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SMR.996 - 27

Lecture III & IV

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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INFLATION AND THEORY OF COSMOLOGICAL PERTURBATIONS

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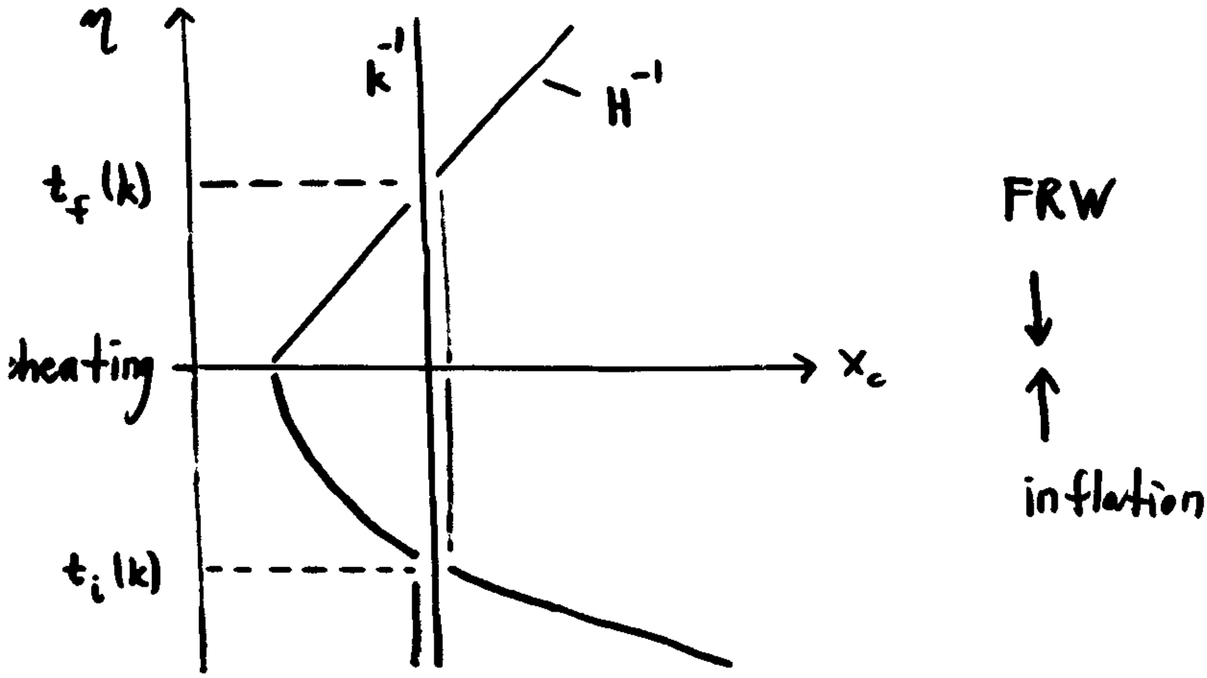
Please note: These are preliminary notes intended for internal distribution only.

R Brandenberger, Lecture 3

Issues :

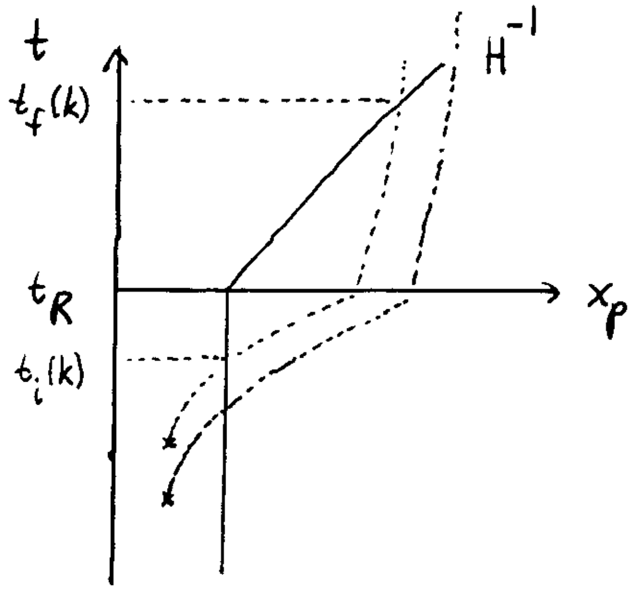


k   wave number of fluctuation

H^{-1} Hubble radius

1. classical evolution of perturbation
2. quantum generation " "
3. quantum → classical transition

First Prediction from Inflation :



$$\frac{\delta M}{M}\left(k, t_i(k)\right) = \text{const}$$

$$\Rightarrow \quad \frac{\delta M}{M}\left(k, t_f(k)\right) = \text{const}'$$

scale invariant (Harrison-Zel'dovich) spectrum

$$\Leftrightarrow \quad \underline{P(k) \sim k} \qquad \text{(primordial spectrum)}$$

2. Basics of Structure Formation

3.1 Gravitational instability

3.2 Newtonian theory

valid when $\lambda \ll t$, $p \ll \rho$

* $\dot{\rho} + \nabla(\rho v) = 0$ Continuity

$\dot{\underline{v}} + (\underline{v}\cdot\underline{\nabla})\underline{v} = -\nabla\phi - \frac{1}{\rho}\nabla p$ Euler

$\nabla^2\phi = 4\pi G\rho$ Poisson

ϕ : Newtonian gravitational potential

* application to an expanding Universe

$\underline{r} = a\underline{x}$

$\underline{v} = \dot{a}\underline{x} + a\dot{\underline{x}}$ $\underline{u} \equiv \dot{\underline{x}}$

expansion peculiar velocity

* linearization ansatz

$\rho(\underline{x},t) = \bar{\rho}(t)(1+\delta(\underline{x},t))$ $\delta \equiv \frac{\delta\rho}{\bar{\rho}}$

gives*:

$\dot{\delta} + \nabla\cdot u = 0$

$\dot{\underline{u}} + 2\frac{\dot{a}}{a}\underline{u} = -a^{-2}(\nabla\delta\phi + c_s^2\nabla\delta)$ $c_s^2 \equiv \frac{dp}{d\rho}$

$\nabla^2\delta\phi = 4\pi G\bar{\rho}a^2\delta$

* For adiabatic perturbations

* Combination

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}\delta - \frac{c_s^2}{a^2}\nabla^2\delta = 0$$

$\downarrow$ Fourier transformation

$$\delta(\underline{x}) = (2\pi)^{-3/2} V^{-1/2} \int d^3k \, e^{i\underline{k}\underline{x}} \delta_{\underline{k}}$$

$$\boxed{\ddot{\delta}_k + 2H\dot{\delta}_k + \left(\frac{c_s^2 k^2}{a^2} - 4\pi G\bar{\rho}\right)\delta_k = 0}$$

N.B. In linear perturbation theory all Fourier modes evolve independently!

A first look at equation:

$$\lambda_J = \frac{2\pi}{k_J} \qquad k_J^2 = \left(\frac{k}{a}\right)^2 = \frac{4\pi G\bar{\rho}}{c_s^2} \qquad \text{Jeans length}$$

physical

$\lambda > \lambda_J \rightarrow$ growth

$\lambda < \lambda_J \rightarrow$ damped oscillation

Some solutions ($t > t_{eq}$):

$$\lambda \gg \lambda_J : \quad \ddot{\delta}_k + \frac{4}{3t}\dot{\delta}_k - \frac{2}{3t^2}\delta_k = 0$$

$$\delta_k(t) = c_1 t^{2/3} + c_2 t^{-1} \qquad \text{i.e. } \underline{\delta_k \sim a}$$

$\lambda \ll \lambda_J$: $\ddot{\delta}_k + 2H\dot{\delta}_k + c_s^2\left(\frac{k}{a}\right)^2\delta_k = 0$

$$\delta_k(t) \sim a^{-1/2}(t)\exp\left\{\pm i c_s k \int dt' a(t')^{-1}\right\}$$

Application : $\hookrightarrow$

Matter perturbations in a smooth relativistic background ($t < t_{eq}$):

$$\ddot{\delta}_k + 2H\dot{\delta}_k - 4\pi G\bar{\rho}_m\delta_k = 0 \qquad \text{if } c_s = 0$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\bar{\rho}_m + \bar{\rho}_r\right)$$

negligible for $t \ll t_{eq}$ $\Rightarrow \delta_k \sim cst$

$$\eta = \bar{\rho}_m / \bar{\rho}_r$$

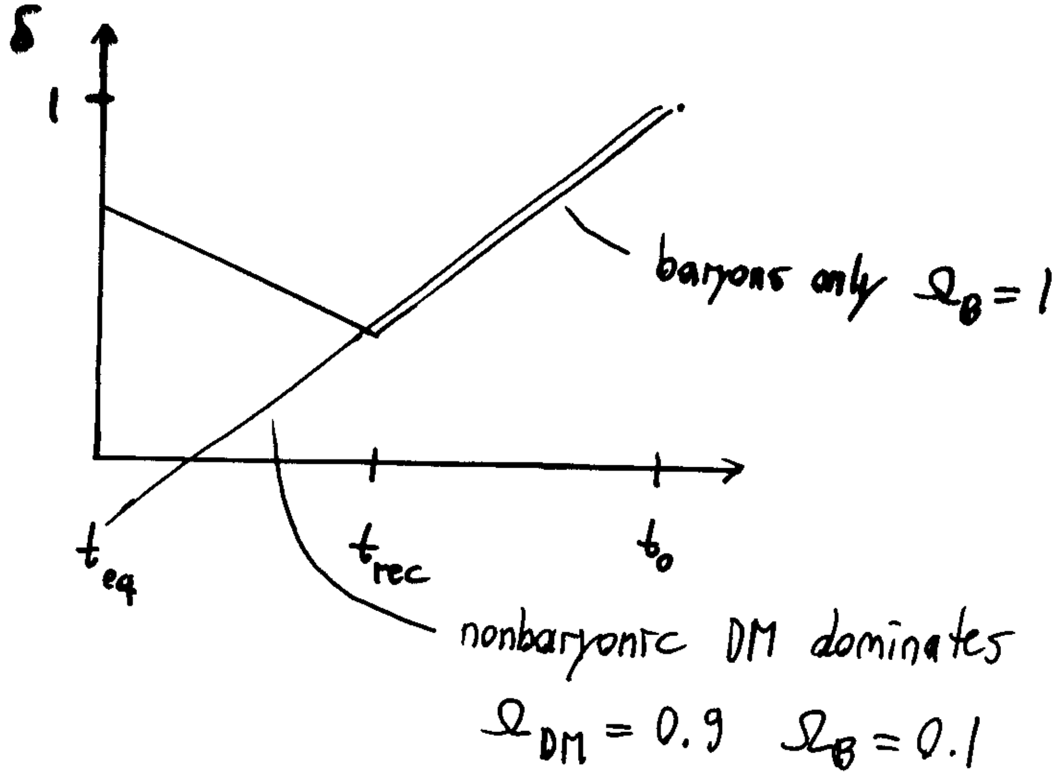
$$\frac{d^2}{d\eta^2}\delta_k + \frac{2+3\eta}{2\eta(1+\eta)}\frac{d}{d\eta}\delta_k = \frac{3\,\delta_k}{2\eta(1+\eta)}$$

$\delta_k \sim 1 + \frac{3}{2}\eta$ growing mode

$\Rightarrow$ $\delta_k \sim cst$ $\quad t < t_{eq}$

structure formation begins at t_{eq}

Application :



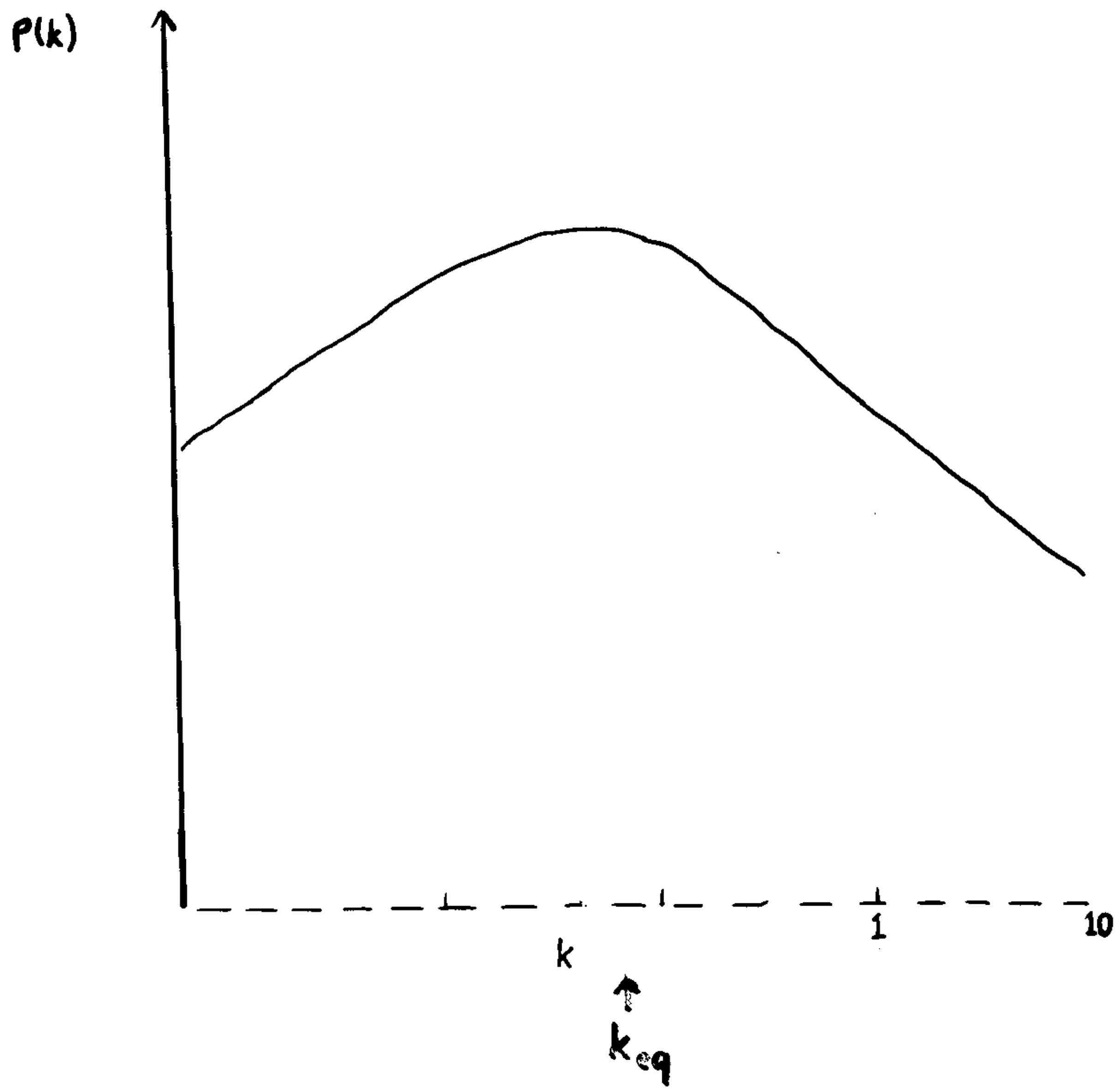
$$\delta(t_{eq})\Big|_B = \left(\frac{a(t_{rec})}{a(t_{eq})}\right)^{3/2} \delta(t_{eq})\Big|_{DM}$$

Fact : $\dfrac{\delta T}{T} \simeq \dfrac{1}{3}\,\delta(t_{eq})$

BDM $\rightarrow$ $\dfrac{\delta T}{T} \sim 10^{-3}$ prediction

DM $\rightarrow$ $\dfrac{\delta T}{T} \sim 10^{-5}$ "

DATA $\Rightarrow$ nonbaryonic DM !


P(k)
1
10
k
k_{eq}

3. Relativistic theory $\lambda > t$, $p \sim \rho$ (classical)

Basic idea:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$
$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$$
$$T_{\mu\nu} = T^{(0)}_{\mu\nu} + \delta T_{\mu\nu}$$

$(g^{(0)}_{\mu\nu}, \bar{T}^{(0)}_{\mu\nu})$ homog. FRW solution

linearization $\downarrow$

$$\delta G_{\mu\nu} = 8\pi G\, \delta T_{\mu\nu}$$

Problem of gauge $\rightarrow$ gauge invariant approach
$\hookrightarrow$

step 1: general scalar metric perturbation

$$\delta g_{\mu\nu} = a^2 \begin{pmatrix} 2\phi & -B_{,i} \\ -B_{,i} & 2(\psi\delta_{ij} + E_{,ij}) \end{pmatrix}$$

step 2: consider small coord. trsf. preserving scalar character of $\delta g_{\mu\nu}$

$$x^{\mu'} = x^\mu + \xi^\mu \qquad \xi^0,\ \xi^i = \xi_{,i}$$

step 3: find induced trsf. of ϕ, ψ, B, E

find gauge invariant combinations

$$\Phi = \phi + a^{-1}[(B-E')a]'$$
$$\Psi = \psi - \frac{a'}{a}(B-E')$$

step 4: derive EOM for Φ & Ψ
easiest in gauge $E=B=0$

step 5: for scalar field matter : $\delta T_i^{\ j} = \delta_i^{\ j}(\cdot\cdot)$

$$\Downarrow$$

$$\Phi = \Psi$$

single gauge invariant variable contains all information about linear gravitational perturbations

for $\lambda \gg t$:

$$\dot{\zeta} = 0 \qquad \zeta = \frac{2}{3}\frac{H^{-1}\dot{\Phi} + \Phi}{1+w} + \Phi \qquad w = \frac{p}{\rho}$$

EOM can be written as a conservation law!

$\Phi = cst$ except during phase transitions

$$\Downarrow$$

$$\boxed{\frac{\Phi}{1+w} = cst}$$

$$\Phi(t_H) \simeq \frac{\delta\rho}{\rho}(t_H)$$

time of Hubble radius crossing

4. Quantum Theory of Relativistic Pert.

.) Quantum generation of fluctuations during inflation :

idea : quantum vacuum fluctuations $\to$ classical perturbs.

semiclassical analysis $\to$ no fluctuations !

$$G_{\mu\nu} = \underbrace{\langle \psi | T_{\mu\nu} | \psi \rangle}_{\text{homogeneous}}$$

quantum analysis $\to$ fluctuations

Given : φ scalar quantum field

$|\psi\rangle$ vacuum state

Prescription : $\varphi_{cl}(\underline{x}, t) = \varphi_0(t) + \delta\varphi(\underline{x}, t)$

$$\delta T^{cl}_{\mu\nu} = \delta T_{\mu\nu}(\varphi_{cl})$$

$$\delta\varphi(\underline{x}, t) = \int d^3k \, \underset{\substack{\uparrow \\ \text{harmonic oscillator}}}{\tilde{\varphi}(\underline{k}, t)} \, e^{ikx}$$

$$|\delta\tilde{\varphi}(k)|^2 = \langle \psi | \, |\tilde{\varphi}(k)|^2 \, | \psi \rangle$$

$$\varphi_0(t)^2 = \langle \psi | \, \varphi(\underline{x})^2 | \psi \rangle$$

use vacuum state wave functional for $|\psi\rangle$

$$\psi[\tilde{\varphi}(k), t] = N \exp\left\{ -\frac{1}{2} \left(\frac{a}{2\pi}\right)^3 \int d^3k \; \omega(k, t) \, |\tilde{\varphi}(k)|^2 \right\}$$

$\downarrow$ $\qquad \omega(k, t_H(k)) = H$

a) $\delta\tilde{\varphi}(k, t) = (2\pi)^{3/2} a^{-3/2} \omega(k, t)^{-1/2} \sim (2\pi)^{3/2} k^{-3/2} H$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \llcorner \; t = t_i(k)$

6) $\varphi_0(t) \simeq (2\pi)^{-1} H^{3/2} t^{1/2}$ after UV & IR renormalization

Combination : $\delta\rho(k, t_i(k)) = \dot{\varphi}_0 \delta\dot{\varphi}_0 + V'(\varphi_0)\delta\varphi_0$

$$\sim k^{-3/2} H^4$$

$$\boxed{\frac{\delta M}{M}(k, t_i(k)) \sim O(1) \frac{H^4}{\rho} \sim 10^{-20}}$$

more precisely using slow rolling approx.

$$\frac{\delta M}{M}(k, t_i(k)) \sim \frac{V'(\varphi(t_i(k))\,\delta\varphi}{\rho} \sim \frac{H\dot{\varphi}_0\,\delta\varphi}{\rho}$$

$$\ddot{\varphi}_0 + 3H\dot{\varphi}_0 = -\frac{\partial V}{\partial \varphi}$$

Classical evolution :

$$\underbrace{\frac{\delta\rho}{\rho}(k, t_f(k))}_{\Phi_H(t_f)} = \underbrace{\frac{1+w(t_f)}{1+w(t_i)}}_{\frac{4/3}{\dot{\varphi}_0^2/\rho}} \underbrace{\frac{\delta\rho}{\rho}(k, t_i(k))}_{\Phi_H(t_i)}$$

$$\boxed{\frac{\delta M}{M}(k, t_f(k)) \sim O(1) \frac{H\,\delta\varphi}{\dot{\varphi}_0}}$$

6) Improved Analysis

Ref: R.B., H. Feldman & V. Mukhanov
Phys. Rep. 215, 203 (92)

- Consistent quantization of linearized metric & matter perturbations in a classical background
- Reduction to gauge invariant variables
- Action becomes action of a single scalar field (free) with time dependent mass
- Canonical quantization
- Mode functions obey classical equations discussed before
- time dependence of mass
  - $\Leftrightarrow$ particle production
  - $\Leftrightarrow$ growth of perturbations
- allows a unified treatment of generation and evolution of fluctuations in inflationary cosmology

3 steps:
1. Action for perturbations
2. Quantization
3. Computation of expectation values

Evaluation

$$V(\varphi) = \lambda \varphi^4$$

slow rolling $\quad 3H\dot{\varphi} = -V'(\varphi) = -4\lambda\varphi^3$

$$\varphi(t_i(k)) \sim m_{pl}$$

$$\frac{\delta M}{M}(k, t_i(k)) \sim \frac{H^2}{\dot{\varphi}} \sim \frac{H^3}{\lambda\varphi^3} \sim \frac{\lambda^{3/2}\varphi^6 m_{pl}^{-3}}{\lambda\varphi^3} \sim \lambda^{1/2} 10^2$$

$$\frac{\delta M}{M}(k, t_f(k)) \stackrel{!}{=} O(1)\, 10^{-4}$$

$$\Rightarrow \quad \lambda \leq 10^{-12}$$

fluctuation problem for inflationary cosmology

Conclusions

1. Inflation gives a theory of structure formation based on fundamental physics

   origin: quantum fluctuations during inflation

2. Relativistic analysis crucial for inflationary cosmology

   gravitational amplification of fluctuations on scales > Hubble radius

3. Predictions:

   * scale invariant spectrum of adiabatic, Gaussian, random phase fluctuations $P(k)$
   * scale invariant spectrum of CMB anisotropies $\frac{\delta T}{T}$
   * amplitudes of $P(k)$ & $\frac{\delta T}{T}$ related

4. Amplitude of $P(k)$?? Fluctuation problem

5. Confrontation with observations

   → need nonbaryonic dark matter

6. Cornerstone: theory of cosmological perturbations

Problems of Inflation

R. Brandenberger, Lecture 4

* reheating problem
* Fluctuation problem
* super Planck scale physics problem
* cosmological constant problem
* Singularity problem

reheating problem: new parametric resonance mechanism

many unresolved technical issues, e.g.

back reaction

effects of noise

fluctuation problem: $\frac{\delta T}{T} \sim 10^{-5} \Rightarrow \lambda < 10^{-12}$

new fine tuning problem from cosmol. to part. physics

missing ingredient

• supergravity [illegible]?

• new principle [illegible] fluctuations.

Super Planck scale physics problem:

modes originate at [illegible] below Planck scale

[illegible] → new effects.

6) Fluctuation problem

observ. $\quad \frac{\delta M}{M}(k, t_H(k)) \sim 10^{-4}$

inflation: $\quad \frac{\delta M}{M}(k, t_i(k)) \simeq \frac{V'\delta\phi}{\rho}$

$$\frac{\delta M}{M}(k, t_H(k)) \simeq \frac{1}{\left(1+\frac{p}{\rho}\right)_{t_i(k)}} \frac{\delta M}{M}(k, t_i(k))$$

theory of cosmol. perturbations

(see e.g. V. Mukhanov, H. Feldman & R.B. 1992)

$$\frac{\delta M}{M}(k, t_H(k)) \simeq \frac{\rho}{\dot\phi^2} \frac{V'\delta\phi}{\rho}$$

$$V(\phi) = \lambda \phi^4$$

$$\Rightarrow \frac{\delta M}{M}(k, t_H(k)) \sim 10^2 \lambda^{1/2}$$

$$\Rightarrow \lambda \lesssim 10^{-10} \quad !!$$

solution (?): asymptotic freedom during inflation

(R.B., V. Mukhanov & A. Sornborger 1993)

2. Problems of Inflationary Cosmology

cosmological constant
fluctuation problem
reheating

a) Cosmological constant

$$\min_{\varphi} V(\varphi) \rightarrow \Lambda_{eff} \qquad (p = -\rho)$$

observations: $\Lambda_{eff} \leq \rho_c$

$$\Rightarrow \frac{\Lambda_{eff}}{M_{pl}^4} \leq 10^{-122} \quad !$$

need: alternative realizations of inflation without scalar fields

i) higher derivative gravity inflation (A. Starobinsky 1980)

ii) condensates (R. Ball & A. Matheson 1992
L. Parker & Y. Zhang 1993
R.B. & A. Zhitnitsky 1996)

Inflation from Condensates

R.B. & A. Zhitnitsky (96)

Idea: condensate $\langle\phi\rangle$

$$\langle\phi\rangle = 0 \qquad t_i > t$$
$$\langle\phi\rangle \neq 0 \qquad t_i < t$$

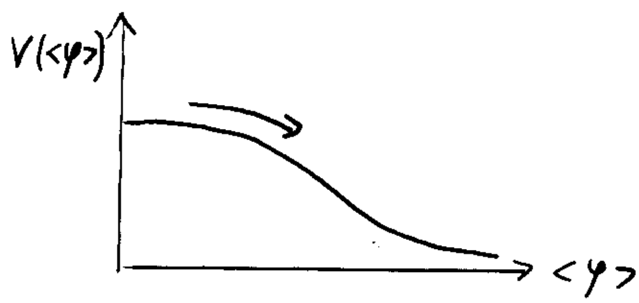
$$\langle H\rangle = \sum_{n=} (-1)^n n!\, a_n \langle\phi\rangle^n$$

Ass: Borel summable

$$\langle H\rangle = \int_0^\infty \frac{f(t)\,dt}{t\,(t m_{pl} + \langle\phi\rangle)}\, e^{-1/t} \equiv V_{eff}(\langle\phi\rangle)$$

$$a_n = \frac{1}{n!}\int_0^\infty dt\, f(t)\, t^{-n-2} e^{-1/t}$$

Ass: IR cutoff $\langle\phi\rangle \Rightarrow \langle\phi\rangle/\varepsilon$



$$\left.\begin{array}{l}\varphi(t)\\ \varepsilon(t)\end{array}\right\}$$ 2 field dynamics

Results: inflation (slow rolling o.k.)

graceful exit

$V_{eff}(\varphi) \to 0$

graceful exit & $V_{eff}(\varphi) \to 0$ linked !!

Some details :

a) single field analysis $(\varepsilon(t) = 1)$

slow rolling conditions : $V' m_{pl} \ll \sqrt{48\pi}\, V$ (1)

$$V'' \ll 24\pi \frac{V}{m_{pl}^2}$$

are satisfied $\forall \varphi$

$\Rightarrow$ graceful exit problem : • inflation never ends (although H decreases)

& cosmol. const. problem : • $V(\varphi)$ never reaches 0

b) double field analysis (ε dynamic)

slow rolling condition (1) becomes

$$\dot{\varepsilon}^2 m_{pl}^2 + \dot{\varphi}^2 \ll 2V(\varphi) \qquad (3)$$

$\varepsilon(t)$: IR cutoff in Planck units

ansatz : $\varepsilon(t) = \varepsilon(0)\,[1 - a(Ht)^p]$

$\varepsilon(0) = \frac{H(0)}{m_{pl}}$, $a \ll 1$, p integer

$a^{\frac{1}{p}} Ht < 1 \rightarrow$ (3) satisfied

$a^{\frac{1}{p}} Ht > 1 \rightarrow$ (3) breaks down

$\Rightarrow$ graceful exit & $V(\varphi) \rightarrow 0$

1. Motivation

a) Cosmology

Fluctuation problem

e.g. $V(\phi) = \lambda \phi^4$

$\frac{\delta T}{T} \sim 10^{-5} \Rightarrow \lambda \leq 10^{-12}$

Needed: New realization of inflation based on fundamental principle (instead of ad hoc small numbers)

b) General Relativity

Singularity problem

- big bang
- black hole

$\rightarrow$ internal inconsistency
practical problems (Cauchy)

Needed: nonsingular theory of gravity

Penrose - Hawking theorems :

$\left.\begin{array}{l}\text{Einstein action} \\ \text{+ weak energy condition}\end{array}\right\} \rightarrow$ geodesic incompleteness

Approaches :

- change matter ×
- change gravity

Justification :

high curvature ⇒ Einstein action breaks down

see e.g. :
- QFT in curved space-time
- perturbative QG
- unified theories

2. Idea

Classical analysis

Q : Is there a class of effective actions for gravity which give nonsingular theory predict inflation ?

This work :

1. construct effective action for gravity in which all solutions with sufficient symmetry are nonsingular.

2. Theory is a higher derivative gravity modification of Einstein theory.

3. It is obtained in analogy with a well established construction in SR.

Important : Theory is asymptotically free !

↑

New principle

P.S. Classical analysis self consistent

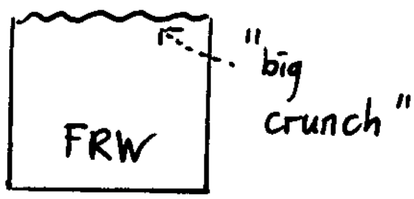
<u>Aim</u>: Construct a theory such that

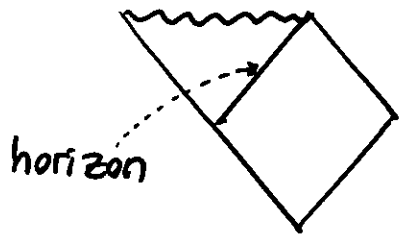
$$g_{\mu\nu} \to g_{\mu\nu}^{DS} \quad \text{as} \quad R \nearrow R_{pl}$$

<u>Consequences</u>:

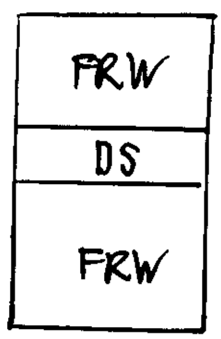
for a collapsing Universe — for a black hole

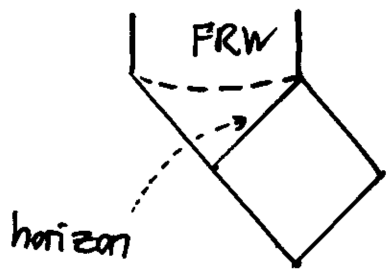
old theory





new theory





3. Analogy

V. Mukhanov & R.B. (92)

Point particle motion in SR

old theory: $S = \int dt \, \frac{1}{2} \dot{x}^2$

$\dot{x}$ unbounded

new theory: $\dot{x}$ bounded

$$S = \int dt \left[\frac{1}{2} \dot{x}^2 + \varphi \dot{x}^2 - V(\varphi) \right]$$

$$\frac{\delta}{\delta \varphi} : \quad \dot{x}^2 = \frac{\partial V}{\partial \varphi}$$

v bounded $\Rightarrow$ $V(\varphi) \sim \varphi$ for $|\varphi| \to \infty$

Newtonian limit $\Rightarrow$ $V(\varphi) \sim \varphi^2$ for $|\varphi| \to 0$

Ex: $V(\varphi) = \dfrac{2\varphi^2}{1 + 2\varphi}$

$$S = \frac{1}{2} \int dt \sqrt{1 - \dot{x}^2}$$

action for point particle motion in SR

4. Construction of Nonsingular Cosmologies

V. Mukhanov & R.B. (92)
R.B., V. Mukhanov & A. Sornborger (93)

Step 1: Find theory with limited R

old theory: $S = \int d^4x \sqrt{-g}\, R$

new theory: $S = \int d^4x \sqrt{-g} \left[R + \varphi R + V(\varphi) \right]$

$V(\varphi)$: as before

$\Rightarrow$ theory with limited R

but: this is insufficient !!

Requirements: 1. all curvature invariants bounded
2. geodesically complete

$\Downarrow$ Limiting Curvature Hypothesis (Markov & Mukhanov)

1. bound one curvature invariant (e.g. R) explicitly
2. ensure that when $R \nearrow R_{max}$

$$g_{\mu\nu} \rightarrow g_{\mu\nu}^{DS}$$

$\Uparrow$ metric of De Sitter space

$\llcorner$ maximally symmetric, nonsingular

Step 2: Implement Limiting Curvature Hypothesis

Idea: a) Find curvature invariant $I_2(g_{\mu\nu})$ such that

$$I_2 = 0 \iff g_{\mu\nu} = g_{\mu\nu}^{DS}$$

b) Make sure that as $R \to R_{max}$ we have

$I_2 \to 0$

(and thus: $g_{\mu\nu} \to g_{\mu\nu}^{DS}$)

using Lagrange multiplier construction

Difficulty: Find I_2 which works in all cases

$\Downarrow$

restrict attention to cases of special symmetry

$$I_2 = \left(4 R_{\mu\nu} R^{\mu\nu} - R^2 + C^2\right)^{1/2}$$

$I_2 = 0 \iff g_{\mu\nu} = g_{\mu\nu}^{DS}$ holds for:

homogeneous & isotropic metrics (i.e. cosmology)

spherically symmetric metrics (i.e. black holes)

homog. & anisotropic

Specific Model:

$$S = \int d^4x \sqrt{-g}\,[R + \varphi_1 R - (\varphi_2 + \sqrt{3}\varphi_1) I_2^{1/2} + V_1(\varphi_1) + V_2(\varphi_2)]$$

$$V_1(\varphi_1) = 12 H_0^2 \frac{\varphi_1^2}{1+\varphi_1}\left(1 - \frac{\ln(1+\varphi_1)}{1+\varphi_1}\right)$$

$$V_2(\varphi_2) = -\sqrt{12}\, H_0^2 \frac{\varphi_2^2}{1+\varphi_2^2}$$

applied to flat collapsing Universe

$$ds^2 = dt^2 - a(t)^2 (dx^2 + dy^2 + dz^2)$$

$\Downarrow$

Variational EOM: $\qquad H = \frac{\dot a}{a}$

(E1) $\qquad H^2 = \frac{1}{12} V_1' \qquad \Rightarrow H(\varphi_1)$

(E2) $\qquad \dot H = -\frac{1}{\sqrt{12}} V_2'$

(E3) $\qquad 3(1-2\varphi_1) H^2 + \frac{1}{2}(V_1 + V_2) = \sqrt{3}\, H (\dot\varphi_2 + 3H\varphi_2)$

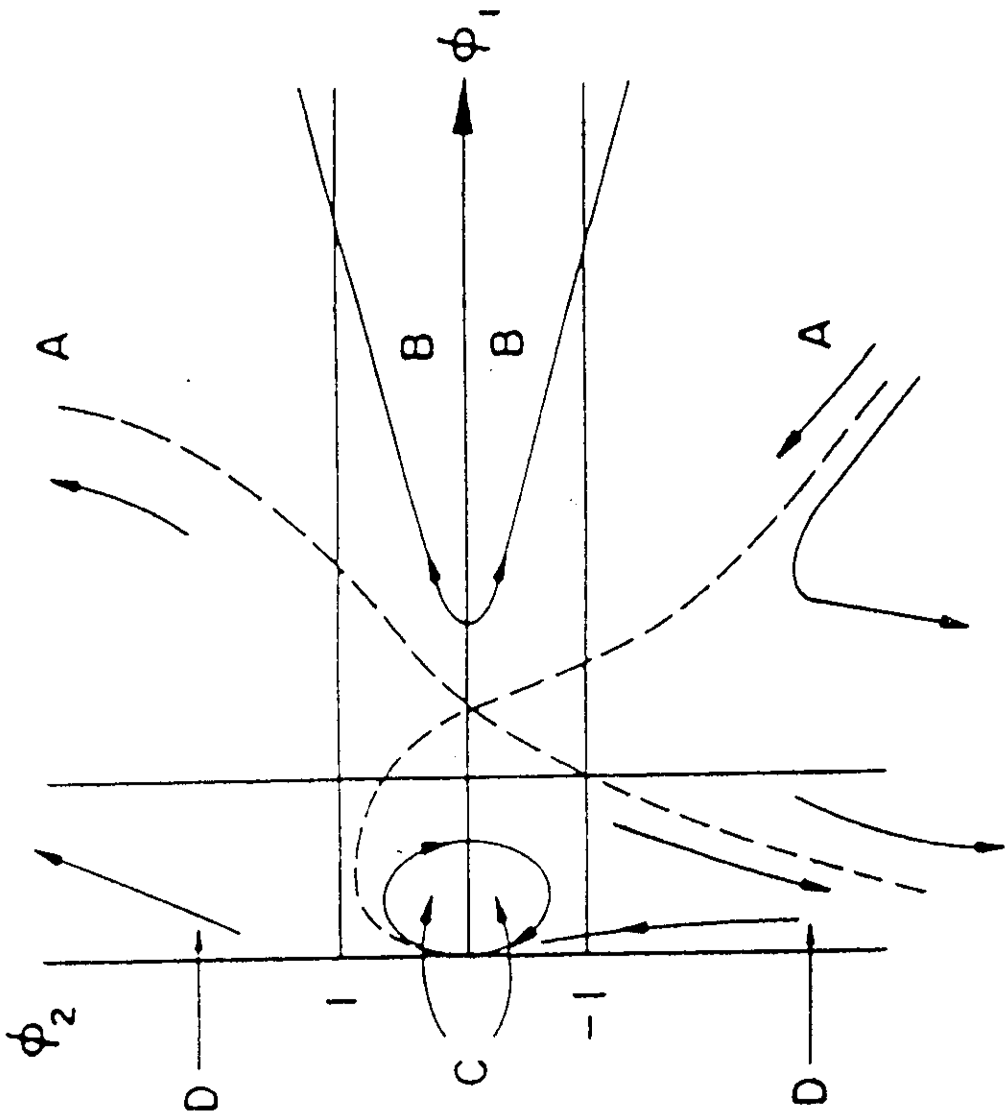
method of solution:

$(E1)^{\cdot}$ & $(E2) \Rightarrow \dot\varphi_1 = \dots$ $\qquad$ (E4)

$E3 \Rightarrow \dot\varphi_2 = \dots$ $\qquad$ (E5)

$(E4)$ & $(E5) \Rightarrow \frac{d\varphi_2}{d\varphi_1} = \dots$

methods:
- analytical phase plane analysis
- numerical



ϕ_1
ϕ_2
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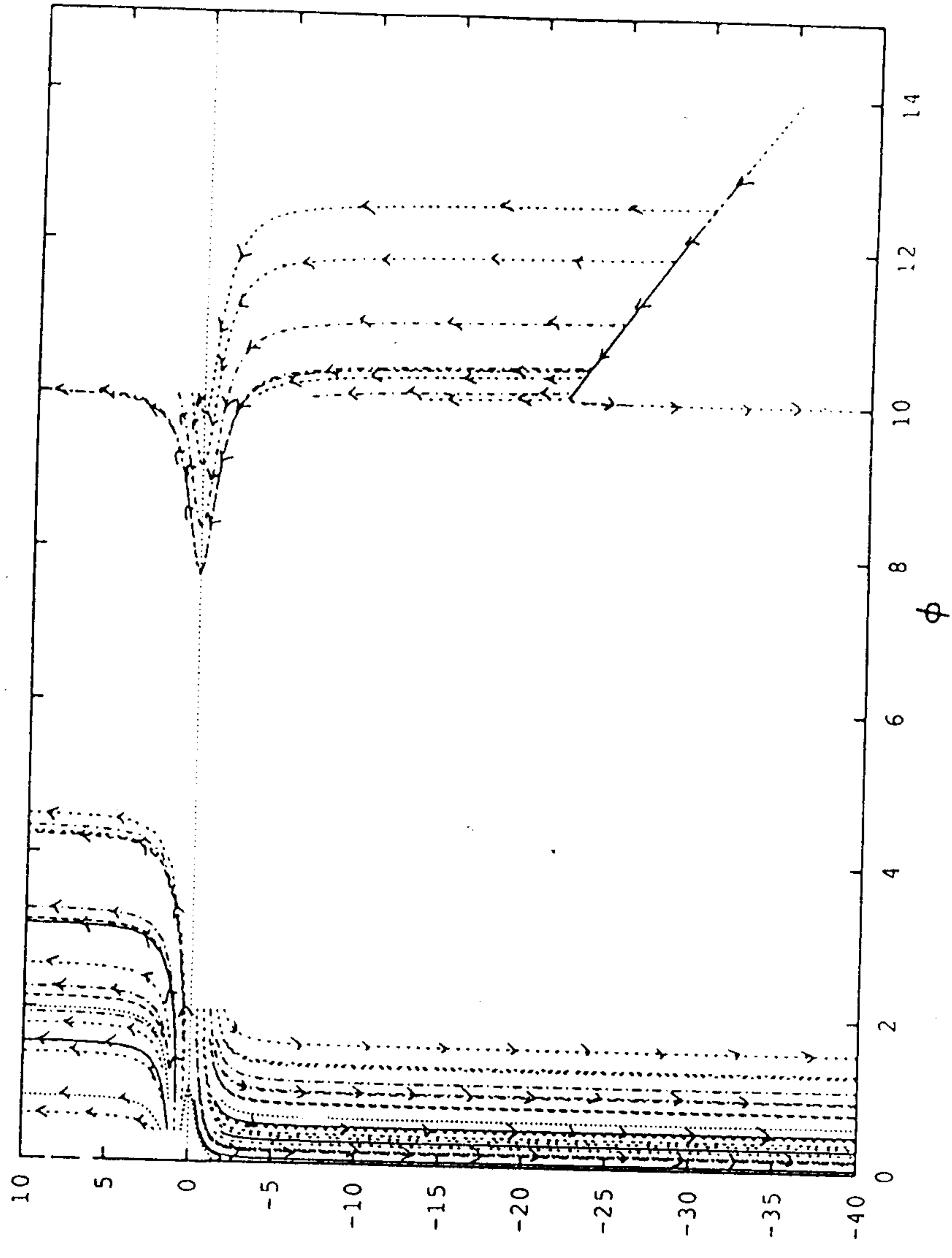
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14
ϕ

Conclusions

It is possible to have nonsingular cosmologies by modifying gravity

Approach 1: higher derivative gravity actions
based on limiting curvature construction

* nonsingular
* geodesically complete
* evolution starts with inflation
* asymptotic freedom during inflation
  $\stackrel{?}{\Rightarrow}$ small perturbations

Approach 2: superstring cosmology
duality → nonsingular

Conclusions

1. Inflationary Universe is an attractive scenerio
   - solves some problems of standard cosmology
   - leads to a predictive theory of structure formation

2. Important unsolved problems of principle remain
   - fluctuation problem
   - cosmological constant problem

3. No convincing realization of inflation
   - connection with modern particle physics & field theory ?

4. New theory of reheating
   - based on parametric resonance
   - leads to high T_R
   - new dark matter candidate