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Lecture I

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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# FLAVOR IN SUPERSYMMETRIC THEORIES

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# FLAVOR IN SUPERSYMMETRIC THEORIES

Lawrence Hall.

## (I) Introduction.

- the flavor parameters.
- the supersym. flavor "problem"

## (II) Flavor Symmetries & Signals.

- symmetries control
  - q/l masses
  - $\tilde{q}/\tilde{l}$ masses
  - mixings V, W.
- U(2) model
- (– slepton oscillations)

## (III) Theories Of Flavor

- origin of small parameters
- the Froggatt-Nielsen mechanism
- scalar masses as probe of high energies.

# AN EXAMPLE OF FLAVOR SYM: QCD $\longrightarrow 0$

Suppose we are interested in SI beneath $m_c$:

- QCD with 3 flavors
- QCD interaction possesses $SU(3)_q \times SU(3)_{q^c} \times U(1)_B$

$SU(3)_q$ on $q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$; $SU(3)_{q^c}$ on $q^c = \begin{pmatrix} u^c \\ d^c \\ s^c \end{pmatrix}$

- This flavor group is broken spontaneously by $\langle q q^c \rangle$:

$$SU(3)_q \times SU(3)_{q^c} \longrightarrow SU(3)_{q+q^c}$$

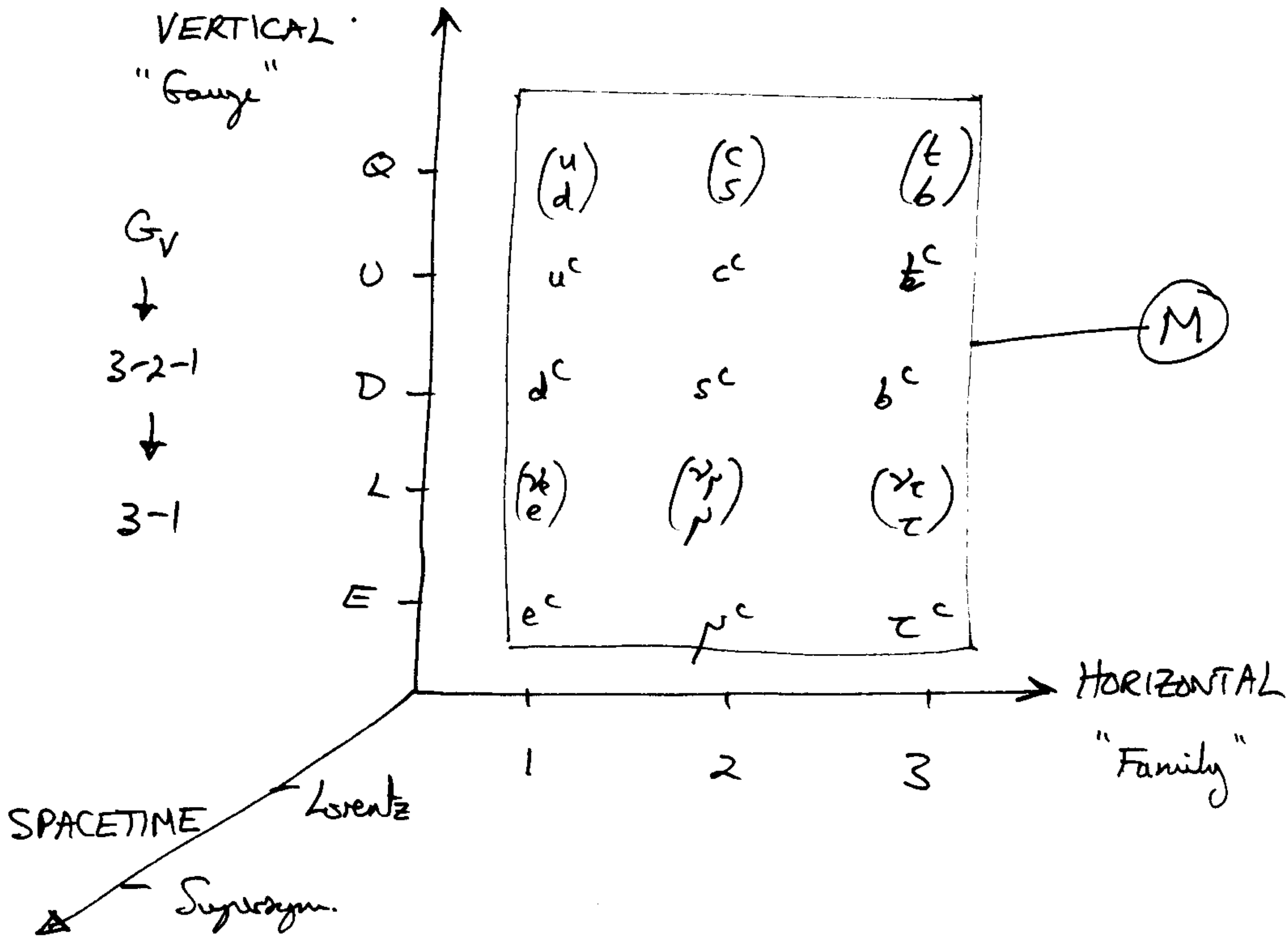
- The SU(3) is weakly broken explicitly

$$\underbrace{SU(3) \xrightarrow{m_s/\Lambda} \underbrace{SU(2)_I \xrightarrow{\frac{m_{u,d}}{\Lambda},\, \alpha} 0}_{\text{brings great order to nuclear physics}}}_{\text{brings great order to hadronic physics.}}$$

Suppose we are above $M_W$; we now have 3 perturbative gauge interactions. What do we mean by flavor & flavor syms? Certainly not isospin!

# WHAT IS FLAVOR?

Think of matter filling out some large multiplet in a 3d symmetry space:




The sheet M gets repeated along the spacetime axi

Lorentz $\quad M \longrightarrow \bar{M}$ the Lorentz-partners.

Supersym. $\quad M \longrightarrow \tilde{M}$ the super-partners.

# THE MAXIMAL FLAVOR GROUP, $G_{f\,max}$, & ITS BREAKING

$G_{f\,max}$ is max. sym. of interactions of $G_V$, $\gamma$ $\therefore$ depends on $G_V$:

| $G_V$ | 1 family | $G_f$ max |
|---|---|---|
| 3-2-1 | Q, U, D, L, E | $U(3)^5$ |
| 5 | T, $\bar{F}$ | $U(3)^2$ |
| 10 | $\psi$ | $U(3)$ |

Experiment tells us that $G_{f\,max}$ is broken by q/l masses and by CKM mixing of weak interactions. ~~This breaking is~~ If this breaking occurs via Yukawa interactions of Higgs, then breaking is

a) ~~At least~~ To $\begin{cases} U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau \\ U(1)_{B-L} \\ 0 \end{cases}$ for $\underline{G_V}$ $\begin{cases} 3-2-1 \\ 5 \\ 10 \end{cases}$

b) Of varying strengths for the different generators.

c) Of order unity only from $m_t$:

| | | | $\underline{G_V}$ |
|---|---|---|---|
| $U(3)_Q \times U(3)_u$ | $\xrightarrow{m_t}$ | $U(2)_Q \times U(2)_u$ | 3-2-1 |
| $U(3)_T$ | $\longrightarrow$ | $U(2)_T$ | 5 |
| $U(3)$ | $\longrightarrow$ | $U(2)$ | 10 |

## THE FLAVOR ISSUES

1. What is the origin of $G_f$ max. ?

2. What are the interactions which break $G_f$ max ?

3. Why is there a hierarchy of breaking ?

## In the STANDARD MODEL :

1. No answer.

2. $\mathcal{L}_{Yuk} = q \lambda_U u^c h + q \lambda_D d^c h^* + l \lambda_E e^c h^*$

3. No answer.

## In the SUPERSYMMETRIZED S.M. :

1. No answer.

2. $h \to h_u, h_d$ $\qquad \mathcal{L}_{Yuk} \longrightarrow [W]_F$

3. No answer.

$$\boxed{\text{IS SUPERSYMMETRY REALLY SO IRRELEVANT TO FLAV}}$$

NO!

( I lied in 2. above )

# THE FLAVOR PARAMS. OF SUPERSYM. 3-2-1

$W = Q \lambda_u U^c H_u + \dots$ $\qquad\qquad \lambda_{U,D,E}$

$V_{soft} = \tilde{q}^+ m_Q^2 \tilde{q} + \dots$ $\qquad\qquad m^2_{Q,U,D,L,E}$

$\quad + \tilde{q}\, \xi_u\, \tilde{u}^c h_u + \dots$ $\qquad\qquad \xi_{U,D,E}$.

BASIC RESULT

$$\boxed{\lambda_{U,D,E} \longrightarrow \lambda_{U,D,E}\ ;\ m^2_{Q,U,D,L,E}\ ;\ \xi_{U,D,E}}$$

$U(3)^5$ transf

$\lambda_u, \xi_u \sim (3, 3, 1, 1, 1)$, etc.

$m_Q^2 \sim (1+8, 1, 1, 1, 1)$, etc.

Comments.

1. I ignore $R_p$ viol. $\lambda'_{ijl} Q_i D_j L_k + \dots$

2. Ignore $\tilde{q}\, \xi'_u\, \tilde{u}^c h_d^* + \dots$ (absent in supergrav.)

3. I ignore higher dim. non-renorm. interactions of effective theory, such as $W_4 = \frac{1}{M} L L H_u H_u$.

# "TRIVIAL" SUPERSYMMETRIC FLAVOR STRUCTURE

$$\boxed{\lambda, m^2, \xi} \quad \begin{array}{l} \rightarrow \lambda \qquad \text{supersym.} \\ \rightarrow \xi, m^2 \qquad \text{supersym. breaking.} \end{array}$$

Assume that supersym. breaking preserves $U(3)^5$ :

$$m^2_{Q,U,D,L,E} = m^2_{0\,Q,U,D,L,E}\, I + \ldots$$

$$\xi_{U,D,E} = 0 + A_{U,D,E}\, \lambda_{U,D,E} + \ldots$$

Examples

1. Minimal supergravity: $m_0^2$, $|A| e^{i\phi_A}$

2. Gauge mediation $m_0{}^2_{Q,U,D,L,E}$ , $A_{U,D,E} = 0$

Comments.

a) These are just boundary conditions, valid at some $\mu_0$.

b) I view $m_0^2$, $|A|$, $\phi_A$ as supersym. br. params (not flavor

c) If $\phi_A \approx O(1)$ then $e^-$ and $n$ edm are perhaps factor 30 too large.

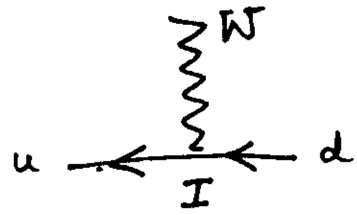
THESE LECTURES : $\boxed{m_0^2 \longrightarrow \underline{\underline{m}}^2}$ (very little on $\xi$)

# THE PHYSICAL FLAVOR PARAMETERS.

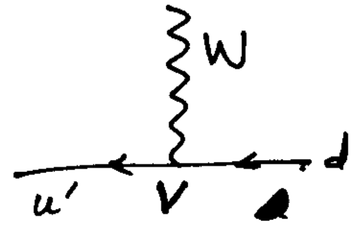
## (A) STANDARD MODEL

### $V_{CKM}$

$$\underline{\underline{\lambda}}_u\, v \xrightarrow{R_u,\,R_{u^c}} \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}, \text{ etc.}$$



$\xrightarrow{R_u,\,R_d}$



$$V = R_u^\dagger R_d$$

### Counting Parameters

In SM it is easy to explicitly redefine fields to show $V \supset 4$ params.
In supersym. this is harder — it is important to first have an algorithm for counting number of physical params. (in particular the phases).

Consider the fields + Lag. of the 3-2-1 invariant theory.

$$N_{Lag} - N_{Redef.} = N_{phys}.$$

| | $N_{Lag}$ | $N_{Redef.}$ | $N_{phys}$ |
|---|---|---|---|
| q sector | $\lambda_U, \lambda_D$ 36 | $q, u^c, d^c$ $U(3)^3/U(1)_B$ 26 | 10 $\begin{cases} m_{u,c,t} \\ m_{d,s,b} \\ \theta_{1,2,3} \\ \phi \end{cases}$ |
| $l$ sector | $\lambda_E$ 18 | $l, e^c$ $U(3)^2/U(1)^3$ 15 | 3 $\{\, m_{e,\mu,\tau}$ |

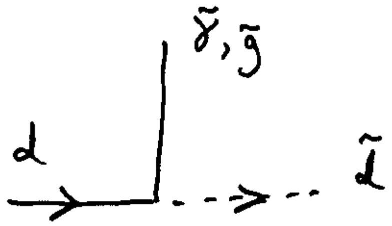
The crucial result is that there is a single phase $\phi$. It has a rephase invariant form

$$J = \mathrm{Im}\left(V_{i\alpha} V_{j\beta} V^*_{i\beta} V^*_{i\alpha}\right) \quad \text{obtained from} \quad \begin{pmatrix} \bullet_{i\alpha} & \bullet_{i\beta}^* \\ \bullet_{j\alpha}^* & \bullet_{j\beta} \end{pmatrix}$$

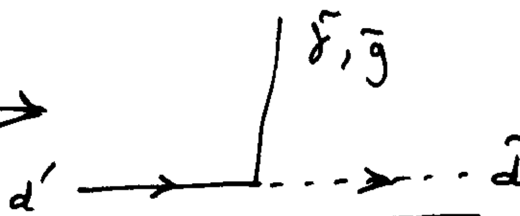
(B) Extension To Supersymm.

There is an analysis, of comparable importance, in all theories with weak scale supersymm. Yet, these results are NOT WELL KNOWN!

• $$m_Q^2 \xrightarrow{\quad R_{\tilde q} \quad} \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix}_{\tilde q}$$




$$\xrightarrow{\quad R_{\tilde d}, R_d \quad}$$




$$\boxed{W^d = R_{\tilde d}^\dagger R_d}$$

There are 6 new flavor mixing matrices:

$$\boxed{W^{u, u^c, d, d^c, e, e^c}}$$

Comments

a) $W^\nu = W^e$ for massless $\nu$.

b) $\xi$ turns scalar diagonaliz$^n$ into $6\times 6$ problem.

c) We should really diag. $m^2 + \lambda^\dagger \lambda v^2$

d) $V_{CKM} = W^{u\dagger} W^d$

e) Special importance for lepton flavor violation. (unless at some scale $m_L^2, m_{e^c}^2 \propto I$)

Conclude.

$$\lambda \rightarrow m_{q,l}\ ; \qquad V \text{ at CC gauge int.}$$

$$m^2 \rightarrow m_{\tilde{q},\tilde{l}}\ ; \qquad W \text{ at NC supergauge int.}$$

COUNTING PARAMS

lepton sector

$$\underset{36}{\lambda_E,\ m_L^2,\ m_E^2} \quad - \quad \underset{17}{\overset{L,E^c}{U(3)^2/U(1)_L}} \quad = \quad 19 \begin{cases} m_{e,\mu,\tau} & 3 \\ m_{\tilde{e},\tilde{\mu},\tilde{\tau}},\ m_{\tilde{e}^c,\tilde{\mu}^c,\tilde{\tau}^c} & 6 \\ (\theta_i)^e,\ (\theta_i)^{e^c} & 6 \\ \phi_i & 4 \end{cases}$$

quark sector

$$\underset{63}{\underset{36}{\lambda_{U,D}}\ \ \underset{27}{m^2_{Q,U,D}}} \quad - \quad \underset{26}{\overset{Q,U^c,D^c}{U(3)^3/U(1)}} \quad = \quad 37 \begin{cases} m_{u,c,t}\ m_{d,s,b} & 6 \\ m_{\tilde{q}_i},\ m_{\tilde{u}^c_i},\ m_{\tilde{d}^c_i} & 9 \\ (\theta_i)^{u,u^c,d,d^c} & 12 \\ \phi_i & 10 \end{cases}$$

We could have written the masses & mixing angles immediately; the non-trivial aspect is that

$$\lambda \text{ (1 phase)} \longrightarrow \lambda, m^2 \text{ (14 phases)}.$$

How should we think of these 14 phases?

[ $\xi_{U,D,E}$ contain a further 54 params. The flavor sector of supersym 3-2-1 contains 110 parameters ]

In 2 gen. supersym. theory similar counting gives $\binom{1}{3}$ phases in $\binom{l}{q}$ sector.

## THE SUPER-KM BASIS.

Perform superfield rotations so that:

- $\lambda_{U,D,E}$ real & diag. (sep. rot. on $u, d$).
- $V$ in a standard form with 1 phase: $\phi_{CKM}$.

At this stage, $W = I$, $\quad m^2_{Q,U,D,L,E} = \begin{pmatrix} \cdot & \alpha & \beta \\ & \cdot & \gamma \\ & & \cdot \end{pmatrix}_{Q,U,D,L,E}$.

### q sector

All phase redefs. used up in getting $V$ (std. form), if try to remove any $(\alpha, \beta, \gamma)$, ~~they~~ while keeping $\lambda$ real, they appear in $V$.

$$\boxed{\phi_{CKM}, \quad (\alpha, \beta, \gamma)_{Q,U,D}}$$

### l sector

Any two phases can be removed from $m^2_{L,E}$ by performing $\dfrac{U(1)_e \times U(1)_\mu \times U(1)_\tau}{U(1)_L}$ rotations.

$$\boxed{\text{Any 4 of } (\alpha, \beta, \gamma)_{L,E}}$$

## MASS BASIS

$$V = \tilde{u}^{+} m_{\tilde{u}}^{2} \tilde{u} + \ldots$$

Let $\tilde{u}^{\bullet} = W^{u+} \tilde{u}'$ so

$$V = \tilde{u}'^{+} \underbrace{\left(W^{u} m_{\tilde{u}}^{2} W^{u+}\right)}_{\longrightarrow \begin{pmatrix} m_{\tilde{u}_1}^{2} & 0 \\ 0 & m_{\tilde{u}_2}^{2} \end{pmatrix}} \tilde{u}'$$

and

$$\tilde{u}^{+} u \tilde{g} \longrightarrow \left(\tilde{u}^{+} W^{u} u\right) \tilde{g}$$

There are now 13 phases in $W^{u,d,u^c,d^c,e,e^c}$, it is useful to write them in a rephase invariant way.

## $\tilde{J}$ (6).

Each W matrix separately contains an invariant (cf $V \to J$).

$$\boxed{\tilde{J} = (\pm)\, \mathrm{Im}\left(W_{i\alpha} W_{j\beta} W_{i\beta}^* W_{j\alpha}^*\right)}$$

applies to the 6 indep. cases of $u, u^c, d, d^c, e, e^c$. But recall that $\tilde{J}^{u,d}$ are not indep. of $J$, because $V_{CKM} = W_u^\dagger W_d$.

There is one $\tilde{J}$ per $m^2$: $\tilde{J} \sim \alpha\gamma\beta^*$, or more precisely:

$$\mathrm{Im}\left(m_{12}^2\, m_{23}^2\, m_{31}^2\right) = \tilde{J}\,(m_1^2 - m_2^2)(m_2^2 - m_3^2)(m_3^2 - m_1^2)$$

$\tilde{J}$, like $J$, require 3 generations. (Note rephase inv. contraction)

## $\tilde{K}$ (8)

Each of the remaining 8 phases must involve more than one W for its definition. They can be written down in terms of a $(W, W^c)$ or $(W, W)$ pair. We <u>choose</u> the 8 indep. phases as:

$$\tilde{K}_{12}^{ee^c,\, dd^c,\, uu^c,\, ud} = \mathrm{Im}\left(W_{21} W_{21}^c\, W_{22}^*\, W_{22}^{c*}\right)$$

which are the 4 physical phases in the 2 generation theory, &

$$\tilde{K}_{13}^{ee^c,\, dd^c,\, uu^c,\, ud} = \mathrm{Im}\left(\;''\quad \text{with } 2 \to 3\right).$$

The scalar indices $^{(i)}$ appear first in $W_{i\alpha}$, & occur as $(i^* i)$ in $\tilde{K}$.
The fermion indices $\alpha$ appear in $\tilde{K}$ as $(\alpha\alpha^c)$, which is rephase invar. since the $\alpha$ fermion mass must be kept real.
The $\tilde{K}_{23}$ are not linearly indep. from $\tilde{K}_{12}, \tilde{K}_{13}$.

## Exercise 1

(a) Consider 2 generation lepton sector.

In basis where $W = I$

$$m^{2\,(c)} = \begin{pmatrix} m_1^2 & m_{12}^2 \\ m_{12}^{c\,2} & m_2^2 \end{pmatrix}^{(c)}$$

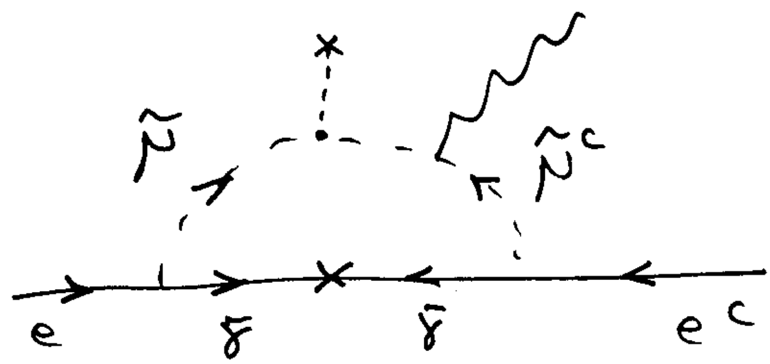
with phase in $m_{12}^{2\,(c)}$

Show that the single rephase invariant is

$$\mathrm{Im}\left(m_{12}^2\, m_{12}^{c\,2*}\right) = -\tilde{K}_{12}^{ee^c}\left(m_1^2 - m_2^2\right)\left(m_1^{c\,2} - m_2^{c\,2}\right).$$

(b) Estimate the size of the $e^-$ edm from $\tilde{K}_{12}^{ee^c}$.

Hint:



Take $(\xi_E)_{22} = A\lambda_\mu$ real.

## (a) 2 GEN : RELATING $\tilde{K}_{12}$ to $m^2$.

$$m^2_{12} = W^+_{1i}\, m_i^2\, W_{i2}$$

But $W^+_{11} W_{12} + W^+_{12} W_{22} = 0$ by unitarity so

$$m^2_{12} = W^+_{12}\left(-m_1^2 + m_2^2\right) W_{22}$$

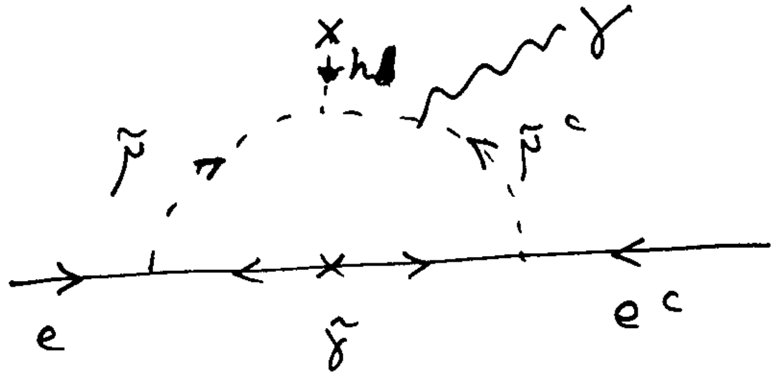
$$\mathrm{Im}\left(m^2_{12}\, m^{c\,2}_{12}\right) = \mathrm{Im}\left(W^*_{21}\, W^{c\,*}_{21}\, W_{22}\, W^c_{22}\right)\left(m_1^2 - m_2^2\right)\left(m_1^{c\,2} - m_2^{c\,2}\right)$$

$$= -\ \tilde{K}_{12}\left(m_1^2 - m_2^2\right)\left(m_1^{c\,2} - m_2^{c\,2}\right)$$

(b) The 1-2 CONTRIBUTION TO EDM OF e : $d_e$

Estimate $\mathcal{L}_{eff} = d_e \, (e \, \sigma^{\mu\nu} e^c) F_{\mu\nu}$

coming from




$$d_e \simeq e \cdot \left(\frac{e^2}{16\pi^2}\right) \cdot \frac{m_{\tilde\gamma}}{\bar m^4} \left(\xi^E_{22} + \lambda^E_{22}\, \mu \tan\beta\right) v_1$$

$$\times \, \mathrm{Im}\left(c s c^c s^c e^{i\phi}\right) \left(\frac{\Delta m^2_{12}}{\bar m^2}\right)\left(\frac{\Delta m^{c\,2}_{12}}{\bar m^2}\right).$$

$$\approx e \cdot 10^{-3} \cdot \left(\frac{m_\mu}{\bar m^2}\right)\left(s s^c \, s_\phi \, \delta_{12} \delta^c_{12}\right).$$

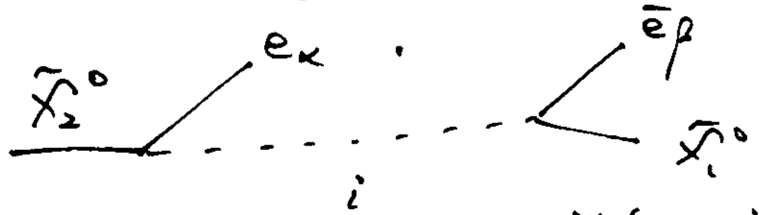
(where $\xi^E_{22} v_1 = A m_\mu$, $A = 1$, $\mu \approx \bar m$, $\tan\beta \approx 1$.

$$\approx e \cdot 10^{-3} \cdot 10^{-5} \, \frac{fm}{5} \, \frac{1}{20} \left(\frac{s s^c}{s_c^2}\right) s_\phi \, \delta_{12} \delta^c_{12}$$

$$\boxed{d_e \approx 10^{-23} \, e\,cm \, \underbrace{\left(\frac{s s^c}{s_c^2}\right) s_\phi \, \delta_{12} \delta^c_{12}}_{\lesssim 4\times 10^{-4}}}$$

## Ex. α  $\tilde{J}$ GIVES CP VIOLATION IN SLEPTON OSCILLATIONS.

Suppose $\tilde{\chi}_2^0$ copiously produced, for example at LHC, & decay leptonically:




$$A_{\alpha\beta}(t) = \sum_i W_{i\alpha} W^*_{i\beta}\, e^{-\Gamma t/2}\, e^{-i m_i t}$$

If $\Delta m_{ij} = m_i - m_j \gg \Gamma$ we can forget $e^{-i m_i t}$, but for $\Delta m_{ij} \approx \Gamma$ (assumed indep $i$) we get slepton oscillation

$$P(e_\alpha \bar{e}_\beta) = \sum_{ij} W_{i\alpha} W_{j\beta} W^*_{i\beta} W^*_{j\alpha} \underline{\int dt\, e^{-\Gamma t}\, e^{-i(m_i - m_j)t}}$$

$$= \sum_{ij} X_{ij} A_{ij} \qquad A_{ij} = \frac{1}{1 + i\frac{\Delta m_{ij}}{\Gamma}}$$

This phenomena could be an excellent probe of $\theta_{ij}$ in $W_{ij}$. More optimistically, it could also probe CP violation:

$$\Delta_{\alpha\beta} = P_{e_\alpha \bar{e}_\beta} - P_{\bar{e}_\alpha e_\beta} = \sum_{ij} (X_{ij} - X^*_{ij}) A_{ij}$$

$$= \sum_{ij} 2i (\mathrm{Im}\, X_{ij}) A_{ij}$$

$$= \sum_{ij} 2i \tilde{J}_{ij} A_{ij}$$

. Since $\tilde{J}_{ij}$ is antisym

$$\boxed{\Delta_{\alpha\beta} = \sum_{ij} 4 \tilde{J}_{ij} \frac{(\Delta m/\Gamma)_{ij}}{1 + (\Delta m/\Gamma)^2_{ij}}}$$