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BLACK HOLES AND D-BRANES

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Black holes and D-branes

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Black holes have an entropy proportional to the area of the horizon. The microscopic degrees of freedom that give rise to the entropy are not visible in the classical theory. String theory, a quantum theory of gravity, provides a microscopic quantum description of the thermodynamic properties of some extremal and near extremal charged black holes. The description uses properties of some string theory solitons called D-branes. In this description, Hawking radiation emerges as a simple perturbative process. The low energy dynamics of particle absorption and emission agrees in detail with the semiclassical analysis in the thermodynamic limit.

1. Introduction

Under a wide variety of conditions General Relativity predicts that singularities will develop [1]. The cosmic censorship hypothesis states that under generic physical situations leading to gravitational collapse the resulting singularities will be covered by an event horizon [2]. This conjecture has not been yet proved but there exists great evidence that it is correct [3]. The area of the horizon is an interesting quantity since it always increases upon classical evolution [4], this looks very similar to the second law of thermodynamics. The analogy became more precise when Hawking showed [5] that quantum mechanics implies that black holes emit thermal radiation with a temperature obeying the first law of thermodynamics $dM = T_H dS$, where the entropy is $S = \frac{A_H}{4G_N \hbar}$ [6], M is the black hole mass and A_H is the horizon area (from now on we set $\hbar = 1$ but keep $G_N \neq 1$). The area increase law becomes the second law of thermodynamics. If one includes Hawking radiation, the black hole mass decreases and so does the area of the horizon, but the total entropy, defined as $S = A_H/4G_N + S_{rad}$, increases. It has always been a puzzle what the degrees of freedom that give rise to this entropy are. It seems clear that some quantum gravity will be necessary to

describe the microstates. String theory [7] is a theory of quantum gravity so one would naturally expect that it should give an answer to this question. But string theory is defined perturbatively and black holes involve strong interactions due to their large mass. Only when some non-perturbative tools became available [8,9] could precise calculations be made [10]. There are, however, rough counting arguments that produce the right scaling for the entropy using just string perturbation theory [11–13]. Our focus will be to explain the calculations that produce precise results, which are valid presently only for a subset of all possible black hole configurations. For other reviews on this subject see [14].

We are going to be treating charged black holes. The cosmic censorship hypothesis gives a bound for the mass of a black hole in terms of its charge $M \geq Q$ (in appropriate units). The black hole with $M = Q$ is called extremal. We will study extremal and near extremal ($M - Q \ll Q$) black holes and we focus on black holes in five space-time dimensions. In four spacetime dimensions the discussion is similar. In section 2 we write down the classical supergravity solutions we are going to describe. In section 3 we present the D-brane description of extremal black holes in five dimensions. In section 4 we study near extremal black holes, their entropy and their decay rates and we compare them to the semiclassical results. In section 5 we compute the greybody factors for emission of massless scalars.

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2. Classical solutions

We consider type IIB string theory compactified on T^5 . The low energy theory is the maximally supersymmetric supergravity theory in five dimensions. It has 32 supersymmetry generators and it is the dimensional reduction of ten dimensional type IIB supergravity. The theory contains 27 abelian gauge fields. The full string theory contains charged objects that couple to each of these gauge fields. These objects are: 5 Kaluza Klein momenta, 5 string winding directions, 5 D-string winding directions, 10 possible D-3-brane wrapping modes, a solitonic NS fivebrane, a D-fivebrane. All these charges are interchanged by U-duality transformations [15] and they are all quantized. Therefore, we measure charges in integer multiples of the elementary units.

We consider a black hole solution which has Q_5 D-fivebranes wrapped on T^5 , Q_1 D-1-branes wrapped on an S_1 (we choose it as the direction $\hat{9}$), and momentum $P = N/R$ also along the direction of the D-string (direction $\hat{9}$). When we mention D-branes in the context of classical solutions we only refer to the charge that the solution is carrying, there are no explicit D-branes in the sense of [9,8] anywhere in spacetime. We choose this set of charges because the string theory description is simpler. Other black holes are related to this by U-duality transformations. Further details about supergravity solutions can be found in R. Khuri's contribution to this volume [16].

We start by presenting the ten dimensional solution [17]

$$e^{-2(\phi-\phi_\infty)} = \left(1 + \frac{r_0^2 \sinh^2 \gamma}{r^2}\right) \left(1 + \frac{r_0^2 \sinh^2 \alpha}{r^2}\right)^{-1}, \quad (1)$$

$$\begin{aligned} ds_{str}^2 = & \left(1 + \frac{r_0^2 \sinh^2 \alpha}{r^2}\right)^{-1/2} \left(1 + \frac{r_0^2 \sinh^2 \gamma}{r^2}\right)^{-1/2} \times \\ & \left[-dt^2 + dx_9^2 + \frac{r_0^2}{r^2} (\cosh \sigma dt + \sinh \sigma dx_9)^2\right. \\ & \left.+ \left(1 + \frac{r_0^2 \sinh^2 \alpha}{r^2}\right) (dx_5^2 + \dots + dx_8^2)\right] \\ & + \left(1 + \frac{r_0^2 \sinh^2 \alpha}{r^2}\right)^{1/2} \left(1 + \frac{r_0^2 \sinh^2 \gamma}{r^2}\right)^{1/2} \times \\ & \left[\left(1 - \frac{r_0^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega_3^2\right]. \end{aligned} \quad (2)$$

Also some components of the Ramond-Ramond three-form field strength $H'_{\mu\nu\rho}$ are nonzero since

the solution carries D1-brane and D5-brane charge. This solution is parameterized by the four independent quantities $\alpha, \gamma, \sigma, r_0$. There are two extra parameters which enter through the charge quantization conditions which are the radius of the 9^{th} dimension R_9 and the product of the radii in the other four compact directions $V \equiv R_5 R_6 R_7 R_8$. The three charges are

$$\begin{aligned} Q_1 &= \frac{V}{4\pi^2 g} \int e^{2\phi_6} * H' = \frac{V r_0^2}{2g} \sinh 2\alpha, \\ Q_5 &= \frac{1}{4\pi^2 g} \int H' = \frac{r_0^2}{2g} \sinh 2\gamma, \\ N &= \frac{R^2 V r_0^2}{2g^2} \sinh 2\sigma, \end{aligned} \quad (3)$$

where $*$ is the Hodge dual in the six dimensions $x^0, \dots, x^5$. For simplicity we set from now on $\alpha' = 1$. All charges are normalized to be integers. We have chosen a convention such that $g \rightarrow 1/g$ under S-duality. Further explanations on the charge quantization conditions can be found in [8,18].

Reducing (2) to five dimensions using the standard dimensional reduction procedure [19], the solution takes the simple and symmetric form:

$$ds_5^2 = -\lambda^{-2/3} h dt^2 + \lambda^{1/3} \left(\frac{dr^2}{h} + r^2 d\Omega_3^2 \right), \quad (4)$$

where

$$\begin{aligned} \lambda &= \left(1 + \frac{r_0^2}{r^2}\right) \left(1 + \frac{r_0^2}{r^2}\right) \left(1 + \frac{r_0^2}{r^2}\right), \\ h &= 1 - \frac{r_0^2}{r^2} \end{aligned} \quad (5)$$

$$r_1^2 = r_0^2 \sinh^2 \alpha, \quad r_5^2 = r_0^2 \sinh^2 \gamma, \quad r_n^2 = r_0^2 \sinh^2 \sigma \quad (6)$$

This is just the five-dimensional Schwarzschild metric with the time and space components rescaled by different powers of λ . The event horizon is at $r = r_0$. Several thermodynamic quantities can be associated to this solution. They can be computed in either the ten dimensional or five dimensional metrics and yield the same answer. For example, the ADM energy is

$$M = \frac{RV r_0^2}{2g^2} (\cosh 2\alpha + \cosh 2\gamma + \cosh 2\sigma). \quad (7)$$

The Bekenstein-Hawking entropy is

$$\begin{aligned} S &= \frac{A_{H_3}}{4G_N^3} = \frac{A_3}{4G_N^3} \\ &= \frac{2\pi R V r_0^3}{g^2} \cosh \alpha \cosh \gamma \cosh \sigma. \end{aligned} \quad (8)$$

where A is the area of the horizon and we have used that the Newton constant is $G_N^{10} = 8\pi^6 g^2$. The Hawking temperature is

$$T = \frac{1}{2\pi r_0 \cosh \alpha \cosh \gamma \cosh \sigma}. \quad (9)$$

The extremal limit corresponds to $r_0 \rightarrow 0$, $\alpha, \gamma, \sigma \rightarrow \infty$ keeping the charges (3) finite. In that limit the entropy (8) becomes

$$S = 2\pi \sqrt{NQ_1 Q_5} \quad (10)$$

and the temperature vanishes. Note that the extremal entropy is independent of any continuous parameters [20,21]. The extremal black hole backgrounds preserve some space time supersymmetries and therefore they are BPS states. In this case the cosmic censorship bound becomes identical to the supersymmetry BPS bound [22].

The near extremal limit corresponds to r_0 small and α, γ, σ large. The relative values of α, γ, σ are related to the total contribution of the different charges to the mass (7). The near extremal black holes that are easiest to analyze in terms of D-branes are those where $\sigma \ll \alpha, \gamma$, or $r_0, r_n \ll r_1, r_5$, which means that the contribution to the mass (7) due to the D-branes is much bigger than the contribution due to the momentum excitations. This limit is called "dilute gas" [23] [24]. In this limit, the mass and entropy of the near extremal black hole become

$$M = \frac{Q_5 R V}{g} + \frac{Q_1 R}{g} + \frac{R V r_0^2}{2g^2} \cosh 2\sigma, \quad (11)$$

$$S = 2\pi \frac{R \sqrt{V} r_0}{g} \sqrt{Q_1 Q_5} \cosh \sigma. \quad (12)$$

Note that the five dimensional Reissner-Nordström solution corresponds to the case of $\alpha = \gamma = \sigma$ which is not included in the dilute gas limit.

All these black hole solutions will be well defined if curvatures are everywhere much smaller than α' , since otherwise α' corrections to the low energy action become important. This generically implies that the sizes of the black hole should obey $r_1, r_5, r_n \gg 1$ (remember that $\alpha' = 1$). The precise condition will be a little more complicated if some scale is very different from the others. If

the compactification sizes are of the order of α' this implies that

$$gQ_1 \gg 1, \quad gQ_5 \gg 1, \quad g^2 N \gg 1 \quad (13)$$

Note that we cannot enforce (13) by setting $Q = 1$ and making $g \gg 1$ since the condition (13) was derived in weakly coupled string theory, if $g \gg 1$ the corresponding bound comes from light D-strings and implies again that $Q \gg 1$. Therefore black holes always involve large values of the charges.

3. D-brane description of extremal 5d black holes

We continue with type IIB string theory on $T^5 = T^4 \times S^1$. We consider a configuration of Q_5 D-fivebranes wrapping the whole T^5 , Q_1 D-strings wrapping the S^1 and momentum N/R_9 along the S^1 , choosing this S^1 to be in the direction 9. All charges N, Q_1, Q_5 are integers. For a review of D-branes see [25]. We take the coupling constant g to be small and the radius R_9 to be large. The total mass of the system is

$$M = \frac{Q_5 R V}{g} + \frac{Q_1 R}{g} + \frac{N}{R} \quad (14)$$

and it saturates the corresponding BPS bound.

We will calculate the entropy of this state in perturbative string theory. This calculation was first done by Strominger and Vafa [10]. Since extremal D-branes are boost invariant along the directions parallel to the branes they cannot carry momentum along S^1 by just moving rigidly. Our first task will be to identify the D-brane excitations that carry the momentum. The BPS mass formula for the whole system implies that these excitations have to be massless and moving along the S^1 since the excitation energy, defined as the total mass of the system minus the mass of the onebranes and fivebranes, is equal to the momentum. If any excitation fails to be massless it would contribute more to the energy than to the momentum and the BPS mass formula would be violated. Excitations of the branes are described by massless open strings. There are many types of open strings to consider: those that go from one 1-brane to another 1-brane, which we

denote as (1,1) strings, as well as the corresponding (5,5), (1,5) and (5,1) strings (the last two being different because the strings are oriented). We want to excite these strings and make them carry momentum in the direction of the S^1 . Exciting some of them makes others massive [18] so we have to find the way to excite the strings so that a maximum number remains massless, since this configuration will have the highest entropy. Let us start by working out the properties of (1,5) and (5,1) strings. The string is described by the usual string action where two of the coordinates have Neumann-Neumann boundary conditions (X^0, X^9), the four extended spatial coordinates have Dirichlet-Dirichlet boundary conditions (X^1, X^2, X^3, X^4) and the other four internal coordinates have Neumann-Dirichlet conditions (X^5, X^6, X^7, X^8). The vacuum energy of the worldsheet bosons is $E = 4(-1/24 + 1/48)$. Consider the NS sector for the worldsheet fermions, the 4 that are in the ND directions will end up having R-type quantization conditions. The net fermionic vacuum energy is $E = 4(1/24 - 1/48)$ and exactly cancels the bosonic one. This vacuum is a spinor under $SO(4)_{5678}$ and obeys the GSO chirality condition $\Gamma^5 \Gamma^6 \Gamma^7 \Gamma^8 \chi = \chi$. What remains is a two dimensional representation. There are two possible orientations and they can be attached to any of the different branes of each type. This gives a total of $4Q_1 Q_5$ different possible states for these strings. Now consider the Ramond sector, the four internal fermions transverse to the string will have NS type boundary conditions. The vacuum again has zero energy and is an $SO(1,5)_{091234}$ spinor and, therefore, a spacetime fermion. Again the GSO condition implies that only the positive chirality representation of $SO(1,5)_{091234}$ survives. When it is also left moving only the 2_+^1 under $SO(1,1)_{09} \times SO(4)_{1234}$ survives. This gives the same number of states as for the bosons. Note that the fermionic (1,5) (or (5,1)) strings carry angular momentum under the spatial rotation group $SO(4)_{1234}$ but the bosonic (1,5) (or (5,1)) strings do not carry angular momentum.

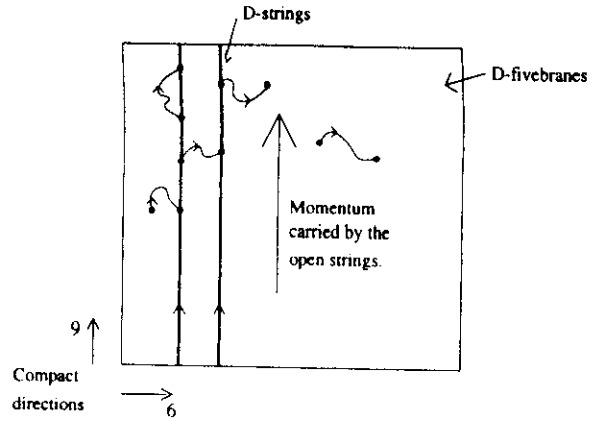


FIGURE 1: Configuration of intersecting D-branes. Open strings go between different branes

The (1,1), (5,5) and (1,5) strings interact among themselves. We are interested in the low energy limit of these interactions. This corresponds to the field theory limit of the system of branes. If we take the size of the S_1 along 9 to be very large this will be a 1+1 dimensional gauge theory. The Lagrangian is then determined by supersymmetry and gauge invariance. The (1,1) and (5,5) strings are $U(Q_1)$ and $U(Q_5)$ gauge bosons respectively and the (1,5) strings (and (5,1) strings) are in the fundamental (anti-fundamental) of $U(Q_1) \times U(Q_5)$. This Lagrangian was studied by Douglas [26] and it turns out that after these interactions are taken into account only $4Q_1 Q_5$ truly massless degrees of freedom remain. In gauge theory terms, one is interested in the Higgs branch of the theory which is $4Q_1 Q_5$ -dimensional. We will spare the details which can be found in [18] (page 52), and conclude that the number of massless states is $4Q_1 Q_5$ (with the same number of bosonic and fermionic states since the theory on the branes is supersymmetric). These massless degrees of freedom are described by a two dimensional conformal field theory [10], which accounts for the low energy excitations of the D-brane system. In fact, it is a (4,4) superconformal field theory, i.e. it has four left moving and four right moving supersymmetry generators. The rotational symmetry $SO(4)_{1234} \sim SU(2)_L \times SU(2)_R$ of the four spatial dimensions acts on this superconformal field the-

ory as the $SU(2)_L \times SU(2)_R$ R-symmetries of the $N=4$ supersymmetry algebra in two dimensions. Notice that the chirality in space becomes correlated with the chirality of the 1+1 dimensional theory. So that $SU(2)_L$ in spacetime acts on the leftmovers and similarly for $SU(2)_R$ [27].

The BPS state that we are interested in has only left moving excitations so the rightmovers are in their ground state $E_R = 0$. The state counting is the same as that of a one dimensional gas of left moving particles with $N_{B,F} = 4Q_1Q_5$ bosonic and fermionic species with energy $E = E_L = N/R_9$ on a compact one dimensional space of length $L = 2\pi R_9$. The standard entropy formula gives [10] [28]

$$S_e = \sqrt{\pi(2N_B + N_F)EL/6} = 2\pi\sqrt{Q_1Q_5N} \quad (15)$$

in perfect agreement with (10), including the numerical coefficient. It might seem surprising that a system could have entropy at zero temperature, but this is a common phenomenon. Consider for example a gas of massless particles in a box with periodic boundary conditions constrained to have a fixed amount of momentum, at $T = 0$ the entropy remains nonzero. This reason is *exactly* the same reason that black hole entropy is nonzero at $T = 0$.

In our previous argument we implicitly took the D-strings and the fivebranes to be singly wound since we were assuming that the excitations carried momentum quantized in units of $1/R$. For large N , $N \gg Q_1Q_5$, the entropy (15) is the same no matter how the branes are wound. However for $N \sim Q_1Q_5$ the winding starts to matter. The reason is that in order for the asymptotic entropy formula to be correct for low N we need to have enough states with small energies [29]. Let us study the effect of different wrappings. We first simplify the problem and consider a set of Q_1 1-branes wrapped on S^1 , ignoring for the time being, the 5-branes. We may distinguish the various ways the branes interconnect. For example, they may connect up so as to form one long brane of total length $R' = RQ_1$. At the opposite extreme they might form Q_1 disconnected loops. The spectra of open strings is different in each case. For the latter case the open strings behave like Q_1 species of 1 dimensional particles, each

with energy spectrum given by integer multiples of $1/R$. In the former case they behave more like a single species of 1 dimensional particle living on a space of length Q_1R . The result [30] is a spectrum of single particle energies given by integer multiples of $\frac{1}{Q_1R}$. In other words the system simulates a spectrum of fractional charges. For consistency the total charge must add up to an integer multiple of $1/R$ but it can do so by adding up fractional charges.

Now let us return to the case of both 1 and 5 branes. By suppressing reference to the four compact directions orthogonal to x^9 we may think of the 5 branes as another kind of 1 brane wrapped on S^1 . The 5-branes may also be connected to form a single multiply wound brane or several singly wound branes. Let us consider the spectrum of (1,5) type strings (strings which connect a 1-brane to a five-brane) when both the 1 and 5 branes each form a single long brane. The 1-brane has total length Q_1R and the 5-brane has length Q_5R . A given open string can be indexed by a pair of indices $[i, j]$ labeling which loop of 1-brane and 5-brane it ends on. As a simple example choose $Q_1 = 2$ and $Q_5 = 3$. Now start with the $[1, 1]$ string which connects the first loop of 1-brane to the first loop of 5-brane. Let us transport this string around the S^1 . When it comes back to the starting point it is a $[2, 2]$ string. Transport it again and it becomes a $[1, 3]$ string. It must be cycled 6 times before returning to the $[1, 1]$ configuration. It follows that such a string has a spectrum of a single species living on a circle of size $6R$. More generally, if Q_1 and Q_5 are relatively prime the system simulates a single species on a circle of size Q_1Q_5R . If the Q 's are not relatively prime the situation is slightly more complicated but the result is the same. A more detailed picture of how this happens is presented in [31].

We can easily see that this way of wrapping the branes gives the correct value for the extremal entropy. As above, the open strings have 4 bosonic and 4 fermionic degrees of freedom and carry total momentum N/R . This time the quantization length is $R' = Q_1Q_5R$ and the momentum is quantized in units of $(Q_1Q_5R)^{-1}$. Thus instead of being at level N the system is at level $N' = NQ_1Q_5$. In place of the original Q_1Q_5

species we now have a single species. The result is

$$S = 2\pi\sqrt{N'} = 2\pi\sqrt{NQ_1Q_5} \quad (16)$$

So we have a long effective string that is moving along the fivebrane. In the extremal case, this effective string picture follows precisely from an analysis of the moduli spaces of BPS states [32]. What one actually has is a sum over multiple string states which one can call “second quantized” strings on a fivebrane [33]. The state in which they are all connected into a single long string is the one having most entropy.

For completeness we will now present the same calculation but in a picture where we start with just D-fivebranes and we build up the charges as excitations of the D-fivebranes. We start with Q_5 D-fivebranes. The low energy theory on the fivebranes is an $U(Q_5)$ supersymmetric Yang Mills with 16 supersymmetries (same amount as $N=4$ in $d=4$). This theory contains BPS string solitons which are constructed as follows: if the fivebranes are along the directions 56789, take an instanton configuration that involves the directions 5678 and the gauge fields along those directions. The corresponding field configuration could be localized in the directions 5678 but will be extended along the direction 9, so it is a string soliton. Notice that, even though we call this solution an “instanton” in the sense that the Yang-Mills fields are self dual solutions of a YM theory in four dimensions (5678), the physical interpretation is that we have a string “soliton” which exists for all times. It turns out that each instanton that lives on the fivebrane world volume carries one unit of D-string charge [26] due to a Chern Simons coupling on the fivebrane of the form

$$\int d^{1+5}x B_2^{RR} \wedge F \wedge F \quad (17)$$

since $F \wedge F$ will be proportional to the instanton number. We are interested in the case that the instanton number is Q_1 . So D-strings dissolve into instantons when they get into fivebranes. In fact, giving an expectation value to the (1,5) strings corresponds to giving a size to the instanton [26,34]. This description makes sense, in principle, only when the mass of the fivebranes

is much bigger than the mass of the D-1-branes in (14), since otherwise the fivebranes would contain so much energy that they would no longer be described by the low energy YM theory. This instanton configuration is characterized by $4Q_1Q_5$ continuous parameters which specify the instanton positions on the branes as well as their relative orientation inside the $U(Q_5)$ gauge group, the space of these parameters is called “moduli space”. When we put some momentum along the direction 9, this momentum can be carried by small oscillations of the instanton configuration. We denote the instanton parameters by ξ^a , $a = 1, \dots, 4Q_1Q_5$. They can be slowly varying functions $\xi^a(t - x^9)$ representing traveling waves moving along the instantons.

These are small oscillations in the parameters specifying the instanton configuration, i.e. oscillations in moduli space. Each bosonic mode has a fermionic superpartner and together they form a (4,4) superconformal field theory with central charge $c = 6Q_1Q_5$. A configuration carrying momentum N corresponds to states in the SCFT with $L_0 = N$ and $\bar{L}_0 = 0$, the entropy of such states can be calculated using the CFT formula $d(N) \sim e^{2\pi\sqrt{Nc/6}}$ for the degeneracy of states at level N . This yields (15) again. The moduli space of instantons is, topologically, a symmetric product: $\mathcal{M} = (T^4)^{Q_1Q_5}/S(Q_1Q_5)$ [35,36]. This is the target space of the SCFT, in other words: $\xi^a(t, x^9)$ defines a map from $R \times S_1$ to $\mathcal{M}$. Since we have twisted sectors there are low lying modes with energies of the order of $1/RQ_1Q_5$ which give rise to the long effective string picture described above. This picture where we start with only one kind of branes is the one that we would naturally use [37,38] if we are working in the M(atric) theory of [39].

We should also address the question of whether the system really forms a bound state, this is a question on the behavior of the zero modes on the moduli space. The analysis of [10] shows that they indeed form a bound state. Note that also for entropic reasons the state would stay together for a long time.

It is interesting that when the momentum is not uniformly distributed along the string (the

9 direction) the D-brane calculation still agrees with the corresponding black hole result [40].

In performing these calculations we have assumed that the coupling was weak. One might naively think that the only condition is $g \ll 1$. However, since there is a large number of branes there could be large $N = Q$ effects which grow as gQ . At low energies these are just the large N effects of the Yang-Mills theory (note that the YM coupling is $g_{YM} = \sqrt{g}$). In string theory they correspond to the possibility of inserting a hole on the worldsheet, each hole has a power of g and a factor of Q coming from the trace over Chan Paton factors. So the above calculations are correct when

$$gQ \ll 1 \quad (18)$$

On the other hand the classical black hole solution is well defined when $gQ \gg 1$. The reason that we expect agreement (and find it) is that we are counting the number of BPS bound states and this number is not expected to change as we vary the coupling constant. Notice that indeed the extremal black hole entropy (10) is independent of the coupling and all other continuous parameters.

4. Near Extremal black holes

We now turn to a discussion of near extremal five-dimensional black holes (4) (11)(12). For the reasons that we have just discussed we might naively not expect agreement in this case. However, in the dilute gas regime, we are very close to a configuration of extremal D1 and D5 branes and supersymmetry non-renormalization arguments do indeed help us [41,42] and explain the agreement that we are going to find. We are going to consider a weakly coupled system of D-branes but we will always restrict to the low energy approximation. We will see that for the near-extremal case the agreement between the two approaches is just as impressive as in the extremal case. The D-brane model is a low energy approximation to the full quantum dynamics of black holes. The energy should be low compared to the scale set by gravitational size of the black hole r , defined as the radius at which the redshift of a static observer becomes of order one, $r_s^2 \sim gQ \sim r_1^2, r_5^2$.

The condition on the energy becomes $\omega r_s \ll 1$, where ω represents the typical energy of the brane excitations, as well as the Hawking temperature of the system [41]. We will not go into the details of the justification of this extrapolation which can be found in [41].

We start with a system of 1D-branes and 5-D branes as before and we add some extra energy and momentum to the system. This energy excites the massless left and right moving modes of the instanton configuration. In the instanton picture we say that we are creating left and right moving excitations on the moduli space, $\xi^I(t, x^9)$. Using the black hole formulas (3),(11) we calculate the energies of the left and right movers

$$M - \frac{Q_5 R V}{g} - \frac{Q_1 R}{g} = \frac{R V r_0^2}{2g} \cosh 2\sigma = \frac{N_L + N_R}{R} \quad (19)$$

$$N = \frac{R^2 V r_0^2}{2g^2} \sinh 2\sigma = N_L - N_R \quad (20)$$

In this fashion we can calculate $N_{L,R}$ in terms of the black hole parameters. The entropy calculation proceeds as in the extremal case. We work in the multiply wound picture with 4 bosons and 4 fermions with effective $N'_{L,R} = Q_1 Q_5 N_{L,R}$. We find that

$$S = \frac{2\pi(\sqrt{N'_L} + \sqrt{N'_R})}{2\pi\sqrt{Q_1 Q_5 N_L} + 2\pi\sqrt{Q_1 Q_5 N_R}} \quad (21)$$

We see that this result agrees with the near extremal entropy in the dilute gas limit (12) once we use (19) (20). This is the simplest case, if we want to consider more general near extremal black holes, including Reissner-Nordström, one has to include other excitations besides the right movers [28,43] and the arguments are not so well justified.

These non-BPS states will decay. The simplest decay process is a collision of a right moving excitation with a left moving one to give a closed string mode that leaves the brane. We will calculate the emission rate for uncharged particles. The basic process is a right moving mode with momentum $p_9 = n/R_9 Q_1 Q_5$ colliding with a left moving one of momentum $p_9 = -n/R_9 Q_1 Q_5$ to give a closed string mode of energy $\omega = 2n/RQ_1 Q_5$. Notice that we are considering the branes to be multiply wound since

that is the configuration that had the highest entropy. If the momenta are not exactly opposite the outgoing string carries some momentum in the 9^{th} direction and we get a charged particle from the five dimensional point of view. Notice that the momentum in the 9^{th} direction of the outgoing particle has to be quantized in units of $1/R_9$, only particles on the branes can have fractional momenta! This means that outgoing charged particles have a very large mass, and that they are thermally suppressed when R_9 is small. All charged particles have masses of at least the compactification scale.

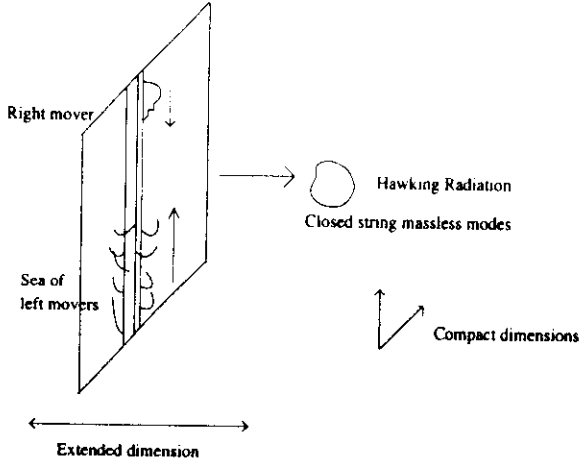


FIGURE 2: D-brane picture of the Hawking radiation emission process.

In other words, we have a very long effective string winding around the compact direction $\hat{9}$, it can oscillate along the other 4 compact dimensions (5678) and it emits gravitational quadrupole radiation. The graviton is polarized along the compact directions and is a scalar from the point of view of the five dimensional observer.

We will calculate the rate for this process according to the usual rules of relativistic quantum mechanics and show that the radiation has a thermal spectrum if we do not know the initial microscopic state of the black hole.

The state of the D-branes is specified by giving the left and right moving occupation numbers of

each of the bosonic and fermionic oscillators. In fact, the near extremal D-branes live in a subsector of the total Hilbert space that is isomorphic to the Hilbert space of a 1+1 dimensional CFT. The initial state $|\Psi_i\rangle$ can emit a closed string mode and become $|\Psi_f\rangle$. The rate, averaged over initial states and summed over final states (as one would do for calculating the decay rate of an unpolarized atom) is

$$d\Gamma \sim \frac{d^4k}{\omega} \frac{1}{F_0^2 R_0^4 V R} \delta(\omega - (p_0^R + p_0^L)) \times \sum_{i,f} |\langle \Psi_f | H_{int} | \Psi_i \rangle|^2 \quad (22)$$

We have included the factor due to the compactified volume RV . The relevant string amplitude for this process is given by a correlation function on the disc with two insertions on the boundary, corresponding to the two open string states and an insertion in the interior, corresponding to the closed string state. We consider the case when the outgoing closed string is a spin zero boson in five dimensions, so that it corresponds to the dilaton, the internal metric, internal $B_{\mu\nu}$ fields, or internal components of RR gauge fields. This disc amplitude, call it $\mathcal{A}$, is proportional to the string coupling constant g and to ω^2 [44]. The reason for this last fact is that it has to vanish when we go to zero momentum, otherwise it would indicate that there is a mass term for the open strings (since one can vary the vacuum expectation value of the corresponding closed string fields continuously). In conclusion, up to numerical factors,

$$\mathcal{A} \sim g\omega^2. \quad (23)$$

Note that performing the average over initial and summing over final states will just produce a factor of the form $\rho_L(n)\rho_R(n)$ with

$$\rho_R(n) = \frac{1}{N_i} \sum_i \langle \Psi_i | a_n^{R\dagger} a_n^R | \Psi_i \rangle \quad (24)$$

where N_i is the total number of initial states and a_n^R is the creation operator for one of the 4 bosonic open string states. The factor $\rho_L(n)$ is similar. Since we are just averaging over all possible initial states with given value of N_R , this corresponds to taking the expectation value of $a_n^\dagger a_n$ in the microcanonical ensemble with total energy

$E_R = N_R/R_9 = N'_R/R_9 Q_1 Q_5$ of a one dimensional gas. Because N'_R is large compared to one, we can calculate (24) in the canonical ensemble. The occupation number is then

$$\rho_R(\omega) = \frac{e^{\frac{\omega}{2T_R}}}{1 - e^{\frac{\omega}{2T_R}}}.$$

We can read off the "right moving" temperature

$$T_R = \frac{1}{\pi} \frac{1}{R} \sqrt{\frac{N_R}{Q_1 Q_5}}. \quad (25)$$

There is a similar factor for the left movers ρ_L with a similar looking temperature

$$T_L = \frac{1}{\pi} \frac{1}{R} \sqrt{\frac{N_L}{Q_1 Q_5}}. \quad (26)$$

In fact it would be more accurate to say that there is only one physical temperature of the gas, which agrees with the Hawking temperature of the corresponding black hole,

$$\frac{1}{T_H} = \frac{1}{2} \left(\frac{1}{T_L} + \frac{1}{T_R} \right) \quad (27)$$

and that $T_{L,R}^{-1} = T_H^{-1} (1 \pm \mu)$ are some natural combination of the temperature and the chemical potential, which gives the gas some net momentum. Using the values for $N_{L,R}$ from (19) (20) we find

$$T_L = \frac{1}{\pi} \frac{r_0 e^\sigma}{2r_1 r_5}, \quad T_R = \frac{1}{\pi} \frac{r_0 e^{-\sigma}}{2r_1 r_5}. \quad (28)$$

The expression for the rate is, up to a numerical constant,

$$d\Gamma \sim \frac{d^4 k}{\omega} \frac{1}{p_0^R p_0^L R V} |\mathcal{A}|^2 Q_1 Q_5 R \rho_R(\omega) \rho_L(\omega) \quad (29)$$

where $\mathcal{A}$ is the disc diagram result. The factor $Q_1 Q_5 R$ is a volume factor, which arises from the delta function of momenta in (22) $\sum_n \delta(\omega - 2n/RQ_1 Q_5) \sim RQ_1 Q_5$. The final expression for the rate is, using (23) in (29),

$$d\Gamma = \frac{\pi^3 g^2}{V} Q_1 Q_5 \omega \frac{1}{e^{\frac{\omega}{2T_L}} - 1} \frac{1}{e^{\frac{\omega}{2T_R}} - 1} \frac{d^4 k}{(2\pi)^4} \quad (30)$$

We have not shown here how to calculate the precise numerical constant in front of (30), this precise calculation was done in [45], and we refer the reader to it for the details.

If we are considering a black hole which is very close to extremality with nonzero momentum $N \sim N_L \gg N_R$ then we find from (26)(25) that $T_L \gg T_R$. Examining the expression for the rate (30) we see that the typical emitted energies are of the order of T_R . Therefore, we can approximate the left moving thermal factor by

$$\rho_L \sim \frac{2T_L}{\omega} \quad (31)$$

and replacing it in (30) we find

$$\begin{aligned} d\Gamma &= \frac{2\pi^2 g^2}{RV} \sqrt{Q_1 Q_5 N} \frac{1}{e^{\frac{\omega}{2T_R}} - 1} \frac{d^4 k}{(2\pi)^4} \\ &= A_H \frac{1}{e^{\frac{\omega}{2T_R}} - 1} \frac{d^4 k}{(2\pi)^4} \end{aligned} \quad (32)$$

where A_H is the area of the horizon. We conclude that the emission is thermal, with a physical Hawking temperature

$$T_H = 2T_R \quad (33)$$

which exactly matches the classical result (9). The area appeared correctly in (30) [46]. Actually, the coupling constant coming from the string amplitude $\mathcal{A}$ is hidden in the expression for the area (area = $4G_N^5 S$). The overall coefficient in (32) matches precisely with the semiclassical result [45].

Notice that if we were emitting a spacetime fermion then the left moving mode could be a boson and the right moving mode a fermion, this produces the correct thermal factor for a spacetime fermion. The opposite possibility gives a much lower rate, since we do not have the enhancement due to the large ρ_L (31).

When separation from extremality is very small, then the number of right movers is small and the statistical arguments used to derive (30) fail. Semiclassically this should happen when the temperature is so low that the emission of one quantum at temperature T causes the temperature to change by an amount of order T . This means that the specific heat is of order one. This

happens when the mass difference from extremality is [47]

$$\delta M_{\min} = M - M_{BPS} \sim \frac{G_N^5}{r_e^4} \quad (34)$$

for a Reissner-Nordström black hole, with r_e being the Schwarzschild radius of the solution. The D-brane approach suggests the existence of a mass gap

$$\delta M_{\min} \sim \frac{2}{Q_1 Q_5 R} \quad (35)$$

which using (3) scales like (34). This is an extremely small energy for a macroscopic extremal black hole. In fact, it is of the order of the kinetic energy that the black hole would have, due to the uncertainty principle, if we want to measure its position with an accuracy of the order of its typical gravitational radius r_s : $\delta M \sim (\Delta p)^2/M$ with $\Delta p \sim 1/r_s$.

Now we calculate the entropy of a rotating black hole in five dimensions [27,48]. The angular momentum is characterized by the eigenvalues on two orthogonal two-planes, J_1, J_2 , for example J_1 corresponds to rotations of the 12 plane and J_2 to rotations of the 34 plane. In terms of the J_3 eigenvalues J_L, J_R of the $SU(2)_L \times SU(2)_R \sim SO(4)$ decomposition of the spatial rotation group we find

$$J_1 = J_L + J_R \quad J_2 = J_L - J_R \quad (36)$$

As we mentioned above J_R, J_L are carried by right and left movers respectively. They are also the eigenvalues of $U(1)$ appearing in the supersymmetry algebra. States carrying $U(1)$ eigenvalue J have conformal weight bigger than $\Delta = 6J^2/c$ where $c = 6Q_1 Q_5$ is the central charge. The states with minimum conformal weight correspond to states $e^{iJ\phi}|0\rangle$, where $j = \frac{c}{12}\partial\phi$ is the $U(1)$ current. In the total left moving energy N_L there is an amount $J_L^2/Q_1 Q_5$ which we are not free to distribute. It is fixed by the condition that the system has angular momentum J_L , so the effective number of left movers that we are free to vary is $\tilde{N}_L = N_L - J_L^2/Q_1 Q_5$. The same is true for the right movers, so that the entropy becomes

$$\begin{aligned} S &= 2\pi\sqrt{Q_1 Q_5}(\sqrt{\tilde{N}_L} + \sqrt{\tilde{N}_R}) = \\ &= 2\pi\sqrt{N_L Q_1 Q_5 - J_L^2} + 2\pi\sqrt{N_R Q_1 Q_5 - J_R^2} \end{aligned} \quad (37)$$

which agrees with the classical entropy formula of a rotating black hole in the dilute gas regime [48]. Actually, in five dimensions we can have rotating BPS black holes by setting $N_R = J_R = 0$, this implies $J_1 = J_2$. Again the corresponding formula agrees with the classical entropy formula but the restriction to the dilute gas regime is no longer necessary since the computation is protected by supersymmetry.

5. Greybody factors

The D-brane emission rate into massless scalars is given by (30). More precisely that is the emission into minimally coupled scalars, scalars that in the supergravity theory are not coupled to the vector fields that are excited in the black hole background. According to the semiclassical analysis the emission rate should be

$$d\Gamma = \sigma(\omega, r_0, \alpha, \gamma, \sigma) \frac{1}{e^{\frac{\omega}{T_H}} - 1} \quad (38)$$

where $\sigma(\omega, r_0, \alpha, \gamma, \sigma)$ is the absorption cross section of the black hole which is a function of the various parameters specifying the black hole solution (4). In the usual Schwarzschild black hole case the only scale in the solution would be the Schwarzschild radius r_s . This emission rate (38) has the same form for any body emitting thermal radiation. The absorption cross section comes in because of detailed balance: in order for the body to be in equilibrium with a bath of radiation it has to absorb as much as it emits. The prefactor in (38) is usually called greybody factor, since it is what makes bodies grey instead of black. At first sight, the semiclassical rate (38) does not seem to be in agreement with the D-brane rate (30) since one has two exponential factors and the other seems to have only one. In order to see whether they really agree we should calculate the greybody factor. It turns out that the greybody factor is precisely such that these two calculations to agree. We now describe this calculation.

We consider the scattering of scalars from a five dimensional black hole in the dilute gas limit $r_0, r_n \ll r_1, r_5$. We also restrict to low energies satisfying $\omega \ll 1/r_1, 1/r_5$ but there is no restriction on $\frac{\omega r_0}{r_1 r_5}$ or $\frac{\omega r_n}{r_1 r_5}$, in other words, no restriction

on ω/T_L , ω/T_R . We follow the notation of [24], where further details of the geometry may also be found. The wave equation in this background becomes

$$\frac{h}{r^3} \frac{d}{dr} r^3 h \frac{d\phi}{dr} + \omega^2 \lambda \phi = 0, \quad (39)$$

where λ, h are defined in (5).

We divide space into a far region $r \gg r_1, r_5$ and a near region $r \ll 1/\omega$ and we will match the solutions in the overlapping region. In the far region, the equation is solved by the Bessel functions

$$\phi = \frac{1}{\rho} [\alpha J_\nu(\rho) + \beta J_{-\nu}(\rho)], \quad (40)$$

with $\rho = \omega r$ and $\nu^2 = 1 - \epsilon$, where $\epsilon = \omega^2(r_1^2 + r_5^2)$ is very small and we keep it to simplify the form of the intermediate equations but will disappear from the final answer. From the large ρ behavior the incoming flux is found to be

$$f_{in} = \text{Im}(\phi^* r^3 \partial_r \phi) = \frac{1}{2\pi\omega^2} |\alpha e^{i\nu\pi/2} + \beta e^{-i\nu\pi/2}|^2. \quad (41)$$

On the other hand, the small ρ behavior of the far region solution is

$$\phi = \frac{1}{\rho} \left[\alpha \left(\frac{\rho}{2}\right)^\nu \left(\Gamma\left(\frac{1}{\nu+1}\right) - \mathcal{O}(\rho^2)\right) + \beta \left(\frac{\rho}{2}\right)^{-\nu} \left(\Gamma\left(\frac{1}{-\nu+1}\right) - \mathcal{O}(\rho^2)\right) \right]. \quad (42)$$

Now we turn to the solution in the near region $r \ll 1/\omega$. Defining $v = r_0^2/r^2$, the near region wave equation is

$$(1-v)^2 \frac{d^2\phi}{dv^2} - (1-v) \frac{d\phi}{dv} + \left(C + \frac{D}{v} + \frac{E}{v^2}\right) \phi = 0 \quad (43)$$

where

$$C = \left(\frac{\omega r_0 r_1 r_5}{2r_0^2}\right)^2, \quad D = \frac{\omega^2 r_1^2 r_5^2}{4r_0^2} + \frac{\nu^2 - 1}{4}, \quad E = -\frac{\nu^2 - 1}{4}. \quad (44)$$

Defining

$$\phi = v^{-(\nu-1)/2} (1-v)^{-i\frac{\omega}{4\pi T_H}} A F \quad (45)$$

with A a constant, we find that the solution to (43) with only ingoing flux at the horizon is given by (45) with the hypergeometric function

$$\begin{aligned} F &= F(a, b, c; 1-v) \\ a &= -\nu/2 + 1/2 + i\frac{\omega}{4\pi T_L} \\ b &= -\nu/2 + 1/2 + i\frac{\omega}{4\pi T_R} \\ c &= 1 + i\frac{\omega}{2\pi T_H} \end{aligned} \quad (46)$$

The behavior for small v can be calculated by expressing the hypergeometric function (46), which depends on $1-v$, in terms of hypergeometric functions depending on v and then expanding in v . Matching this with (42) we find

$$\alpha/2 = A \left[\frac{\Gamma(1+i\frac{\omega}{2\pi T_H})}{\Gamma(1+i\frac{\omega}{4\pi T_L})\Gamma(1+i\frac{\omega}{4\pi T_R})} \right], \quad (47)$$

$$\beta \ll \alpha$$

The absorbed flux is

$$f_{abs} = \text{Im}(\phi^* h r^3 \partial_r \phi) = \frac{\omega r_0^2}{2\pi T_H} |A|^2. \quad (48)$$

The absorption cross section for the radial problem is given by the ratio of the two fluxes (48) (41). The plane wave cross section is obtained by multiplying by $4\pi/\omega^3$

$$\sigma_{abs} = \pi^3 r_1^2 r_5^2 \omega \frac{e^{\frac{\omega}{T_H}} - 1}{(e^{\frac{\omega}{T_L}} - 1)(e^{\frac{\omega}{T_R}} - 1)} \quad (49)$$

where the exponential terms come from the gamma functions in (47). We see that it has precisely the right form to make the D-brane result (30) agree with the semiclassical calculation (38).

These greybody factor calculations have been generalized to various cases. One possible generalization is to consider the emission of scalars that are not minimally coupled, in some cases precise agreement is found [49] [50]. From the D-brane point of view the difference between these scalars and the one that we have been considering is in the conformal weight of the operator on the effective SCFT that they couple to. The minimally coupled scalars couple to operators of dimension (1,1) (like $\partial X \bar{\partial} X$) while the scalars in [49] couple to operators of conformal weights (2,2) or (1,2) in [50]. There is still some puzzling disagreement for the case of operators of weight (3,1) [51], which hopefully will be resolved soon!. Another generalization is to consider the emission of higher partial waves [52,54,53]. These calculations of greybody factors shows that some of the features of the near extremal geometry are encoded in the dynamics of the 1+1 dimensional gas (or the CFT). Since the wavelengths of the particles we scatter are much bigger than the size of the black holes it is hard to get precise information about the

metric. A more direct way to obtain information about the metric is by using D-brane probes [55]. In that approach one starts from D-branes in flat space and by integrating out the massive stretched open strings one obtains an action for the probe D-branes that is at some distance from the rest of the branes. This action is then interpreted as the action of a D-brane in the presence of some classical supergravity background. This works to one loop [55,56] but the status of the higher loop contributions is unclear.

The four dimensional black holes have a similar description [57–59].

These results have clear implications in the information loss problem.

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Supergravity solutions

• D-5-brane solution

• In 10-d. string metric

$$-ds_{10, \text{str}}^2 = f_5^{-1/2} [-dt^2 + dx_5^2 + dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2] + f_5^{3/2} [dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2]$$

$$f_5 = 1 + \frac{g Q_5}{x_1^2 + x_2^2 + x_3^2 + x_4^2} = 1 + \frac{g Q_5}{r^2}$$

$$\therefore e^{-2\phi_{10}} = f_5$$

$$\text{---} H_{ijk} \approx \frac{1}{2} \epsilon_{ijkl} \partial_l f_5 \quad i, j, k, l = 1, \dots, 4$$

• $\frac{1}{2}$ SUSY

$$S(\text{gravitino}) = [\text{Bosonic fields}, \epsilon_L, R$$

$$\Rightarrow \text{vanishes when } \epsilon_L^\infty = \not{p}^0 \not{p}^1 \not{p}^2 \not{p}^3 \not{p}^4 \epsilon_R^\infty$$

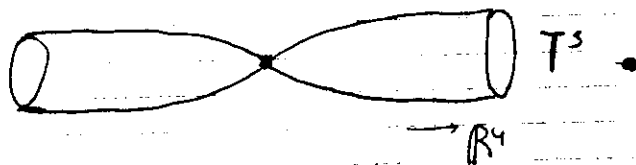
$\rightarrow$ value of the spinors at $r \rightarrow \infty$

- Behavior of the solution as $r \rightarrow 0$

$\cdot G_{\parallel}^{rs} \rightarrow 0$ longitudinal directions shrink

$\cdot G_{\perp}^{rs} \rightarrow \infty$ perpendicular directions expand

Physical understood as the 5-brane exerting some tensions or pressures on the radii of the compact dimensions



~~coupling~~ coupling constant $\rightarrow e^{\phi} \rightarrow 0$ at $r=0$

- Any solution in 10-d which is translationally invariant ^{along 5-dimensions} will be also a solution of the compactified theory.

- There is a precise algebraic relation between the field in $d=1+$ and the 10-dimensional fields

e.g. $G_{\mu\nu}^{(10)} = G_{\mu\nu}^{(10)} - G_{\mu I}^{(10)} G_{\nu J}^{(10)} G^{IJ}$; $e^{-2\phi_5} = e^{-2\phi_{10}} \sqrt{det}$

etc. (See Maldaena-Schwartz-)

- Once we are in $d=5$ we can go to the 5-dimensional Einstein metric

$$\int \sqrt{G_{(4)}} e^{-2\phi_5} [R + \dots] \rightarrow$$

$$\rightarrow \frac{1}{16\pi G_5} \int \sqrt{g_E} R_E + \dots$$

$$\text{if: } G_{(4)}^{(5)} = e^{\frac{4\phi_5}{3}} g_E$$

• All this eventually gives

$$ds_{5,E}^2 = -\lambda^{-\frac{2}{3}} dt^2 + \lambda^{\frac{1}{3}} [dx^2 + r^2 d\Omega_3^2]$$

$$\lambda = f_5 = 1 + \frac{gQ_5}{r^2}$$

• Horizon, singularity - $r=0$, $A_H=0$

• There are scalar fields which change (or go to zero) at the horizon
e.g. - deformation, size of internal manifold, etc.

• Add one more charge and momentum

$$ds_{10,etc}^2 = f_1^{-\frac{1}{2}} f_5^{-\frac{1}{2}} [-dt^2 + dx_9^2 + K(dt + dx_9)^2] + \\ + f_1^{\frac{1}{2}} f_5^{-\frac{1}{2}} [dx_5^2 + \dots + dx_8^2] + f_1^{\frac{1}{2}} f_5^{-\frac{1}{2}} [dx_1^2 + \dots + dx_4^2]$$

$$\begin{aligned} f_5 &= 1 + \frac{gQ_5}{r^2} & f_1 &= 1 + \frac{gQ_1}{r^2 V} & K &= \frac{g^2 N}{R^2 V \pi^2} & f_n &= 1 + K \\ f_5 &= 1 + \frac{r_s^2}{r^2} & f_1 &= 1 + \frac{r_1^2}{r^2} & f_n &= 1 + \frac{r_n^2}{r^2} \end{aligned}$$

Now all scalar fields are balanced:

e.g. $G_{55} \longrightarrow \frac{1+K}{f_1^{1/2} f_5^{1/2}} \underset{r \rightarrow 0}{\sim} \frac{N}{\sqrt{Q_1 Q_5}}$



Brane-tensions are balanced by the pressure of the KK Mon

- Values of the scalar fields at the Horizon minimize the mass in 5-d Planck units. (general property of BPS black holes).

$$M_E = M_{\text{str}} \cdot G_H^{1/3}$$

↓

$$M_{\text{str}} = \frac{Q_5 R V}{g} + \frac{Q_1 R}{g} + \frac{V}{R} = M_{\text{BPS}} \quad \text{Mass of the black hole at extremality}$$

- 5-d Einstein metric

$$ds^2 = -\lambda^{-2/3} dt^2 + \lambda^{1/3} [dr^2 + r^2 d\Omega_3^2]$$

$$\lambda = f_1 f_2 f_m \quad , \quad f_m = 1 + K$$

~~A_H~~ $f_i = 1 + \frac{r_i^2}{r^2}$

$r_i^2 \rightarrow$ distance scales characterizing the solution.

-horizon is at $r=0$

$$A_H = \lim_{r \rightarrow 0} \lambda^{1/2} r^2 \cdot 2\pi^2 = 2\pi^2 r_m r_1 r_5$$

$$S_{\text{BH}} = \frac{A_H}{4G_N}$$

We use that $G_N = \frac{1}{4R^2V} \pi g^2$

And we get

$$S_{BH} = 2\pi \sqrt{N Q_1 Q_5}$$

- All dependence on continuous parameters has disappeared
(or it should if we are going to argue that the number of BPS states is independent of the values of these parameters).
- The charges can be quantized ~~also~~ by summing 1 of the quantization conditions (for one of the charges) and using U-duality the rest are determined, (without using string theory)
- Solution is smooth at the horizon. (no diverging curvatures or scalar fields).
- Supergravity solutions make sense when
$$R_i \gg \alpha' = 1$$
$$R \sim V^{-1/3} \rightarrow gQ \gg 1 \quad g^2 N \gg 1$$
$$\Rightarrow Q \gg 1, N \gg 1$$

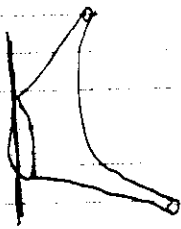
$\rightarrow$ Classical solutions correspond to large values of the quantized charges.
- 5-d Reissner Nordstrom solution $\rightarrow \pi_1 = \pi_5 = \pi_m$

The D-brane description is valid when:

$$g \ll 1$$

$$\text{or } gQ \ll 1, \quad g^2 M$$

large Q effects $\rightarrow$ Adding holes to a string diagram:



each hole $\rightarrow g \cdot Q$

• If $gQ \ll 1 \rightarrow$ we can keep the 1st order in perturbation theory

• Near extremal solutions

$\rightarrow$ Add some mass:

$$M \geq M_{\text{BPS}} = \frac{Q_5 R V}{g} + \frac{Q_1 R}{g} + \frac{\mu}{R}$$

$$ds_5^2 = -\lambda^{-2/3} h dt^2 + \lambda^{1/3} \left[\frac{d\pi^2}{h} + \pi^2 d\Omega^2 \right]$$

$$\lambda = t_s b_s b_m, \quad h = 1 - \pi_0^2 / \pi^2$$

$$\pi_1^2 = \pi_0^2 \sin^2 \alpha$$

$$\pi_s^2 = \pi_0^2 \sin^2 \gamma$$

$$\pi_m^2 = \pi_0^2 \sin^2 \phi$$

$$Q_1 = \frac{\pi_0^2 \sin^2 \alpha}{2g} \cdot V$$

$$Q_5 = \frac{\pi_0^2 \sin^2 \gamma}{2g}$$

$$M = \frac{\pi_0^2 \sin^2 \phi}{2g^2} R^2 V$$

$$M = \frac{RV\pi_0^2}{2g^2} (\chi_2 \alpha + \chi_2 \gamma + \chi_2 \epsilon)$$

$$S = \frac{A_{10}}{4G_0^{10}} = \frac{A_5}{4G_0^5} = \frac{2\pi RV\pi_0^3}{g^2} \chi_2 \chi_2 \gamma \chi_2 \epsilon$$

Extremal limit $\pi_0 \rightarrow 0$, $\alpha, \gamma, \epsilon \rightarrow \infty$ with $\pi_0 e^\alpha, \pi_0 e^\gamma, \pi_0 e^\epsilon$ fixed.

~~Dilute gas limit is defined by $\pi_0^2 \ll \pi_0^2$~~

• Near extremal limit:

$$M - M_{BPS} \ll M_{BPS} \longrightarrow \pi_0 \ll \max\{\pi_\alpha, \pi_\gamma, \pi_\epsilon\}$$

• We will concentrate in the dilute gas limit defined by

$$\pi_0, \pi_\alpha \ll \pi_\gamma, \pi_\epsilon$$

$$M \approx \frac{Q_5 RV}{g} + \frac{Q_1 R}{g} + \frac{RV\pi_0^2}{2g^2} \chi_2 \epsilon$$

$$S \approx \frac{2\pi R \sqrt{V} \pi_0}{g} \chi_2 \epsilon \sqrt{Q_1 Q_5}$$

defining N_L, N_R by

$$N_{L,R} = \frac{R^2 V \pi_0^2}{4g^2} e^{\pm 2\epsilon}$$

we find

$$M = \frac{Q_5 RV}{g} + \frac{Q_1 R}{g} + \frac{N_L}{R} + \frac{N_R}{R}$$

$$S = 2\pi \sqrt{Q_1 Q_5} (\sqrt{N_L} + \sqrt{N_R})$$

