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Lecture II & III

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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FLAVOR IN SUPERSYMMETRIC THEORIES

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Please note: These are preliminary notes intended for internal distribution only.

(II) FLAVOR SYMMETRIES.

L.J. Hall.

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THE SUPERSYM. FLAVOR/CP "PROBLEM"

SM $\lambda \rightarrow 13$ flavor params.

Supersym. $\lambda, m^2 \rightarrow 56$ flavor params.

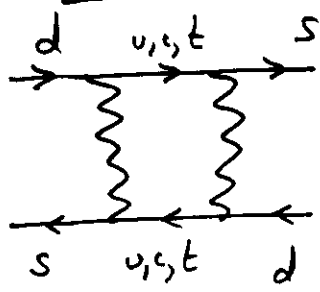
Experiments on FCNC/CP $\Rightarrow$ many of 42 dimensionless parameters of m^2 are necessarily small.

ϵ_K

Gives the most powerful constraints.

Others: d, e , $\mu \rightarrow e$, $b \rightarrow s$, ...

S.M. recall GIM



$$\propto \left(V_{2i} + \frac{k + m_i}{k^2 - m_i^2} V_{i1} \right)^2$$

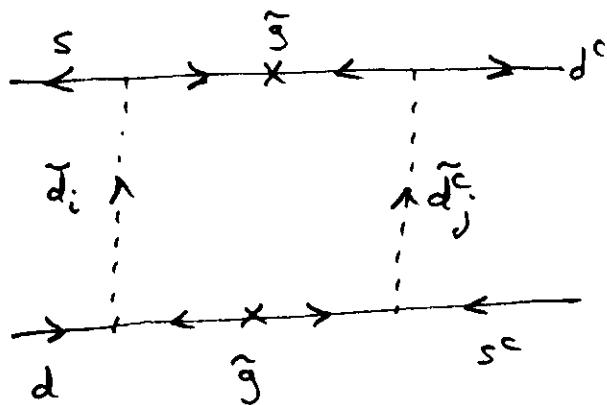
$$\epsilon_K^{SM} \propto G_F^2 m_t^2 f \left(\frac{m_t}{M_W} \right) \text{Im} (V_{td} V_{ts}^*)^2$$

GIM: 12 problem solved by small m_q
 3 problem solved by small δ_{ij} .

STRONG SUPER-BOX FOR E_K : need SGIM

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Considers only 2 gen. theory - cannot argue that this contrib: small due to small W_{31}, W_{32} . LR box has largest matrix element:



$$\propto \left(W_{2i}^+ \frac{1}{k^2 - m_i^2} W_{i1} \right) \left(W_{ij}^{c+} \frac{1}{k^2 - m_j^{c2}} W_{j2} \right)$$

In superKM basis each m^2 has 1 phase $m^2 = \begin{pmatrix} m_1'^2 & m_3'^2 \alpha \\ m_3'^2 \alpha^* & m_2'^2 \end{pmatrix}$, $m^{c2} = \dots$

Hence mass basis obtained first by rel. rot. α between d and s phases, & then 2×2 real rotation:

$$W^d = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \alpha \end{pmatrix}, \quad W^{d^c} = \begin{pmatrix} c^c & s^c \\ -s^c & c^c \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \alpha^c \end{pmatrix}$$

$$E_K^{\text{imag}} \propto \text{Im}(sc s^c c^c \alpha^* \alpha^c) \frac{m_2^2 - m_1^2}{(k^2 - m_1^2)(k^2 - m_2^2)} \cdot \frac{m_2^{c2} - m_1^{c2}}{(k^2 - m_1^{c2})(k^2 - m_2^{c2})}$$

$$\approx \frac{1}{\bar{m}^2} (cc^c ss^c \text{Im} \alpha^* \alpha^c) \frac{\Delta m_{12}^2}{\bar{m}^2} \frac{\Delta m_{12}^{c2}}{\bar{m}^2}$$

Hence we must compare

$$G_F^2 m_t^2 \underbrace{\text{Im}(V_{td}^* V_{ts})^2}_{\leq 10^{-7}} \quad \text{to} \quad \frac{1}{\bar{m}^2} \underbrace{cc^c ss^c \text{Im} \alpha^* \alpha^c \frac{\Delta m_{12}^2}{\bar{m}^2} \frac{\Delta m_{12}^{c2}}{\bar{m}^2}}_{\leq 10^{-6} \text{ for } \bar{m} = 350 \text{ GeV.}}$$

THE POSSIBLE DIRECTIONS.

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The supersym. FC/CP problem is a useful guide to constructing theories of flavor.

1. $\bar{m} \gg 300 \text{ GeV}$

Regain fine tuning.

5 TeV OK for "cousins" but only $(15)^2$.

2. $S, S^c \ll S_c$

Apparently outrageous:

$$W = \tilde{R}^+ R \xrightarrow{\quad} \mathcal{O}_c.$$

"Alignment" possibility

$$R_d = I, R_u = V_{CKM}.$$

3. $\Delta m \propto \ll 1$

Apparently outrageous:

3 gen. level

$$W = \tilde{R}^+ R \xrightarrow{\quad} \bar{f}$$

How about $\Delta m_K, \mu \rightarrow e \gamma$.

4. $\frac{\Delta m_{12}^{2(c)}}{\bar{m}^2} \ll 1$

Original idea (DG), & still the best.

WHY DEGENERATE?

① Gauge Mediation

Flavor signals are sparticle spectrum. ~~W~~ W are "trivial". ~~At~~ Does not address g/l masses.

② Flavor Symmetries

These lectures.

1. Choose a flavor group $G_f = G_{f, \max} = (u/3)^5$
2. Write down most general G_f invariant Lag. with all dimensionless couplings of order unity.
3. At this point: light q/l are massless
 $\tilde{q}/\tilde{l}$ of $1/2$ are degenerate

~~small mixing~~
 small mixing angles, eg V_{cb}, W_{23} are zero.

4. Allow G_f to be broken by a set of ~~dim. params.~~ —
 at least some of which are small

$$G_f \xrightarrow{\epsilon} G \xrightarrow{\epsilon} G' \xrightarrow{\epsilon'} \dots$$

*** $\epsilon, \epsilon', \dots$ determine $\left\{ \begin{array}{l} \text{light } q/l \text{ masses} \\ \text{small mixing angles} \\ \text{small } \tilde{q}/\tilde{l} \text{ mass splittings} \end{array} \right.$

5. Advantages:

- i) few ϵ, ϵ' for many derived small quantities.
- ii) we can construct theories for small SB quantities.

$$\epsilon = \frac{\langle \phi \rangle}{M}$$

$$\underbrace{G_f = u/3)^5}_{\text{Hall-Randall 1990}}$$

Hall-Randall 1990¹⁵

1. $G_f = u/3)^5$
2. $\lambda, \xi = 0$; $m_{Q,U,D,L,E}^2 = m_0^2_{Q,U,D,L,E} I$
3. $m_{q,l} = \theta_i = 0$. q/l degenerate with weak universality
4. Assume: $E_u(3,3,1,1,1)$, $E_D(3,1,3,1,1)$, $E_E(1,1,1,3,3)$

$$\lambda_u = E_u + E_D E_D^\dagger E_u + \dots$$

$$\lambda_D = E_D + E_U E_U^\dagger E_D + \dots$$

$$\lambda_E = E_E + E_E E_E^\dagger E_E + \dots$$

$$\rho_{U,D,E} = E_{U,D,E} + \dots$$

} with $O(1)$
coeffs
understood.

Thus weak universality $\rho_{U,D,E} = A_{U,D,E} \lambda_{U,D,E} + \dots$
(except for t)

$$m_Q^2 = I + E_U^\dagger E_U + E_D^\dagger E_D + O(\epsilon^4)$$

Comments.

1. Δ Compactly solves μ -decay. $\Delta m/CP$ problem.

$$\frac{\Delta m_{12}^2}{m^2} \approx \lambda_c^2 \approx 10^{-4}, \text{ etc.}$$

2. No progress on understanding q/l masses.

CAN A SMALLER G_f STILL SOLVE FC PROBLEM
BUT HAVE FEW ENOUGH SB PARAMS. TO MAKE PROGRESS
ON q/l MASSES?

$$G_f = SU(2)$$

Dine, Leigh, Kagan 1993 ¹⁶

$\tilde{d}/\tilde{s}$ degenerate because members of a $SU(2)$ doublet
why 12 years?!

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \quad \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \quad \begin{pmatrix} l_1 \\ l_2 \end{pmatrix} \quad \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \quad \text{or} \quad \psi_a \quad a=1,2.$$

$$q_3 \quad u_3 \quad d_3 \quad l_3 \quad e_3 \quad \psi_3$$

Disaster:

$$W \supset q_a d_b \epsilon^{ab} h = (q_1 d_2 - q_2 d_1) h$$

$$\lambda_D \approx \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + o(\epsilon) \quad \text{no light } q/l !!$$

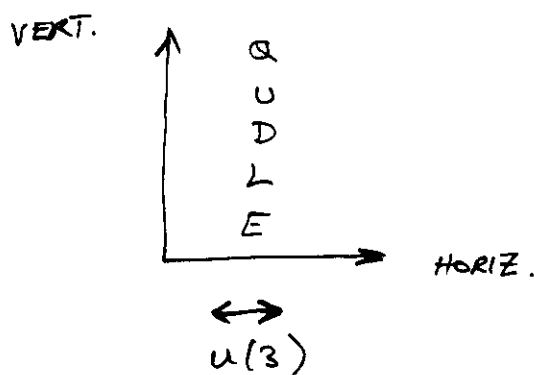
Lesson: Too ambitious, $SU(2)$ too small for G_f .

$SU(2)^5$
 $q_a, d_a, \dots$ transform
under different $SU(2)$

Too large.

$SU(2) \times U(1)$

What $U(1)$?



$u(3) \xrightarrow[\text{top}]{\text{large}} SU(2) \times U(1).$

$$u(3) \longrightarrow u(2).$$

$$3 \times 3 = \bar{3} + 6$$

$$q_A \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{AB} u_B h$$

$$A, B = 1, 2, 3$$

$$u(3) \left\{ \begin{array}{l} \begin{pmatrix} 1 & -1 & 0 \end{pmatrix} \\ \begin{pmatrix} 1 & 1 & -2 \end{pmatrix} \\ \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \end{array} \right\} \longrightarrow \left\{ \begin{array}{l} \begin{pmatrix} 1 & -1 & 0 \end{pmatrix} \\ \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \end{array} \right\} u(2)$$

$$\begin{array}{ccccc} q_a & u_a & d_a & l_a & e_a \\ q_3 & u_3 & d_3 & l_3 & e_3 \end{array}$$

← lower index means charge +1.

$$\underline{b/t, \tau/t}$$

$u(3)$ and its breaking cannot explain all of q/l masses.

Small $b/t, \tau/t$ may be due to a Higgs effect:

$$W = (q u) h_u + (q d + l e) h_d$$

Either i) $v_u \gg v_d$: large $\tan \beta$.

or ii) All couplings to h_d suppressed: Higgs take part in flavor.

THE U(2) THEORY OF FLAVOR

See Barbieri, Hall, Romanino hep-ph/9702315 & ref. cited.

$$\rightarrow \begin{pmatrix} \psi_a \\ \psi_3 \end{pmatrix} \quad \psi = q, u, d, l, e.$$

Yukawa couplings:

$$\frac{1}{M} (\psi_a \ \psi_3) \begin{pmatrix} S^{ab} + A^{ab} & \phi^a \\ \phi^b & 1 \end{pmatrix} \begin{pmatrix} \psi_b \\ \psi_3 \end{pmatrix} h_{ud} + O\left(\frac{1}{M^2}\right)$$

g/l mass hierarchy suggests:

$$u(2) \xrightarrow{\epsilon} u(1) \xrightarrow{\epsilon'} 0$$

Most general 1st stage is

$$\phi^a = \begin{pmatrix} 0 \\ V \end{pmatrix}, \quad S^{ab} = \begin{pmatrix} 0 & 0 \\ 0 & V \end{pmatrix}$$

with $\frac{V}{M} \xrightarrow{\epsilon} 0 = \epsilon$

Most general 2nd stage is

$$S^{ab} = \begin{pmatrix} V' & V' \\ V' & V \end{pmatrix}, \quad A^{ab} = \begin{pmatrix} 0 & V' \\ -V' & 0 \end{pmatrix}$$

with $\frac{V'}{M} \xrightarrow{\epsilon'} 0 = \epsilon'$

Study of alignment of S using a general ^{form of} potential shows
only A breaks final $u(1)$.

Yukawa couplings:

$$\lambda = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & \epsilon & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix} \quad \text{---} \quad \textcircled{1}$$

The S matrices have same sym. therefore same form.

The m^2 matrices follow from

$$\begin{pmatrix} \tilde{\Psi}_a^+ & \tilde{\Psi}_3^+ \end{pmatrix} \begin{pmatrix} 1 + S^\dagger S + \phi^\dagger \phi + A^\dagger A & \phi_a \\ \vdots & \vdots \\ \phi^{+b} & 1 \end{pmatrix} \begin{pmatrix} \tilde{\Psi}_a \\ \tilde{\Psi}_3 \end{pmatrix}$$

Relevant terms:

$$m^2 = \begin{pmatrix} m_1^2 & 0 & \vdots & 0 \\ 0 & m_1^2(1+\epsilon^2) & \vdots & \epsilon m_4^{2*} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \epsilon m_4^2 & \vdots & m_3^2 \end{pmatrix} \quad \text{---} \quad \textcircled{m^2}$$

flav. param.

$$\begin{aligned} \lambda &\longrightarrow m_{q1l}, \quad V : & 13 &\longrightarrow 11 \\ m^2 &\longrightarrow m_{\tilde{q}1\tilde{l}}, \quad W : & 43 &\longrightarrow 25 \end{aligned}$$

205 — 10 scalar masses, 5 R-mass splittings
 — 5 $\mathcal{O}_{23}$ for W
 — 5 phases for W $\xrightarrow[\text{in lepton sector } \mu\tau]{\text{Rephases.}}$ $\tilde{K}_{23} \sim ee^c, uv^c, dd^c, ud$
 ($\tilde{K}_{12}, \tilde{J} = 0!$) 35

← THE HIGGS MYSTERY →

I left out 2 chiral superfields of MSSM: H_u, d .
They do not fit in, in fact they are vectorlike & should be heavy: $M H_u H_d$.

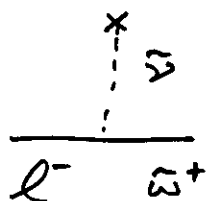
REMOVE HIGGS & SUPERFIELDS!

Break EWS via $\langle \tilde{S} \rangle$.

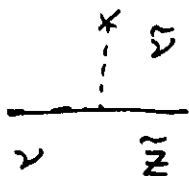
supersym. would provide
dream of Higgs-matter
unific

Problems

(a)



Need 4th generation.



(b)

$$W = (\bar{L} \lambda_E E^c) L_4 + (\bar{Q} \lambda_D D^c) L_4$$

where does m_L come from. This is the killer.

(c)

Successful gauge unificⁿ REQUIRES light H_u, d .

Is $H_{u,d}$ PART OF FLAVOR?

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$$G_{fmax} = U(3)^5 \times U(1)_{H_u} \times U(1)_{H_d}$$

Basic problem: there is no 3-2-1 gauge distinction between L and H_L :

$$G_{fmax} = U(3)^4 \times U(4)_L \times U(1)_{H_u}$$

$$W = Q U H_u + Q D L + L E L + \mu L H_u$$

where L runs over 4 species.

Disallows p decay. $\Rightarrow$ need some symmetry ~~XXXX~~
eg R_p .

Important Question

Is X part of G_f that explains small param. of $m, \tilde{m}^2$

(III) THEORIES OF FLAVOR

L. Hall

- 1.) The origin of small dimensionless θ terms.
- 2.) The $\Rightarrow$ see-saw : heavy-light mixing.
- 3.) The charged fermion see-saw : Froggatt-Nielsen mechanism.
- 4.) The Dimopoulos-Pomarol effect.
- 5.) U(2) Froggatt-Nielsen theories.

III / ORIGIN OF SMALL FLAVOR SYMM. BREAKING

1) We interpreted $\left\{ \begin{array}{l} m_e/m_p \dots \\ V_{cb} \dots \\ \frac{\Delta m_{12}^2}{m^2} \dots \end{array} \right\}$ as small SB params ϵ ~~etc.~~

2) In field theories, with all dimensionless couplings $O(1)$, there are 2 origins for ϵ :

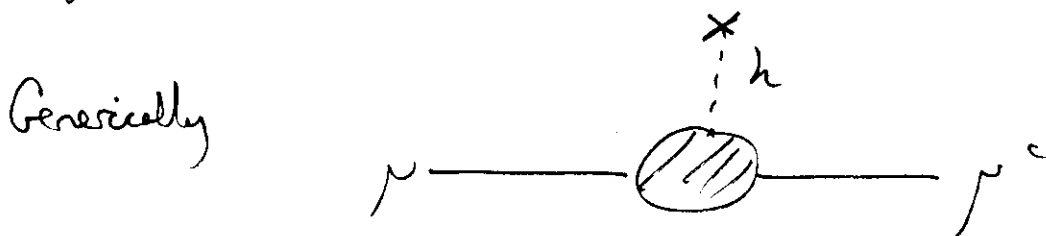
(i) Perturbative loop factors: $1/16\pi^2$.

(ii) Ratio of mass scales: m/M .

(i) RADIATIVE FERMION MASS HIERARCHIES.
are very plausible.

$$m_\tau : m_\mu : m_e = 1 : \frac{1}{16\pi^2} : \left(\frac{1}{16\pi^2}\right)^2$$

Any such theory has new exotic particles & interactions:



Basic problem

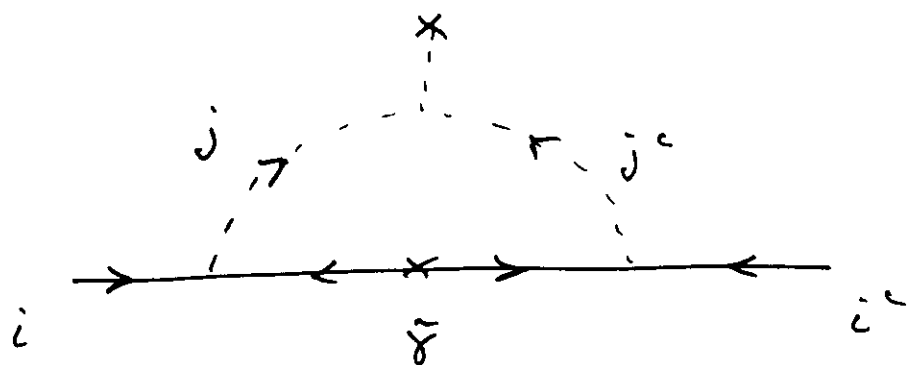
a) Predictive theories have not been constructed.

b) No constraint on mass M in loops.

Supersymmetric theories offer new hope because

b) Superpartners with mass $O(v_{EW})$ in loop.

a) Hope to measure superpartner masses & couplings so that supersymmetry really can be computed.



Could be zero.
At scales $\gg v$
fermion mass
info. is in
($\tilde{g}, m^2$)

$$m_i = \frac{e^2}{16\pi^2} \cdot \frac{m_{\tilde{g}}}{m_{\tilde{g}}^2} \cdot (\tilde{g}_{jj^c} + \mu \lambda_{jj^c} \tan \beta) v_i W_{ji} W_{ji}^c$$

However, these things are closely related to $j \rightarrow i\gamma$.

$$\tau \rightarrow \mu \gamma \quad \text{forbids} \quad \frac{m_\mu}{m_\tau} = O(\alpha)$$

$$\mu \rightarrow e \gamma \quad \text{forbids} \quad \frac{m_e}{m_\mu} = O(\alpha)$$

$$\tau \rightarrow e \gamma \quad \text{CLEED measurement of 1996 "excludes" } \frac{m_e}{m_\tau} \sim \frac{\alpha}{4\pi}$$

$\Rightarrow$ Superpartner loops at weak scale are apparently NOT generating dominant contrib: to light fermion masses. Basic problems a) and b) remain for radiative ferm. masses.

ii) $\epsilon = m/M$

3

Where do mass scales come from?

a) Definition of unit: M_{Pl} . M could be M_{Pl} .

b) Dimensional Transmutation

— $\frac{dg}{dt} = -\frac{bg^3}{16\pi^2}$, $\frac{\Lambda}{M_0} = e^{-\frac{8\pi^2}{bg_0^2}}$. In QCD flavor symmetries are broken at Λ .

— $\Rightarrow$ DSB
 m^2 run -ve in supersym.

— $\frac{dm_i^2}{dt} = \lambda^2 m_i^2$ with $V = m_i^2(\phi) |\phi|^2$

Gives $\langle \phi \rangle \sim \mu_0$ where $m_i^2(\mu_0) = 0$. In this case D.T. occurs by perturbative dynamics.
(need D.T. for supersym. breaking for origin of m^2 !).

— Supersymmetric QCD with $N_f = N_c$

Compling gets strong generating Λ

Mesons $M_{ij} = \bar{q}_i q_j$

Baryons $B = \epsilon_{ijk} \dots q_i q_j q_k \dots$

Constrained by: $\det M = \Lambda^{N-3} (\bar{R} B) = \Lambda^N$

$\Rightarrow$ There are a variety of ways in which dynamics generates a scale by D.T. & vevs form at this scale.

NEUTRINO MASSES:

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1. With minimal matter content, 3-2-1 gauge interaction free ν masses to arise only at $d=5$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_R + \frac{g}{M} l l h h \quad \text{--- (1)} \quad l = \begin{pmatrix} \nu \\ e \end{pmatrix} \quad h = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$

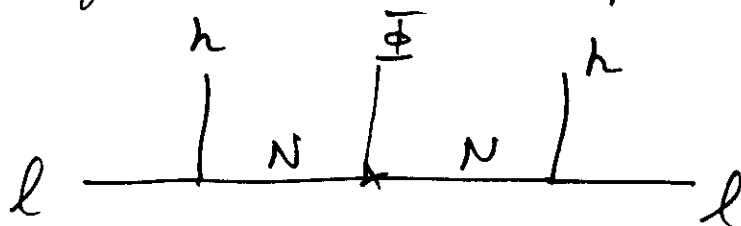
For $M = M_p$, too small of interest.

Interesting to consider $M < M_p$, & new physics ^{thing} which produces ~~these~~ such M .

2. Introduce RH. ν : N . They are right under 3-2-1 and acquire some large mass. This could occur by some field $\langle \Phi \rangle \approx M$ breaking lepton number via a D.T.

$$\mathcal{L} = \bar{\Phi} N N + \lambda l h N$$

3. Integrate out N to obtain effective theory beneath M



This generates $\mathcal{L}_{\text{eff}}$ of (1), with $g = \lambda^2$.

Inserting $\langle h \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$ gives $m_\nu = \frac{\lambda^2 v^2}{M}$ --- (55)

This is known as see-saw: as $M \uparrow$ so $m_\nu \downarrow$.

THE SEE-SAW AS HEAVY-LIGHT MIXING.

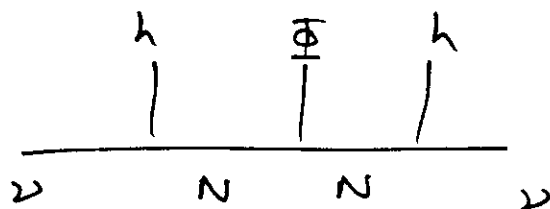
1/5

The result (55) is the approx. result of diagonalizing the mass matrix

$$\frac{1}{2} (\nu, N) \begin{pmatrix} 0 & \lambda v \\ \lambda v & M \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix}$$

$$\begin{pmatrix} \nu' \\ N' \end{pmatrix} = \begin{pmatrix} c & -s \\ +s & c \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix} \quad \text{with} \quad \frac{s}{c} = \sqrt{\frac{m_{\nu'}}{M_{N'}}} \xrightarrow[\text{small } \theta]{} \frac{\lambda v}{M}$$

One may worry about whether



really gives the right answer, because we should integrate out N' not N . We can check by:

$$W = \frac{M}{2} (N N) + \lambda v (\nu N)$$

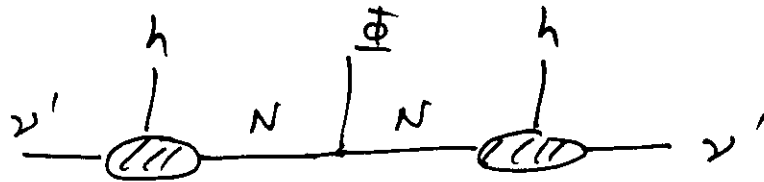
$$= \frac{M}{2} (N' - \theta \nu')^2 + \lambda v (\nu' + \theta N') (N' - \theta \nu')$$

$$\supset \underbrace{\left(\frac{M}{2} \theta^2 \right)}_{(A)} \nu'^2 + \underbrace{(-\lambda v \theta)}_{(B)} \nu' = -\frac{\lambda^2 v^2}{M} \frac{\nu' \nu'}{2}$$

Thus effective theory does reproduce correct result to leading order in $1/M$.

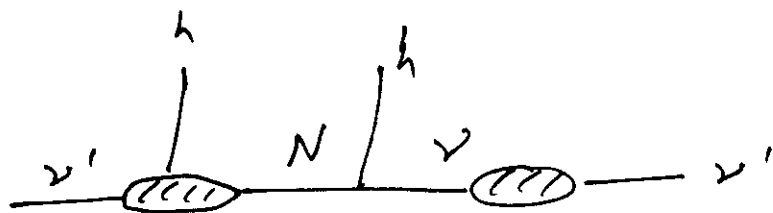
We can understand (A) & (B) diagrammatically by writing down the initial mass terms and using mixing.

(A)



$$+\left(\frac{M}{2}\right) \cancel{(-0)}^2$$

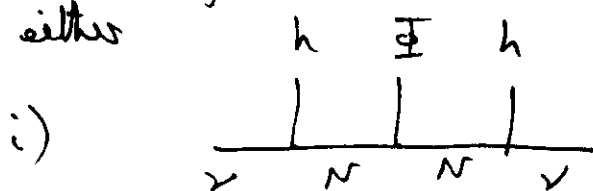
(B)



$$+2\nu(-0)(1)$$

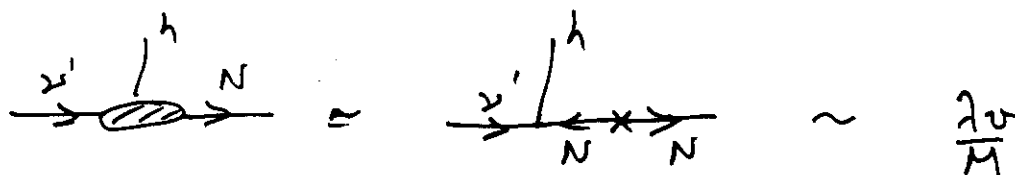
In these diagrams we are NOT integrating anything out — we are just doing the rotation of $N \leftrightarrow N$ and $N h \nu$ couplings from (ν, N) to (ν', N') basis & looking at terms with external $\nu' \nu'$.

Thus to leading order in $1/M$, we get correct answer by either



pretending N heavy
& ν light.

ii) Heavy - light mixing.



"FROGGATT-NIELSEN MECHANISM"

Cannot be identical to ν : $m_\nu \propto \langle h \rangle^2$

want $m_e \propto \langle h \rangle$.

Ingredients: 0) G_f to explain absence of lel (for ν it was $su(2)$)

1) ~~heavy~~ heavy charged particles with $su(2) \times u(1)$ invariant mass

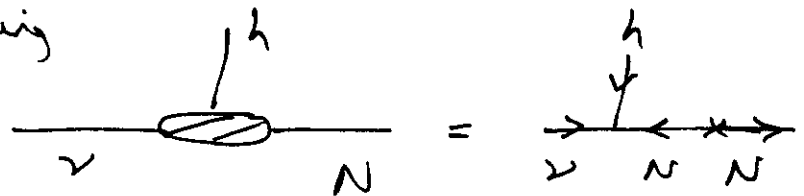
for leptons

$$W \supset \Phi L \bar{L} + \Phi E^c \bar{E}^c$$

2) Heavy - light mixing

$$W \supset \bar{L} \phi_l l + \bar{E}^c \phi_e e^c$$

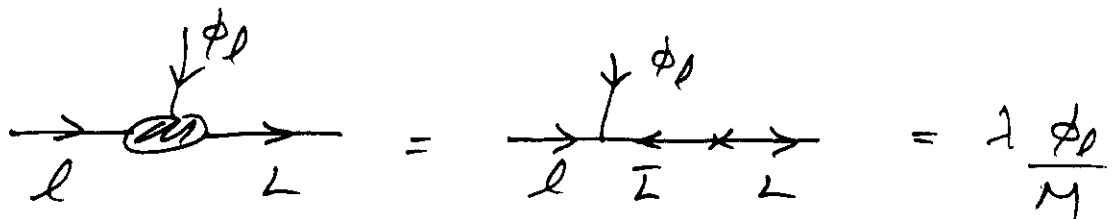
Thus ν are light because heavy-light mixing occurs via $su(2)$ breaking



$$= 2\nu/M$$

Charged fermions are light

because heavy-light mixing occurs via G_f breaking



Clearly L and l transform differently under G_f .

3) G_f invariant interactions of h of form

$L E^c h$, $\bar{L} E^c h$, $L e^c h$ (depending on model)

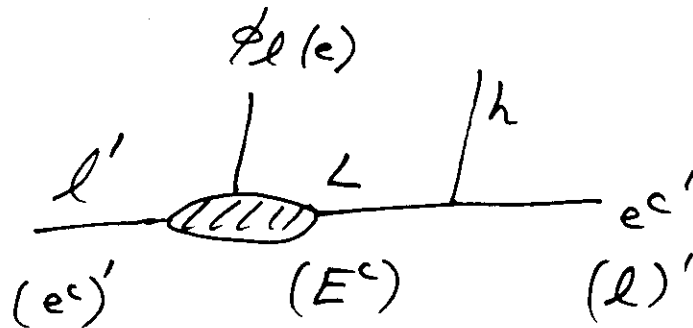
while G_f forbids $\bar{L} e^c h$.

Heavy-light mixing now produces small lepton masses:

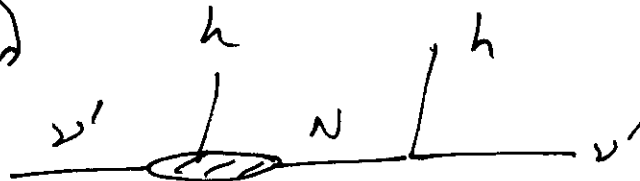
Type B

$$\boxed{\frac{m_e}{v} \approx \frac{\phi_{Le}}{M}}$$

1st order FN



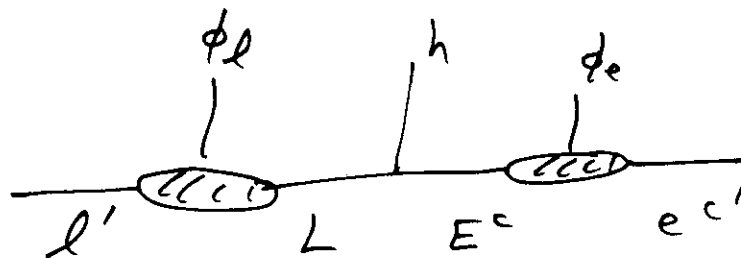
Conjugate:



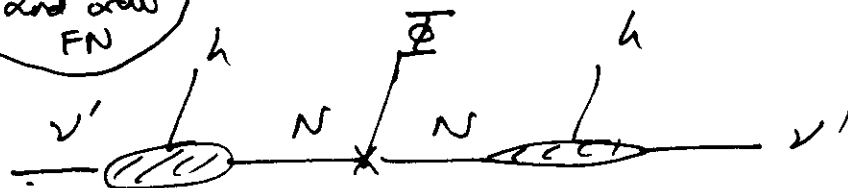
Type A

$$\boxed{\frac{m_e}{v} = \frac{\phi_L \phi_e}{M^2}}$$

2nd order FN



Conjugate



Notice inversion of roles of fields which violate flavor & those which violate $(B-L)$

Write down the most general G_f invariant set of m^2 parameters:

$$V_{\text{soft}} = (\ell^* \ L^*) \begin{pmatrix} m_\ell^2 & 0 \\ 0 & m_L^2 \end{pmatrix} \begin{pmatrix} \ell \\ L \end{pmatrix}$$

0 by G_f . $\nearrow$

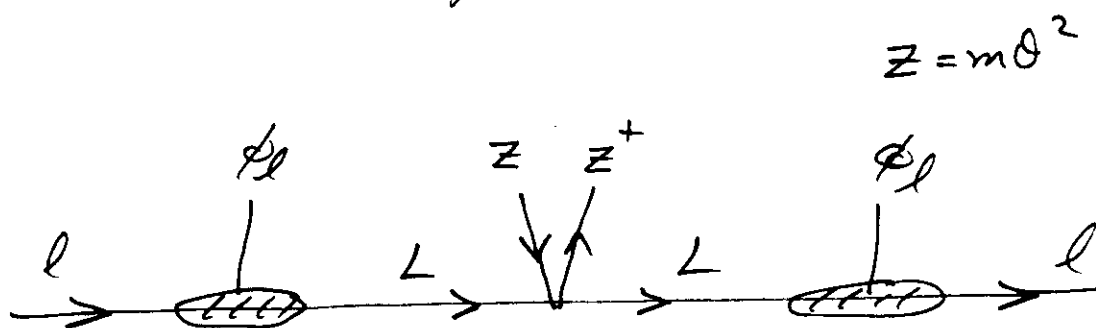
Do superfield rotation to diagonalize (heavy-light)

* super mass terms:

$$\begin{pmatrix} \ell' \\ L' \end{pmatrix} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} \ell \\ L \end{pmatrix}$$

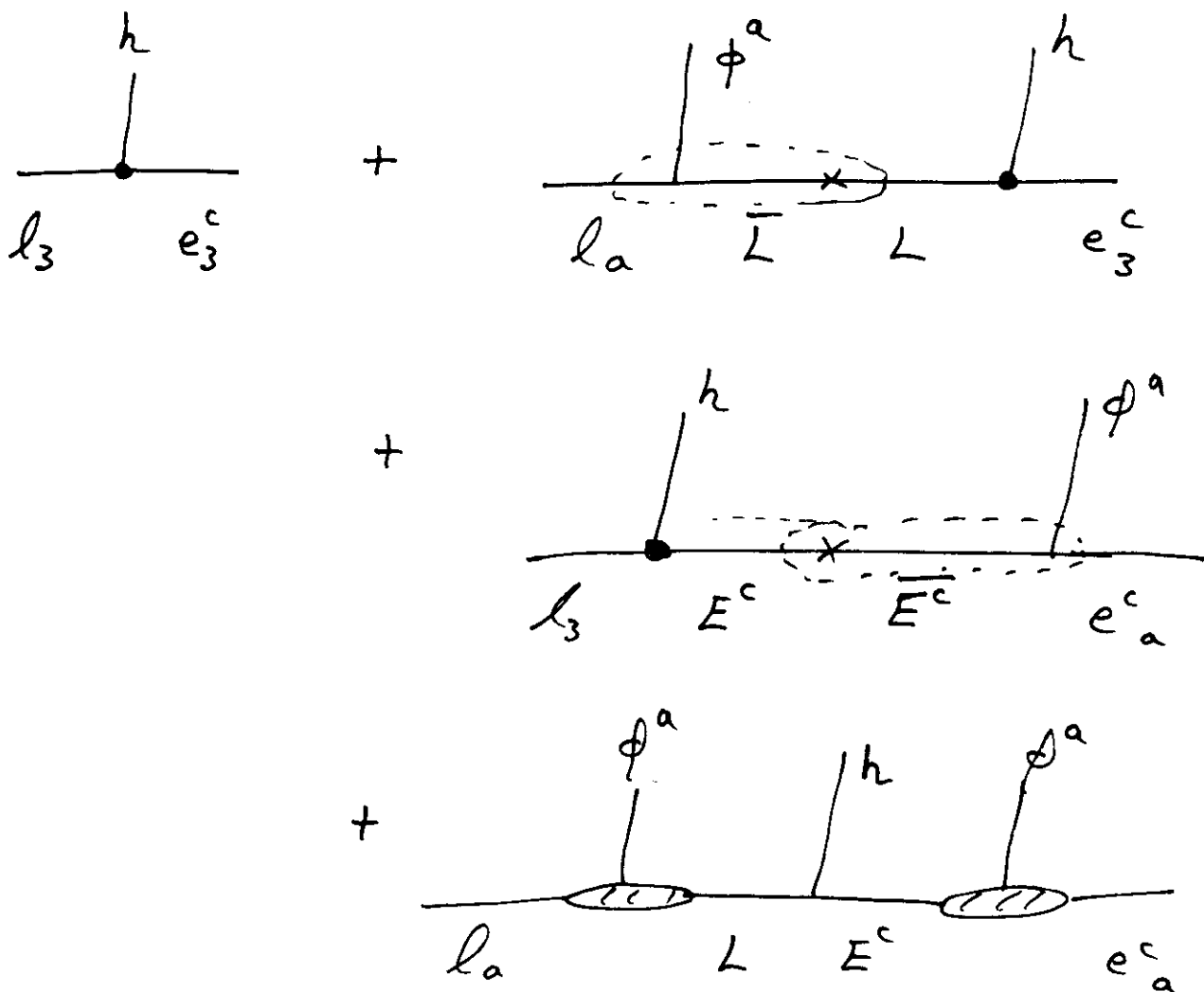
$$V_{\text{soft}} \rightarrow (\ell'^* \ L'^*) \begin{pmatrix} c^2 m_\ell^2 + s^2 m_L^2 & \dots \\ \dots & \dots \end{pmatrix} \begin{pmatrix} \ell' \\ L' \end{pmatrix}$$

This is just a quadratic FN :



$$\Delta m_\ell^2 \sim \Theta^2 m_L^2 \sim \left(\frac{\phi_l}{M}\right)^2 m_L^2$$

Choose heavy vector leptons as $U(2)$ singlets:

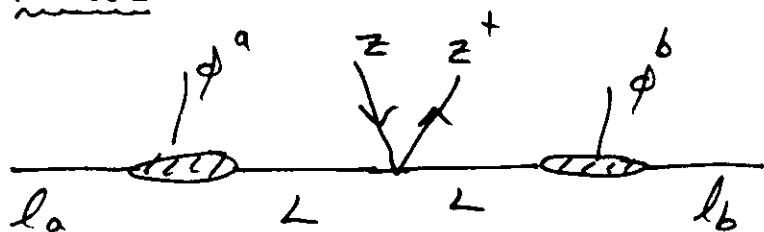


$$\chi_E \sim \begin{pmatrix} \epsilon^2 & \epsilon \\ \epsilon & 1 \end{pmatrix}$$

$$\epsilon = \phi / M$$

Interesting

However



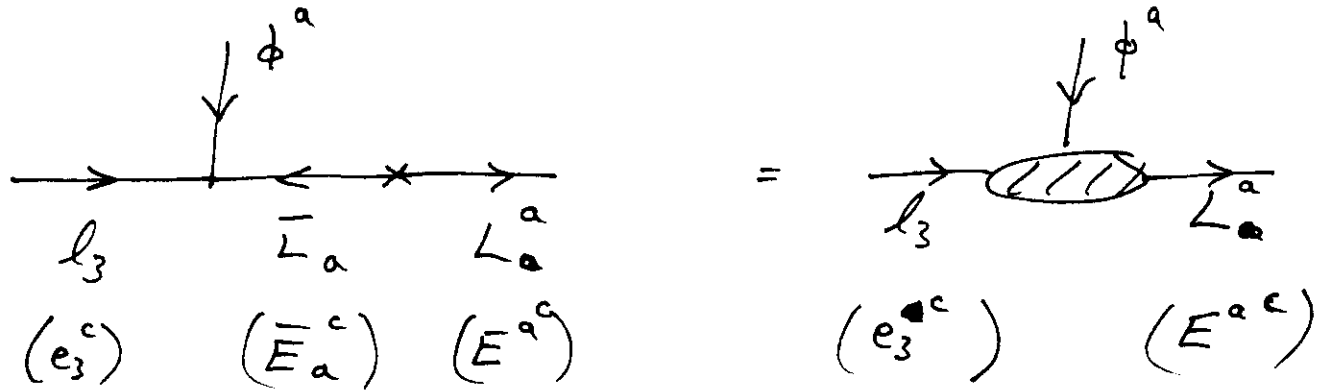
$$\Rightarrow \begin{pmatrix} m_1^2 & m_1^2 + \epsilon^2 m_2^2 & m_2^2 \\ m_1^2 + \epsilon^2 m_2^2 & m_2^2 & m_3^2 \\ m_2^2 & m_3^2 & m_3^2 \end{pmatrix}$$

$\frac{m_\mu}{m_e} \sim \frac{1}{17}$

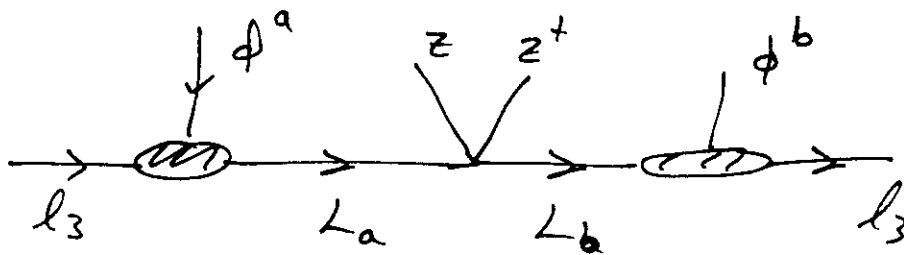
$U(2)$ doublet heavy leptons: $M(L^a \bar{L}_a + E^{ca} \bar{E}_a^c)$

Note that l_a / L^a do transform differently.

The simplest heavy-light mixing is via ϕ^a :

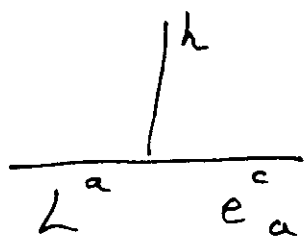


Thus the DP scalar effect shifts mass of $\tilde{l}_3, \tilde{e}_3^c$

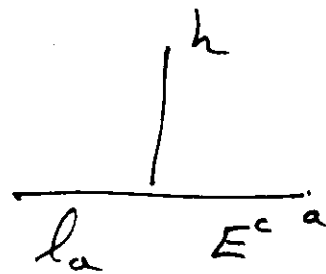


$$m^2 \longrightarrow \begin{pmatrix} m_1^2 & & \\ & m_1^2 & \\ & & m_3^2 + \epsilon^2 m_2^2 \end{pmatrix} \quad \text{SAFE!}$$

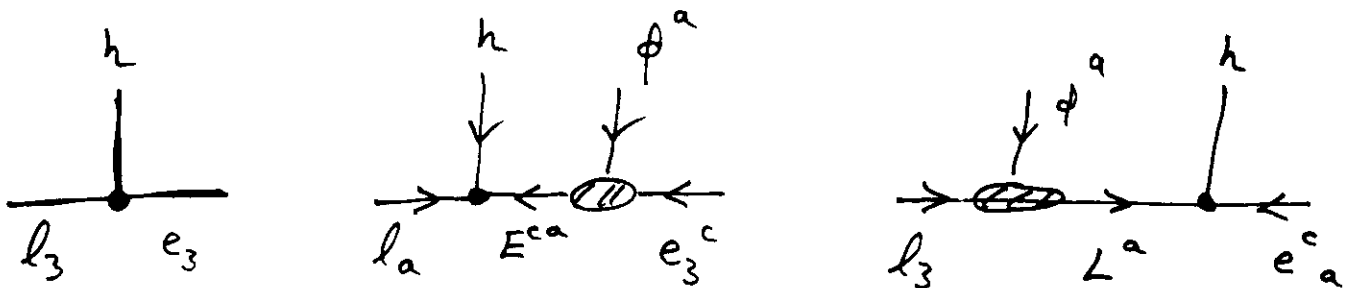
The h couplings are:



and

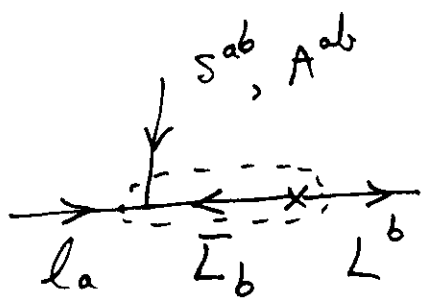


so the fermion diagrams are



Generating $\lambda = \begin{pmatrix} & e \\ e & 1 \end{pmatrix}$.

Introduce S^{ab}, A^{ab} to get more heavy light mixings



giving



$$\langle \frac{S^{ab}}{M} \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \epsilon \end{pmatrix}$$

$$\langle \frac{A^{ab}}{M} \rangle = e^{ab} \left(\frac{\epsilon'}{M} \right) \text{ gives}$$

$$\lambda = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & \epsilon & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix}$$

D.P. effect now arises from S and gives $\Delta m_{12}^2 / m^2 = \epsilon^2 = \left(\frac{m_\mu}{m_\tau} \right)^2$