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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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D-BRANES AND BLACK HOLES

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Please note: These are preliminary notes intended for internal distribution only.

D- BRANES AND BLACK HOLES

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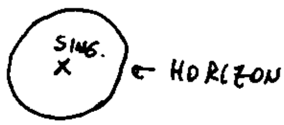
PART I

. Introduction

- General Relativity $\rightsquigarrow$ gravitational collapse
- Classically $\to$ Singularity covered by event horizon



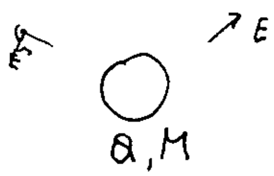
$\downarrow$ PHYSICS IS PREDICTIVE FOR THE OUTSIDE OBSERVER

- $\Delta A_H \geq 0$ Area of the horizon increases
- Quantum fields on the classical background $\longrightarrow$

  $\to$ Hawking Radiation

  $T_H \sim \frac{\hbar}{r_s}$ ← Schwarzschild radius. (in $d=4$ $r_s \propto G_N M$)

- Charged black holes



- No naked singularities $\Rightarrow$ $M \geq Q$

  If $M = Q$ $\to$ Extremal

  $\searrow$ Electric and gravitational forces cancel.

  (notice that an electron violates this bound!)

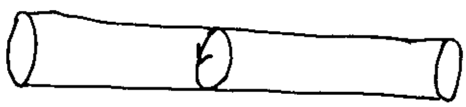
- Five dimensional case.

  - IIB string theory on T^5 $\longrightarrow$ 32 SUSYs

    S (• Fundamental closed strings
        • D-1-branes

    S G. D-3-branes

    S (• D-5-branes
        • NS-5-branes



$T^5 \to$ compact dimensions

$\to \mathbb{R}^4$: extended spatial dimensions

wrapping an extended object on the compact dimensions produces a localized charged particle in the 4 extended spatial dimensions.

e.g. • fundamental string winding in the direction 9

- $M = T_F\, 2\pi R_9$
- Charge $\to$ $B_{\mu 9}$ ~ gauge field

$$\int_{\Sigma_2} B = \int dt\, dy\, B_{\mu 9} = 2\pi R_9 \int dt\, B_{09} \sim q \int A_0$$

Number of different charges: Couple to:

5 - Kaluza Klein momenta $\longrightarrow$ $G_{\mu I}$ $\mu = 0,1,2,3,4$ $I = 5,6,7,8,9$

5 - Fundamental strings charges $B^{NS}_{\mu I}$

5 - D-1-branes $B^{RR}_{\mu I}$

10 D-3-brane winding modes $A^{RR}_{\mu IJK}$

1 D-5 brane A_μ defined by $\underbrace{H^{RR}_{\mu\nu\delta} \sim (dA)_{\varepsilon\delta}\, \varepsilon^{\varepsilon\delta}}_{\text{all 1+4 dim. indices}}$

1 NS-5-brane $\tilde{A}_\mu$ defined by $H^{NS} \sim {}^{*}_{5} d\tilde{A}$

---

total: 27 charges.

U-duality $(E_{6,6}(\mathbb{Z}))$ exchanges all the charges.

→ fundamental 27 of $E_{6,6}$

(Since $E_{6,6}$ is non compact → the charges are real)

- All these branes break (individually) $\frac{1}{2}$ of SUSY

e.g. A D-5 brane preserves the SUSY

$$\varepsilon_L = \pm \Gamma^0\Gamma^5\Gamma^6\Gamma^7\Gamma^8\Gamma^9 \varepsilon_R$$

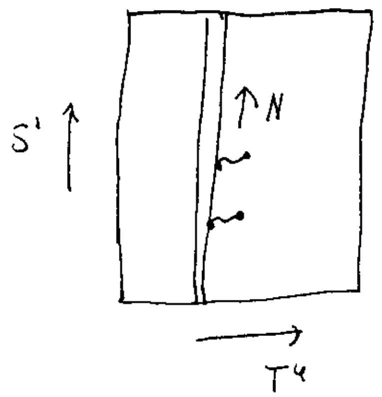
$\uparrow$

Sign of the charge

$\varepsilon_L, \varepsilon_R$ → 16-component Majorana-Weyl spinors in $d = 1+9$

- We want to study a configuration of D-5-branes, D-1 branes and momentum

Counting. The idea:

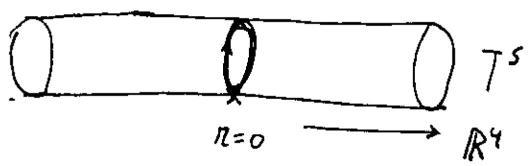


Q_5 - D-5-branes

Q_1 - D-1-branes

$P = \frac{N}{R}$ = momentum

each break ½ of SUSY → preserve $\frac{1}{8}$ of SUSY



$4Q_1Q_5 = N_B = N_F$ = number of bosonic and fermionic massless open string modes.

Momentum: $N = \sum_{\{\text{open strings}\}} n_i = \sum_{n=1}^{\infty} n N_n$; N_n = number of open strings with momentum n.

Entropy → entropy of a 1+1 dimensional gas of massless particles.

$$S = 2\pi \sqrt{\frac{(N_B + \frac{1}{2}N_F)}{6} \cdot E \cdot R}$$ = formula for the entropy for a 1+1 dim gas of massless, leftmoving particles with Energy $E = P = \frac{1}{R}$ living on a periodic circle of length $L = 2\pi R$.

$$S = 2\pi \sqrt{Q_1 Q_5 N}$$

$\boxed{\alpha' = 1}$

(Polchinski)

open strings

$\downarrow$ low energies

massless states $\to$ transverse fluctuations of the branes
$\searrow$ gauge fields on the D-brane

$\downarrow$

Q_5 - D-5 branes.
along $\hat{5}, \hat{6}, \hat{7}, \hat{8}, \hat{9}$

$U(Q_5)$ gauge theory

$\uparrow$
dimensional reduction of $d = 1+9$ Super Y.M.

$$S^{10-d}_{YM} = \frac{1}{g^2_{YM}} \int \frac{1}{4} \mathrm{Tr}\, F_{NM} F^{NM} + \text{fermions}$$

go to $1+5-d$

$$A_M \longrightarrow \begin{cases} A_\alpha & \alpha = 0,5,6,7,8,9 \\ A_I & I = 1,2,3,4 \end{cases}$$

$M = 0, \dots, 9$

$$S^{1+5}_{YM} = \frac{1}{g^2_{YM}} \int \frac{1}{4} \mathrm{Tr}\, F_{\alpha\beta} F^{\alpha\beta} + \frac{1}{2} \mathrm{tr}\, D_\alpha A_I D^\alpha A^I + \frac{1}{4} \mathrm{tr}\, [A_I A_J]^2 + \text{ferm}$$

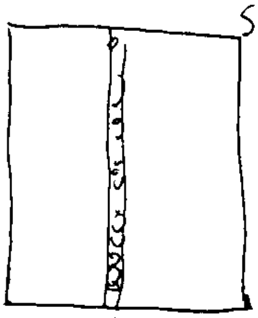
$g^2_{YM} = g$ = string coupling. (closed string coupling)

This theory contains string-like solitons constructed as follows.

- Consider an instanton in the directions 5, 6, 7, 8

$$A_{5 \dots 8}(x^5, \dots, x^8) \quad , \quad A_9 = 0$$

$\uparrow$
Usual instanton solution of 4 (Euclidean!) dimensional Y.M.



- localized instanton.

- Energy: $E_{inst} = \frac{N_{inst} \cdot R_9}{g^2_{YM}} = \frac{N_{inst} R_9}{g}$

- Break $\frac{1}{2}$ of worldvolume SUSY's $\rightarrow \varepsilon = \Gamma^{5678} \varepsilon$

- This description is reasonable if the energy of the instanton is much smaller than the energy of the 5-brane:

$$E_{inst} << M_5 = \frac{Q_5 R_9 V_4}{g} \quad ; \quad V_4 = R_5 R_6 R_7 R_8$$

and $V_4 >> 1$

since we are using the low energy excitations (massless fields).

Can an observer far away from the D-5-brane see if it has instantons?

$\rightarrow$ YES (Douglas)

because there is a coupling

$$\int \underbrace{d^{1+5}x}_{\text{worldvolume of the 5-brane}} \; \underbrace{B^{RR}}_{\text{bulk RR-2-form potential}} \wedge \underbrace{\mathrm{tr}(F \wedge F)}_{\text{gauge fields on the D-brane}} \longrightarrow N_{inst} \int d^{1+1}x \; B^{RR}_{09}$$

Instantons $\longrightarrow$ D-1-brane charge.

$Q_1 = N_{inst}$ (the precise coefficient works out this way).

. The whole configuration preserves $\frac{1}{4}$ of the SUSY.

. We need to add one more charge

. Note that the system is boost invariant along the direction $\hat{9}$

$\Downarrow$

excitations moving along $\hat{9}$ are massless

. Put momentum $P = \frac{N}{R_9}$ along $\hat{9}$

. We need to understand what modes carry the momentum.

. The instanton configuration is characterized by some moduli ξ^a

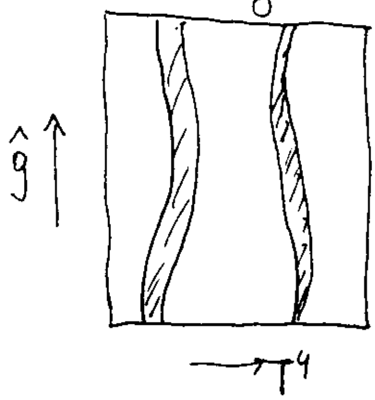
$A_m(x^5, .., x^8, \xi^a)$

$\xi^a \rightsquigarrow$ span a space so called the "moduli space of instantons" $\equiv \mathcal{M}$

The number of parameters is $4Q_1Q_5$, this is a mathematical result for Q_1 instantons in $SU(Q_5)$

(the formulas here and in what follows are true to leading order in Q_1, Q_5, for large $Q_1, Q_5 \gg 1$).

. Moduli can oscilate $\simeq \xi^a(t, x^9) \simeq$ oscillations of the instanton configuration oscillating instantons



$$S = \int dx^{1+1} \int d^4x \, \mathrm{tr} F^2 = \int dx^{1+1} G_{ab}(\xi) \partial^\alpha \xi^a \partial_\alpha \xi^b$$

• Only left movers : $\xi^a(t+x^5) \longrightarrow \begin{cases} L_0 = N \\ \bar{L}_0 = 0 \end{cases}$

Entropy $\rightarrow d(N) = e^S = e^{2\pi\sqrt{\frac{c}{6} L_0}}$

$$C = N_B + \tfrac{1}{2} N_F = 6 Q_1 Q_5$$

• $\boxed{S = 2\pi\sqrt{Q_1 Q_5 N}}$

• Number of BPS states with these quantum numbers → same for all values of g, R, V, etc.

• However we implicitly assumed that $N \gg Q_1 Q_5$

• Effective "temperature" $T_L^{-1} \sim \frac{\partial S}{\partial E} \Rightarrow T_L \sim \frac{1}{R}\sqrt{\frac{N}{Q_1 Q_5}}$ ← mean energy of particles.

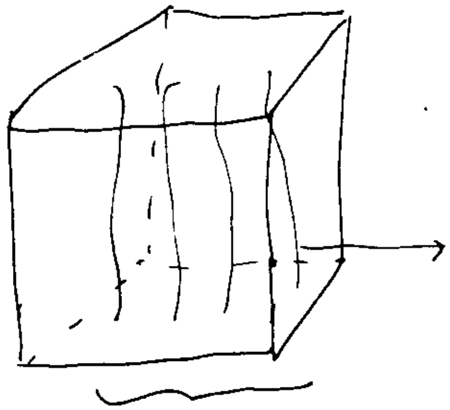
↑ mean energy

we want $T_L \gg \frac{1}{R}$ to apply the continuum formula $\Rightarrow N \gg Q_1 Q_5$

• For lower T → Understand better the structure of the moduli space.

• $\mathcal{M} = (T^4)^{Q_1 Q_5} / S(Q_1 Q_5)$ topologically

• "Fractional" instantons , $Q_1 \longrightarrow Q_1 \cdot Q_5$, each with $N^{inst}_{frac} \sim 1/Q_5$



effective strings are "fractional"

$Q_1 Q_5$ times .

$\Rightarrow$ can connect to form a long string $\Rightarrow$ state with maximum entropy

$$E_{gap} = \frac{1}{R\, Q_1 Q_5} \ll T_L \sim \frac{1}{R}\sqrt{\frac{N}{Q_1 Q_5}}$$

Same as estimated from semiclassical arguments .....(we will see)

References: Strominger & Vafa : hep-th/9601029
J. Maldacena hep-th/9608235