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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

2 June - 4 July 1997

COSMIC STRINGS: EVOLUTION AND APPLICATIONS

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Cosmic Strings

Introduction

What are cosmic strings?

String formation

Initial density – how many are formed?

Length distribution – long strings?

Evolution of strings

Scaling?

Cosmological implications

Can strings explain

CMB anisotropy + large-scale structure? **No!**

baryogenesis?

highest energy cosmic rays?

magnetic field?

dark matter?

⋮

Conclusions

References

Reviews of Cosmic Strings:

A. Vilenkin + EPS Shellard
Cosmic strings and other topological defects. CUP 1994

MB Hindmarsh + TWB Kibble
Cosmic strings. Rep Prog Phys 58 (94) 477

W.H. Zurek
Cosmological experiments in condensed matter systems, Phys. Rep. 276 (96) no. 4

Experiments:

P.C. Hendry et al Nature 368 (94) 315

C. Bauerle et al Nature 382 (96) 332

VMH Ruutu et al Nature 382 (96) 334

Big-Bang Phase Transitions

Planck time ↑ quantum gravity

$$\sqrt{\frac{\hbar G}{c^5}} \simeq 10^{-45}\,\mathrm{s} \qquad kT \sim 10^{19}\,\mathrm{GeV}$$

Grand unification

$t \simeq 10^{-36}\,\mathrm{s}$ $\qquad kT \sim 10^{16}\,\mathrm{GeV}$

:

? supersymmetry breaking ?

⋮

Electro-weak

$t \simeq 10^{-10}\,\mathrm{s}$ $\qquad kT \sim 100\,\mathrm{GeV}$

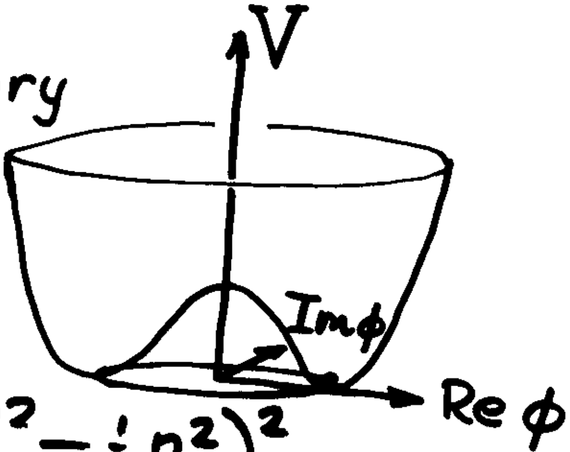
Quark-hadron

$t \simeq 10^{-4}\,\mathrm{s}$ $\qquad kT \sim 100\,\mathrm{MeV}$

Phase transitions ⇒ { inflation
defects
baryogenesis

Symmetry Breaking

U(1) theory



$\phi \rightarrow \phi e^{i\alpha}$

$$V = \frac{1}{2}\lambda\left(|\phi|^2 - \frac{1}{2}\eta^2\right)^2$$

Degenerate ground state: $\langle\phi\rangle = \frac{\eta}{\sqrt{2}} e^{i\alpha}$

$T > T_c \sim \eta$ $\quad$ $(c = \hbar = k_B = 1)$

Symmetric phase $\langle\phi\rangle \simeq 0$

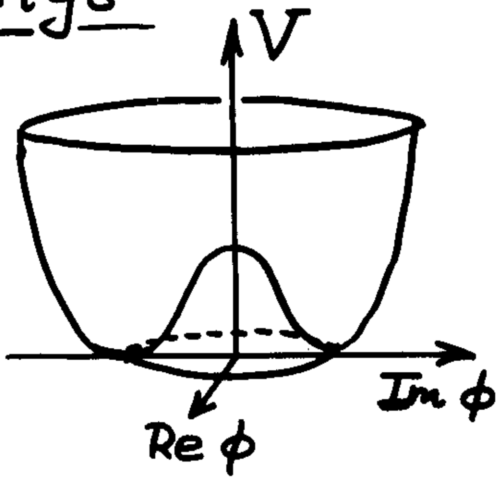
$T < T_c$

ordered phase $\langle\phi\rangle \neq 0$

As T falls below T_c, ϕ chooses random phase α – different in different regions $\Rightarrow$ defects

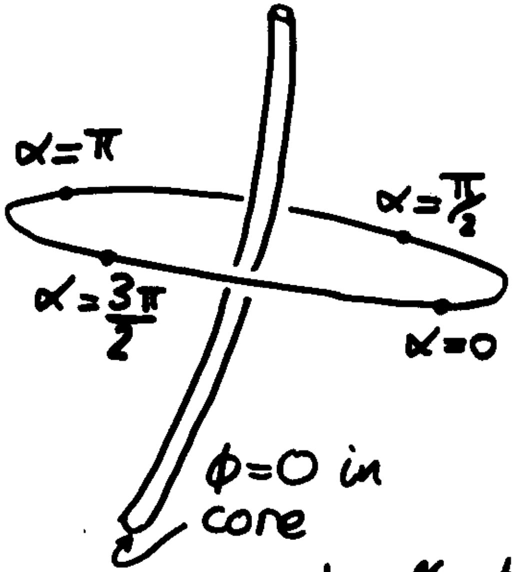
Cosmic Strings

$$V_0 = \frac{1}{2}\lambda\left(\phi^*\phi - \frac{1}{2}\eta^2\right)^2$$



symmetry $\phi \rightarrow \phi e^{i\alpha}$ broken below T_c.

As T ↘ below T_c, ϕ chooses phase: $\langle\phi\rangle = \frac{\eta}{\sqrt{2}} e^{i\alpha}$



cosmic string is topologically stable.

$$\frac{\text{energy}}{\text{length}} = \text{tension} = \mu \sim \eta^2$$

$(c = \hbar = k_B = 1)$

Gravitational effects depend on

$$G\mu \sim \left(\frac{T_c}{E_{Pl}}\right)^2 \sim 10^{-6} \text{ for GUT strings}$$

Types of Cosmic Strings

Local – breaking of local gauge symmetry
- dynamics is local, described by Nambu-Goto action

Global – breaking of global symmetry
- long-range interactions
- radiate Goldstone bosons (eg axions)

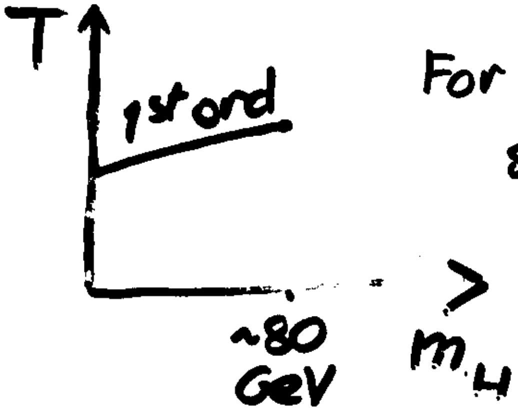
Superconducting
- bosonic charge carriers or fermions trapped in transverse zero modes
- interactions with charged particles and magnetic fields
- loops may be stabilized by angular momentum and currents (vortons)

Caveat

Phase transitions in non-Abelian gauge theories may not show true symmetry breaking.

e.g. Electroweak theory

Kajantie et al



For $m_H \gtrsim 80$ GeV: smooth crossover, no transition.

Not true for U(1) theory* – e.g. superconductor. [* needs to be confirmed]

Defect formation may be different in non-Abelian theories.

Condensed Matter Examples

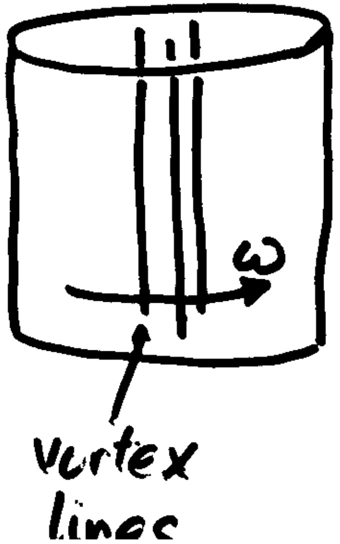
Liquid ^{4}He

Below $T_\lambda \sim 2{\cdot}18\,\mathrm{K}$, condensate $\phi(\underline{x})$ (macroscopic occupation)

$\rho_{sup.fl} \sim |\phi|^2$ $\qquad \phi = |\phi| e^{i\alpha}$

$\underline{v}_{sup.fl} \simeq \frac{\hbar}{m_4} \underline{\nabla} \alpha$

Rotating container:



global strings:

vortex lines where $|\phi| = 0$ in core

quantized vorticity

$\kappa = \frac{2\pi\hbar}{m}$

Type II superconductors

BCS theory:

e's form Cooper pairs $(\underline{k}\uparrow, -\underline{k}\downarrow)$ (^1S state)

Condensate of Cooper pairs, wave function $\phi(\underline{r})$

Landau-Ginsberg model

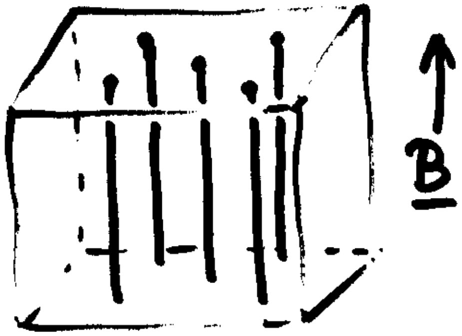
$$F = \frac{1}{2m_4} \underline{D}\phi^* \cdot \underline{D}\phi + \tfrac{1}{4}\lambda(\phi^*\phi)^2 + a\phi^*\phi$$

Type II: $\lambda > e^2$

Flux tubes:

$\phi = 0$ in core

quantized flux $= \frac{\pi\hbar}{e}$



Liquid Crystals

Nematic phase:

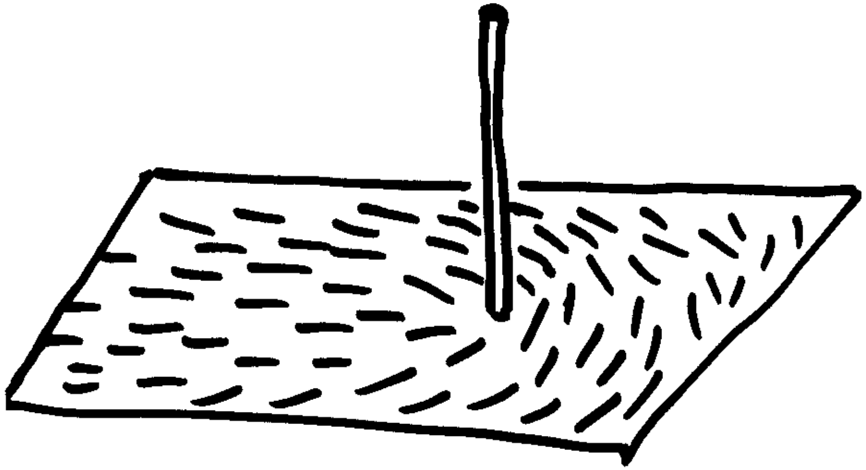
director

$\overrightarrow{\underline{n}} \equiv \overleftarrow{-\underline{n}}$

No translational order

Long-range orientational order

Disclinations:



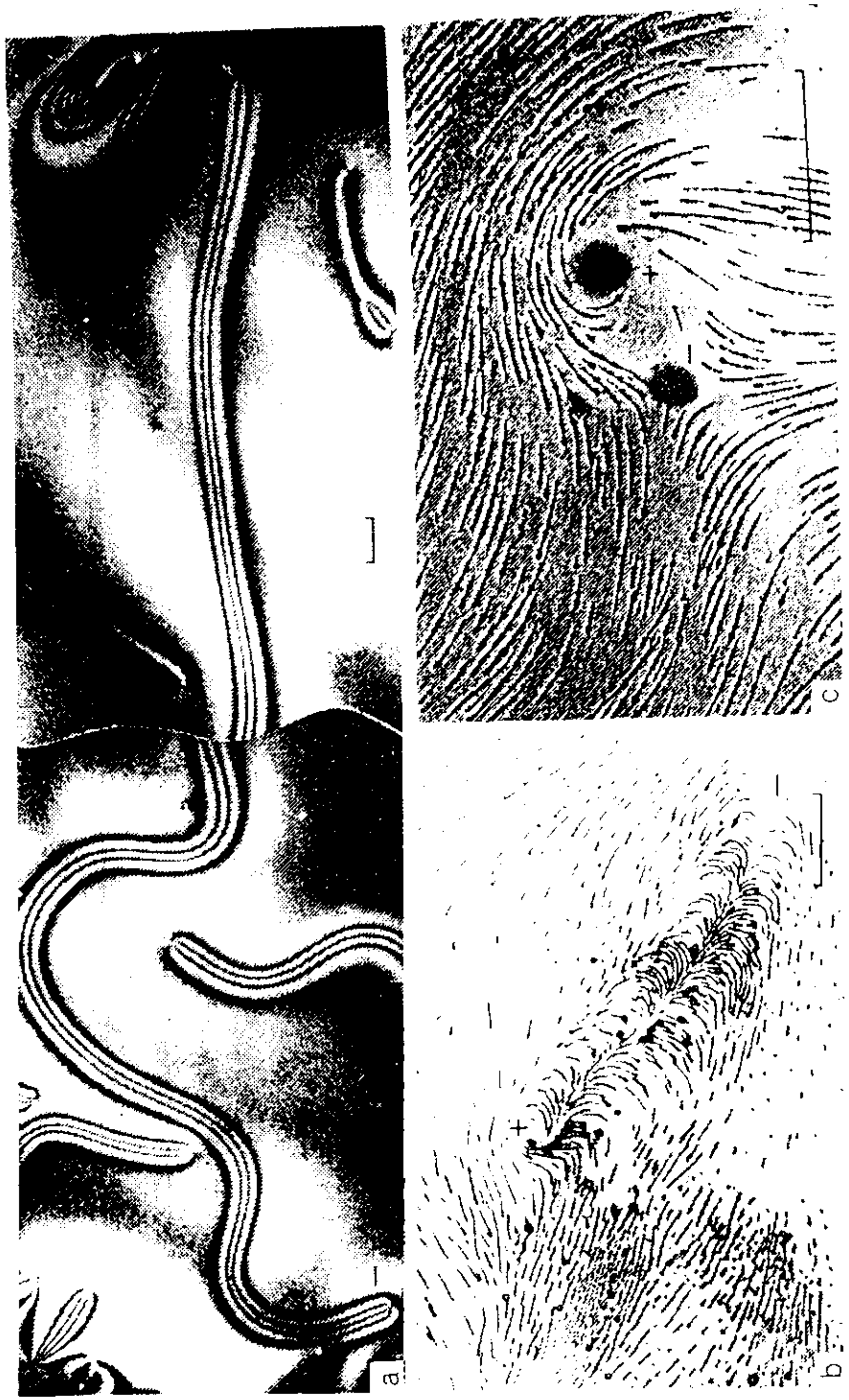


FIG. 1. a,b—Strings connecting a boojum and an antiboojum in a thin layer of a nematic liquid crystal ($h \simeq 10\ \mu m$) with a hybrid orientation of **n**; a—texture of the liquid crystal in polarized light (the ends of the same string, 11 mm long, are shown); b—decorated sample (the chains run along lines of the director in the *T* plane); c—pair of boojums in a thick sample ($h \simeq 60\ \mu m$) (there is no string). The length of the horizontal bar in parts a–c is 200 μm.

Lavrentovich + Rozhkov 1988 JETP!

Cosmological Effects

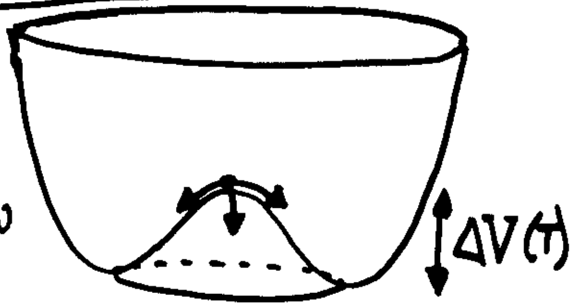
To predict cosmological effects we need to know:

1. What defects are formed and how many?
2. How defects evolve?
3. How do they affect matter and radiation?

For (1) and to some extent for (2) we can test ideas by applying them to analogous condensed matter examples.

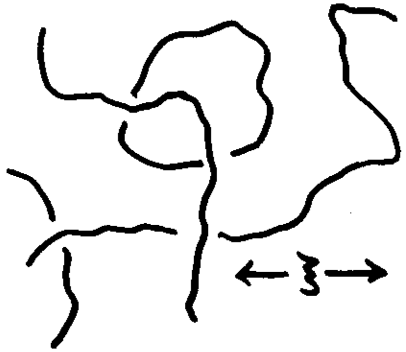
Defect Formation

Take U(1) model:



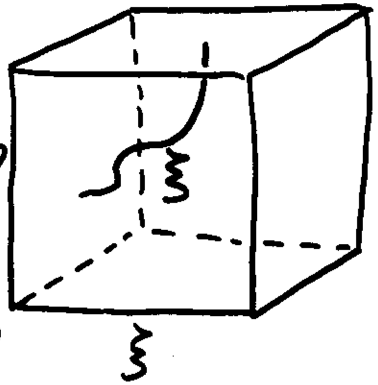
As T falls below T_c, $\langle \phi \rangle \neq 0$

chooses phase α varying randomly.

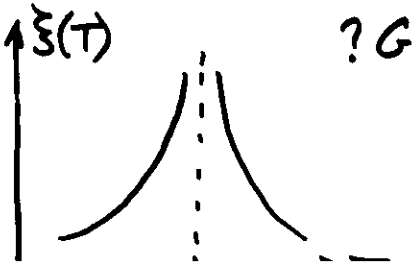


$\Rightarrow$ random tangle of string, scale ξ

What sets scale ξ_{string}?



Correlation length
$\xi(T)$ of ϕ? At what T?



? Ginzburg temp T_G at which

$$\Delta V(T)\, \xi(T)^3 \sim kT$$

Ginzburg Length

Ginzburg temperature T_G

$$\xi = \frac{\xi_0}{\sqrt{1-\frac{T^2}{T_c^2}}}$$

$$\Delta f \simeq \tfrac{1}{2}\lambda \phi_0^4(T), \qquad \phi_0(T) = \tfrac{\eta}{\sqrt{2}}\sqrt{1-\frac{T^2}{T_c^2}}$$

T_G def. by $\quad \Delta f(T).\,\xi^3(T) \simeq T$

$T_c \sim \eta$

$$\Rightarrow$$

$$1-\frac{T_G^2}{T_c^2} \sim \lambda \qquad (\text{if } \lambda \ll 1)$$

Ginzburg length

$$\xi_G = \xi(T_G) \simeq \frac{\xi_0}{\sqrt{\lambda}}$$

T_G is temperature above which strings are wiggly on scale of string width

Zurek Criterion

["Equilibrium" established out to ξ_{hor}?]

Causal horizon

$$\xi_{hor} = u.\,(t-t_c)$$

$u = c$ in early universe

$u =$ speed of 2nd sound in liquid He

He: $\quad \xi_{eq} = \dfrac{\xi_0}{\sqrt{\varepsilon}} \qquad u = u_0\sqrt{\varepsilon}$

or more precisely $\xi_{eq} = \xi_0 \varepsilon^{-\nu}, \quad u = u_0 \varepsilon^{1-\nu}$

$\nu_{RG} \simeq \frac{2}{3}$

$$\varepsilon = 1 - \frac{T}{T_c} \simeq \frac{t-t_c}{\tau_Q}$$

$$\frac{1}{\tau_Q} = \text{quench rate} = -\left.\frac{\dot{T}}{T}\right|_{T=T_c}$$

Better: $\quad \xi_{hor} = \displaystyle\int_{t_c}^{t} u(t')\,dt' = \frac{u_0 \tau_Q}{2-\nu}\,\varepsilon^{2-\nu}$

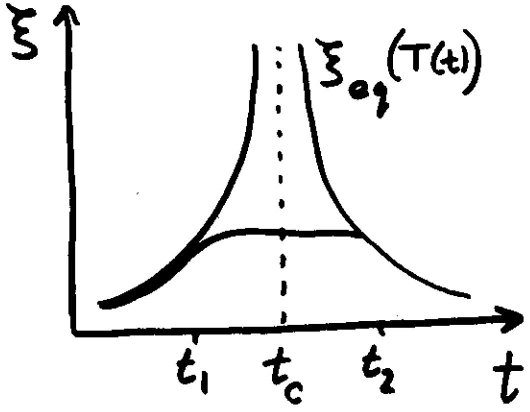
Zurek criterion is: t_Z is time at which

$$\xi_{hor} = \xi_{equ}(T)$$

Zurek Criterion (alt.)

This is dynamical problem: of rates.

T_G relevant to creation of small loops, and wiggliness of long strings, not to creation of long strings.



Causality

$$\Rightarrow \dot{\xi} < c$$

[Rel: c = light speed

NR: c = char. speed
(e.g. second sound)

This suggests: $\xi \sim \xi_{eq}(T(t))$

up to point where $\dot{\xi} \sim c$, (t_1),

thereafter $\xi \sim$ const. up to t_2.

So $\xi_{str}(t_2) \simeq \xi_{eq}(T(t_1)) \simeq \xi_{eq}(T(t_2))$

Alternatively: compare quench rate with

Thermal Field Theory

Calculation of String Density

(A Gill + R. Rivers), G. Karra + R. Rivers
B. Halperin

String is locus of zeros of field, density can be calculated from

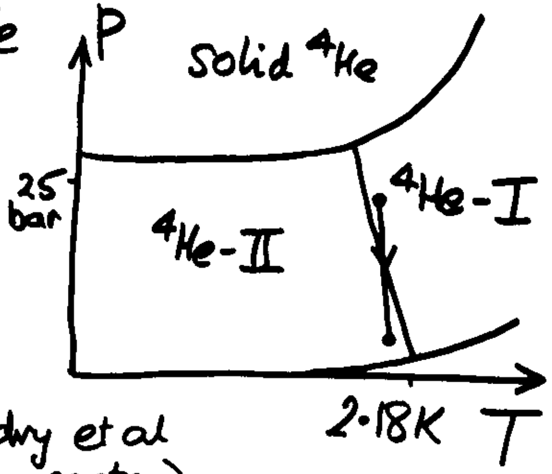
$$W(\underline{x}-\underline{y};t) = \langle \phi(\underline{x},t)\, \phi^*(\underline{y},t) \rangle.$$

To eliminate 'fluctuation zeros', smooth over some length scale M^{-1}. String density is only meaningful if dependence on M is weak.

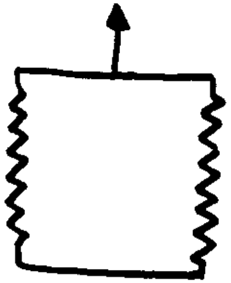
Results depend on whether $m_\phi \tau_Q$ is of order 1, $\ln\frac{1}{\lambda}$, or $\frac{1}{\lambda}$. Generally speaking, string density agrees with Zurek estimate but with correction factors of form $\left(\ln\frac{1}{\lambda}\right)^\beta$ — numerically corrections are not large unless λ is very small.

Experimental Test in ^4He

Zurek proposed experiment to test vortex formation scenario in ^4He



Pressure quench



Transition induced by sudden pressure drop ⇒ vortices formed

Vortices detected by attenuation of 2nd sound

Results consistent with Zurek

But not conclusive: vorticity can be generated in other ways, eg hydro-

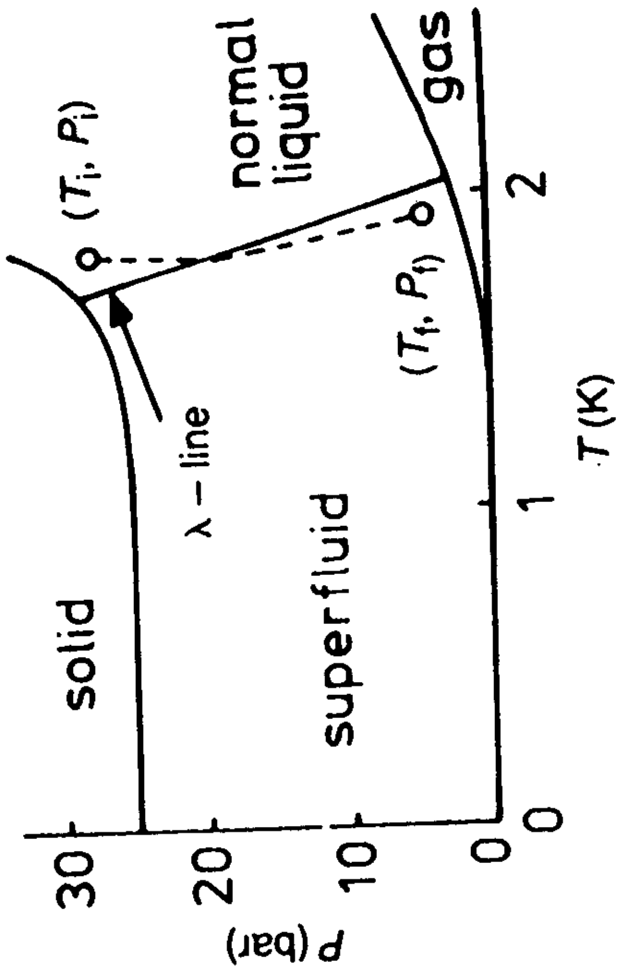
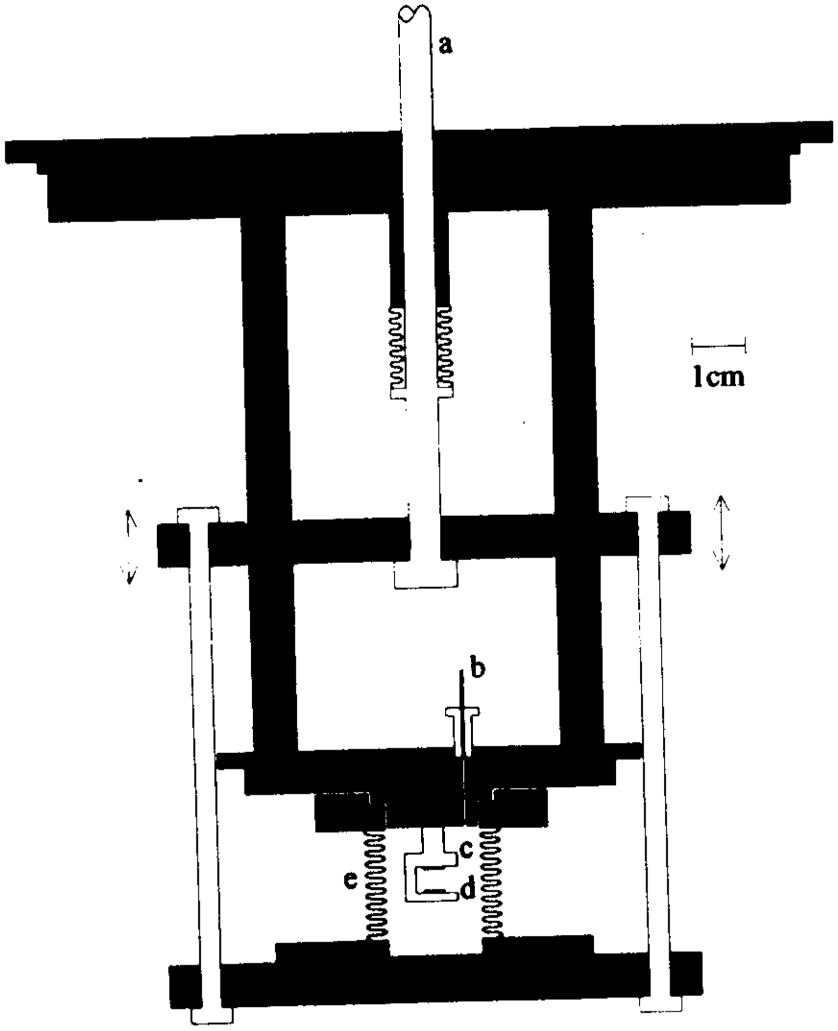


FIG. 1 Sketch of expansion trajectory (dashed) through the λ-transition on the ^{4}He pressure–temperature (P–T) phase diagram, from initial values (T_i, P_i) to final values (T_f, P_f).

Expansion Cryostat



a: pull-rod, b: fill tube, c: heater, d: bolometer e: phosphor-bronze bellows

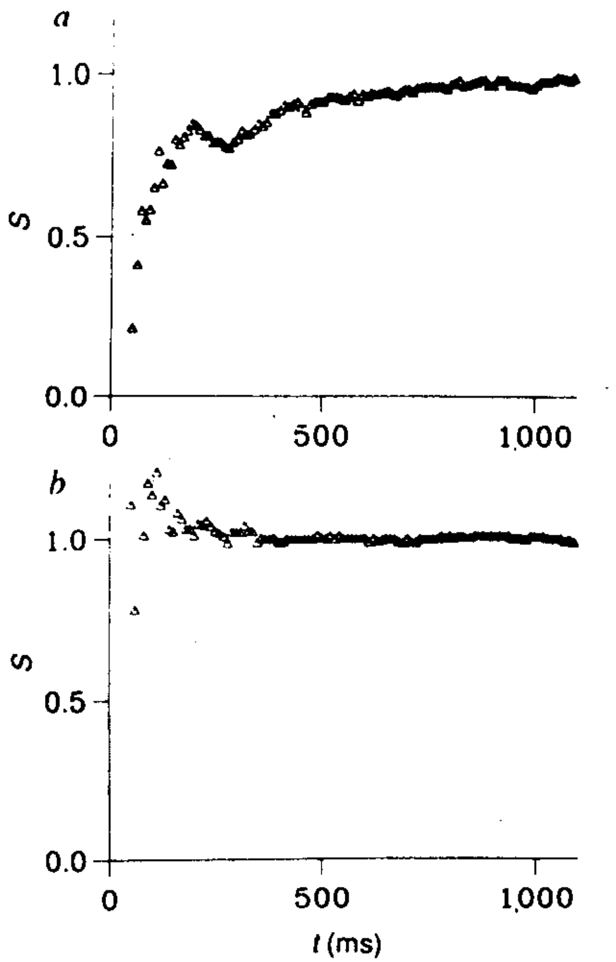


FIG. 3 Evolution of the normalized second sound signal amplitude S as a function of time *t*. *a*, Following an expansion through the λ-transition from starting temperature and pressure $T_i = 1.81$ K, $P_i = 29.6$ bar to final values of $T_f = 2.04$ K, $P_f = 6.9$ bar. *b*, Following a test expansion, not through the λ-transition, but lying wholly below it, with $T_i = 1.58$ K; $P_i = 23.0$ bar; $T_f = 1.74$ K; $P_f = 4.0$ bar.

Liquid ^{3}He

Fermi liquid

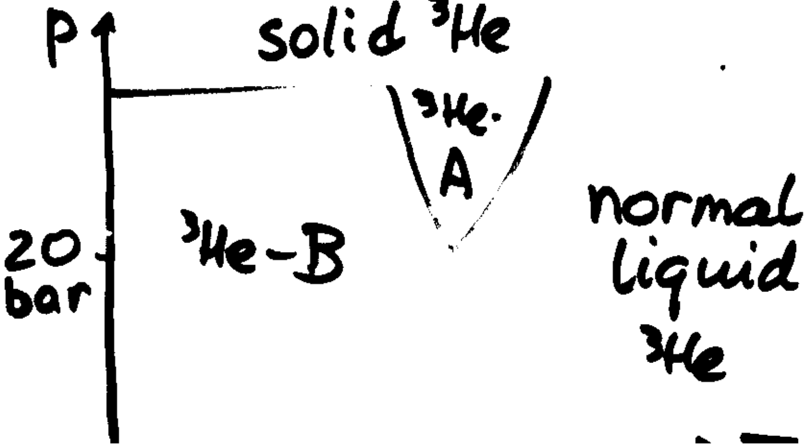
cf. BCS: Cooper pairs in ^1S state

Pairing of ^{3}He atoms in ^3P state: $L=1$, $S=1$

Order parameter

$$A_{\sigma m} \quad \sigma, m = 1, 0, -1$$

⇒ very complex defects

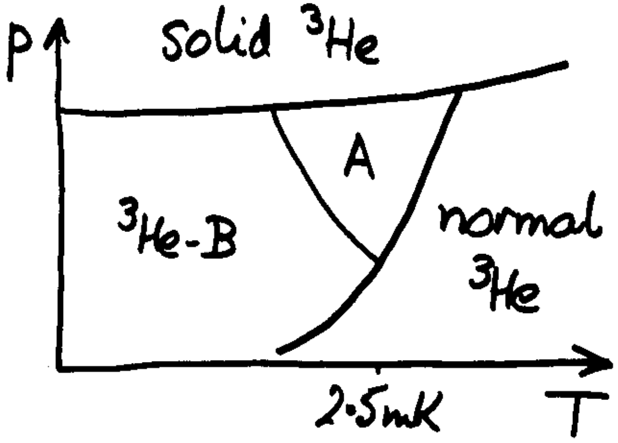


Experimental Test in ^{3}He

^{3}He: fermi liquid

superfluid below ~3mK

Forms Cooper pairs in $L=1$, $S=1$ state.



Order parameter: $A_{m\sigma}$ (3x3 complex)

⇒ very rich defect structure

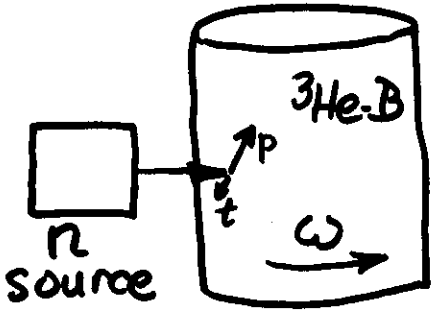
Advantages:

- $S \neq 0$ ⇒ we can use NMR, count individual vortices
- $\xi \sim 100$ nm ($\gg \xi(^4He)$)
  ⇒ Landau-Ginsburg theory good
  ⇒ more energy need to create vortex, so easier to prevent extrinsic formation
- we can arrange T-driven transition

"Mini-Big Bang" (Helsinki)

Ruutu et al: Nature 382 (96) 334

Ruutu, Krusius, Xu } Helsinki
Eltsov, Makhlin, Volovik } Helsinki + Moscow
Gill, Kibble } Imperial
Plaçais ENS

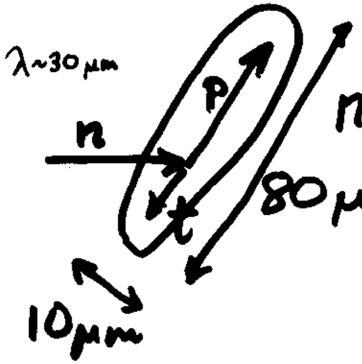


$\omega > \omega_c \sim 2{\cdot}8$ rad/s

$\Rightarrow$ spontaneous vortex formation.

$\omega < \omega_c$, no n

$\Rightarrow$ no vortices (superfluid stationary $\Rightarrow$ counterflow $\underline{v}_s - \underline{v}_n$)



$n + {}^3He \rightarrow p + t + 760$ keV

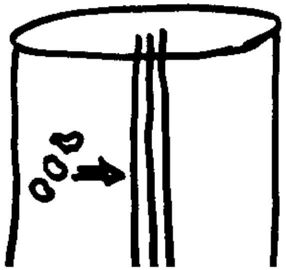
raises T above T_c;

region cools in $\sim 10^{-6}$ s

$\Rightarrow$ vortex formation.

$\underline{v}_s - \underline{v}_n \Rightarrow$ Magnus force

loops above min. size (dep. on ω) stretched,



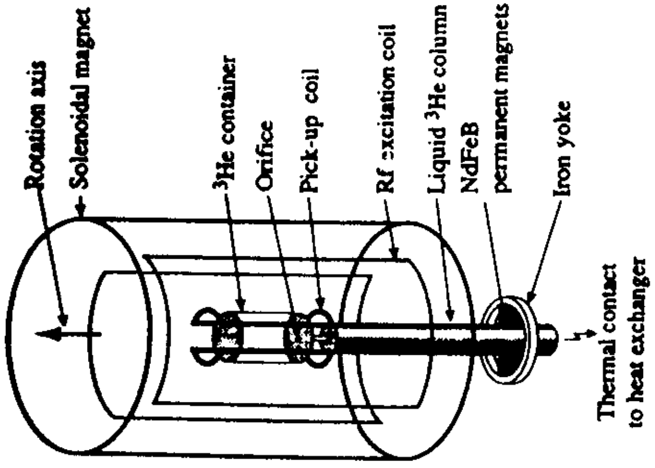


Figure 8. ³He tower with sample container and NMR coils. The cylindrical NMR cell is connected via a 60 mm long liquid ³He column to a large ³He volume with a sintered heat exchanger for thermal contact to the copper nuclear refrigeration stage, located below the parts shown here. A system of superconducting orthogonal coils is used for NMR. The pick-up and rf excitation coils are saddle-shaped, generating fields along the x and y axes. The axially oriented steady field is produced with an end-compensated solenoid. The pick-up coil is fixed on an epoxy coil former which is thermalized to the copper body of the ³He cell while the other coils are thermally and mechanically connected to the mixing chamber, situated above the parts shown here. The actual dimensions of the large outer coils are not drawn to scale. The entire ³He cell with the NMR coils is inside a superconducting Nb shield, to avoid interference from the demagnetization field for cooling and temperature stabilization. This Nb jacket is on the heat shield which is fixed to the mixing chamber.

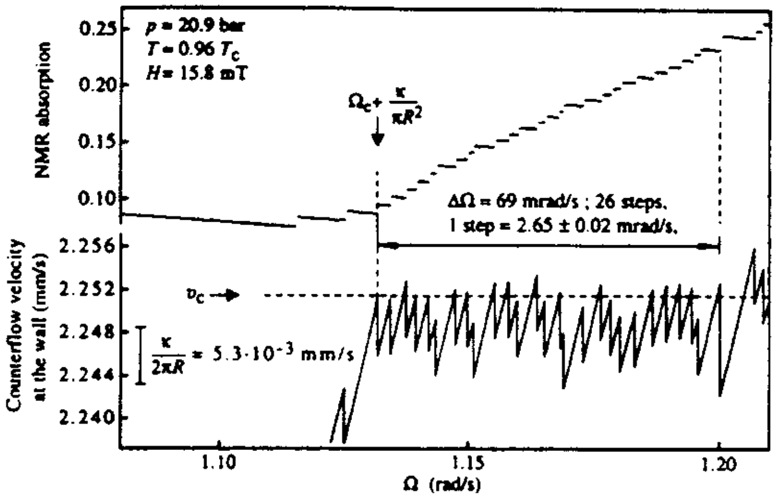
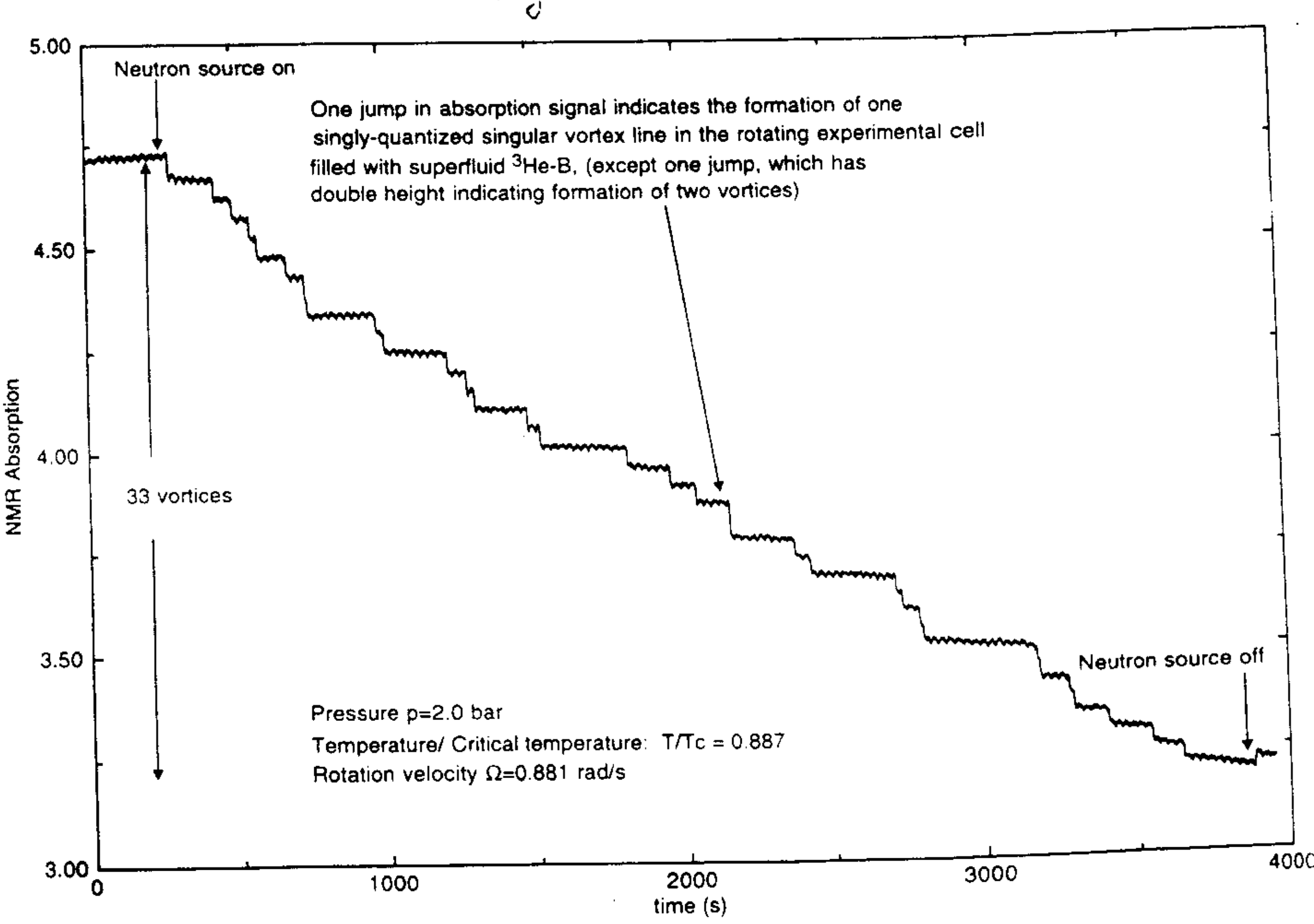
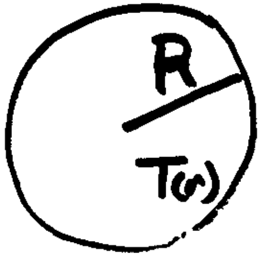


Figure 14. Single-quantum nucleation: Nucleation of vortices as a function of Ω during acceleration ($d\Omega/dt = 2.4 \cdot 10^{-4}$ rad/s^2) in container #2. *Top:* The vertical axis denotes the height of the Larmor peak, normalized to its value in the nonrotating state. Vortex formation starts with the first step-like increase at 1.115 rad/s, but the nucleation threshold Ω_c is identified from the third step (dashed vertical line), where the critical flow velocity v_c reaches a stable value (dashed horizontal line). On the far right at 1.200 rad/s there is a step which corresponds to 2 circulation quanta. *Bottom:* The corresponding counterflow velocity $v = (\Omega - \Omega_c^*)R$ at the cylindrical wall, with a discontinuous reduction equivalent to one circulation quantum $\kappa = h/(2m_3) = 0.0662$ mm^2/s at each step.



Cooling of Bubble



Approximate as sphere

E_0 = energy deposited

bulk: T_0

$$T(r,t) - T_0 \simeq \frac{E_0}{(4\pi D t)^{3/2} C_v} e^{-r^2/4Dt}$$

$$D = v_F \ell \sim 10^{-4} m^2 s^{-1}$$

ℓ = m.f.p.

$$T(R_b, t) = T_c$$

$$\Rightarrow R_b \simeq \left(\frac{E_0}{C_v T_c}\right)^{1/3} \left(1 - \frac{T_0}{T_c}\right)^{-1/3} \sim 10 \mu m$$

Cooling time

$$\tau_Q \sim \frac{R_b^2}{D} \sim 10^{-6} s$$

Dependence on v_s (or ω)

$$v_s = |\underline{v}_s - \underline{v}_n| = \omega R$$

Loops are captured if

$$r > r_0(v_s) = \frac{\kappa}{4\pi v_s} \ln \frac{r_0}{\xi}$$

$$\kappa = \frac{\pi \hbar}{m_3}$$

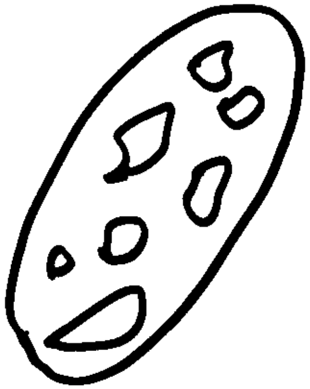
No. of loops formed of length ℓ to $\ell + d\ell$ is $n(\ell)d\ell = \frac{C}{\xi^{3/2} \ell^{5/2}} d\ell$ [Analytic + computer studies]

Linear size $r \sim (\xi \ell)^{1/2}$, whence

$$n(r)dr = \frac{2C}{r^4} dr$$

No. of loops captured
= no. formed in volume V_b
with lengths such that

$$r_0 < r < 2R_b$$



No. of loops captured

after each neutron absorption event

$$N = V_b \int_{r_0}^{2R_b} n(r)\,dr$$

$$= \frac{4\pi}{3} R_b^3 \cdot \frac{C}{3}\left(\frac{1}{r_0^3} - \frac{1}{(2R_b)^3}\right)$$

$$= \frac{\pi C}{9}\left[\left(\frac{2R_b}{r_0}\right)^3 - 1\right] = \frac{\pi C}{9}\left[\left(\frac{v_s}{v_{cn}}\right)^3 - 1\right]$$

where

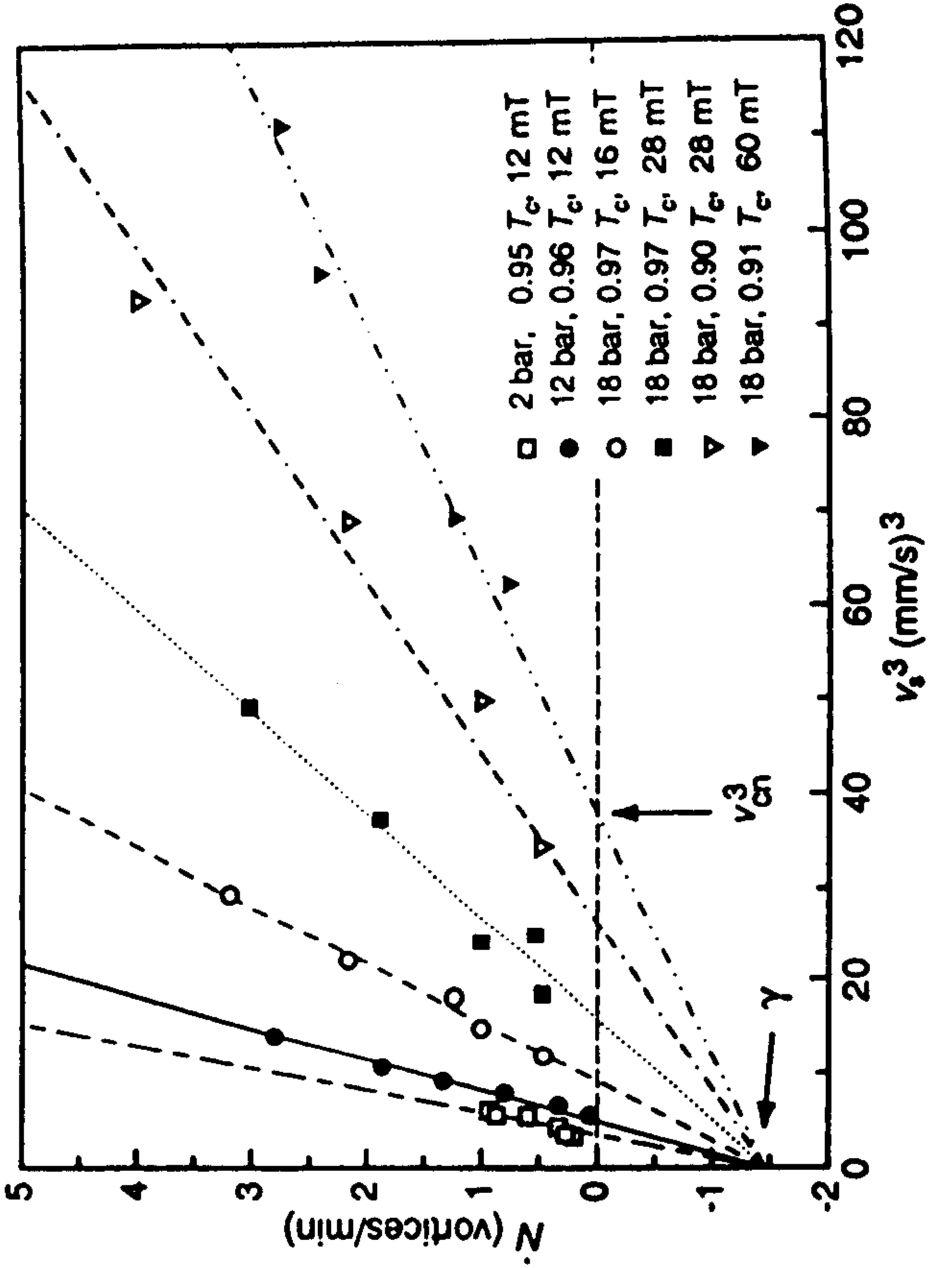
$$v_{cn} = \frac{\kappa}{8\pi R_b} \ln \frac{2R_b}{\xi}$$

$$\propto \frac{1}{R_b} \propto \left(1 - \frac{T_0}{T_c}\right)^{1/3}$$ T_0 = bulk temperature

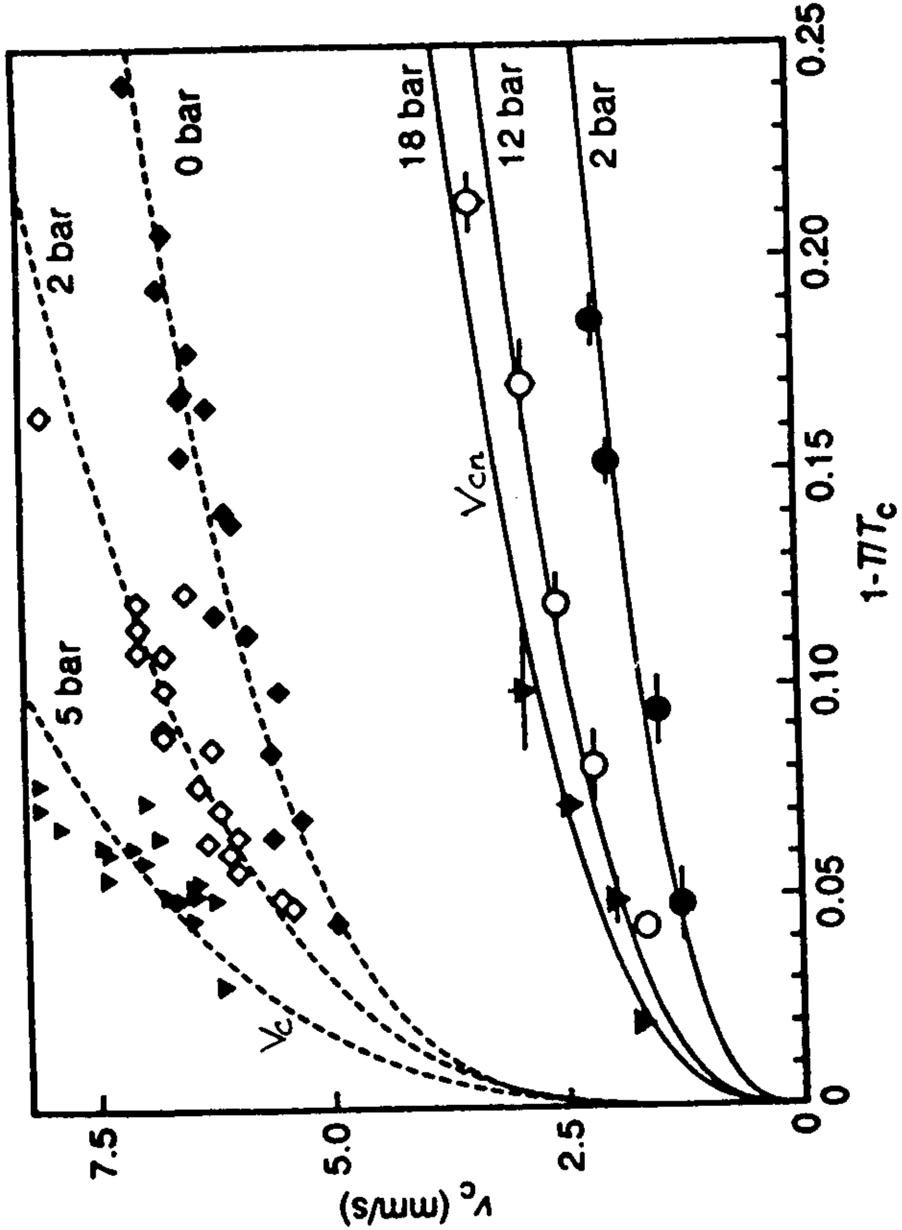
(critical vel. for n-induced vortex formation)

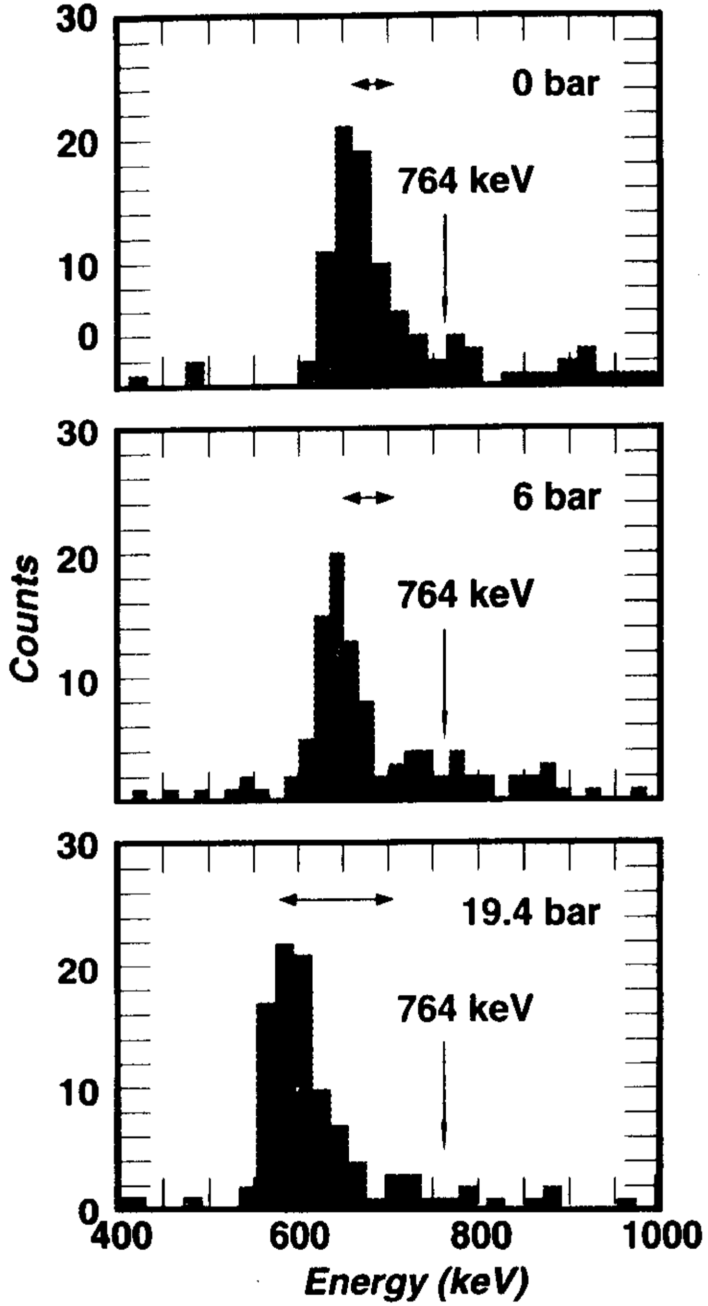
Cf: $$v_c \propto \left(1 - \frac{T_0}{T_c}\right)^{1/4}$$

(critical vel. for spontaneous vortex formation at walls)



20 Bäuerle et al Nature July 1996





Defect Formation - Summary

String defects may be formed at a phase transition where a U(1) symmetry is broken [or more generally in breaking G to H if G is simply connected and H is disconnected].

Zurek estimate of initial defect density works well for ^{3}He and ^{4}He. It should work in the cosmological context too.

We need tests in gauge theories -

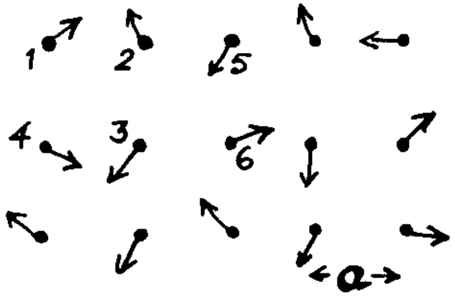
? superconducting thin films.

Also: we need to resolve the question of defect formation in non-Abelian gauge theories.

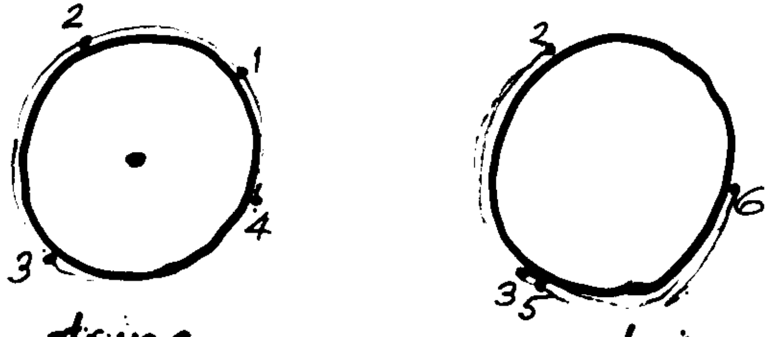
Initial Length Distribution

Vachaspati - Vilenkin simulation

- cubic lattice.
- choose phases randomly at lattice sites



'geodesic rule': phase interpolates by shortest path



string　　no string

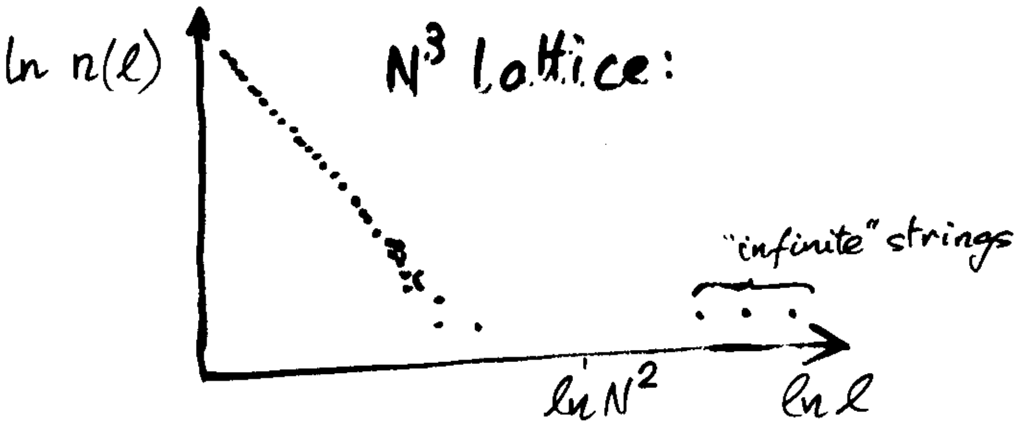
$$\xi_{str} \sim a$$

Loops and Long Strings

Population of loops:

$$n(\ell)\,d\ell \propto \ell^{-5/2}\,d\ell$$

and "infinite" strings



VV: 20% loops, 80% infinite string

Tetrahedral lattice (no ambiguity in joining strings): 37% loops, 63% infinite string

First-Order Transitions

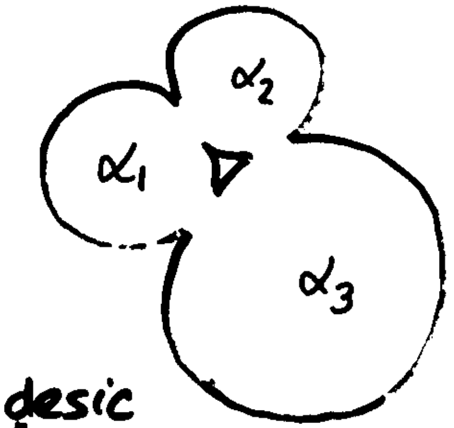
Bubble nucleation

rate/space-time vol = $\gamma \sim T^4 e^{-S_3/T}$

bubbles expand, fill space

$\Rightarrow$ no. bubbles/vol. $\sim \gamma^{3/4}$

Random phase choice α in each bubble



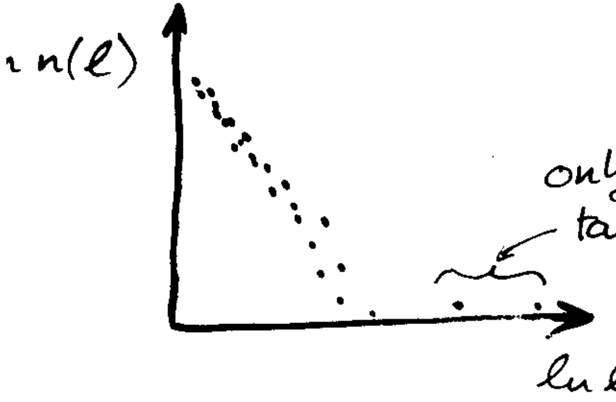
strings form where bubbles meet

"Geodesic rule:

prob. of string formation = $\frac{1}{4}$: $\xi \sim \gamma^{-1/4}$

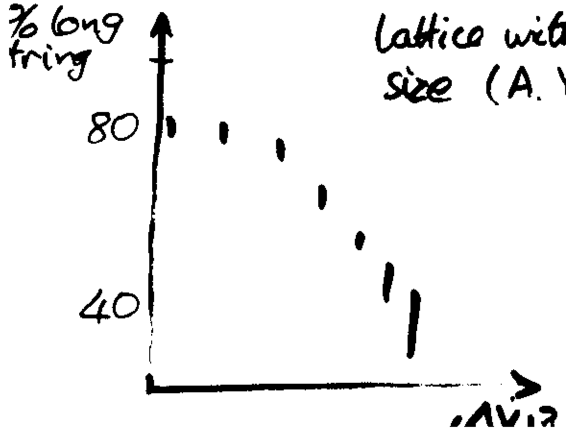
Are there any long strings?

Borrill: nucleated bubbles at random, chose random phases



only enough to watch tail of $\ell^{-5/2}$ dist.
— so no ∞ strings?

Why? - there is no lattice
- large variance of domain volumes



lattice with variable domain size (A. Yates + TK)

Does % long string → 0 for very large $\frac{\langle \Delta V \rangle}{\langle V \rangle}$?

Long strings and the field correlation function

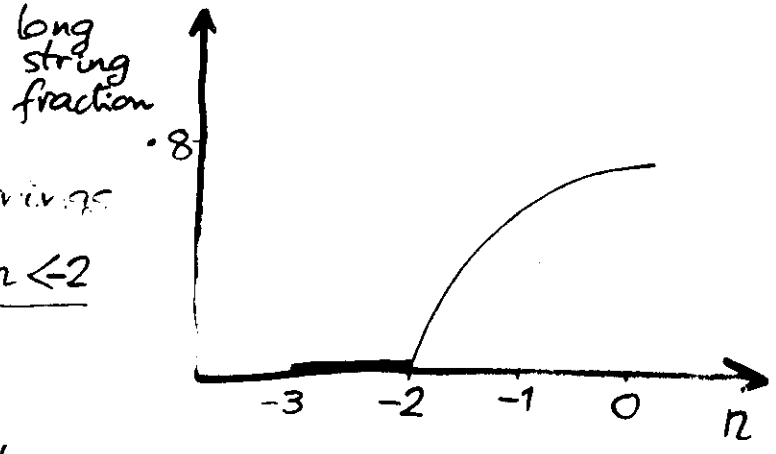
J. Robinson and A. Yates:

Choose random $\phi(\underline{x})$ as Gaussian random variable with assigned power spectrum

$$\langle \tilde{\phi}(\underline{k})\, \tilde{\phi}^*(\underline{k}') \rangle = (2\pi)^3 P(k)\, \delta(\underline{k}-\underline{k}')$$

$$P(k) \propto k^n$$

Fraction of long string depends strongly on n



No long strings for $\underline{-3 < n < -2}$

Note:

Causality ⇒ $n = 0$ on large scales

$n < -3$ ⇒ divergence at small x

Evolution of Strings

Initial configuration is random tangle with length scale $\xi_{(str)}$

$$\xi \ll t$$

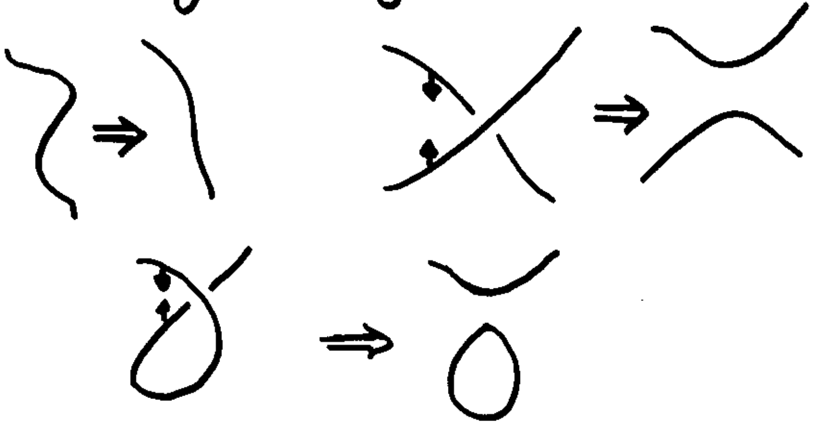
How do strings evolve as universe expands?

Is the initial fraction of long string important?

[illegible] if strings don't explain

Early Evolution

Initially heavily damped



$\xi \nearrow$ $\quad t_d \simeq \dfrac{\mu}{\sigma\rho}$ $\quad \mu \sim \eta^2$

$$\rho = \frac{3}{32\pi G t^2}$$

$$\sigma \simeq \frac{\pi^2}{T}\left(\ln(RT)\right)^2$$

$$t_d \sim t^{3/2} \Rightarrow \frac{1}{\xi}\frac{d\xi}{dt} \sim \frac{t_d}{\xi^2}$$

$$\Rightarrow \xi \propto t^{5/4}$$

$\xi \sim t \sim t_d$ at $T = T_* \simeq G\mu\, m_P$

cf $T_c \simeq \sqrt{G\mu}\, m_P$

For $t > t_*$, strings move freely, relativistically.

Chuang, Durrer, Turok + Yurke 1991 Science 251, 1336

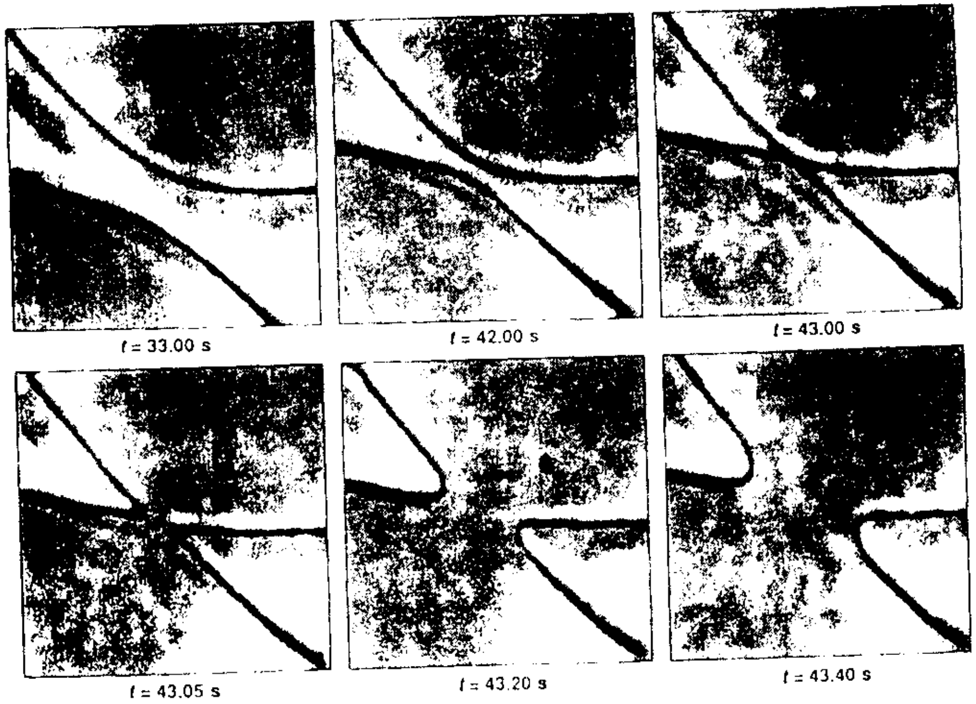


Fig. 1. String intercommutation sequence, showing two type-$\frac{1}{2}$ strings crossing each other and reconnecting the other way. Each picture shows a region 140 μm in width. Note that the two strings lie almost in the same plane—the intercommutation occurs after the strings move toward each other under their mutual attraction.

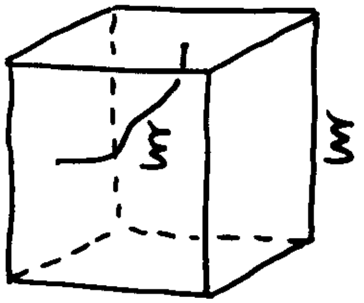
Later evolution - Scaling?

Two possible final outcomes:

1. $\xi \propto t$ (scaling)
2. $\xi/t \searrow \Rightarrow$ string domination.

Def. of ξ:

$$\rho_{str} = \frac{\mu}{\xi^2}$$



1. Scaling $\Rightarrow \rho_{str} \propto \frac{1}{t^2}$

But $\rho_{rad} \propto T^4 \propto \frac{1}{R^4} \propto \frac{1}{t^2}$ $(R \propto t^{1/2})$

$$\Rightarrow \frac{\rho_{str}}{\rho_{rad}} \simeq \text{constant}$$

2. $\xi/t \searrow \Rightarrow \frac{\rho_{str}}{\rho_{rad}} \nearrow$

$\Rightarrow$ string-dominated universe (not)

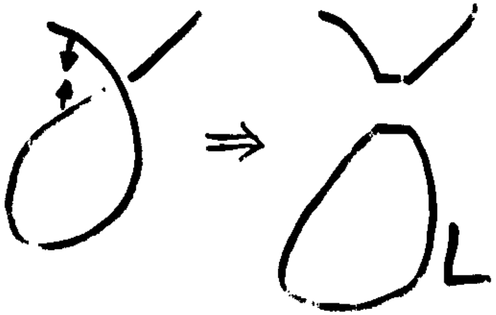
Chuang, Durrer, Turok + Yurke 1991

Fig. 4. A coarsening sequence showing the strings visible in our 230-µm-thick pressure cell containing K15 nematic liquid crystal, at t = 1.0, 1.7, 2.9, and 4.8 seconds after a pressure jump of ΔP = 4.7 MPa from an initially isotropic state in equilibrium at approximately 33°C and 3.6 MPa. The evolution of the string network shows self-similar or "scaling" behavior. Each picture shows a region 360 µm in width.

$$\xi \propto \sqrt{t}$$

Energy loss mechanism

Loop formation and decay by gravitational radiation



Loop, length L has period L/2, emits gravitational waves with power

$$P = \Gamma G\mu^2 \quad (\Gamma \simeq 50\text{, depending on shape})$$

$$\Rightarrow \frac{dL}{dt} = -\Gamma G\mu \Rightarrow \text{Lifetime} \simeq \frac{L}{\Gamma G\mu}$$

$$\sim 10^4 L \text{ for GUT strings}$$

Limit from ms pulsar timing

Gravitational waves ⇒ fluctuations in timing

⇒ limit on gravitational wave density

$\Rightarrow G\mu \lesssim 10^{-6}$ (depending on H)

Vortons

Davis + Shellard
Carter + Martin
Martin + Peter
Brandenberger et al

Superconducting strings –
two conserved charges

N – winding number (topologically conserved)
defines current

Q – charge (dynamically conserved)

$$N \neq 0 \;\&\; Q \neq 0 \Rightarrow J \neq 0$$

The loop may be stable (a vorton)

Stable loops could come to dominate ρ

$$\Omega_{\text{vortons}} < 1 \Rightarrow \eta \approx T_C \lesssim 10^9\ \text{GeV}$$

If $\eta \sim 10^9$ GeV, vortons could be dark matter.

Vortons could be accelerated in active galactic nuclei and constitute extreme high-energy cosmic rays

Bonazzola + Peter

– most efficient way of extracting energy

Numerical Simulations of String Evolution

Confirm scaling of long-string network

Albrecht + Turok
Bennett + Bouchet
Allen + Shellard

$$\frac{2t}{\xi} \rightarrow \gamma \quad \text{(constant)}$$

$\gamma \sim 7$ (BB, AS)

But there is a lot of small-scale structure on strings.

Typical interkink distance $\zeta \ll \xi$

Typical loop size $\ll \xi$

In simulations while $\xi \propto t$, $\zeta \sim$ const.

Does the small-scale structure eventually scale?

Shellard & Allen.

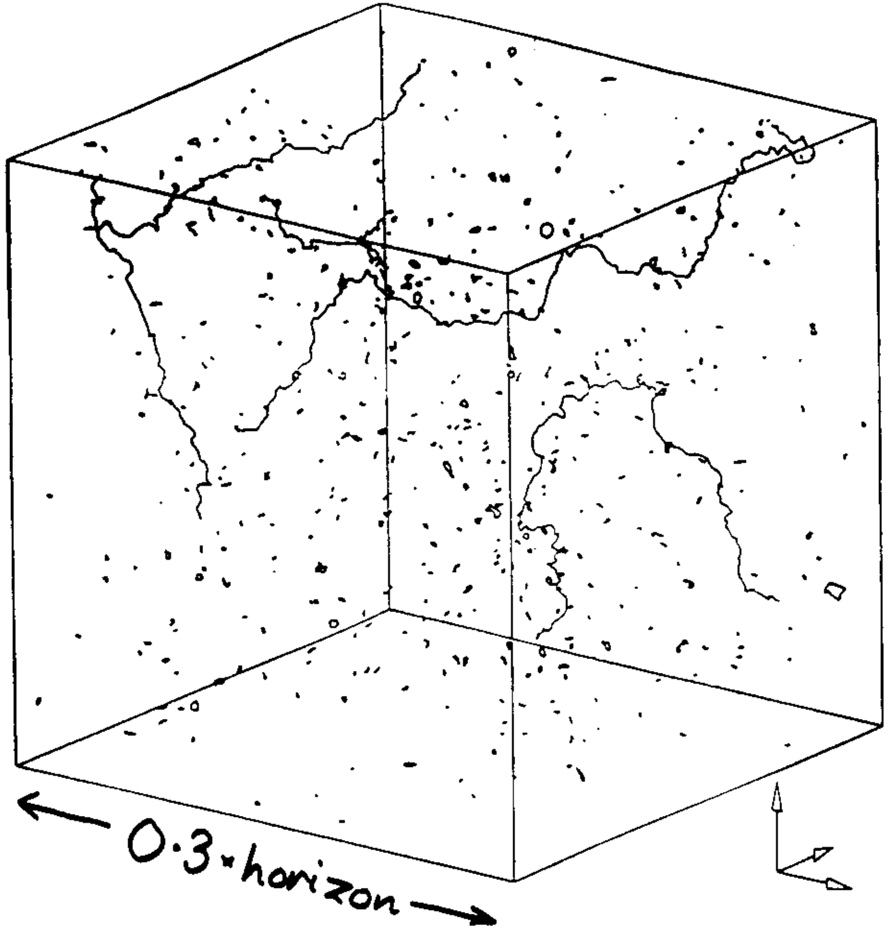


Fig. 3. Portion of the numerical box showing the evolved cosmic string system in a small 18^3 simulation. The horizon at this time is approximately 3.5 times the box sidelength. Note the large number of kinks and short wavelength modes and the predominance of small loops on scales much less than the horizon. The curved segment in the foreground is the result of a fairly recent intercommuting between uncorrelated segments. It exhibits enhanced substructure and is actively forming small loops as it contracts.

Analytic Study

D. Austin, E. Copeland, TK

based on three length scales

ξ: $\rho_{str} = \frac{\mu}{\xi^2}$ inter-string distance

$\bar{\xi}$: correlation length along string

ζ: inter-kink distance

Derive rate equations for $\xi, \bar{\xi}, \zeta$ or for scaling variables $\gamma = \frac{1}{H\xi}$, $\bar{\gamma} = \frac{1}{H\bar{\xi}}$, $\varepsilon = \frac{1}{H\zeta}$.

$H = \frac{\dot{T}}{T} = \frac{1}{pt}$ $\quad p = \begin{cases} 2 & (RD) \\ 3/2 & (MT) \end{cases}$

$$pt\frac{\dot{\gamma}}{\gamma} = p + \left(\frac{E}{4} + \frac{\chi}{2} - \frac{\text{[illegible]}}{2}\right) - \frac{c}{2}\bar{\gamma} - \frac{\Gamma G\mu}{2}\varepsilon$$

$$pt\frac{\dot{\bar{\gamma}}}{\bar{\gamma}} = p + \left(\frac{E}{2} - \text{[illegible]}\right) + \frac{\chi\gamma^2}{\omega\bar{\gamma}} - \frac{I}{2}\bar{\gamma} - \Gamma G\mu\varepsilon$$

$$pt\frac{\dot{\varepsilon}}{\varepsilon} = p + \left(\frac{1+C-3}{2}E - \text{[illegible]}\right) + \frac{\chi\gamma^2}{\varepsilon} - kc\bar{\gamma} - \Gamma G\mu\hat{C}\varepsilon$$

stretching inter-commuting loop form. gravit. radiation

Scaling ⇒ fixed point (all RHS = 0)

Transient Scaling Regime

Seen in simulations

$$\xi, \bar\xi \propto t \qquad \zeta \not\propto t$$

$$\gamma, \bar\gamma \rightarrow \text{consts} \qquad \epsilon \nearrow$$

Initially grav. rad. negligible

$$pt\frac{\dot\gamma}{\gamma} = p + \left(\frac{F}{4} + \frac{\bar\alpha_{ne}}{2} - \frac{3}{2}\right) - \frac{c}{2}\bar\gamma \qquad = 0$$

$$pt\frac{\dot{\bar\gamma}}{\bar\gamma} = p + \left(\frac{F}{2} - \bar\alpha_{ne} - \bar G\right) + \frac{\chi\gamma^2}{\omega\bar\gamma} - \frac{I}{2}\bar\gamma = 0$$

(essentially equations of
EC, TK + DA, PR D45, R1000 (92))

$\Rightarrow$ scaling values of $\gamma, \bar\gamma$.

RD era: BB $\Rightarrow$ $\gamma = 7{\cdot}2 \pm 1{\cdot}4$
$c\bar\gamma = 1{\cdot}14 \pm {\cdot}04$

Our equations suggest

$$\gamma \simeq 4 \quad \bar\gamma \simeq 6 \quad c \simeq 0{\cdot}2$$

Full Scaling

Transient regime

$$\gamma, \bar\gamma \sim \text{const}, \qquad \epsilon \nearrow$$

continues until $TG\mu\epsilon \sim 1$.*

Then: do we reach regime where $\gamma, \bar\gamma, \epsilon$ <u>all</u> consts?

Depends on magnitudes of $\hat C$, k: we require

$$\hat C > \hat C_{cr} > k$$

$$\hat C_{cr} = \frac{p - \bar\alpha_{ne} - \frac{3}{2}(1-4c)F}{2p - 3 + \bar\alpha_{ne} + \frac{1}{2}F} \simeq \begin{cases} 1{\cdot}8 & \text{RD} \\ 2{\cdot}8 & \text{MD} \end{cases}$$

Problem: $\hat C$, k are very hard to estimate.

*<u>But</u> for different values of $\hat C, k$, it <u>is</u> possible to have

Is there any small-scale structure?

Vincent, Hindmarsh + Sakellariadou

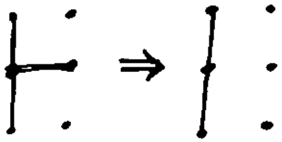
Simulations in flat space (Smith-Vilenkin algorithm) — exact solution of Nambu-Goto equations, with loops above a minimum size removed when formed.

⇒ • loops are formed at the smallest size allowed

- good agreement with analytic model of Austin et al. (ACK)
- if there is no lower cutoff of loop size, small-scale structure disappears. (consistent w. ACK)

Is this what happens in real universe?

It may be an artefact of the Lattice (which allows exact backtracking.



Need a simulation including effect of gravitational radiation.

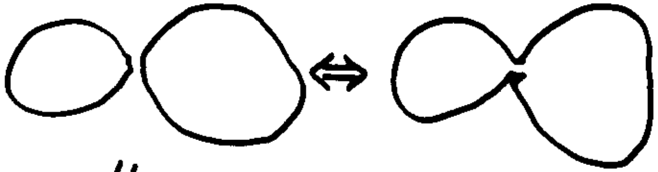
Evolution of Loops Alone

D. Steer + T.K.

Suppose that the initial string distribution does contain only loops. How would they evolve?

Derive evolution equation for number density $n(\ell)\,d\ell$ of loops in range $\ell \sim \ell + d\ell$, including:

- expansion
- gravitational radiation
- processes



Preliminary results:

radiation era: → scaling $n(\ell) \sim \dfrac{C}{t^{3/2}\ell^{5/2}}$

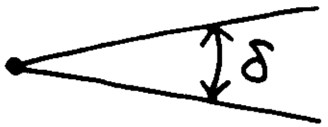
matter era: solution becomes unstable and all strings disappear in finite time

Observable Effects

Gravity:

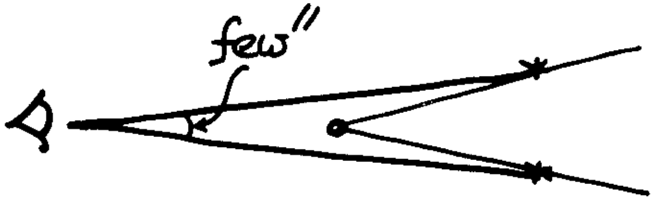
$\dot{\underline{g}} = \underline{0}$

(straight, static string)



$\delta = 8\pi G\mu$

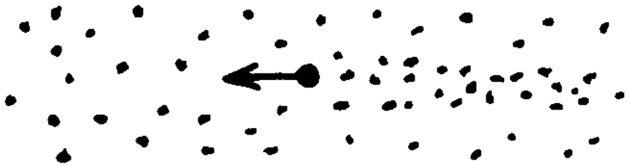
Gravitational Lensing



double images

$\Rightarrow$ arrays of lensed pairs

Wakes



density perturbations $\frac{\delta\rho}{\rho} \propto G\mu$

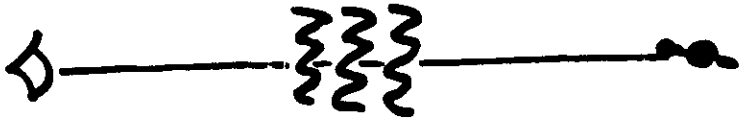
$\Rightarrow$ galaxy formation

Gravitational Radiation

Oscillating loops emit grav. waves $P = \Gamma G\mu^2 \quad \Gamma \sim 50$

Long strings also radiate

Limit from ms pulsar timing:



Grav. waves $\Rightarrow$ fluctuations in timing

Limit on fluctuations

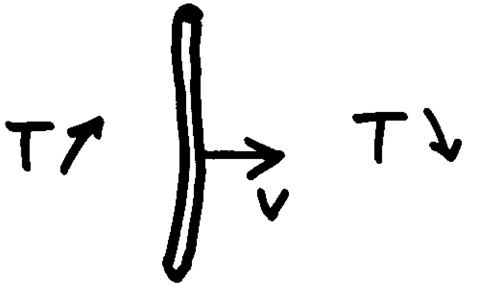
$\Rightarrow$ limit on grav. wave energy density

$\Rightarrow G\mu <$ few $\times 10^{-6}$

(more stringent for large H)

(limit $\propto H^{-7/2}$)

Cosmic Microwave Background



$$\frac{\Delta T}{T} \simeq 8\pi G\mu v \sim 10^{-5}$$

Compatible with COBE, etc.

Inflation vs. Cosmic Strings

Both ⇒ viable models of CMBR, density perturbations

Calculations in string model difficult because of uncertainties in evolution.

Look for robust signals — best prospect is small angular scale CMBR distortions

CMB Anisotropy

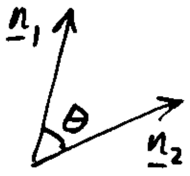
Measure $T_{CMB}(\theta,\varphi)$.

$$\frac{\Delta T}{T_0} = \frac{T(\theta,\varphi) - T_0}{T_0} = \sum_{\ell,m} a_{\ell m} Y_{\ell m}(\theta,\varphi).$$

Angular power spectrum:

$$C_\ell = \langle |a_{\ell m}|^2 \rangle \qquad C_\ell^{1/2} \sim 10^{-5}.$$

Correlation function:

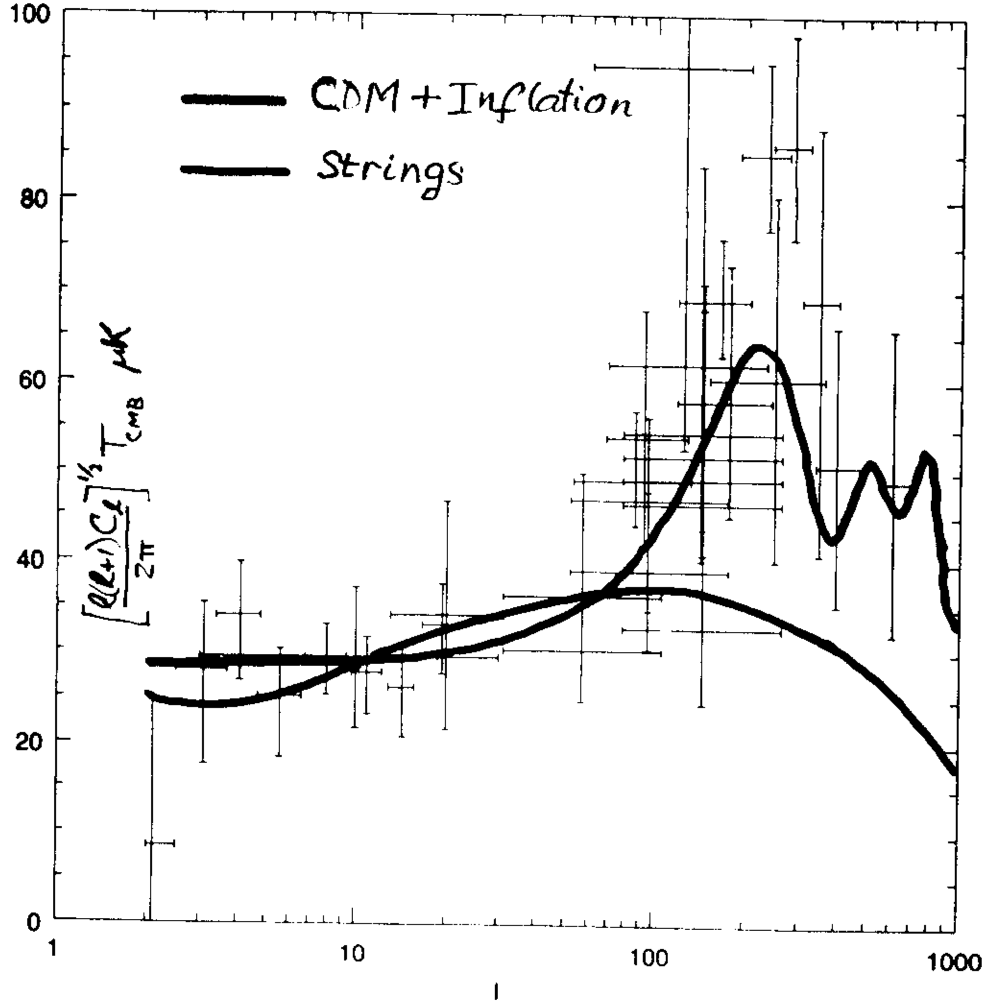


$$C(\theta) = \left\langle \frac{\Delta T(\underline{n}_1)}{T_0} \; \frac{\Delta T(\underline{n}_2)}{T_0} \right\rangle$$

$$= \frac{1}{4\pi} \sum_\ell (2\ell+1) C_\ell \, P_\ell(\underset{\cos\theta}{\underset{\shortparallel}{\underline{n}_1 \cdot \underline{n}_2}})$$

C_1: motion of Earth rel. to CMB ~ 600 km/s

CMB Anisotropy



$$\Delta T(\theta,\varphi) = \sum_{l,m} a_{lm} Y_{lm}(\theta,\varphi)\, T_0$$

$$\langle |a_{lm}^2| \rangle = C_l$$

Scalar, Vector + Tensor Modes

$$g_{\mu\nu} = \underset{\overset{\uparrow}{\text{FRW}}}{{}^0g_{\mu\nu}} + \underset{\overset{\uparrow}{\text{small}}}{h_{\mu\nu}}$$

Synchronous gauge $h_{0\mu} = 0$

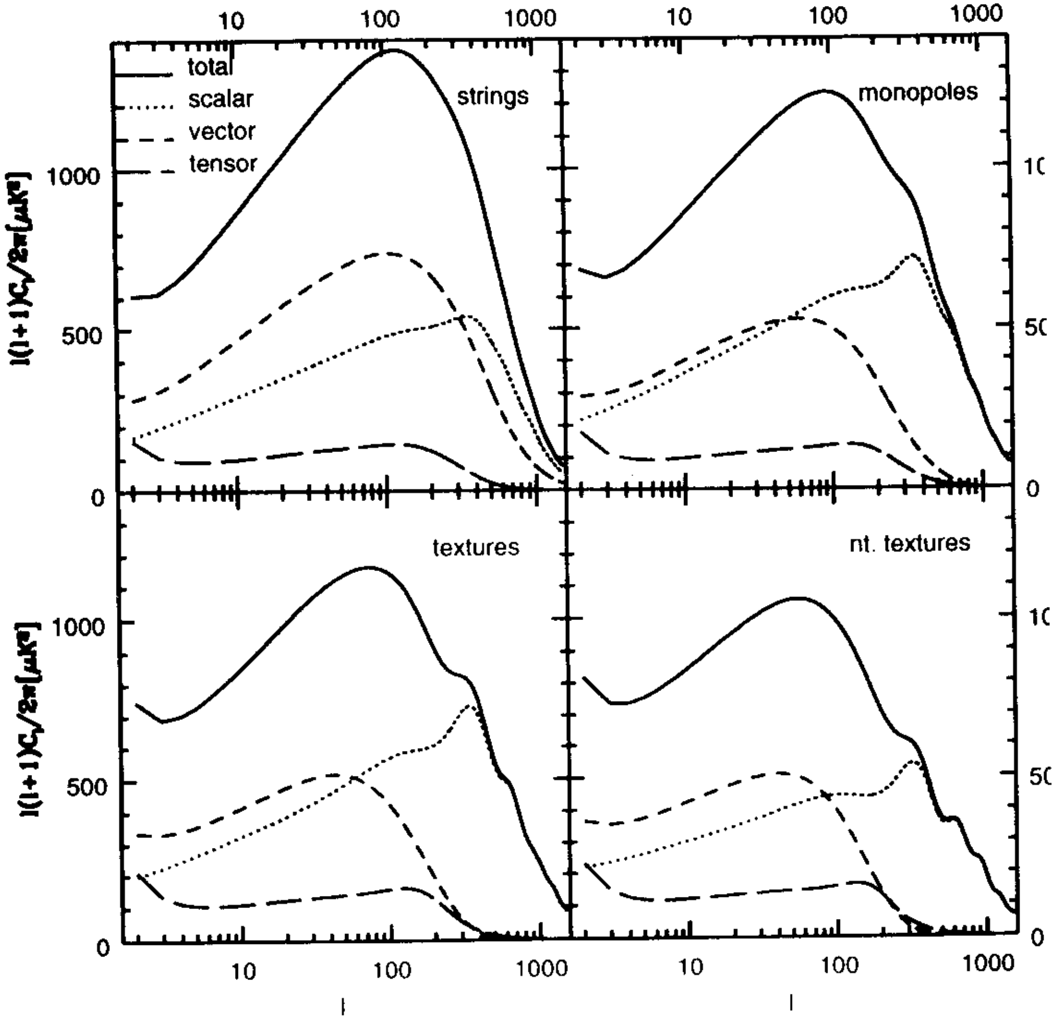
$$\tilde{h}_{ij} = \overset{\text{tensor}}{t_{ij}} + \overset{\text{vector}}{(\hat{k}_i v_j + v_i \hat{k}_j)}$$

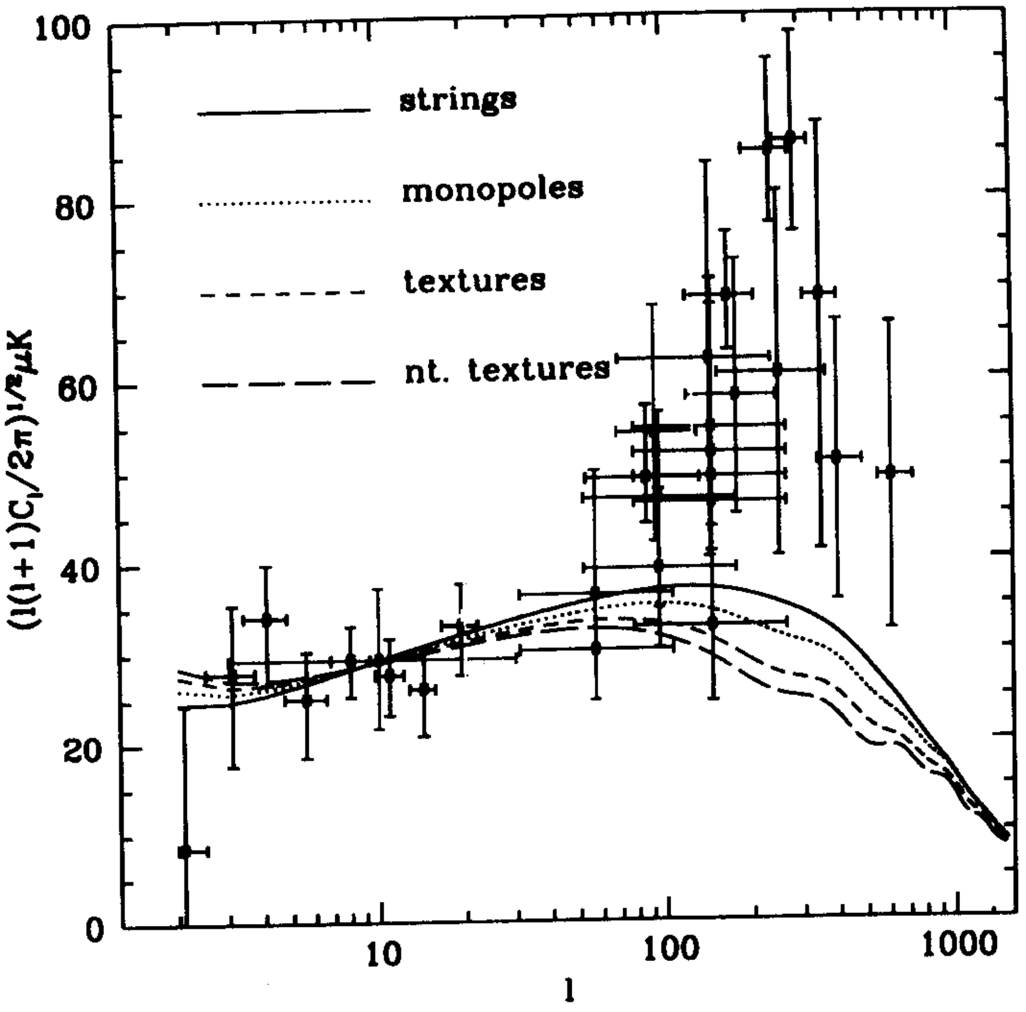
$$+ \underset{\text{scalars}}{\tfrac{1}{2}(\delta_{ij} - \hat{k}_i \hat{k}_j) s + \tfrac{1}{2}(3\hat{k}_i \hat{k}_j - \delta_{ij}) s'}$$

$$s = \tilde{h}_{ii} \qquad s' = \hat{k}_i \hat{k}_j h_{ij}$$

Scalar, vector + tensor modes evolve separately

- density perturbations coupled to scalar modes
- velocity perturbations to vector modes
- gravitational waves are tensor modes





Pen, Seljak + Turok 1997

Density Perturbations

$$\frac{\rho(\underline{r},t)}{\bar{\rho}(t)} - 1 = \sum_{\underline{k}} \delta(\underline{k}t)\, e^{i\underline{k}\cdot\underline{r}}$$

$\underline{r}$: comoving dist.

$\underline{k}$: comoving wave vector

Power spectrum

$$P(\underline{k}) = |\delta(\underline{k})|^2 \propto k^n$$

$$-3 < n \leqslant 4$$

Large scales $(k \to 0)$: $P(k) \propto k^4$.

Inflation: solve

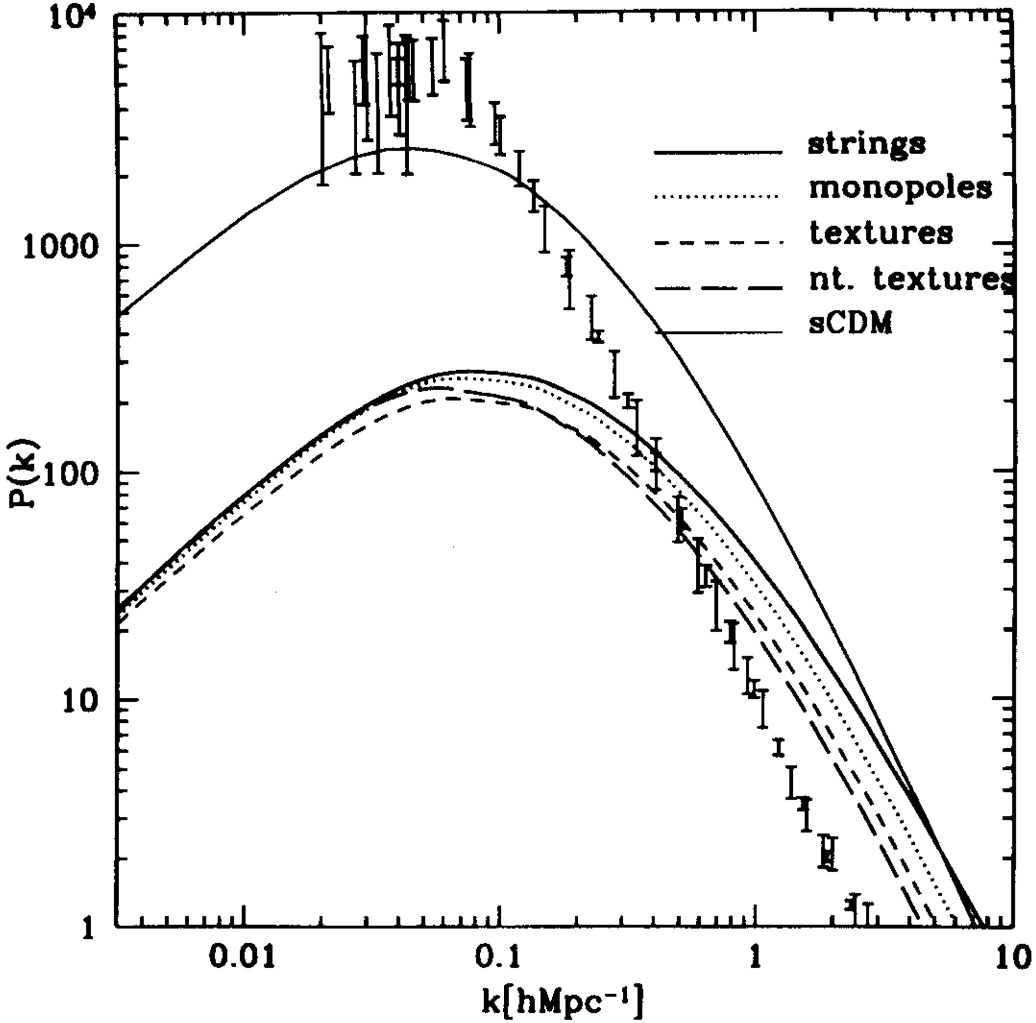
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}^{\text{matter}}$$

to find how δ evolves.

Defect models (including strings): solve

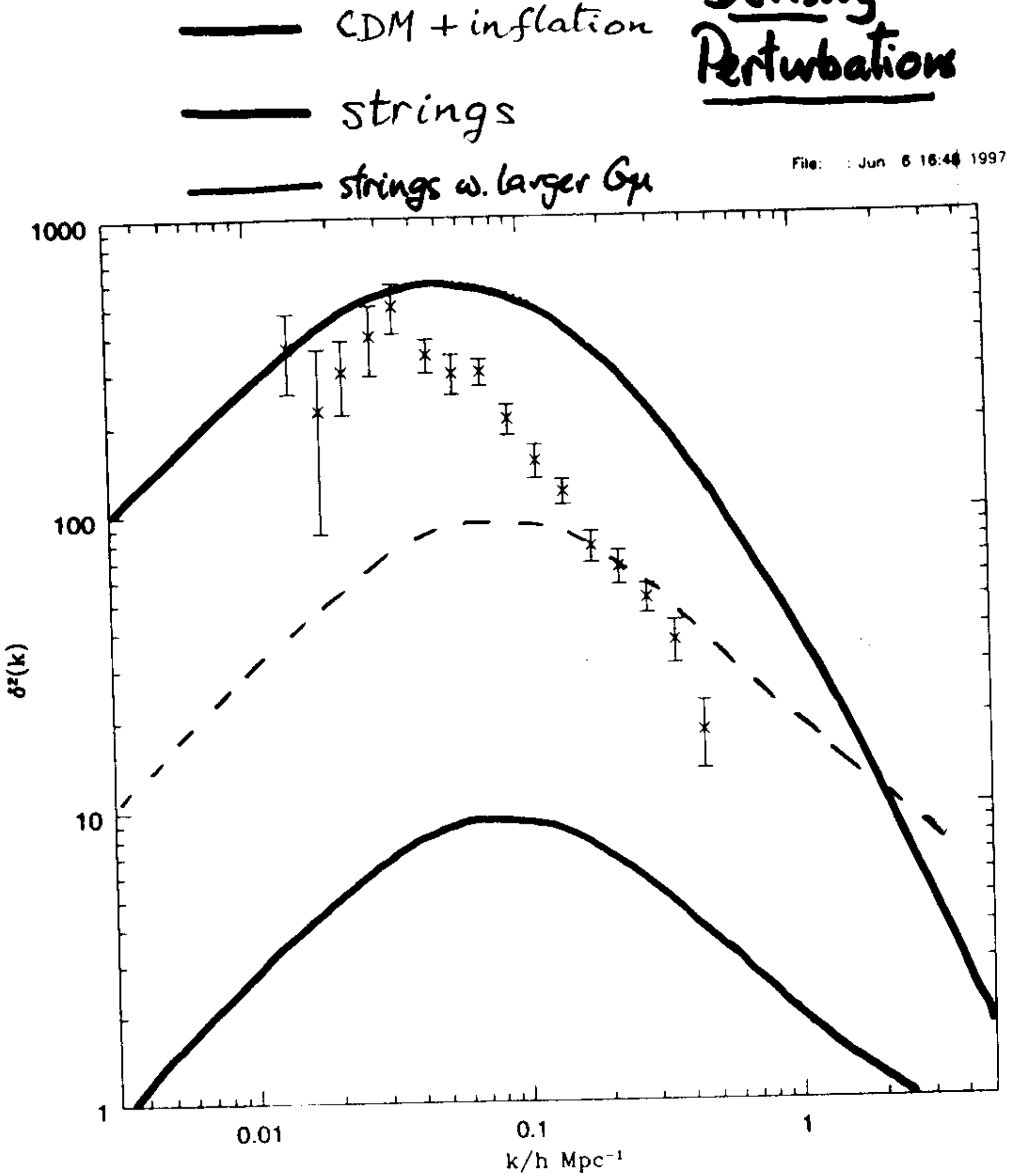
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G \left(T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^{\text{defects}}\right)$$

$\Rightarrow$ 'active' perturbations.



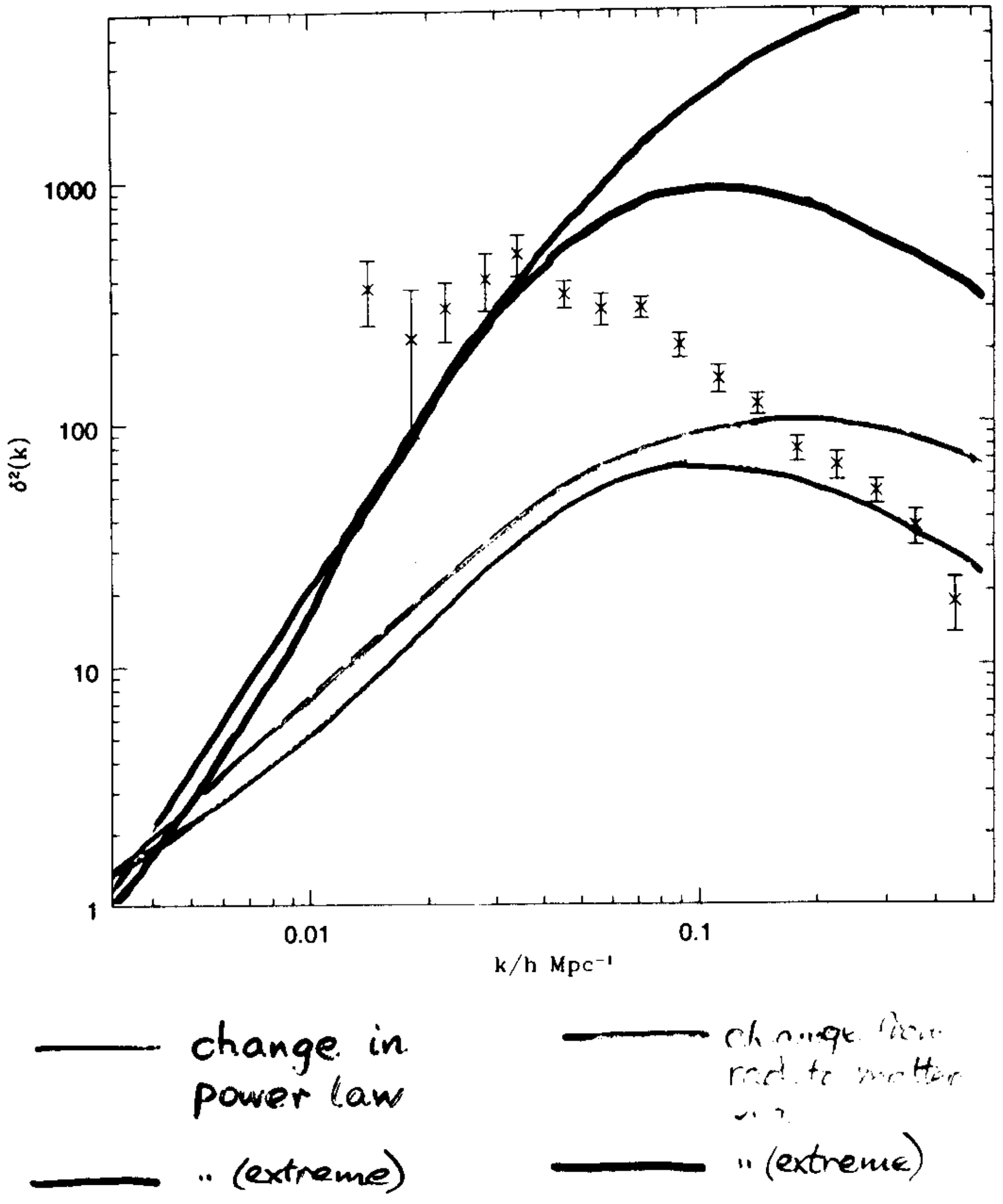
Pen, Seljak + Turok 97

Density Perturbations



Albrecht,
Magueijo
Ro

Deviations from Scaling



Conclusions

To fit both
CMB – Cobe normalization
&
Large-scale structure
would require very artificial changes

Cosmic strings do not provide an acceptable theory of structure formation.

but ...

what is "artificial"?
Loops only might help.
etc

Also:

CDM and inflation doesn't work either

– we need mixed dark matter or $\Lambda \neq 0$ or ...

and

Cosmic strings could explain very high energy cosmic rays ($E > 10^{20}$ eV) and could provide a candidate for CDM.