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Lecture I

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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PHASE TRANSITIONS AND DEFECTS IN COSMOLOGY

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Please note: These are preliminary notes intended for internal distribution only.

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Phase Transitions & Defects in Cosmology

Introduction

SSB in condensed matter + particle physics.

∴ phase transitions in cosmology

Phase transitions in condensed matter → defects.

Phase transitions in cosmology → defects

eg: GUTs ⇒ magnetic monopoles.

Some ~~defects~~ defects may have unpleasant consequences

⇒ new cosmology eg. inflation.

Other defects may be beneficial or benign

⇒ structure formation, or, window to very early universe

eg. superheavy cosmic strings; light superconducting strings or light magnetic monopoles

These lectures:

~~[illegible]~~

1) Introduction to phase transitions
2) ~~[illegible]~~ Classification of defects
3) Lattice Studies of defect formation
4) ~~[illegible]~~ Ongoing work on defect formation
5) Directions

Magnetic system : "spins at $\vec{x}$".
spin density $= s(\vec{x})$.

Constant External magnetic field $= H$ ~~(const~~

Partition function $= Z(H, \beta) = \int Ds.\exp\left[-\beta\int d^3x \left(\mathcal{H}[s] - H.s(x)\right)\right]$

where $\beta = \dfrac{1}{kT}$, $\mathcal{H}[s]=$ Hamiltonian density.

~~Thus:~~
$$\cancel{-\frac{\partial F}{\partial H}\Big|_\beta = \frac{1}{\beta}\frac{\partial}{\partial H}\ln Z}$$

Then: $Z = e^{-\beta F(H,\beta)}$

where: $F(H,\beta) =$ Helmholtz free energy.

$$-\frac{\partial F}{\partial H}\Big|_\beta = \frac{1}{\beta}\frac{\partial}{\partial H}\ln Z = \frac{1}{Z}\int d^3x \int Ds.\, s(x).e^{-\beta\int dx(\mathcal{H} - Hs)}$$
$$= \int d^3x \langle s(x)\rangle$$
$$\equiv M \quad (\text{magnetization})$$

Refs: M.Peskin & Schroeder ; A.Linde, Rep. Prog. Phys., Vol. 42 (1979), 25 ; S Coleman & E.Weinberg, PRD 7 (1973) 1888 ; L.Dolan & Jackiw, PRD 9 (1974) 3320, S Weinberg, PRD 9 (1974) 3357, ...

Gibbs free energy $= G = F + M.H.$ $\qquad G = G(M,\beta).$

Then: $$\left.\frac{\partial G}{\partial M}\right|_\beta = \left.\frac{\partial F}{\partial M}\right|_\beta + H + M.\left.\frac{\partial H}{\partial M}\right|_\beta.$$

$$= \left.\frac{\partial F}{\partial H}\frac{\partial H}{\partial M}\right|_\beta + H + M\left.\frac{\partial H}{\partial M}\right|_\beta$$

$$= H.$$

So: at given H, one can find M by solving $\left.\frac{\partial G}{\partial M}\right|_\beta = H.$

In particular, if $H=0$, $\left.\frac{\partial G}{\partial M}\right|_\beta = 0$ gives equilibrium magnetization.

Quantum Field Theory

$T=0$ first.

generating functional $\Big\} = Z[J] = \int D\varphi.\ \exp\left[i \int_{0<t<T} d^4x. \left(\mathcal{L}[\varphi] + J(x).\varphi(x)\right)\right]$

$$= \langle \Omega | e^{-iHT} | \Omega \rangle \qquad \text{vacuum} = |\Omega\rangle.$$

$$\equiv e^{-E[J]} \qquad E[J] = \text{"energy}^{*}\text{ functional"}$$

$$\frac{\delta E[J]}{\delta J(x)} = - \frac{\int D\varphi\, e^{i\int(\mathcal{L}+J\varphi)} \cdot \varphi(x)}{\int D\varphi.\ e^{i\int(\mathcal{L}+J\varphi)}} = -\langle \Omega | \varphi(x) | \Omega \rangle_J$$

$$\equiv -\varphi_{cl}(x).$$

Next:

$$\Gamma[\varphi_{cl}] = -E[J] - \int d^4y.\ J(y).\varphi_{cl}(y).$$

$$= \text{effective action.}$$

And:
$$\frac{\delta \Gamma[\varphi_{cl}]}{\delta \varphi_{cl}(x)} = -J(x).$$

$\therefore$ for given $J(x)$, the equilibrium $\varphi_{cl}(x)$ can be found by solving $\dfrac{\delta\Gamma[\varphi_{cl}]}{\delta\varphi_{cl}(x)} = -J(x)$.

* energy is a misnomer. eg. dimensions of $E[J]$ not of energy.

In particular, if $J=0$, the solution of $\frac{\delta\Gamma}{\delta\varphi_{cl}}=0$ gives the "equilibrium" (vacuum expectation value) φ_{cl}.

So far $\varphi_{cl} = \varphi_{cl}(x)$.

Further progress possible if we take φ_{cl} to be independent of x. i.e. φ_{cl} = constant

Then: $\Gamma[\varphi_{cl}] \equiv -(V.T).\, V_{eff}(\varphi_{cl})$.

where:

VT = four volume.

$V_{eff}(\varphi_{cl})$ = effective potential.

Example:

$$L = \frac{1}{2}(\partial_\mu \varphi)^2 + \frac{\mu^2}{2}\varphi^2 - \frac{\lambda}{4}\varphi^4.$$

Let: $\varphi = \varphi_{cl} + \chi$ $\qquad \varphi_{cl}$ = constant (independent of x).

Then:

$$L = \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2}(3\lambda\varphi_{cl}^2 - \mu^2)\chi^2 - \lambda\varphi_{cl}\chi^3 - \frac{\lambda}{4}\chi^4$$
$$+ \frac{1}{2}\mu^2\varphi_{cl}^2 - \frac{\lambda}{4}\varphi_{cl}^4 - \varphi_{cl}(\lambda\varphi_{cl}^2 - \mu^2)\chi.$$

Tree level: $\qquad [\varphi_{cl}]_{tree} = \pm\frac{\mu}{\sqrt{\lambda}} \equiv \varphi_{cl,0}$

Next - perturbation theory:

$L = L_0 + L_{int}$

with

$$L_0 = \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2}(3\lambda\varphi_{cl}^2 - \mu^2)\chi^2 + \frac{\mu^2}{2}\varphi_{cl}^2 - \frac{\lambda}{4}\varphi_{cl}^4$$

$$L_{int} = -\lambda\varphi_{cl}\chi^3 - \frac{\lambda}{4}\chi^4.$$

$V_{eff}(\varphi_{cl}) = -\frac{\mu^2}{2}\varphi_{cl}^2 + \frac{\lambda\varphi_{cl}^4}{4} +$  $+$  $+$  $+ \dots$

$$= -\frac{\mu^2}{2}\varphi_{cl}^2 + \frac{\lambda}{4}\varphi_{cl}^4 + \frac{1}{2(2\pi)^4}\int d^4k \ln(k^2 + m^2(\varphi_{cl})) + \dots$$

where: $m^2(\varphi_{cl}) = 3\lambda\varphi_{cl}^2 - \mu^2$

To one loop:

$$V(\varphi_{cl}) = -\frac{\mu^2}{2}\varphi_{cl}^2 + \frac{\lambda}{4}\varphi_{cl}^4 + \frac{1}{2(2\pi)^4}\int d^4k \ln\left(k^2 + m^2(\varphi_{cl})\right)$$

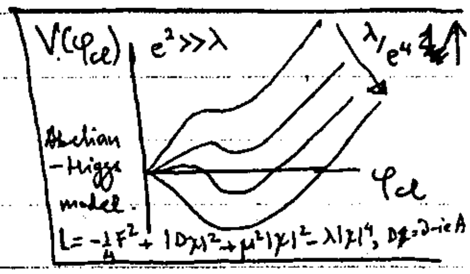
$$= -\frac{\mu^2}{2}\varphi_{cl}^2 + \frac{\lambda}{4}\varphi_{cl}^4 + \frac{1}{(2\pi)^3}\int d^3k \left(\vec{k}^2 + m^2(\varphi_{cl})\right)^{1/2}$$

Interpretation: $V(\varphi_{cl})$ = tree level potential + energy of all fluctuations about $\varphi = \varphi_{cl}$

Needs renormalization.
Introduce counterterms, renormalization conditions.
Finally,

$$V(\varphi_{cl}) = -\frac{\mu^2}{2}\varphi_{cl}^2 + \frac{\lambda}{4}\varphi_{cl}^4 + \frac{1}{64\pi^2}(3\lambda\varphi_{cl}^2 - \mu^2)^2 \ln\left(\frac{3\lambda\varphi_{cl}^2 - \mu^2}{2\mu^2}\right)$$
$$+ \frac{21\lambda\mu^2}{64\pi^2}\varphi_{cl}^2 - \frac{27\lambda^2}{128\pi^2}\varphi_{cl}^4$$

$$\equiv V_0(\varphi_{cl}) + V_1(\varphi_{cl}).$$



Note: Renormalization conditions chosen to keep $\varphi_{cl} = +\sqrt{\frac{\mu^2}{\lambda}}$ and $\frac{d^2V}{d\varphi^2} = 2\mu^2$ at $\varphi = \varphi_{cl}$. (Linde, '79).

Other conditions. (For $\overline{MS}$ see Peskin & Schroeder, pg. 377).

Example:

$$\mathcal{L} = \frac{1}{2} D_\mu \varphi_i D^\mu \varphi_i - V(\varphi) - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \mathcal{L}_F$$

with:
$$D_\mu \varphi = (\partial_\mu - ieA^a_\mu T^a)\varphi$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + e f_{abc} A^b_\mu A^c_\nu$$

$$\mathcal{L}_F = i\bar{\Psi}\gamma^\mu D_\mu \Psi - \bar{\Psi}\Gamma_i \Psi \varphi_i$$

One loop correction: [Coleman-Weinberg, '73

$$V_1(\varphi) = \frac{1}{64\pi^2}\left[\mathrm{Tr}\left(\mu^4 \ln \frac{\mu^2}{\sigma^2}\right) + 3\,\mathrm{Tr}\left(M^4 \ln \frac{M^2}{\sigma^2}\right) - 4\,\mathrm{Tr}\left(m^4 \ln \frac{m^2}{\sigma^2}\right)\right]$$

where:

σ = renormalization scale

$$\left.\begin{array}{l} \mu = \text{scalar mass} \\ M = \text{vector mass} \\ m = \text{spinor mass} \end{array}\right\} \text{matrices.}$$

$T \neq 0$:

<u>Prescription:</u>

Find $V_{eff,T}(\varphi)$ as we did $V_{eff,q}(\varphi)$ but with:*

(i) $k_0 \longrightarrow i k_0$

(ii) $k_0 = \begin{cases} 2n\pi T & \text{bosons} \\ (2n+1)\pi T & \text{fermions} \end{cases}$

(iii) $\int dk_0 \longrightarrow 2\pi T \sum_{n=-\infty}^{\infty}$

eg.

Recall:

$$V_{eff,q}(\varphi) = -\frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4 + \frac{1}{2(2\pi)^4}\int d^4k \ln(k^2+m^2)$$

Then:

$$V_{eff,T}(\varphi) = -\frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4 + \frac{T}{2(2\pi)^3}\sum_{n=-\infty}^{\infty} d^3k \ln\left[(2\pi n T)^2 + \vec{k}^2 + m^2\right]$$

$$= -\frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4 - \frac{\pi^2}{90}T^4 + \frac{m^2(\varphi)}{24}T^2 + O(\lambda^2) \qquad (T \gg m)$$

with $m^2(\varphi) = 3\lambda\varphi^2 - \mu^2$.

~~*If the number of particles N in the system is large (N >> 1) we have~~ $\cancel{\langle a^\dagger_{k=0} a \rangle \sim N \gg 1 \quad \therefore [a_0, a_0^\dagger] = 1 \ll N \sim \langle a_0^\dagger a_0 \rangle}$

Alternate derivation:

eg. $L = \frac{1}{2}(\partial_\mu \varphi)^2 + \frac{\mu^2}{2}\varphi^2 - \frac{\lambda}{4}\varphi^4$.

Eqn. of motion: $\Box\varphi + \mu^2\varphi - \lambda\varphi^3 = 0$.

Let: $\varphi = \varphi_{cl} + \chi$.

Then: $\{\Box\varphi_{cl} + \mu^2\varphi_{cl} - \lambda\varphi_{cl}^3\}$

$$+ \Box\chi + \mu^2\chi - \lambda(3\varphi_{cl}^2\chi + 3\varphi_{cl}\chi^2 + \chi^3) = 0$$

Ensemble average:

$$\Box\varphi_{cl} + \mu^2\varphi_{cl} - \lambda\varphi_{cl}^3$$

$$+ 0 + 0 - \lambda(0 + 3\varphi_{cl}\langle\chi^2\rangle_T + 0) = 0$$

Used:

$$\langle\chi\rangle_T = 0 = \langle\chi^3\rangle_T$$

$$\therefore \quad \Box\varphi_{cl} + \mu^2\varphi_{cl} + \lambda 3\langle\chi^2\rangle_T\varphi_{cl} - \lambda\varphi_{cl}^3 = 0.$$

$$\langle \chi^2 \rangle = \frac{1}{(2\pi)^3} \int \frac{d^3p}{2\omega_p} \left(2\langle a_p^\dagger a_p \rangle + 1 \right).$$

$$= \frac{1}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\sqrt{p^2 + m^2(\varphi_{cl})}} \frac{1}{\left[e^{\sqrt{p^2+m_{cl}^2}/T} - 1 \right]} + \text{constant (indep. of T.)}$$

Used: $\int d^3p \longrightarrow 4\pi \int p^2 dp$ (isotropic)

$$\omega_{\vec{p}} = \sqrt{\vec{p}^2 + m^2(\varphi_{cl})}$$

$$n_\omega = \frac{1}{e^{\omega/T} - 1} \quad \text{(Bose distribution).}$$

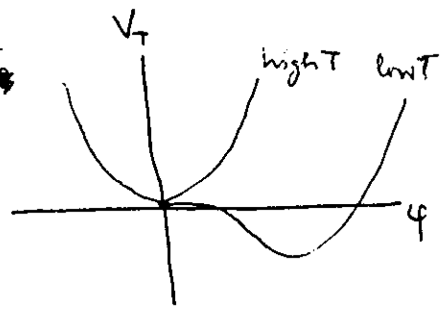
For $T \gg m_\varphi$: $\langle \chi^2 \rangle = \frac{T^2}{12}$

Then:

$$\Box \varphi_{cl} + \mu^2 \varphi_{cl} - \frac{3T^2}{12} \lambda \varphi_{cl} - \lambda \varphi_{cl}^3 = 0.$$

or, $\Box \varphi_{cl} + \left(\mu^2 - \frac{\lambda}{4} T^2 \right) \varphi_{cl} - \lambda \varphi_{cl}^3 = 0.$

$\therefore V_T(\varphi) = -\frac{1}{2}\left(\mu^2 - \lambda \frac{T^2}{4} \right) \varphi^2 + \frac{\lambda}{4} \varphi^4$

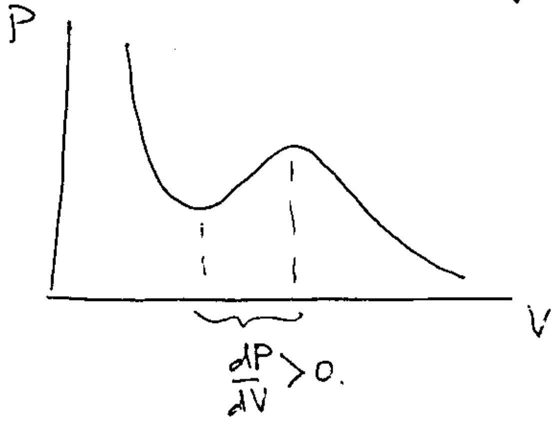


Maxwell Contruction:

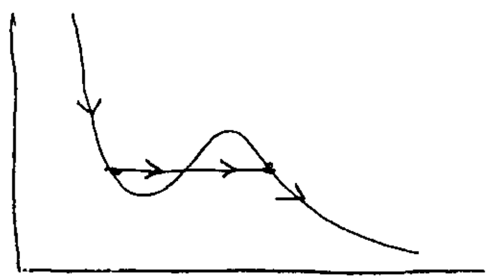
Recall thermodynamics of phase transitions.

If $\frac{dP}{dV} > 0$ then unstable system.

eg. Van der Waal's eqn of state for a gas

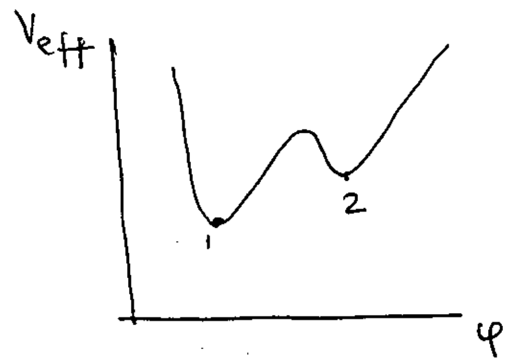


Maxwell's construction:



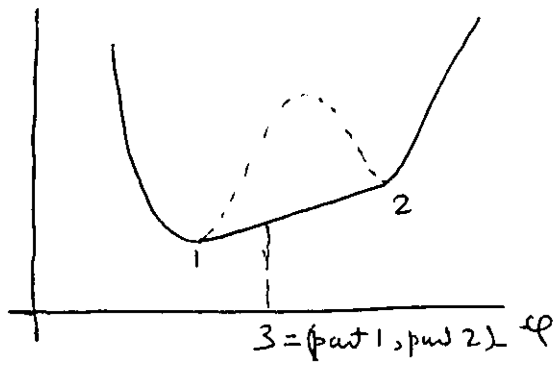
Reason: Assumption of uniform phase not valid. Maxwell's construction uses a coexistence of two phases.

Similarly: if



— this is valid only for constant φ i.e. coexistence of phases has been excluded.

Maxwell's construction — part of the system is in phase 1 and part in phase 2. Linear Coexistence of phases gives lower potential than V_{eff}.



Order of the phase transition:

eg. $$L = \frac{1}{2}(\partial_\mu \varphi)^2 + \frac{\mu^2}{2}\varphi^2 - \frac{\lambda}{4}\varphi^4.$$

The role of the thermodynamic potential is played by the effective potential.

$$V_T(\varphi) = -\frac{1}{2}\left(\mu^2 - \frac{\lambda}{4}T^2\right)\varphi^2 + \frac{\lambda}{4}\varphi^4$$

Phases are given by the minima of V_T.

$$\frac{\partial V_T}{\partial \varphi} = 0 = -\left(\mu^2 - \frac{\lambda}{4}T^2\right)\varphi_0 + \lambda\varphi_0^3$$

$$\Rightarrow \quad \varphi_0 = \begin{cases} \sqrt{\frac{1}{\lambda}\left(\mu^2 - \frac{\lambda}{4}T^2\right)} & \left(T < \frac{2\mu}{\sqrt{\lambda}}\right) \\ 0 & \left(T > T_c = 2\mu/\sqrt{\lambda}\right). \end{cases}$$

$\therefore$ Phase ($\varphi = 0$) : $\quad V_T = 0$
(Phase 1)

Phase ($\varphi = \varphi_0$): $V_T = -\frac{1}{2}\left(\mu^2 - \frac{\lambda}{4}T^2\right)\frac{1}{\lambda}\left(\mu^2 - \frac{\lambda T^2}{4}\right) + \frac{\cancel{\lambda}}{4}\sqrt{\frac{1}{\lambda}\left(\mu^2 - \frac{\lambda}{4}T^2\right)}\,\frac{1}{\cancel{\lambda}}\left(\mu^2 - \frac{\lambda}{4}T^2\right)$
(Phase 2)

$$= \frac{1}{\lambda}\left(\mu^2 - \frac{\lambda T^2}{4}\right)^2\left[-\frac{1}{2} + \frac{1}{4}\right] = -\frac{1}{4\lambda}\left(\mu^2 - \frac{\lambda T^2}{4}\right)^2$$

$$= -\frac{1}{4\lambda}\cdot\frac{\lambda^2}{16}\left(T_c^2 - T^2\right)^2 = -\frac{\lambda}{64}\left(T^2 - T_c^2\right)^2$$

$$\therefore \quad \tilde{V}_T \Big|_{\substack{\text{phase 1} \\ T=T_c}} = 0 = \tilde{V}_T \Big|_{\substack{\text{phase 2} \\ T=T_c}} \qquad \tilde{V}_T = V_T(\varphi_0(T))$$

$$\cancel{\frac{\partial V_T}{\partial T}} = \frac{d\tilde{V}_T}{dT}\Big|_{\substack{\text{phase 1} \\ T=T_c}} = 0 = \frac{d\tilde{V}_T}{dT}\Big|_{\substack{\text{phase 2} \\ T=T_c}}$$

$$\frac{d^2\tilde{V}_T}{dT}\Big|_{\substack{\text{phase 1} \\ T=T_c}} = 0$$

$$\frac{d^2\tilde{V}_T}{dT}\Big|_{\substack{\text{phase 2} \\ T=T_c}} = -\frac{\lambda}{64}\, 2\, \frac{d}{dT}\left[(T^2 - T_c^2)\, 2T\right]\Big|_{T=T_c}$$

$$= -\frac{\lambda}{16}\left(3T^2 - T_c^2\right)\Big|_{T=T_c} = -\frac{\lambda}{8} T_c^2 \neq 0.$$

$$\therefore \quad \frac{d^2\tilde{V}_T}{dT^2}\Big|_{\substack{\text{phase 1} \\ T=T_c}} \neq \frac{d^2\tilde{V}_T}{dT^2}\Big|_{\substack{\text{phase 2} \\ T=T_c}}.$$

$\therefore$ 2nd order phase transition (Ehrenfest classification

Comment: Cooling can also lead to symmetry restoration.

Classification of Topological Defects

Consider a Lagrangian L that is invariant under transformations of a group G.* So:

$$L(\varphi) = L(D(g)\varphi).$$

When $\varphi_{cl} \neq 0$, the symmetry gets spontaneously broken to $H \subset G$. The subgroup H is determined by the condition:

$$D(h) \cdot \varphi_{cl} = \varphi_{cl} \qquad \forall\, h \in H.$$

Next note that

$$D(g)\left[D(h)\,\varphi_{cl}\right] = D(g)\,\varphi_{cl}.$$

$\therefore$ the action of an element g of G on φ_{cl} is identical for all the elements $\{gh_i : h_i \in H\} \equiv gH$. This is called a (left) coset.

The "vacuum manifold" is given by the action of gH on φ_{cl}. since, if $\varphi_{cl} = \varphi_0$ minimizes the (effective) potential, so does $D(gH)\varphi_0$.

$\therefore$ the topology of the vacuum manifold is given by the topology of the manifold $G/H = \{gH\}$.

* In principle, the action should be ~~chosen to~~ invariant under the symmetry group ~~transformations~~.

Topology of the vacuum manifold is important because non-trivial topology implies non-trivial classical solutions.

Example 1:

$$L = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{\lambda}{4}(\varphi^2 - \eta^2)^2 \qquad \varphi = \text{real scalar field}$$

Symmetry: $\varphi \longrightarrow \pm\varphi \qquad \therefore G = \mathbb{Z}_2.$

$\varphi_{cl} = \pm\eta$

Then: $H = \mathbb{1}$.

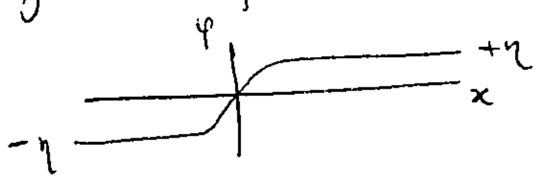
$G/H = \mathbb{Z}_2.$

Manifold: $\underset{-1}{\bullet} \quad \underset{+1}{\bullet}$

Non-trivial manifold because it has disconnected components.

Zeroth homotopy group $= \pi_0(G/H) = \mathbb{Z}_2.$

Topological defect = domain wall



There exists an 'x' for which $\varphi(x) = 0$ if $\varphi(\pm\infty) = \pm\eta$.
But $\varphi = 0$ is <u>outside</u> the vacuum manifold and hence the configuration has non-zero energy.

Bogomolnyi's method:

$$E = \int dx \left[\frac{1}{2}\left(\frac{d\varphi}{dx}\right)^2 + \frac{\lambda}{4}(\varphi^2-\eta^2)^2\right]$$

$$= \frac{1}{2}\int dx.\left[\left\{\frac{d\varphi}{dx} + \sqrt{\frac{\lambda}{2}}(\varphi^2-\eta^2)\right\}^2 - 2\sqrt{\frac{\lambda}{2}}(\varphi^2-\eta^2)\frac{d\varphi}{dx}\right]$$

$$= -\sqrt{\frac{\lambda}{2}}\int dx.\frac{d}{dx}\left(\frac{\varphi^3}{3}-\varphi\eta^2\right) + \frac{1}{2}\int dx\left\{\frac{d\varphi}{dx}+\sqrt{\frac{\lambda}{2}}(\varphi^2-\eta^2)\right\}^2$$

$$= -\sqrt{\frac{\lambda}{2}}\left[\frac{\varphi^3}{3}-\varphi\eta^2\right]_{x=-\infty}^{+\infty} + \frac{1}{2}\int dx\left\{\frac{d\varphi}{dx}+\sqrt{\frac{\lambda}{2}}(\varphi^2-\eta^2)\right\}^2$$

$$\geqslant -\sqrt{\frac{\lambda}{2}}\left[\frac{\varphi^3}{3}-\varphi\eta^2\right]_{-\infty}^{\infty}$$

If $\varphi(\pm\infty) = \pm\eta$

$$E \geqslant -\sqrt{\frac{\lambda}{2}}\,\eta^3\left[\left(\frac{1}{3}-1\right)-\left(-\frac{1}{3}+1\right)\right] = -\sqrt{\frac{\lambda}{2}}\,\eta^3\left[-\frac{2}{3}\times 2\right] = \frac{4}{3}\sqrt{\frac{\lambda}{2}}\,\eta^3.$$

And $E = \frac{4}{3}\sqrt{\frac{\lambda}{2}}\,\eta^3$ if

$$\frac{d\varphi}{dx} = \sqrt{\frac{\lambda}{2}}(\eta^2-\varphi^2).$$

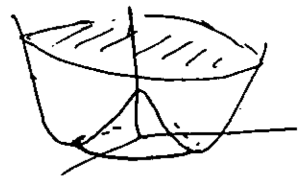
$$\therefore \quad \int\frac{d\varphi}{\eta^2-\varphi^2} = \sqrt{\frac{\lambda}{2}}\,x. \qquad \text{or,} \quad \varphi = \eta\tanh\left(\sqrt{\frac{\lambda}{2}}\,x\right).$$

Example 2:

$$L = \frac{1}{2}|\partial_\mu \varphi|^2 - \frac{\lambda}{4}(|\varphi|^2 - \eta^2)^2 \qquad \varphi = \text{complex scalar field.}$$

Symmetry: $\varphi(x) \longrightarrow e^{i\alpha}\varphi(x)$ $\quad \alpha = $ constant in x.

$\therefore \quad G = U(1)_{\text{global}}$



$H = \mathbb{1}$

Manifold $= S^1 \qquad (|\varphi|^2 = \eta^2)$.

$\pi_1(G/H) = \mathbb{Z}$

Solution = global cosmic string = vortex

Example 3:

$$L = \frac{1}{2}|\partial_\mu \vec{\varphi}|^2 - \frac{\lambda}{4}(\vec{\varphi}\cdot\vec{\varphi} - \eta^2)^2 \qquad \vec{\varphi} = \text{real triplet}$$

$G = O(3)$

$H = O(2)$.

Manifold $= S^2 \qquad (\vec{\varphi}\cdot\vec{\varphi} = \eta^2)$.

$\pi_2(G/H) = \mathbb{Z}$.

Global monopole.

Example 4: Gauge above examples. Manifold is unchanged. $\therefore$ solutions survive. Global string becomes local string, global monopole becomes magnetic monopole ('tHooft - Polyakov). (Bogomolnyi method applicable)