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Lecture I & II

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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ELECTROWEAK PHASE TRANSITION AND BARYOGENESIS

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Please note: These are preliminary notes intended for internal distribution only.

I. Electroweak phase transition

1. Effective potential : vacuum case

Action of Standard Model (as an example)

$$S = \int d^4x \left[(D_\mu \varphi)^+ D_\mu \varphi - V_0(\varphi) + \text{gauge terms} + \text{fermions} \right]$$

$$V_0(\varphi) = -\mu^2 \varphi^+ \varphi + \frac{\lambda}{2} (\varphi^+ \varphi)^2$$ - scalar potential

$\langle \varphi \rangle$: minimum of $V(\varphi)$

This is true at classical (= tree) level only.

Is $V_0(\varphi)$ the whole story?

No: loop corrections.

One loop: $V_{eff}(\varphi) = V_0(\varphi) + V_1(\varphi)$

$V_1(\varphi)$ = zero point energy (density) of all spieces of particles in background field φ.

$m_{gauge} = g\varphi$, $m_{fermion} = h\varphi$, etc

$$V_1(\varphi) = \left[\sum_{bosons} \sum_{states} \frac{1}{2} E - \sum_{fermions} \sum_{States} E \right] / \text{Volume}$$

i.e.

$$V_1(\varphi) = \sum_{\text{bosons}} \sum_{\text{spins}} \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \sqrt{\vec{k}^2 + m_i^2(\varphi)}$$

$$- \sum_{\text{fermions}} \sum_{\text{spins}} \int \frac{d^3k}{(2\pi)^3} \sqrt{\vec{k}^2 + m_f^2(\varphi)}$$

Sometimes non-trivial contribution (Coleman, E. Weinberg)

More systematic way

$$e^{-E(\varphi)(\text{time})} = \int d(\text{fields})\, e^{-S[\varphi,\ \text{other fields}]}\, \delta\Big[\int \varphi\, d^4x - (\text{Volume})(\text{time})$$

$\varphi \to \varphi + \delta\varphi$; expand integral in loops

One loop

$$e^{-E(\varphi)\cdot \text{time}} = e^{-V_0(\varphi)(\text{volume})(\text{time})} \times$$

$$\times \prod_{\text{bosons}} \left[\text{Det}\left(-\partial^2 + m_i^2(\varphi)\right)\right]^{-1/2}$$

$$\times \prod_{\text{fermions}} \left[\text{Det}\left(\not{\partial} + m_f(\varphi)\right)\right]$$

$$\Downarrow$$

$$V_1(\varphi) = \sum_{\text{bosons}} \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \ln(k^2 + m_i^2) - \sum_{\text{fermions}} \int \frac{d^4k}{(2\pi)^4} \ln(k^2 + m_f^2)$$

Actually, this is the same zero-point energy. (Integrate over k^0), up to divergent terms to be renormalized anyway.

$\langle \varphi \rangle$: minimum of $V_{eff}(\varphi)$!

2. Temperature-dependent effective potential = free energy density in background field φ

"One loop" : free energy of particles in plasma; their interactions neglected

$$V_T(\varphi) = V_{T=0}(\varphi)$$

$$+ \Big[\sum_{\text{bosons}} \sum_{\text{states}} T \ln\left(1 - e^{-\beta E}\right) \qquad \beta = \frac{1}{T}$$

$$+ \sum_{\text{fermions}} \sum_{\text{states}} T \ln\left(1 + e^{-\beta E}\right) \Big] / \text{Volume}$$

(textbooks on STATISTICAL MECHANICS)

$$= V_{T=0}(\varphi) + \sum T \int \frac{d^3k}{(2\pi)^3} \ln\left(1 \mp e^{-\frac{1}{T}\sqrt{\vec{k}^2 + m_i^2(\varphi)}}\right)$$

$\mp$: bosons / fermions

$\sum$: sum over spiecees and spins

Generally: RATHER complicated function of φ and T

- High temperature expansion: „$T \to \infty$"

Bosonic contribution to leading order:

$$T \int \frac{d^3k}{(2\pi)^3} \cdot \frac{1}{2} \ln\left[\vec{k}^2 + m_i^2(\varphi)\right]$$

NB: same thing as one-loop contribution to eff. potential <u>at zero temperature</u> in <u>three</u> dimensions.

Anyway, MSM with $\sin\theta_W = 0$:

$$V_T(\varphi) = \underbrace{-\frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4}_{V_0(\varphi)} + \frac{3T^2}{32} g^2 \varphi^2 - \frac{3g^3}{32\pi}\varphi^3 \cdot T$$

+ "unimportant" terms

also top Yukawa, with positive sign

$g = SU(2)_{weak}$ gauge coupling.

First order phase transition always.

Remember: this is one loop approximation!

Structure of 1-loop $V_T(\varphi)$ is generic, only coefficients change (MSSM, two-Higgs-doublet,..

Second critical point : φ^2 term in V_T disappears at $\frac{3}{16} g^2 T^2 = \mu^2 \Rightarrow \boxed{T = \sqrt{\frac{8}{3}} \frac{m_H}{g}}$

in fact $\frac{m_H}{\sqrt{g^2 + h_{top}^2}}$

At this point

$$V_T(\varphi) = \frac{\lambda}{4}\varphi^4 - \frac{3}{32\pi^2} g^3 T \varphi^3$$

Minimum at $\varphi = \frac{9}{32\pi} \frac{g^3}{\lambda} T = \frac{3}{2\pi} \frac{g^2}{\lambda} \mu$

Compare to zero temperature $\langle \varphi \rangle_{vac} = \frac{\mu}{\sqrt{\lambda}}$

At critical temperature $\varphi \ll \langle \varphi \rangle_{vac}$

As temperature drops down, φ ~~too~~ moves to $\langle \varphi \rangle_{vac}$.

All this is true only in 1-loop approximation. Whether this approximation is valid, remains to be understood!

4. More systematic approach:
euclidean ~~fit~~ field theory with finite time.

Static properties

$$\mathrm{Tr}\left[\hat{O}\, e^{-\frac{H}{T}}\right] \quad ; \quad \hat{O} \text{ depends only on } \vec{x}.$$

Example: free energy

$$e^{-F(T)/T} = \mathrm{Tr}\, e^{-\frac{H}{T}}$$

etc.

Note: 1) $e^{-H/T}$ = euclidean evolution operator for time $1/T \equiv \beta$.
imaginary time

~~2) $e^{-H/T}(\varphi(\beta); \varphi(0)) = \int_{\varphi}$~~

Its kernel in generalized coordinate representation (bosonic fields ϕ only for simplicity)

$$\left(e^{-H/T}\right)\left(\Phi_f(\vec{x}); \Phi_i(\vec{x})\right) = \int\limits_{\substack{\Phi(\tau=0)=\Phi_i \\ \Phi(\tau=\beta)=\Phi_f}} d\Phi\; e^{-\int_0^\beta \mathcal{L}(\tau)\, d\tau}$$

Trace : set $\Phi_i = \Phi_f = \phi$ and integrate over ϕ

$$\Downarrow$$

$$e^{-F(T)/T} = \int d\phi \, e^{-\int_0^\beta \mathcal{L}(\tau) d\tau}$$

periodic fields:
$\phi(\beta) = \phi(0)$

Similarly,

$$\mathrm{Tr}\left[\hat{O} e^{-H/T}\right] = \int_{\text{periodic } \phi} d\phi \; O(\phi) \, e^{-\int_0^\beta \mathcal{L}(\tau) d\tau}$$

Fermions: ANTI - periodic in euclidean time.

Now, $\phi(\vec{x},\tau) = \sum_{n=0,\pm1,\dots} e^{i \cdot 2\pi n T\tau} \phi_n(\vec{x})$ - bosons

$$\psi(\bar{x},\tau) = \sum_{n=0,\pm1,\dots} e^{i\, 2\pi(n+\frac{1}{2})T\cdot\tau} \psi_n(\vec{x}) \text{ - fermions}$$

$\phi_n(\bar{x})$, $\psi_n(\bar{x})$ = Matsubara modes

$$\Downarrow$$

$$\int d\phi \, e^{-\int_0^\beta d\tau \int d\bar{x} \, L(\bar{x},\tau \,|\, \phi)} = \int \prod_n d\phi_n \, e^{-\beta \int d\bar{x} \, \tilde{\mathcal{L}}(\bar{x};\phi_n)}$$

$\Rightarrow$ Dimensional reductio[n] to 3dim!

Most non-trivial : "static" fields

$$S = \int_0^\beta d\tau \int d^3x \quad (\partial_0 \phi)^2 + (\partial_i \phi)^2 + V(\varphi) + \ldots$$

$$\int d^3x \Big\{ \beta \left[\partial_i \phi_{n=0} \right]^2 + V_0(\phi_{n=0})$$
$$+ \sum_{n \neq 0} \beta \left\{ (\partial_i \phi_n)^2 + (2\pi T)^2 \cdot n^2 \phi_n^2 + \ldots \right\} \Big\}$$

heavy at high T

$\phi_{n=0}$: light at high T. NB Fermions always heavy!

Ignore heavy fields (integrate them out)

$\Downarrow$

get 3d theory! - at zero temperature

$$S_{3d} = \frac{1}{T} \int d^3x \left[(D_i \varphi)^\dagger (D_i \varphi) + \frac{1}{4} (F_{ij}^a)^2 + V(\varphi) \right]$$

(plus A_0 terms!)

Scale T out

$$S_{3d} = \int d^3x \left[|D\varphi|^2 + \frac{1}{4} F_{ij}^2 + V(\varphi) + A_0\text{-terms} \right]$$

with three-dim couplings:

gauge $g_3 = g\sqrt{T}$

Higgs : $\lambda_3 = \lambda T$

A_0 massive due to Debye screening! $m_{A_0} \sim gT$

5. Infrared problem.

Massless 3-dim gauge theories are "sick" in Pert. Theory in infrared!

Gauge coupling is dimensionfull:

n-th loop $\propto (g_3^2)^{N-1}$

If 3-d fields are massive ("broken phase") $\Rightarrow$ series in $\frac{g_3^2}{M^2}$ [if external momenta $\vec{k} \neq 0$ $\Downarrow$ $\frac{g_3^2}{\vec{k}^2}$]

If massless $\Rightarrow$ infrared divergencies.

Free energy in unbroken phase infrared divergent in four loops ($F \propto T^3$ by dimensions

$$F_{4\text{-loop}} \propto (g_3^2)^3 = g^6 T^3 \cdot \log \Lambda_{infrared}$$

Higher loops: linear, quadraic, ... divergencies in infrared.

Good behavior: $M^2 \gg g_3^2 = g^2 T$

Recall at critical temperature: one loop result

$$\varphi \propto \frac{g^3}{\lambda} T \Rightarrow M_W \propto \frac{g^4}{\lambda} T$$

Need $\frac{g^2}{\lambda} \gg 1$. Not valid at large λ, i.e., at large m_H!

Intermediate conclusions:

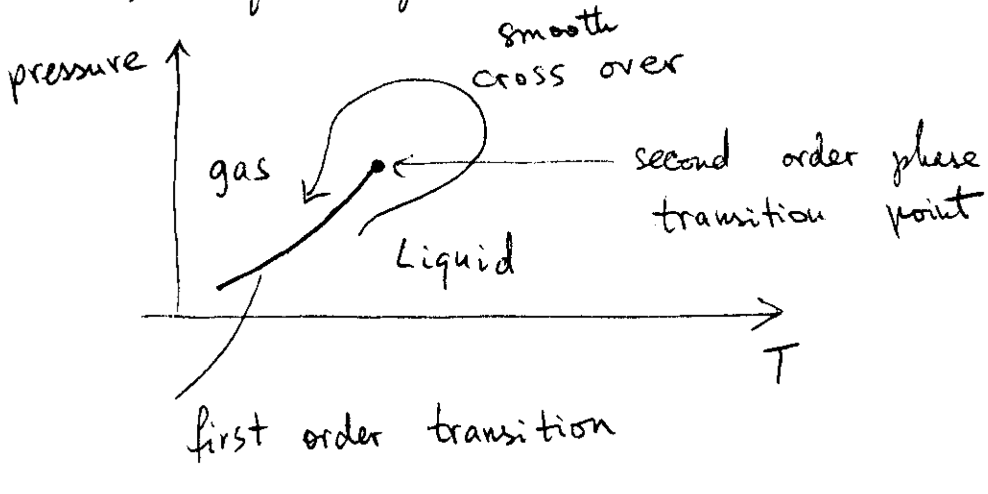
* Small m_H: first order phase transition; perturbation theory reliable, except nearby $\varphi = 0$, at critical temperature and at lower T

* Large m_H: perturbation theory unreliable at critical temperature. Order of phase transition ~~…~~ cannot be found perturbatively.

* Above critical temperature: perturbation theory unreliable at $\vec{k} \lesssim g^2 T$

<u>Complication</u>. No order parameter in MSM or MSSM ("local", gauge invariant)

cf: liquid - gas transition



Gauge "symmetry" is <u>not</u> quite a symmetry

Higgs mechanism is <u>not</u> symmetry breaking

6. What to do?

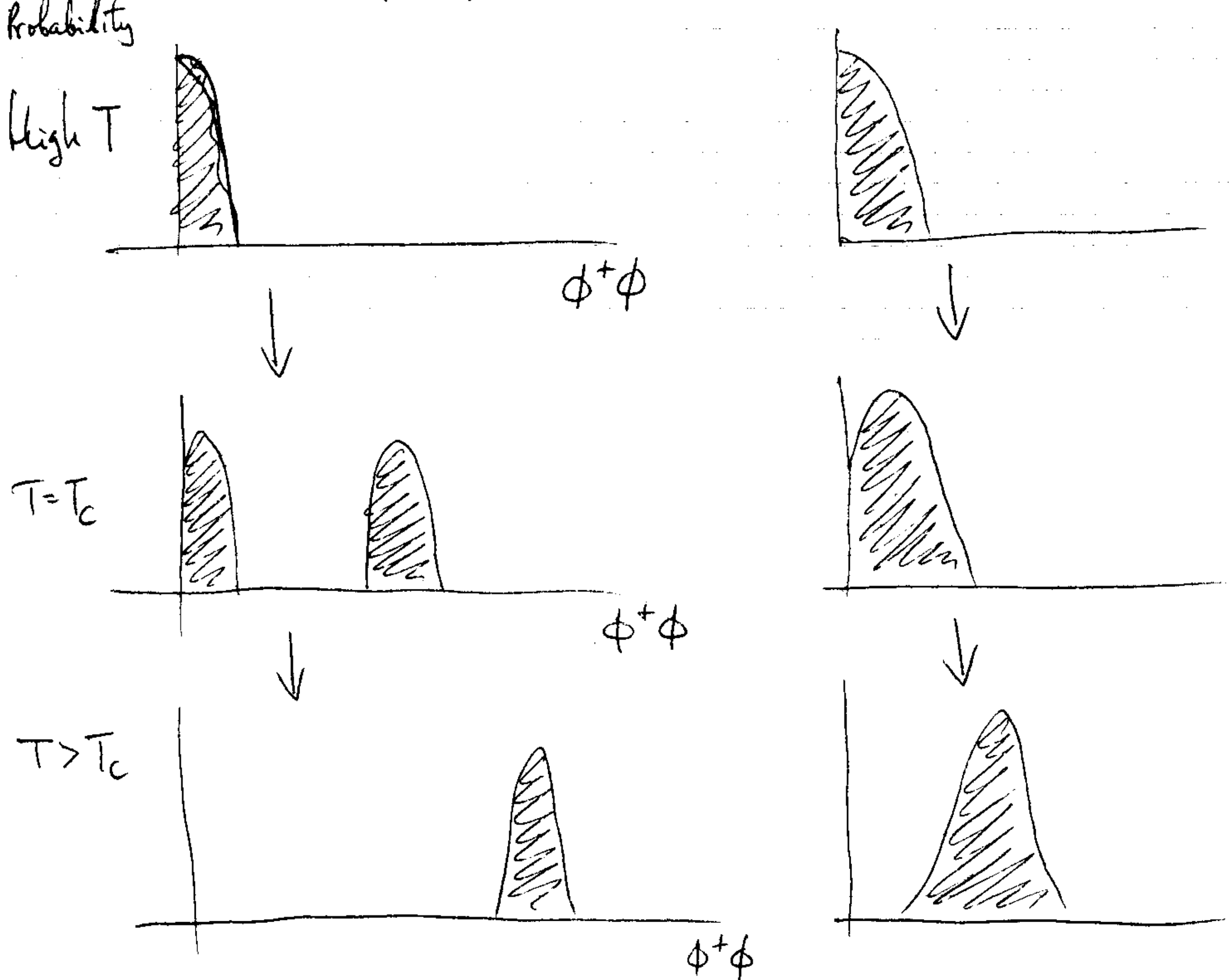
Non-perturbative studies : lattice

full 4d simulations

Ilgenfritz et. al.
Karsch et. al.

3d simulations
with analytic treatment
of heavy modes
Kajantie et. al.

Example: calculate probability of observing $\langle \phi^+\phi \rangle$ on lattice



First order transition:
coexistence of phases at $T=T_c$

Second order
or
cross over

Similarly: latent heat, surface tension.

Simplification in 3d simulations:
3d theory is the same (near phase transition) for all $SU(2) \times U(1)$ theories (MSM, MSSM, ...).
Because
Only one scalar field becomes massless at phase transition temperature

OUTCOME FOR MSM:

Phase transition is first order at $M_H \lesssim 60$ GeV
For realistic Higgs masses smooth crossover, second order or very weakly first order.

MSSM: Depends on $tg\beta$, etc., but mostly 2nd order, cross over or very weakly first order

Perturbation theory works quite well until at critical point $m_W(\varphi) \approx gT$

II. Electroweak baryon number non-conservation.

1. Simple example of non-conservation of fermion quantum numbers due to level crossing: massless fermions in (1+1) dimensions.

Free Dirac equation in (1+1) dimensions: fermions = two-component spinors

$$\psi = \begin{pmatrix} \chi \\ \eta \end{pmatrix}$$

Dirac equation

$$i\gamma^\mu \partial_\mu \psi = 0$$

$$\gamma^0 = \tau^1 \;, \quad \gamma^1 = i\tau^2$$

$$\Downarrow$$

$$i(\partial_0 - \partial_1)\chi = 0$$

$$i(\partial_0 + \partial_1)\eta = 0$$

$\chi = \chi(x_0 + x_1)$ moves left with speed of light

$\eta = \eta(x_0 - x_1)$ moves right with speed of light

Concentrate on right fermions η.

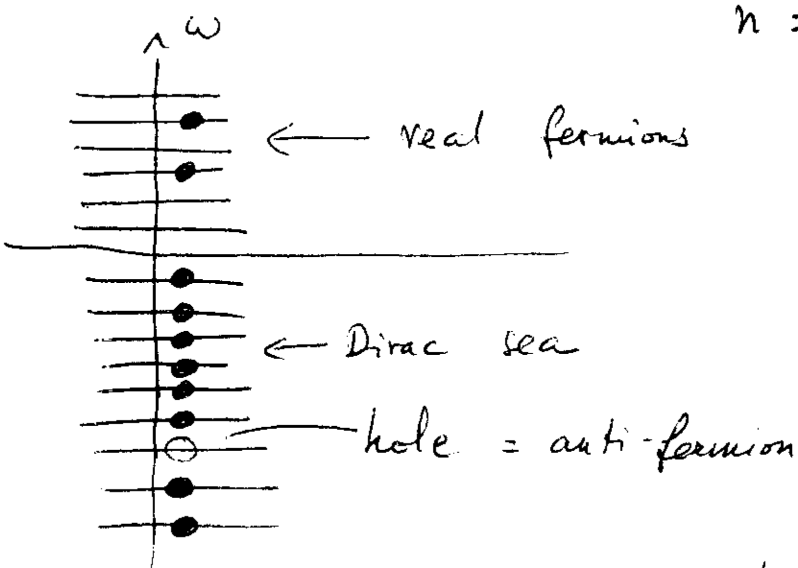
Levels of free right fermions

Space = circle of length L (with periodic boundary conditions)

Eigenfunctions of Dirac hamiltonian

$$\eta_\omega = e^{-i\omega x^0 + ikx^1} \quad ; \quad k = \omega = \frac{2\pi}{L} n$$

n = integer



Suppose now that fermions are electrically charged

Impose background electric field for a while (in positive = right direction)

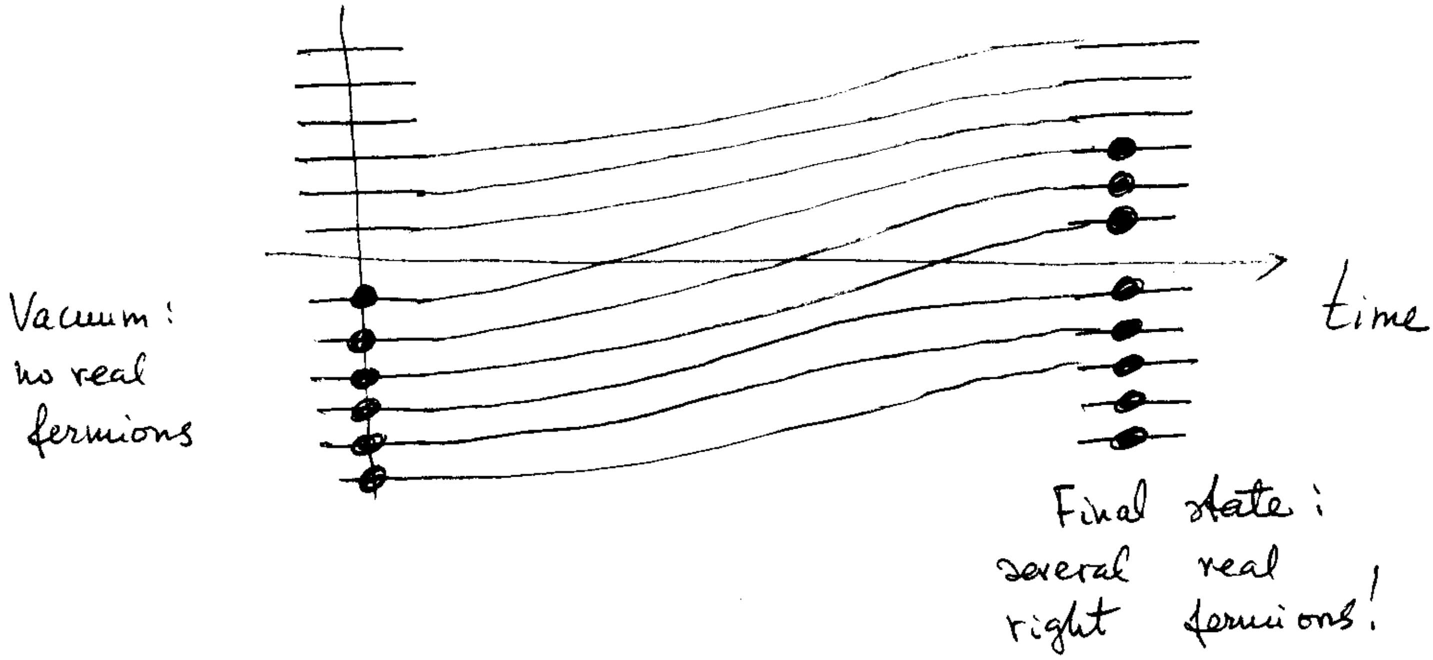
Fermions accelerate in this field

No ~~ferm~~ right fermions move left $\Rightarrow$ no decelerated right ~~electr~~ fermions.

Energy of all right fermions increases.

Negative energy levels become positive energy!

When electric field is switched off, fermions remain on their levels $\Longrightarrow$ real fermions are born from Dirac sea! Number of right fermions is changed!



How many?

Dirac equation in background electric field; $A_0 = 0$ gauge

$$i\partial_0 \eta = -i(\partial_1 - i\cancel{e}A_1)\eta$$

include into A_1

Adiabatically changing field $A_1(x,t)$ [for a moment]

$$\eta = e^{-i\int^t \omega(t)dt} \eta_{\omega(t)}(x^1)$$

—

$$-i\left[\partial_1 - ieA_1(x^1,t)\right]\eta_{\omega(t)} = \omega(t)\,\eta_{\omega(t)}$$

$$\Downarrow$$

$$\eta_{\omega(t)} = e^{\,i\,\omega(t)x^1 + i\int^{x^1} A_1(x^1,t)\,dx^1}$$

Periodicity : $\omega_n(t)\cdot L + \int_{-L/2}^{L/2} A_1(x^1,t)\,dx^1 = 2\pi n$

$$\Downarrow$$

$$\omega_n(t) = \frac{2\pi n}{L} - \frac{1}{L}\int_{-L/2}^{L/2} A_1(x^1,t)\,dx^1$$

Now, electric field $E = -\partial_0 A_1$

If E is turned on for a while, than

$A_1(t\to-\infty) = 0$; $A_1(t\to+\infty) \neq 0$

$$\Downarrow$$

$\omega_n(t\to-\infty) = \frac{2\pi n}{L}$ **but** $\omega_n(t\to+\infty) = \frac{2\pi}{L}(n+q)$

with

$$q = -\frac{1}{2\pi}\left[\int A_1\,dx^1\right]_{t\to+\infty}$$

Level # n becomes level # $(n+q)$

Number of levels crossing zero from below $= q$

$\Downarrow$

Change in number of right fermions

$$\Delta N_R = q$$

NB1 Adiabaticity is not essential:
in non-adiabatic case only pair creation of right fermions and anti-fermions can occur.

$$\Delta N_R = \begin{pmatrix} \text{\# of right} \\ \text{fermions} \end{pmatrix} - \begin{pmatrix} \text{\# of their} \\ \text{anti-fermions} \end{pmatrix}$$

changes by $\Delta N_R = q$

NB2 If there are also left fermions in theory, then

$$\Delta N_L = -q$$

Total $N_F = N_L + N_R$ is conserved

Chirality $Q^5 = N_L - N_R$ is not conserved.

Meaning of $q \equiv -\frac{1}{2\pi}\int_{-L/2}^{L/2} A_1\, dx^1$

topologically distinct vacua.

Vacuum (classical) = pure gauge

$$A_1 = \frac{1}{i}\omega\, \partial_1\, \omega^{-1} \qquad \underline{\underline{A_0 = 0}}\ ;\ \omega = \omega(x^1)$$

$$\omega = e^{i\alpha(x^1)} \in U(1)$$

ω = periodic in space $\Rightarrow$ $\alpha\left(+\frac{L}{2}\right) - \alpha\left(-\frac{L}{2}\right) = 2\pi q$

q = integer = topological # of vacuum =

= degree of mapping $\underset{\text{space}}{S^1} \longrightarrow \underset{\text{gauge group}}{U(1)}$

$$-\frac{1}{2\pi}\int_{-L/2}^{L/2} A_1\, dx^1 = -\frac{1}{2\pi i}\int_{-L/2}^{L/2} (-i\partial_1\alpha)\, dx^1$$

$$= \frac{1}{2\pi}\left[\alpha\left(\frac{L}{2}\right) - \alpha\left(-\frac{L}{2}\right)\right]$$

$$\Downarrow$$

$q = -\frac{1}{2\pi}\int_{-L/2}^{L/2} A_1\, dx^1$ = topological # of final vacuum (if $q = 0$ in initial vacuum).

So, in this simple model:

1) $\Delta N_R = q \neq 0$

2) ΔN_R does not depend on form of background gauge field, but only on its topology

3) Selection rule: ΔN_R is the same for all ~~fermion~~ right fermion spieces; $\Delta N_L = -q$ for all left spieces.

NB 1. For process considered, define

$$Q = -\frac{1}{4\pi}\int \varepsilon_{\mu\nu} F_{\mu\nu}\, d^2x = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dt \int_{-L/2}^{L/2} E\, dx^1$$

Since $E = -\partial_0 A_1$, this is full derivative integral $\Rightarrow$ $Q = -q$

$$\Delta N_R = -Q \ ; \ \Delta N_L = Q \quad \text{for each spieces.}$$

NB2 Euclidean zero mode

Consider euclidean Dirac (Weyl) equation ($A_0 = 0$ gauge)

$$-\partial_0 \eta = H_D(\tau)\eta$$

where levels of H_D move like above

In adiabatic case: solution

$$\eta(\tau, x^1) = e^{-\int \omega(\tau)\, d\tau}\, \eta_{\omega(\tau)}(x^1)$$

$\eta_{\omega(\tau)}$ = same eigenfunctions as above.

If $\omega(\tau \to -\infty) < 0$

$\omega(\tau \to +\infty) > 0$

Then $\eta(\tau, x^1)$ decays both as $\tau \to -\infty$ and $\tau \to +\infty$

$$\Downarrow$$

Euclidean Dirac operator has ~~eigenmode~~ eigenfunction with zero eigenvalue

$$\partial_0 \eta + H_D(\tau)\eta = 0 \Leftarrow$$ euclidean Dirac equation has normalizable solution in euclidean space-time

In fact, # of normalizable zero eigenvalues of euclidean Dirac operator, # $\mathrm{Ker}(\not{D})$, also depends only on topology; namely

$$\Delta N_R = \mathrm{Ker}(\not{D}_{right}) - \mathrm{Ker}(\not{D}^{+}_{right}) = -Q = q$$

This is another way to study fermion number non-conservation: count # of fermion zero modes in euclidean space-time (in fact, it is original way of $\to$ $\boxed{\text{'t Hooft}}$)

2. Vacuum structure of non-abelian gauge theories in (3+1) dimensions

Classical vacuum = pure gauge

$A_0 = 0$ gauge

$\vec{A} = \omega \vec{\partial}\, \omega^{-1}$ $\qquad\qquad \omega(\vec{x}) \in$ Gauge group

$\varphi = \omega \varphi_0$ (Higgs field)

Take space to be a large sphere S^3 (not essential!)

ω : mapping $S^3 \to$ Gauge group

$\cancel{\pi_3(S^3) \in G}$

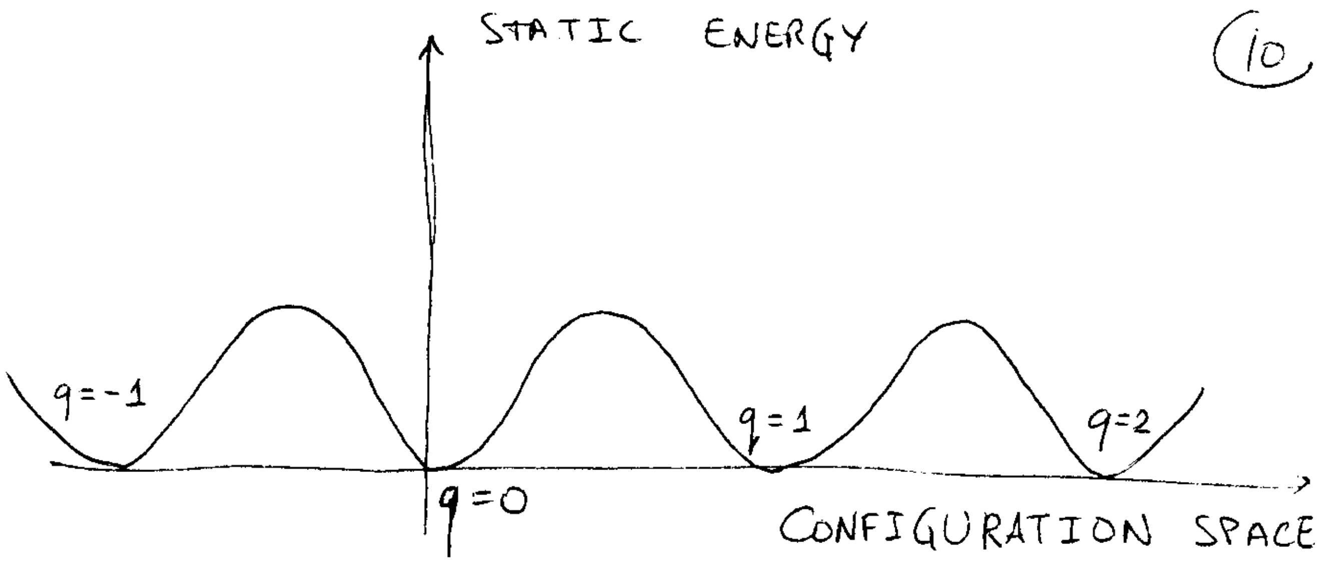
For simple gauge groups, $\pi_3(G) = \mathbb{Z}$

(say, $G = SU(2)$ - 3-sphere)

$\Downarrow$

Topologically distinct vacua, numbered by $q = 0, \pm 1, \pm 2,$

If one moves from one vacuum to another, one should pass configurations with non-zero static energy.



If a configuration $\vec{A}(\vec{x}, t)$ interpolates between different vacua, then

$$q_{final} - q_{initial} = Q = -\frac{1}{16\pi^2} \int d^4x \; Tr\left(F_{\mu\nu} \tilde{F}_{\mu\nu}\right)$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\lambda\rho} F_{\lambda\rho}$$

Exactly in the same manner as in $(1+1)d$

$$\Delta N_L = Q$$

$$\Delta N_R = -Q$$

for fundamental fermions in $SU(N)$.

For each spieces!

Fermion quantum numbers are not conserved iff one travels from ~~one~~ a vacuum to topologically distinct one.

<u>NB</u> Nothing like this for <u>abelian</u> theories in $(3+1)$ dimensions.

3. Electroweak theory: selection rules.

- Only left-handed fermions interact with SU(2) gauge fields.
- U(1) does not produce violation of fermion numbers.

$$\Delta N = q \qquad \underline{\text{For each spieces}} \text{ (doublet)}$$

In physical terms

$$\cancel{\Delta N_e = \Delta N_\mu = \Delta N_\tau = q} \qquad \Delta L_e = \Delta L_\mu = \Delta L_\tau = q$$

$$\cancel{\Delta B} \qquad \Delta B = \underbrace{3}_{\text{color}} \cdot \underbrace{3}_{\text{generation}} \cdot \underbrace{\tfrac{1}{3}}_{\text{B of each quark}} q = 3q$$

$$\Downarrow$$

1) B and L_i are not conserved separately

2) $(B-L)$; $\left(L_e - \frac{1}{3}B\right)$; $\left(L_\mu - \frac{1}{3}B\right)$; $\left(L_\tau - \frac{1}{3}B\right)$ are all conserved; $(L_e - L_\mu)$, etc also.

Out of four B, L_e, L_μ, L_τ <u>three</u> are conserved after non-perturbative effects are taking into account.

<u>NB</u> QCD with massless quarks: $U_A(1)$ is not conserved $\Rightarrow$ no <u>9th</u> light pseudoscalar ~~boson~~ meson.

4. Rates.

a) $T=0$: tunneling;

$$\Gamma \propto e^{-16\pi^2/g_2^2} \sim e^{-160} \qquad \text{- forget.}$$

g_2 = SU(2) gauge coupling

b) Intermediate temperatures : thermal jumps

- Most naive estimate : Boltzmann

$$\Gamma \propto e^{-\frac{\text{Hight of barrier}}{T}}$$

Saddle point between vacua = static unstable solution to field equations = sphaleron

Hight of barrier = $E_{sph} = \frac{2m_W}{\alpha_W} B\left(\frac{m_H}{m_W}\right) \sim 10\,\text{TeV}$ $\quad \alpha_W = \frac{g_2^2}{4\pi}$

Most naively $\quad \Gamma \propto e^{-\frac{E_{sph}}{T}}$

- Less naively : ~~$m_W = W$~~ $\langle\varphi\rangle = \langle\varphi\rangle(T) \Rightarrow$

$m_W = m_W(T) \Rightarrow E_{sph} = E_{sph}(T)$

Reasonable estimate!

$$\Gamma = (\text{Pre-exponent}) \cdot e^{-\frac{2m_W(T)}{\alpha_W T} B\left(\frac{m_H}{m_W}\right)}$$

Pre-exponent calculated, but not greatly important.

c) High temperatures

$\alpha_W T \gtrsim M_W(T) \Rightarrow$ no exponential suppression

Semiclassical analysis not applicable.

This applies, in particular, to "unbroken phase".

"Unbroken phase": coupling constant $= g_3 = g\sqrt{T}$

T always comes with $g^2 \simeq \alpha_W$

Sphaleron rate / unit time · unit volume

$$\Gamma \propto T^4 \Rightarrow \Gamma = \text{const}\,(\alpha_W T)^4$$

Shaposhnikov

Not necessarily true! (Arnold, Son, Yaffe), maybe, extra α_W.

Numerical simulations must decide.

Early Universe: compare rate to rate of evolution of Universe.

High T:

$$\underbrace{\text{const}}_{1\div 0.1\,?} \cdot \alpha_W^4 T \gtrsim \frac{T^2}{M_{pl} \cdot N_*} \Rightarrow T \lesssim 10^{12}\ \text{GeV}$$

(10^{10} GeV if α^5)

Baryon & lepton numbers definitely not conserved before EW phase transition.

After phase transition:

Sphaleron rate small if

$$\frac{2\, m_W(T)}{\alpha_W T}\, B\!\left(\frac{M_H}{M_W}\right) > \ln \frac{M_{Pl} N_*}{T}$$

OK if $\langle \varphi \rangle$ is large after phase transition $\Rightarrow$ strongly first order phase transition

MSM: $m_H < 50$ GeV - ruled out experimentally

MSSM: corner(s) of parameter space.

Two-Higgs-Doublet models: part of parameter space.

Generically, sphaleron processes are in equilibrium immediately after phase transition