$$

later

e.values are $\lambda_1 \cong -\dfrac{m_u^2}{M}$, $\chi_{1L} \sim \nu_{eL} + \varepsilon \chi_L$

$m_1 = |\lambda_1|$ $\begin{Bmatrix} \chi_{1L} = \nu_{eL} + \varepsilon \chi_L \\ \chi_{2L} = \chi_L - \varepsilon \nu_{eL} \end{Bmatrix}$ $\lambda_2 \cong M + \dfrac{m_u^2}{M}$. $\chi_{2L} \sim \chi_L - \varepsilon \nu_{eL}$

mixing ~ $\underline{\varepsilon \sim m_u/M}$.

Depending on precise value of M,

m_1 can be 10^{-5} eV | 10^{10} GeV

$\boxed{1 \text{ eV} \mid 10^5 \text{ GeV}}$ unlikely

<u>Summarise</u>

In Electro-weak theory or GUTS ν_e is essentially massless or the mass is ~~very~~ <u>very small</u> (precise value not predicted) and it is <u>Majorana</u>

Some Scenarios

(1) No $I=0$ ν_R exists.

Only $I=1$ m_L allowed.

e.g. G-R model with Majoron.

m_{ν_i} arbitrary. $\boxed{g_{\nu_i \nu_i M} \quad \nu_L \nu_L M}$

or $I=1$ induced higher order.

(2) ν_R exists [SO(10), E_6 ...]

(a) $m_L(I=1)=0$, $m_R(I=0) \neq 0$

CMP etc. <u>usual See-Saw</u>

(b) $m_L \neq 0$, $m_R \neq 0$.

General Case.

m_1, m_2 may be close!

θ " " large.

6×6 mixing for 3 flavors.

In Lepton Sector, identical - - -
(For simplicity assume Dirac mass for ν's).

Then.

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L = U_\nu^\dagger \begin{pmatrix} \nu_{e0} \\ \nu_{\mu 0} \\ \nu_{\tau 0} \end{pmatrix} \qquad \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L = U_\ell^\dagger \begin{pmatrix} e_0 \\ \mu_0 \\ \tau_0 \end{pmatrix}_L$$

Leptonic Charged Weak Current is

$$J_\mu^c = \overline{(\nu_1\, \nu_2\, \nu_3)}_L \gamma_\mu U \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}.$$

where $U = U_\nu U_\ell^\dagger$ = 3×3 unitary matrix

= 3 angles + 1 phase.

In the limit of 2 flavors.

$$J_\mu^c = \overline{(\nu_1\, \nu_2)}_L \gamma_\mu \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e \\ \mu \end{pmatrix}_L$$

ν_1, ν_2, e, μ are mass e.states with masses m_1, m_2, m_e, m_μ.

It is convinient to define effective flavor eigenstates by writing the current as

$$J_\mu^c = \overline{(\nu_e \; \nu_\mu)}_L \gamma_\mu \begin{pmatrix} e \\ \mu \end{pmatrix}_L$$

Then.
$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}.$$

or.
$$\nu_e = \cos\theta\, \nu_1 + \sin\theta\, \nu_2$$
$$\nu_\mu = -\sin\theta\, \nu_1 + \cos\theta\, \nu_2 .$$

The state produced with $(\mu)e$ in a weak CC process (β-decay) is (ν_μ) "ν_e". (With some caveats!)

ν Mixing & Oscillations (Flavor)

If $m_{\nu_i} \neq 0$ & no degeneracy

then weak e·states $\neq$ mass e·states

(e.g. $K_L, K_s) \neq (K_0, \bar{K}_0)$. (in general)

$$\nu_e = \cos\theta\, \nu_1 + \sin\theta\, \nu_2 \qquad (m_2 > m_1)$$

$$\nu_\mu = -\sin\theta\, \nu_1 + \cos\theta\, \nu_2$$

If $\Psi(0) = \nu_e = c\nu_1 + s\nu_2$

$$\Psi(t) = e^{-iEt/\hbar}\left[c\nu_1 e^{-i m_1^2 t/2E} + s\nu_2 e^{-i\frac{m_2^2}{2E}t}\right]$$

$$\left.\begin{array}{l} E \gg m \\ E_i = E + \dfrac{m_i^2}{2E} \\ E \sim p \end{array}\right\} \quad A(\nu_e, t) = \langle \nu_e | \Psi(t) \rangle$$

$$= e^{-iEt/\hbar}\left[c^2 e^{-i\frac{m_1^2}{2E}t} + s^2 e^{-i\frac{m_2^2}{2E}t}\right]$$

Survival Probability

$(t = L/c)$ $\quad P(\nu_e, t) = |A|^2 = \left[1 - \sin^2 2\theta \sin^2 \dfrac{\delta m^2 L}{4E}\right]$

Appearance Prob.

$$P(\nu_\mu, t) = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}.$$

If $\dfrac{\delta m^2 L}{4E} \gg 1$, $\quad \left\langle \sin^2 \dfrac{\delta m^2 L}{4E} \right\rangle \approx 1/2$

Non-Orthogonality of "Massless" ν's

[old idea 1977] Sugawara et al.

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}.$$

Suppose
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$m_1 = 0$, $m_2 = 0$, $m_3 \sim$ large.
say 20 GeV.

Then "ν_e" in $n \to p e^- \bar{\nu}_e$
is "ν_e" $= U_{11}\nu_1 + U_{12}\nu_2$

"ν_μ" in $\pi \to \mu \nu_\mu$
is "ν_μ" $= U_{21}\nu_1 + U_{22}\nu_2$

& $\langle$"ν_e"$|$"ν_μ"$\rangle = \underbrace{-U_{13}^* U_{23}}_{\text{new}} \neq 0$

So "ν_e" & "ν_μ" from β decay & π-decay <u>Not Orthogonal</u> <u>although</u> massless!!

But ν_e & ν_μ in W decay (almost) orthogonal!!

$W \to e\nu_e$
$\to \mu\nu_\mu$

Generalise to ν_e, ν_μ, ν_τ almost massless

ν_L = 4th Generation ? SU(2) singlet

<u>Current</u> or Limits

Langacker et al.	$\lambda^{e\mu}$	$\lambda^{e\tau}$	$\lambda^{\mu\tau}$
	10^{-3} ($\mu \to e\gamma$)	0.11 (unitarity)	0.045 (osc.)

CP Violation in ν Oscillations

With 3 flavors it's possible

Suppose at $t=0$, a ν_e is created.

$$\psi(0) = \nu_e$$

Then $\psi(t) = \sum_i U_{ei} \nu_i \, e^{-iE_i t}$

Amplitude to find ν_α at t is:

$$A^{(t)}_{\nu_e \to \nu_\alpha} = \sum_i U_{ei} U^*_{i\alpha} \, e^{-iE_i t}$$

Probability for finding ν_α at t:

$$P^{(t)}_{\nu_e \to \nu_\alpha} = \left| A^{(t)}_{\nu_e \to \nu_\alpha} \right|^2$$

$$= \sum_{i,j} \Big\{ |U_{ei}|^2 |U_{i\alpha}|^2 + 2\,\mathrm{Re}\,(U_{ei} U^*_{i\alpha} U^*_{ej} U_{j\alpha}) \cos \delta E_{ij} t$$

$$+ \underbrace{2\,\mathrm{Im}\,(U_{ei} U^*_{i\alpha} U^*_{ej} U_{j\alpha}) \sin \delta E_{ij} t}_{=0 \text{ if CP is conserved.}} \Big\}$$

$$\boxed{\delta E_{ij} t \equiv \frac{\delta m^2_{ij} L}{2 E_\nu}}$$

- Q value $\gg m_i$
- coherence preserved.

Other tests of CP Conservation

$$P(\nu_\alpha \to \nu_\beta, t) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta, t) = 0$$

if CP Conserved

$$P(\nu_\alpha \to \nu_\beta, t) - P(\nu_\beta \to \nu_\alpha, t) = 0$$

if CP conserved

Fourier Analysis of $P(\nu_\alpha \to \nu_\beta, t)$

$= \ldots\ldots B\cos\alpha t + C\sin\alpha t + \cdots$

If both $B \neq 0$, $C \neq 0 \Rightarrow$ CP Not Conserved

For 3 flavors: If

$$P(\nu_e \to \nu_e, t \to \infty) = \frac{1}{3}$$

Then

$$P(\nu_\mu \to \nu_\mu, t \to \infty) = \frac{1}{2}$$

$$P(\nu_\tau \to \nu_\tau, t \to \infty) = \frac{1}{2}$$

And if $\sum_{\alpha = e, \mu, \tau} P(\nu_\alpha \to \nu_\alpha, t \to \infty) < 4/3$

$\Rightarrow$ either CP Violated or $N_\nu > 3$!

Lesson

ν_e, ν_μ, ν_τ . . . .

process dependent.

i.e. 1) composition different
2) coupling to e, μ, τ different.

"ν_e", "ν_μ" in β-decay, π-decay
do not couple with strgth 1.

ν_e in W-decay does.

e.g. if 4th Generation
m_{ν_4} large
& $\nu_\tau \nu_4$ mixing large

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_L \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \theta \\ 0 & 0 & -\theta & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$$

$\theta \sim \theta_c$

$m_{\nu_4} > m_\tau$.

ν Properties (Helsinki, ν96).

$m_{\nu_e} < 3.5$ eV Troitsk

$m_{\nu_\mu} < 160$ KeV

$m_{\nu_\tau} < 23.1$ MeV (Aleph)

$m^2_{\nu_e}$:

$-22 \pm 17 \pm 14$	Mainz
-1 ± 6.3	Troitsk
$-24 \pm 48 \pm 61$	Zurich
-54 ± 30	(PDG)

0ν Double Beta Decay $\Longrightarrow \langle m^{M}_{\nu_e} \rangle < 0.7\,\mathrm{eV}$

^{76}Ge (Heidel. – Moscow) $\tau_{1/2} > 9.6 \cdot 10^{24}\,y$

Magnetic Moments

$\mu_{\nu_e} < 1.8 \cdot 10^{-10} \mu_B$

$\mu_{\nu_\mu} < 7.4 \cdot 10^{-10} \mu_B$

$\mu_{\nu_\tau} < 5.4 \cdot 10^{-7} \mu_B$

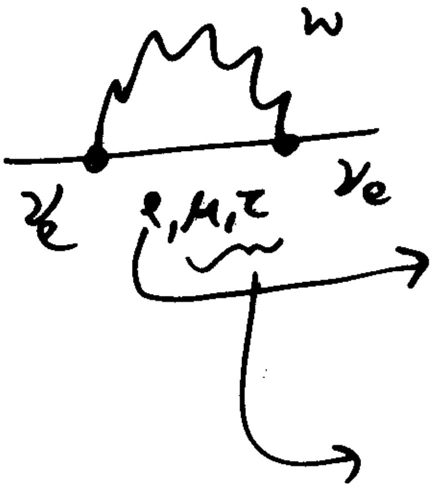
(SM, Dirac ν, $\mu_{\nu_e} \sim \left(\frac{G_F m_\nu m_e}{\cdots \pi^2} \mu_B \right) \sim 10^{-19} \left(\frac{m_\nu}{eV} \right) \mu_B$)

Magnetic Dipole Moment of ν_e

- Either ν_e is a Dirac part.
- Or mdm is a transition moment.

$$\nu_{e_L} \longrightarrow \nu^c_{\alpha R} \quad (\alpha \text{ can be } \mu \text{ or } \tau).$$

"Standard Model"



$$\mu_{\nu_e} = \frac{3\, m_e G_F}{4\sqrt{2}\,\pi^2}\, m_{\nu_e}\, \mu_B$$

$$\mu = 3 \cdot 10^{-19}\, \mu_B \left(m_\nu / 1\,\mathrm{eV}\right)$$

$$\mu = 3 \cdot 10^{-19}\, \mu_B \left(\frac{m_{\nu_e}}{1\,\mathrm{eV}}\right)\left(\frac{m_\tau}{m_e}\right) |U_{\tau e}|^2$$

4th Gen.

$$m_L < 100\ \mathrm{GeV}$$

$$|U|^2 < 10^{-2}$$

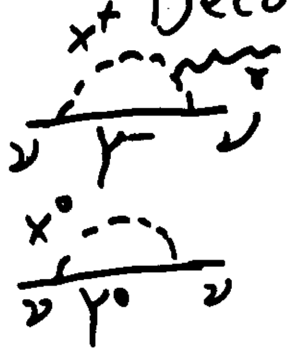
$$\rightarrow \mu < 10^{-14}\, \mu_B.$$

Neutrino Magnetic Moment (Theory)

SM $\rightarrow$ $\mu_{\nu_e} \sim 10^{-19} \left(\frac{m_{\nu e}}{1\,eV}\right) \mu_B$

Large $\mu_\nu \longleftrightarrow$ Large m_ν ?

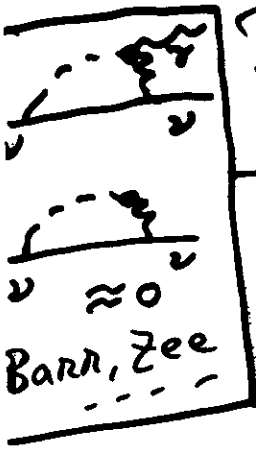
Decoupling via Voloshin Symmetry (improved)



$\nu_{eL} \longleftrightarrow \cancel{(\nu_{eR})^c}$

$\longrightarrow (\nu_{\mu R})^c \equiv \nu_{\mu L}$.

SU(2) or Discrete Subgroup.



Realizations: Chang, Keung, - - -
(Vysotsky, Stepanov) Leurer, - - -
Babu, Mohapatra, - - -
Ecker, Grimus, - - -
Sarkar, - - -
George, - -

Simplest Case: $(\nu_e, \bar{\nu}_\mu)$ make a Dirac Particle á la ZKM.

Then $\delta m^2 = 0$!

But $\delta m^2 \sim 10^{-7} eV^2$ experimentally! more interesting

S. Pakvasa

ν- Decays

Radiative

$\nu_\alpha \to \nu_\beta + \gamma.$

Standard Model.

$$\Gamma = \frac{G_F^2 m_\alpha^2}{128\pi^4}\left(\frac{9}{16}\right)\frac{\alpha}{\pi}\left|\sum_i \frac{m_i^2}{m_W^2} U_{i\alpha} U_{i\beta}^*\right|^2$$

$$\left(1 - m_\beta^2/m_\alpha^2\right)$$

$$\left(1 + m_\beta^2/m_\alpha^2\right).$$

Z-exch.

GIM ~~Flavor~~ violation ε

$\nu_\alpha \to \nu_\beta \, \nu_i \bar{\nu}_i$

$$\Gamma = \frac{\varepsilon^2 G_F^2 m_\alpha^5}{128\pi^3} \quad (p.5).$$

$$= \frac{G_F^2 m_\alpha^5}{128\pi^3} \frac{\alpha^2}{8\pi^2}\left|\sum \frac{m_j^2}{m_W^2} U_{j\alpha} U_{\beta j}^*\right|^2$$

(Radiative)

m_{ν_e} Experiments

β-Decay $\quad (A, Z) \longrightarrow (A, Z+1) + e^- + \bar{\nu}_e$

For allowed transitions

$$\frac{dN}{dE} = c\,p\,E\left[(E_m - E)^2 - m_\nu^2\right]^{1/2}(E_m - E)\,F(E)$$

$$= c\,p\,E\left[(Q-T)^2 - m_\nu^2\right]^{1/2}(Q-T)\,F(E)$$

F = Fermi function (Coulomb eff.)

$Q = T_{max}$ if $m_\nu = 0$.

$T = K.E.$

$$\frac{dN}{dT} = c\,p\,E\,(Q-T)^2\,F(E) \quad [\text{if } m_\nu = 0]$$

Slope of dN/dT at $T = Q$ or $T = Q - m_\nu$

$= 0$ for $m_\nu = 0$

$= \infty$ for $m_\nu \neq 0$.

Kurie Plot

$$K(T) = \sqrt{\frac{dN/dE}{pE\,cF(E,Z)}}$$

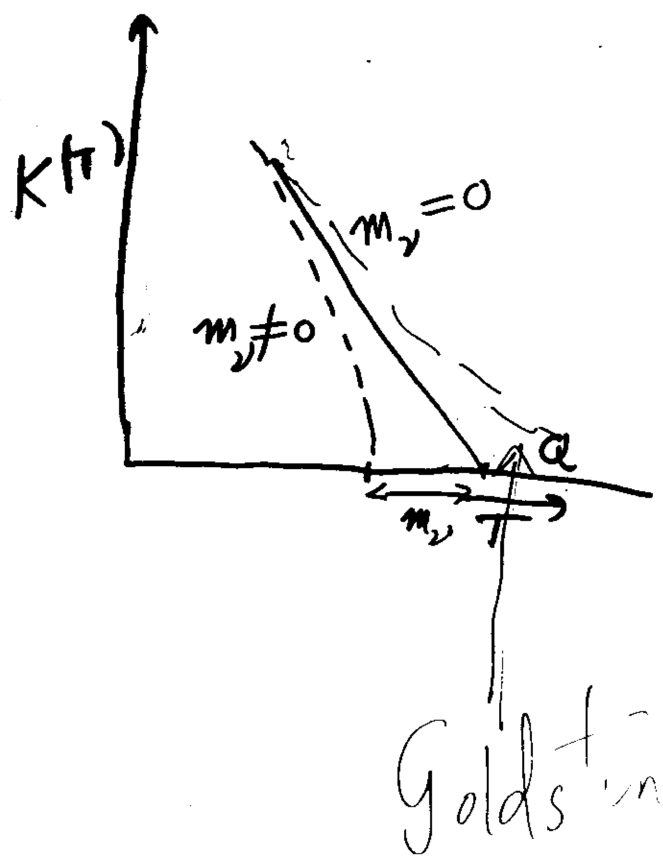
$$= \sqrt{(Q-T)^2 - m_\nu^2}\,(Q-T)^{1/2}$$

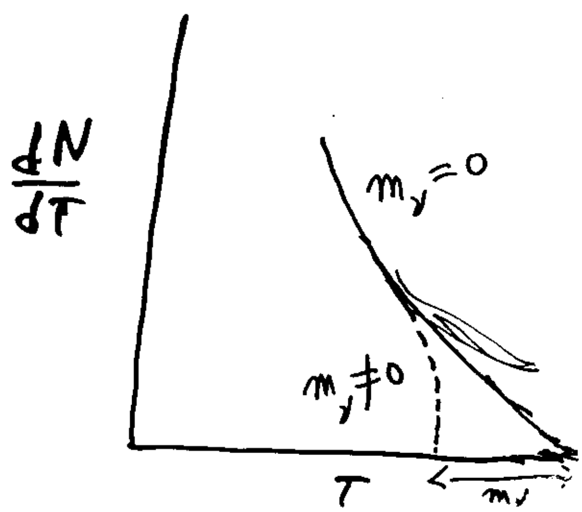
$$= Q-T \qquad \text{if } m_\nu = 0.$$

Slope of $K(T)$ at $Q=T$, $m_\nu=0$

$= -1.$

$= \infty \qquad m_\nu \neq 0$





Effect of m_ν change of slope of Kurie plot at end point. But # events v. small there. Typically for $\Delta E/T_m \sim 0.01$

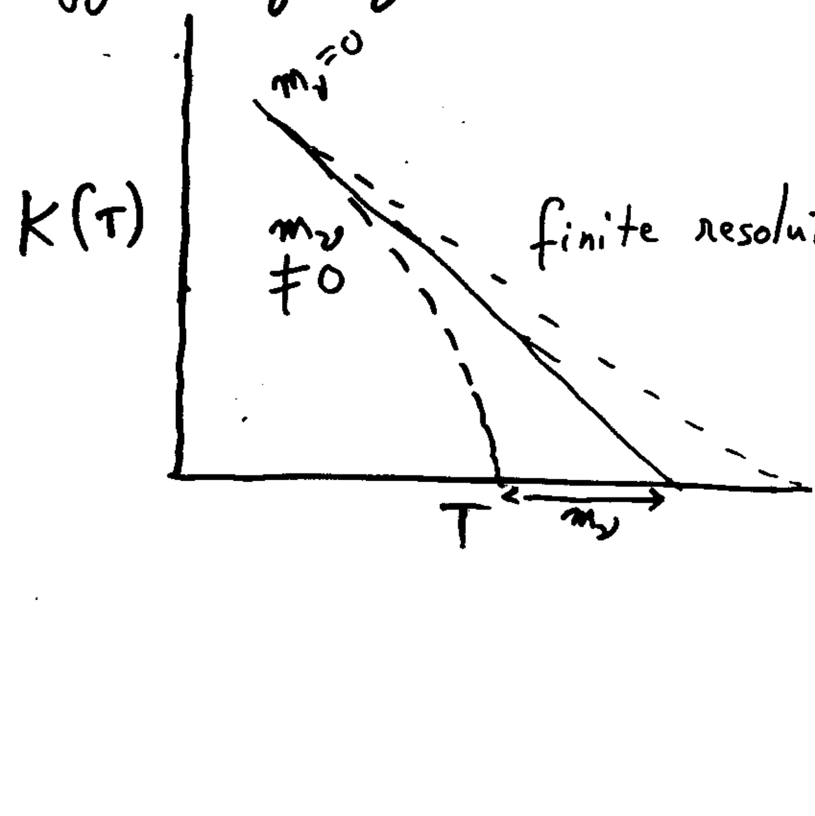
$\#(\beta) \lesssim 10^{-5} \cdot (\Delta E^3)$.

So need a source of $\tau_{1/2}$ short enough to give events.

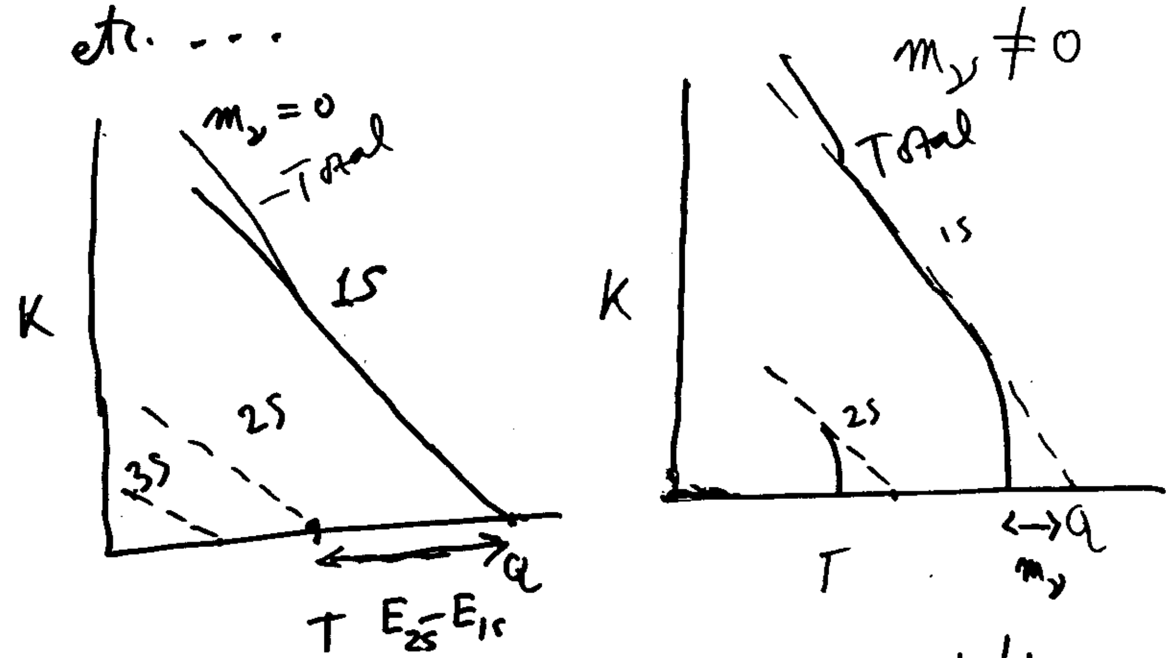
but Q-value should not be too large

<u>Compromise</u> T^3, $\left\{ \begin{array}{l} T_m = Q \sim 18.6 \text{ keV} \\ \tau_{1/2} \sim 12 \text{ y}. \end{array} \right.$

Effect of finite resolution



Final He_3^+ has 1 e^-. This may be left in ground state, 1st excited state etc. . . .



For atomic He_3^+ calculations indicate 70% in 1S, 25% in 2S, 5% in others.

For solid source or molecular T_3 need to know other energy shifts

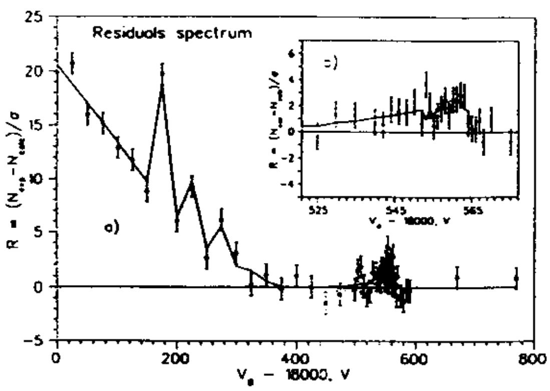


Fig. 3. Residuals from the fit of the tritium spectrum. The residual for each point is the difference between the measured value and the calculated one divided by the corresponding error. The zero line is the standard spectrum ($m_\nu^2 = 0$) fitted with fixed background and $E_{low} = 18350$ eV and extrapolated to 18000 eV. The solid line is fitted spectrum with variation of m_ν^2, ΔN_{step}, EM_0^C, P^{MC} and of standard parameters. Jumps in the curve are due to the difference in measurement times of different points.

one can get rid of the low-energy anomaly by the restriction $E_{low} \geq 18350$ V. Taking this into account, we have made a fit with $E_{low} = 18350$ V and varied only two parameters: the normalization factor and the end-point energy, fixing the background as average between the points 18570 and 18770 V. The residual spectrum of this fit, extrapolated down to 18000 V, is shown in Fig. 3 and part of the fitted spectrum is presented in Fig. 2 (line 5).

The χ^2 value for this fit equals 73 for 42 degrees of freedom but from Fig. 2 and Fig. 3 one can see that the main contribution to χ^2 comes from the area in the vicinity of the end point. The difference between the experimental and fitted spectra consists in an excess of counts observed at about 7 eV below the end point and spreading into the lower energy region, gradually sinking in the increasing statistical errors. The simplest representation of such an anomaly appears to be a step-like structure corresponding to the spike- or bump-like structure in the differential spectrum.

The slope of the step is comparable to the energy resolution (~ 4 eV) which is an argument in favor of the monochromaticity of this excess in the differential spectrum.

The low-energy anomaly of the experimental spectrum is a rise in the counting rate at energies below 18300 V. A similar deviation has been observed earlier in the experiment of the Mainz group [3], where it was found that it can be represented as a certain partial branch of the beta decay, has been missed in the calculation of the final state spectrum, with the end-point energy around 18500 eV and relative intensity of about 4%. In our case the end point of this missed component should be around 18400 eV.

Both anomalies give rise to a negative m_ν^2.

To confirm that the step-like structure can be the reason for the unphysical negative value of m_ν^2, a fit was made with the standard spectrum summed up with a step-like spectrum in the form of a θ-function, which depended on two variable parameters: ΔN_{step} and E_{step}. It was taken that $\Delta N_{step} \neq 0$ for $E \leq E_{step}$ and $\Delta N_{step} = 0$ for $E > E_{step}$.

The fit with these additional variables gives $\Delta N_{step} = 2.90 \pm 0.60$ mH (this is $(6.3 \pm 1.3) \times 10^{-11}$ of the total decay rate) and $E_{step} = 18566.1 \pm 0.8$ eV for $E_{low} = 18350$ eV and practically the same for all E_{low} up to 18500 eV. The corresponding χ^2/d.o.f has proved to be ~ 0.8 which is (6–10)% less than in our first fit.

To compensate for the effect of a negative m_ν^2 one can also try to vary some other parameters which were previously fixed. First of all, these are the transmission factor and the final state spectrum. By making a fit with m_ν^2 and transmission factor regarded as variables, one obtains a χ^2 distribution centered at $X = 0.31 \pm 0.09$, whereas experimentally it has been found that $X = 0.31 \pm 0.03$.

To obtain a zero value of m_ν^2 one should take $X = 0.55$. The experimental error for X given above completely excludes this possibility.

The variation of the final state spectrum appears to be a rather arbitrary procedure since the calculations made in [5] and, recently, in [6] do not leave any room for such a variation up to an accuracy of $m_\nu^2 \sim 1$ eV, as claimed by the authors. One can only note that this spectrum has never been measured experimentally.

The variation of the population of the ground state and of the three first excited states (28–45 eV) was made separately for the initial experimental spectrum and for the spectrum with a subtracted step function that corresponds to the standard spectrum. In the first case the χ^2 distribution has given $\Delta p = (-0.8 \pm$

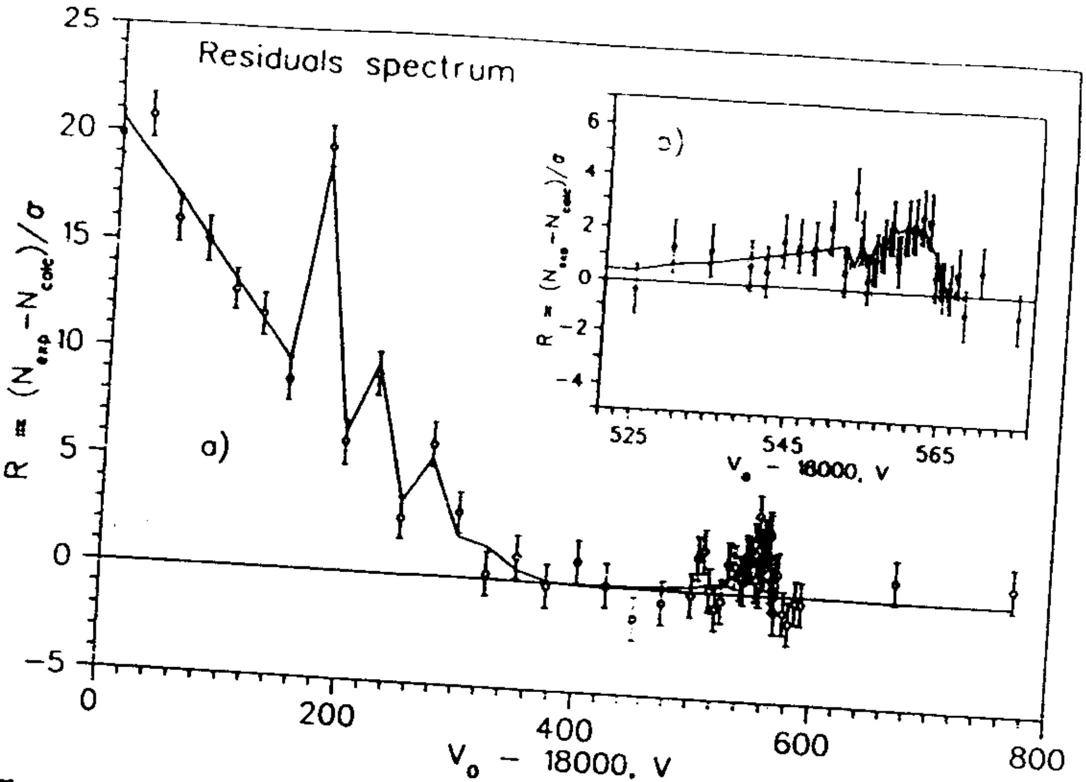


Fig. 3. Residuals from the fit of the tritium spectrum. The residual for each point is the difference between the measured value and the calculated one divided by the corresponding error. The zero line is the standard spectrum ($m_\nu^2 = 0$) fitted with fixed background and $E_{low} = 18350$ eV and extrapolated to 18000 eV. The solid line is fitted spectrum with variation of m_ν^2, ΔN_{step}, EM_0^C, P^{MC} and of standard parameters. Jumps in the curve are due to the difference in measurement times of different points.

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15–25 eV/c^2 the upper limit $P_{H_\nu} < 5\%$ (95% C.L.), thus refuting the statement made in [8].

5. Conclusion

The study of the tritium β-spectrum near its end point with the TROITSK ν-MASS setup has revealed the effect of a negative m_ν^2 with high statistical confidence when fitting the experimental spectra. This effect was found to be caused by certain anomalies in the β-spectrum, one of which, located about 7 eV below the end point, can be represented phenomenologically as a nearly monochromatic bump-like structure with integral intensity of $\sim(6.3 \pm 1.3) \times 10^{-11}$ of the total decay rate.

The other anomaly can be represented with a partial β decay into an unknown final state with an excitation energy of 173 ± 14 eV and relative probability $(4.2 \pm 0.4)\%$ (errors are statistical only). This anomaly appears to be similar to the effect observed earlier by the Mainz University group [3] with an excitation energy of 90 eV and approximately the same probability (for more details see [7]). The difference in excitation energy (if it exists) can be attributed to the difference in the phase state of the tritium sources in the two experiments (frozen T_2 in Mainz and gaseous T_2 in Troitsk) or to some unknown systematics. New measurements of the Mainz group demonstrate a significant variation of this effect from run to run, so the origin of the low-energy anomaly is quite mysterious [9].

Both anomalies cannot be understood at present basing on the known properties of the tritium decay or explained by systematic effects, unless one considers a very arbitrary modification of the final state spectra.

The next runs with an upgraded setup will enable us to resolve some ambiguity concerning the latter point.

By treating both effects phenomenologically and describing them with a minimal set of variable parameters it proved to be possible to eliminate the problem with the negative value of m_ν^2, to define an upper limit on the neutrino mass:

$$m_\nu < 4.35\ eV/c^2 \text{ at } 95\% \text{ C.L.}, \tag{6}$$

and to obtain an upper limit on the possible mixing with a heavy neutrino in the mass range of 15–25 eV/c^2:

$$P_H < 5\% \text{ at } 95\% \text{ C.L.} \tag{7}$$

The limit on the lightest neutrino mass is now the most restrictive one and has been obtained with accounting of the systematic effect represented by an anomaly which was never seen before.

Acknowledgements

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[1] V.M. Lobashev and P.E. Spivak, Moscow Preprint INR P-0291 (1983); Nucl. Instrum. Methods A 240 (1985) 305.

Some thing to keep in mind

ν_e MASS SQUARED

The tritium experiments which yield the best limits for $m(\nu_e)$ actually measure mass squared. Any effort to combine their results to obtain an improved limit, therefore requires use of the mass squared results shown here. Note that we exclude the results of BORIS 87 because of controversy over the possible existence of large unreported systematic errors, see BERGKVIST 85B, BERGKVIST 86, SIMPSON 84, and REDONDO 89. For a review see ROBERTSON 88.

VALUE (eV^2)	CL%	DOCUMENT ID		TECN	COMMENT
-107 ± 60 OUR AVERAGE					
$-65 \pm 85 \pm 65$	95	[9] KAWAKAMI	91	CNTR	$\overline{\nu}_e$, tritium
$-147 \pm 68 \pm 41$	95	[10] ROBERTSON	91	CNTR	$\overline{\nu}_e$, tritium
$-11 \pm 63 \pm 178$		[11] FRITSCHI	86	CNTR	$\overline{\nu}_e$, tritium

Are $m^2_{\nu_e} < 0$, $m^2_{\nu_\mu} < 0$?

If ν's Tachyons

$\longrightarrow p \rightarrow$

ν_μ MASS

Applies to ν_2, the primary mass eigenstate in ν_μ. Would also apply to any other ν_j which mixes strongly in ν_μ and has sufficiently small mass that it can occur in the respective decays. (This would be nontrivial only for $j \geq 3$, given the ν_e mass limit above.)

VALUE (MeV)	CL%	DOCUMENT ID		TECN	COMMENT
<0.27	90	[1] ABELA	84	SPEC	$m^2 = -0.097 \pm 0.072$

[1] ABELA 84 used the PDG 84 value for $\pi^\pm$ mass, in conjunction with μ momentum measurement in $\pi \rightarrow \mu \nu_\mu$ decay to obtain $m < 0.25$ and $m^2 = -0.16 \pm 0.08$. The values shown here for mass and m^2 are corrected values obtained by BECKERMAN 86 from the ABELA 84 data using the more accurate $\pi^\pm$ mass of BECKERMAN 86.

PDG 1992

- $m^2_{\nu_e} < 0$ in Tritium (End Point) Experiments
- $m^2_{\nu_\mu} < 0$ in $\pi \longrightarrow \mu \nu_\mu$ (?)

Neutrino as Tachyon.

- Theory? (QFT)
- Exptl. Tests (Kostelecky)

1. - $p \longrightarrow n e^+ \nu_e$ $\quad (T_p = T^e_{th})$
   - $\mu^- \longrightarrow \pi^- \nu_\mu$ $\quad (T_\mu = T^\mu_{th})$.

   $E_{threshold}$ in function of m^2_ν.

2. ν velocity decreases (to c) as E_ν increases!

$$\frac{E_2}{E_1} = \frac{v_1^2 - 1}{v_2^2 - 1}.$$

2. IBEC. $e^- + (Z,A) \longrightarrow (Z-1,A) + \nu_e + \gamma$

[de-Rujula] Study the end point of 3 body γ-spectrum just as in β-decay. Event rate v. small. Ho^{163} etc.

Does not look promising. Current limit from IBEC studies of Ho^{163}

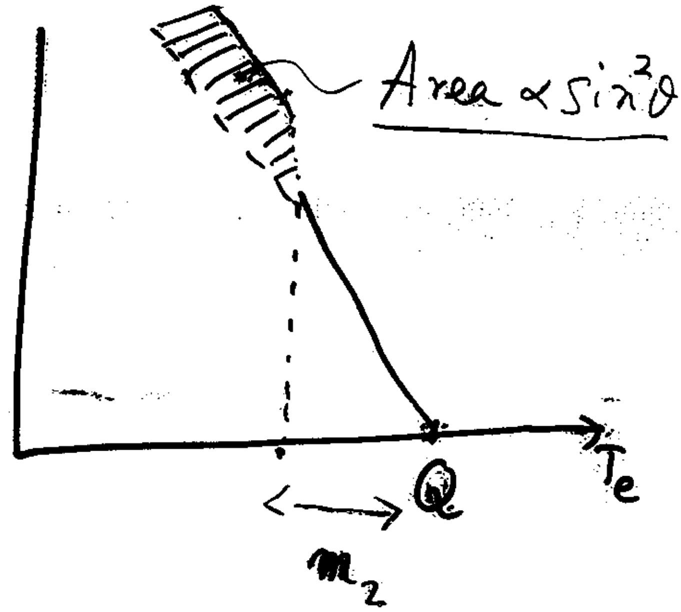
$$m_{\nu_e} < \cancel{1.3\ \text{KeV}}\ 550\ \text{eV} \quad \left\{ \begin{array}{l} \text{CERN} \\ \text{KEK} \end{array} \right.$$

Mixing & Kinks (17 KeV ν)

If $\nu_e = \begin{Bmatrix} \text{mixture of light } \nu_1 \\ \& \text{ heavy } \nu_2 \end{Bmatrix}$

β-decay phase space has Kinks

e.g. Kurie Plot $\sqrt{\frac{1}{p_e T_e}\frac{dN}{dT_e}}$



Simpson: $\begin{Bmatrix} m_2 \sim 17 \text{ KeV} \\ \sin^2\theta \sim 10\% \end{Bmatrix}$
1985
1990

Many experiments: lot of confusion, Yes, No.

Many (more) theory papers!!

1992: Two beautiful & convincing experiments
- Tokyo
- Argonne

$\Big\}$ $\sin^2\theta < 0.073\%$ at 17 KeV

Conclusion: No 17 KeV ν.

Postscript: Hime, Jelley, Norman found the artifacts in apparatus.

Simpson

$m_{\nu_H} \sim 17$ KeV 1985

$|U_{e H}|^2 \sim 0.8$ to 1%

1989-90-91
Hime-Simpson
Hime-Jelley
Sur et al.

Caltech
Moscow
Tokyo
Princeton
Bombay

} NO. (1985-6)

¡ Yes!!

1990-91.
Caltech
Oklahoma } No!!

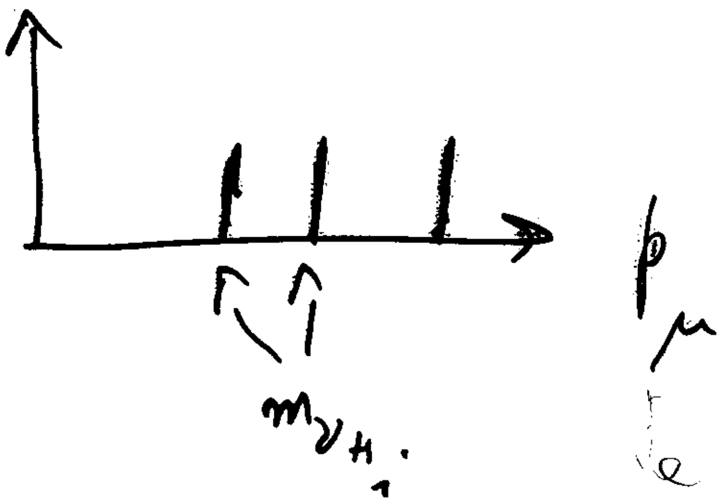
Simpson. YES.

??

"Kinks" in 2 Body Decay.

$\pi \to \mu \nu_\mu$, $e \nu$

$\to \mu \nu_H$



$m_{\nu_{H_i}}$.

$\beta\beta0\nu$

- Best current results – Heidelberg-Moscow collaboration – Gran Sasso Laboratory.
  - Semiconductor grade Ge.
  - Operates as both source and detector.
  - Source is isotopically enriched ^{76}Ge (from 7.8% to 86%).
  - Energy resolution of about 2-3 keV at the end point of 2,039 keV.

detector number	total mass [kg]	active mass [kg]	enrichment in ^{76}Ge [%]	FWHM at 1332 keV [keV]
enr#1	0.980	0.920	85.9 ± 1.3	2.22 ± 0.02
enr#2	2.906	2.758	86.6 ± 2.5	2.43 ± 0.03
enr#3	2.446	2.324	88.3 ± 2.6	2.71 ± 0.03
enr#4	2.400	2.295	86.3 ± 1.3	2.14 ± 0.04
enr#5	2.781	2.666	85.6 ± 1.3	2.55 ± 0.05

Table 1: *Technical parameters of the five enriched detectors*

detector	statistical significance [kg·y]	statistical significance without first 200 days [kg·y]	background 2000–2080 keV [counts/keV·y·kg]
enr#1	3.15	2.18	0.18 ± 0.03
enr#2	7.87	7.02	0.20 ± 0.02
enr#3	5.40	4.40	0.21 ± 0.02
enr#4	0.93	—	—
enr#5	0.35	—	—
Σ	17.70	13.60	[illegible]

Table 3: *Full data of the experiment and used data after ending of initial activities for the evaluation of the $0\nu\beta\beta$ decay*

$\lambda_1 \cos^2\theta + \lambda_2 \sin^2\theta$

$= -\lambda_1 \cos^2\theta + \lambda_2 \sin^2\theta$

$= a.$

So if a is small e.g. 0 !

One can have cancellation.

Can this be checked?

In principle, yes. e.g. if

$m_1 \sim$ few eV, $m_2 \sim$ few MeV.

Cancellation varies with nucleus.

e.g. if "$m^{M}_{\nu_e}$" (Te) $\equiv 0$

then "$m^{M}_{\nu_e}$" (Ca) > 13 eV.

"$m^{M}_{\nu_e}$" (Se) > 5 eV.

etc.

Mixing & Oscillations with Sterile Neutrinos

(first considered by Pontecorvo)

Recall the case of Neutrino Mass when both Dirac & Majorana mass terms present. Another way to look at it is Doublet-Singlet Mixing. E.g. consider

$$\begin{pmatrix} \nu_{e_0} \\ e_0 \end{pmatrix}_L \qquad \eta_{e_0 L} \longrightarrow \text{LH Singlet}$$

hence has no SM interactions!

For simplicity let $e_{0L} \equiv e_L$.

RH fields are $\nu_{e_R}^{0c}$ & $\eta_{e^0 R}^{c}$

Hence SM Higgs doublet can give Dirac mass terms

$$d\left[\overline{\nu_{e_L}^{0}}\,\eta_{e^0 R}^{c} + \overline{\eta_{e^0 L}}\,\nu_{e_R}^{0c}\right]$$

If there are scalar doublets $I=1$ ((complex) & $I=0$ (or bare mass term)

Then can get Majorana ~~mass~~ terms

$I=1$: $\quad a \quad \overline{\nu^0_{e_L}}\, \nu^{0\,c}_{e_R}$

$I=0$: $\quad b \quad \overline{\eta^0_{e_L}}\, \eta^{0\,c}_{e_R}$.

The $\nu_e - \eta_e$ mass matrix is

$$\overline{(\nu^0_{e_L} \ \ \eta^0_{e_L})} \begin{pmatrix} a & d \\ d & b \end{pmatrix} \begin{pmatrix} \nu^{0\,c}_{e_R} \\ \eta^{0\,c}_{e_R} \end{pmatrix} + h.c.$$

When the mass matrix is diagonalised $\mathcal{M} = \begin{pmatrix} a & d \\ d & b \end{pmatrix}$. — e.v. m_1, m_2

by an orthogonal matrix R. (if no phases)

and the mass eigenstates are

$$\left.\begin{aligned} \nu_{1_L} &= \cos\phi\, \nu^0_{e_L} + \sin\phi\, \eta^0_{e_L} \\ \nu_{2_L} &= -\sin\phi\, \nu^0_{e_L} + \cos\phi\, \eta^0_{e_L} \end{aligned}\right\} \quad \begin{aligned} \nu^0_{e_L} &= \cos\phi\, \nu_{1_L} - \sin\phi\, \nu_{2_L} \\ \eta^0_{e_L} &= \sin\phi\, \nu_{1_L} + \cos\phi\, \nu_{2_L} \end{aligned}$$

The charged current is

$$J^c_\mu = \overline{\nu^0_{e_L}}\, \gamma_\mu e_L = \overline{\nu_{e_L}}\, \gamma_\mu e_L$$

where $\nu_{e_L} = \cos\phi\, \nu_{1_L} - \sin\phi\, \nu_{2_L}$

In the most general ~~case~~ there could be mixing amongst six states $\nu_e, \eta_e, \nu_\mu, \eta_\mu, \nu_\tau, \eta_\tau$. described by a 6×6 matrix.

Different models "predict" different scenarios for # of states, for pattern of mixing etc.

Oscillations of Massless ν's

- Flavor violating Gravity

Halprin, Leung, Pantaleone

ν_1 couple to gravity : $f_1 \phi_{gr} E$

ν_2 " " : $f_2 \phi_{gr} E$

$$\nu_e = c\nu_1 + s\nu_2 \qquad [\delta f = 2\delta\gamma]$$
$$\nu_\mu = -s\nu_1 + c\nu_2$$

$$P_{ee} = 1 - \sin^2 2\theta_{gr} \sin^2\left\{\frac{\delta f \phi E L}{2}\right\}$$

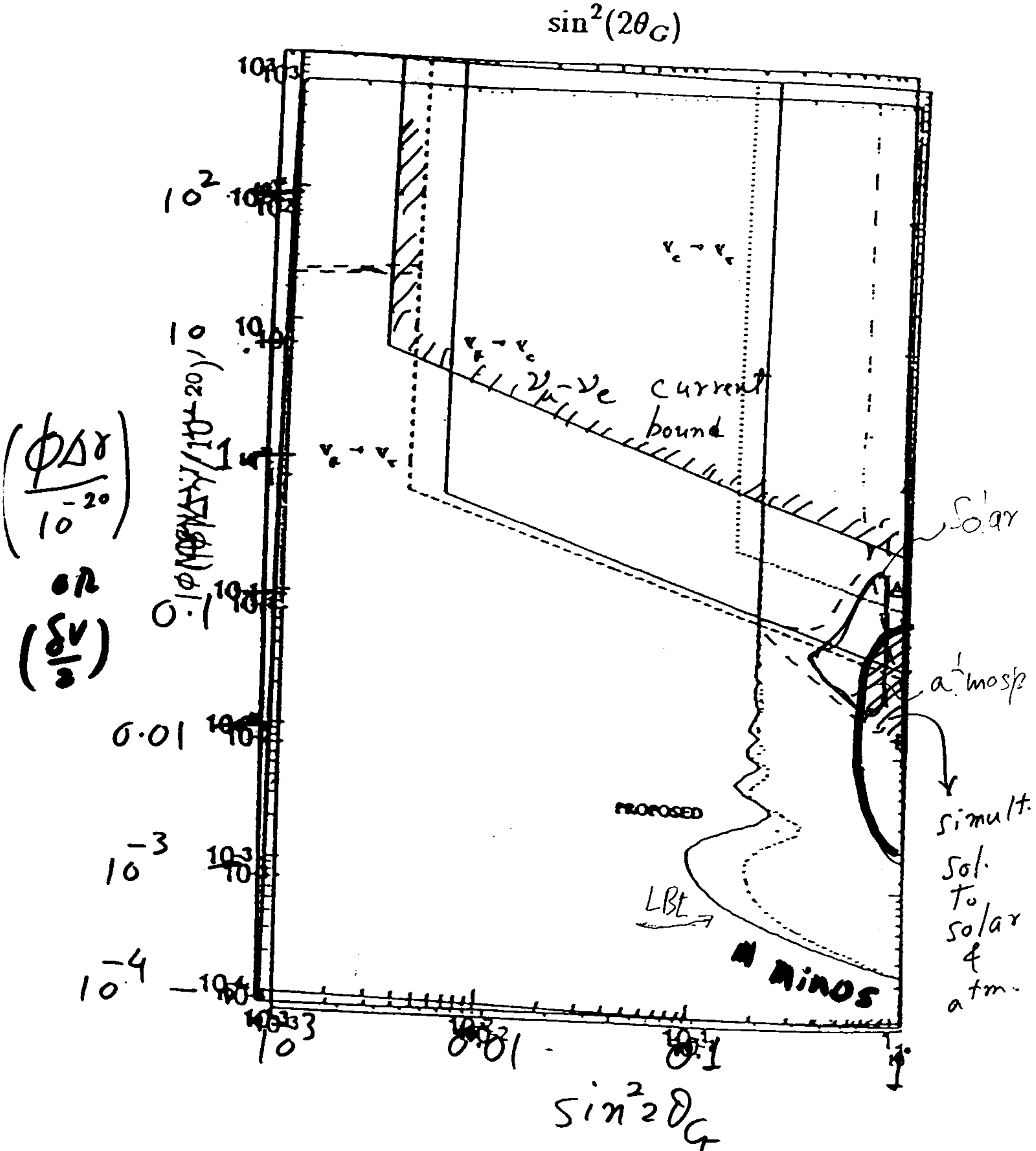
etc.

- Lorentz Invariance Violation ($\neq c$)

Coleman, Glashow

- each particle has its own max. speed
- $\nu_1 \to v_1$ $\nu_2 \to v_2$ ("velocity" eigenstates)
- $E_1 = v_1 p$, $E_2 = v_2 p$ $(p \approx E)$
- $\delta E = \delta v p$

$$P_{ee} = 1 - \sin^2 2\theta_v \sin^2\left\{\frac{\delta v E L}{2}\right\}$$



Halprin, Leung, Pantaleone (95)

Important feature to be checked:

Does Oscillating term

go as $\sin^2\left(\frac{\delta m^2}{4}\right)(L/E)$

or $\sin^2\left(\frac{\delta V}{2}\right)(LE)$?