UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL ATOMIC ENERGY AGENCY
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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SMR.996 - 9

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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

2 June - 4 July 1997

ORIENTIFOLDS AND DUALITY

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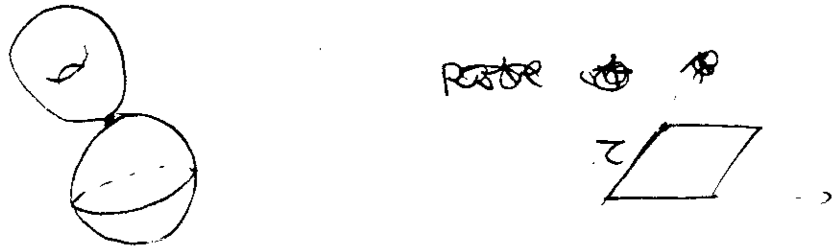
We see thus that starting with some basic duality like $SL(2,\mathbb{Z})$ of IIB with ~~10~~ 32-supercharges we can get all the structure of the more interesting duality between IIA on K3 ~~and~~ Heterotic on T4 with 16-supercharges. This is quite an explicit construction, and all we need to do is to keep track of a few discrete symmetries.

I shall now describe one more duality with 16 supersymmetries, before moving onto theories with 8 supercharges. Kachru will discuss theories with exclusively 4 supercharges.

F-theory on K3 $\leftrightarrow$ Heterotic on T_4

Before discussing F-theory, let us note that the K3 orbifold $T^4/\mathbb{Z}_2$ that we constructed can be viewed as an elliptic fibration on $\mathbb{C}P^1$.

i.e. a torus as a fiber on S^2

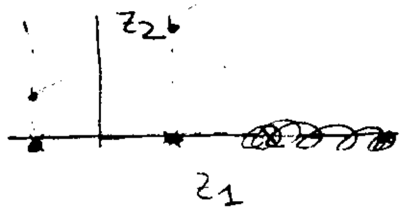


The parameters of the torus in general are allowed to vary. There can be points where the torus degenerates by pinching handles on S^2.

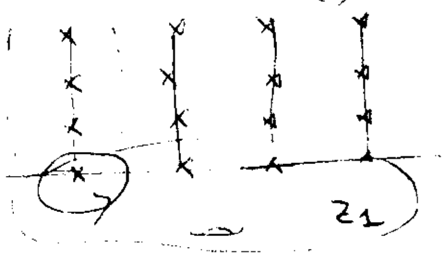
Let's note that $T^2/\mathbb{Z}_2$ is S^2

$\mathbb{Z}_2$: $z_1 \to -z_1$ There are four fixed points each with deficit angle $180^\circ = \pi$. so the total deficit angle is 4π giving us $\int R = 4\pi$ i.e the correct Euler class of 2.

$$\frac{T^4}{(\mathbb{Z}_2)} = \frac{T^2_{(1)} \times T^2_{(2)}}{\{1, I_1 I_2\}}$$



So locally, away from the fixed points of I_1 It looks like $T^2_{(2)} \times S^2 = T^2_{(1)}/\{1, I_1\}$



If you go around a fixed point of I_1 then the parameters of the $T^2_{(2)}$ are ~~trans~~ twisted by I_2. i.e has $\mathbb{Z}_2$ monodromy.

$(z_1, z_2) \sim (-z_1, -z_2)$

at a fixed point of I_1 (e.g. $z_1 = 0$), If we go on a closed loop we can come back with $z_2 \to -z_2$

consider Type-I theory on T^2. This is dual to Heterotic on $T2$ by the Heterotic – Type-I duality. Now consider two T-dualities

T_{89} : $\quad \Omega \rightarrow \Omega(-1)^{F_L} I_{89}$. ⊛

Type-I on $T2$ is thus dual to another orientifold on $IIB^{\text{on } T^2} / \{1, \Omega(-1)^{F_L} I_{89}\}$

Now recall that $\Omega(-1)^{F_L} \in SL(2,\mathbb{Z})$ i.e. it is the perturbative element $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ of $SL(2,\mathbb{Z})$.

Thus the theory has the structure of

IIB on $T_2 / \{1, I_{89}R\}$ i.e IIB on $S^2\ (z)$

$R = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

The fields ~~too~~ belong to some representation of $SL(2,\mathbb{Z})$ $\tau(z)$, $B_{\mu\nu}(z)$ $B_{\mu\nu}(z)$ etc.

and as we go around the fixed point they are twisted by $\mathbb{Z}_2$.

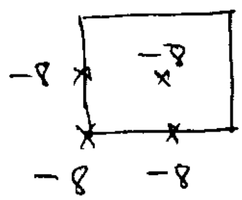
We thus have a $\mathbb{Z}_2$ bundle on S^2

More generally we can have $SL(2,\mathbb{Z})$ bundle on S^2 Such a compactification is called F-theory.

⊛ $(\Omega(-1)^{F_L} I_{89})^2 = \Omega(-1)^{F_L}\, \Omega(-1)^{F_R} = \Omega^2 = 1$, $\quad I_{89}^2 = (-1)$, $\quad \Omega(-1)^{F_L} = (-1)$

Before disussing F-theory, let us look at the orientifold.
$$\frac{\text{IIB on } T2}{\{1, (-1)^{F_L} I_{89}\Omega\}}$$

I_{89} on T_2 has four fixed points. So, each is an orientifold plane. Note the $\Omega(-1)^{F_L} I_{89} \xrightarrow{T_{89}} \Omega$ as we saw before. So the tadpole calculation is the same as in Type-I that we did before. The total charge is -32 distributed equally among the four orientifold plan



We need 32 7-branes

Orientifold 7-planes, charge can be neutralized by putting 8 7-branes at each fixed plane, giving $SO(8)^4$.

This is after all T-dual to Type-I $SO(32)$ string. $SO(32)$ is broken to $SO(8)^4$ by Wilson lines.

F-theory:

Most string compactifications ~~...~~ until recently, basically solve Einstein equations in the low-energy limit for some compact manifold K, (~~..~~ sometimes with torsion)

$R_{ij} = 0.$ (sometimes w/ nonzero ~~...~~ B_{ij})

So, you could compactify on K only if K was Ricci-flat, In particular with vanishi c_1, to get SU(N) holonomy, to get unbroken susy

But one can imagine, more general possibilit in which other fields are excited.

In particular in ~~.....~~ IIB string we have dilaton-axion field $\tau = a + ie^{-\phi}$ a - RR field

so one can have,

$$R_{ij} = T_{ij}^{\phi} \qquad \begin{array}{l} \partial\bar{\partial}\tau + \dfrac{2\,\partial\tau\,\bar{\partial}\bar{\tau}}{(\bar{\tau}-\tau)} = 0. \\ \Box\tau = 0 \end{array}$$ more or less

We can allow $SL(2,\mathbb{Z})$ jumps in τ.

~~For simplicity~~ ~~we~~ consider

$$\int \cancel{R} + d^4x\sqrt{g}\left(R - \frac{1}{2}\frac{\partial_\mu\bar{\tau}\,\partial^\mu\tau}{(\mathrm{Im}\,\tau)^2}\right)$$

which is the action of IIB with other fields set to zero.

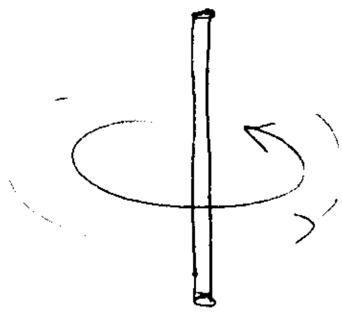
For concreteness, consider Heterotic compactified on $T^4 \times T^2$ to 4-dimensions. which has the same action, if we excite the complex-structure modulus of the torus T^2 only

τ 

$T: \tau \to \tau + 1$ is a symmetry.

$S: \tau \to -1/\tau$.

Consider a cosmic string which is like a vortex line in a superfluid helium.



~~As we go around the~~ ~~string, θ,~~

A cosmic string is a topological defect in which the phase angle θ of the order-parameter field has winding number.

$$\cancel{\theta(x,y)} \qquad \theta(x_1, x_2) \longrightarrow \theta(x_1, x_2) + 2\pi$$

$$z = x_1 + i x_2 \qquad \theta(z) \longrightarrow \theta + 2\pi$$

$\theta/2\pi \sim$ a field

We can have
$$\left.\begin{aligned} \tau(z) &\longrightarrow \tau(z) + 1 \\ \text{or } \tau &\longrightarrow -1/\tau \end{aligned}\right\} \text{"stringy" cosmic string.}$$

A cosmic-string has some deficit angle because it has energy density at the core.

$\tau(z)$

For stringy cosmic string the deficit angle is $\frac{\pi}{6}$, so locally a cone.

If we put <u>24</u> ~~cosmic~~ ~~stringy~~ stringy cosmic strings then the deficit angle is 4π or Euler character 2 to get a compact base manifold, S^2.

The internal torus is fibered over this S^2, its modulus τ varies over S^2, $\tau(z)$, $\tau \xrightarrow{SL(2,\mathbb{Z})} \tau$ is allowed because it defines the same torus (upto conformal rescalings).

We thus obtain a ~~elliptically~~ ~~fiber~~ compact four-dimensional manifold that is elliptically fibered over S^2.

Torus = elliptic curve (complex 1-dimensional) that arose in the efforts to understand elliptic functions.

K3 As an elliptic fibration

$$y^2 = x^3 + fx + g$$

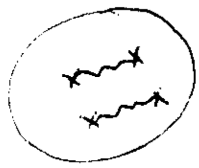
f and g = complex #'s.

This is a torus. Write it as

$$wy^2 = x^3 + fw^2x + gw^3 \qquad \cancel{w \neq 0}$$

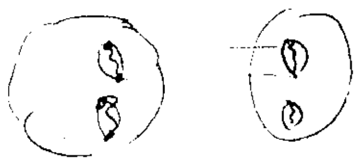
This is a cubic in $\mathbb{C}P^2$, so its the simplest Calabi–Yau, = 1-fold with $c_1 = 0$ (No curvature).

More geometrically, consider $x \in \mathbb{C}P^1$

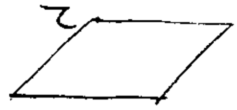


$$y = \pm\sqrt{\prod (x - c_i)^3}$$

take double cover



gives a torus.



τ

τ depends on f and g.

Now, we can take f and g to be functions of $z \in \mathbb{C}P^1$. Then for every value of z i.e. at each point, we get a torus. As we move around in z, f and g and consequently τ changes.

= elliptic fibration.

Now arrange things such that the resulting 4-d manifold is K3, i.e. a Calabi-Yau.

This is easily achieved by taking

f = polynomial in z of degree 8
g = " " z of degree 12.

To see that its K3, add one more coordinate $f(z,w) = z^8 + z^4w^4 + w^8$ etc.

$$y^2 = x^3 + f(z,w)x + g(z,w)$$

This is a polynomial of degree 12 in $W\mathbb{P}^3_{(1,1,4,6)}$ (w, z, x, y)

$\sum$ weights = degree = 12

$\Rightarrow c_1(K) = 0$.

~~The~~ We can set $w=1$ by the equivalence relation

$$(w, z, x, y) \sim (\lambda w, \lambda z, \lambda^4 x, \lambda^6 y)$$

by choosing $\lambda = \frac{1}{w}$ when $w \neq 0$. $w=0$ is just the "point at infinity" needed to get a compact space.

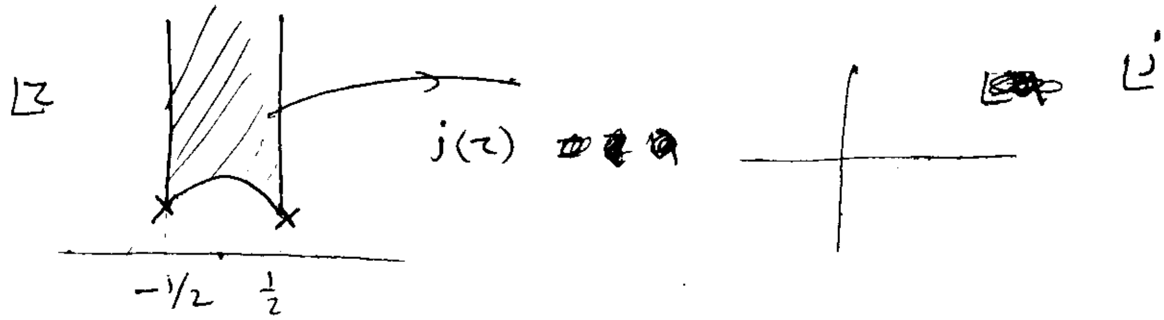
We thus have,

$$y^2 = x^3 + f(z)\,x + g(z) \qquad \frac{dx}{y}$$

$z \in \mathbb{CP}^1$ so we have a torus for every z. The modular parameter τ of the torus is given by,

$$j(\tau) = \frac{4\cdot(24\, f)^3}{27\, g^2 + 4f^3}$$

where $j(\tau)$ is the well known j-function, that gives a 1-1 map between the ~~moduli space~~ fundamental domain ~~of the~~ (the moduli space of the torus) to $\mathbb{CP}^1$



$$j(\tau) = \frac{\left[\theta_1^8(\tau) + \theta_2^8(\tau) + \theta_3^8(\tau)\right]^3}{\eta(\tau)^{24}}$$

θ_i ~ Jacobi θ-functions

η ~ Dedekind η-function.

because, it's a 1-1 map we get

$$\tau(z) = a(z) + i e^{-\phi(z)}$$

Now consider a zero of $\Delta \equiv 4f^3 + 27g^2$ i.e. the denominator of j. There will be in general $\underline{24}$ zeroes, Near every zero z_i, we have

$$j(\tau) \sim \frac{c}{(z - z_i)} \sim \frac{1}{q}$$

Now ~~j(τ)~~ ~~→ e^{2πiτ}~~ ~~as~~ $\mathrm{Im}\,\tau \to \infty$ i.e. $q = e^{2\pi i \tau} \to 0$ is mapped under $j(\tau)$ to ∞

$$\tau(z) \simeq \frac{1}{2\pi i} \log(z - z_i)$$

As we go around $(z - z_i) \to e^{2\pi i}(z - z_i)$

$$\tau \to \tau + 1 \quad \text{or} \quad a \to a + 1 \quad \phi \to \phi$$

Thus we have 24 "stringy-cosmic strings" as promised <u>except</u> that we are no in 10 dimensions instead of 4, so we really have a 7-brane instead of 1-brane (string): The extra 6-dimensions that dont participate just take 1+1 ~~string~~ 1-brane world volume to ~~?~~ 7+1 7-brane worldvolume

The metric near such a point z_i can be found explicitly, and has the form

$$ds^2 = \frac{dz\, d\bar{z}}{|z - z_i|^{1/6}} \sim r^{-\frac{1}{6}} (dr^2 + r^2 d\varphi^2)$$
$$0 \leq \varphi \leq 2\pi$$

$$r^{-2\lambda} (dr^2 + r^2 d\varphi^2) \qquad \frac{r^{1-\lambda}}{(1-\lambda)} = t \qquad dt = r^{-\lambda} dr$$

$$\Rightarrow \quad dt^2 + t^2 (1-\lambda)^2 d\varphi^2$$

$\Rightarrow$ $\varphi' = (1-\lambda)\varphi$ goes from 0 to $2\pi - 2\pi\lambda$

$\Rightarrow$ deficit angle $= 2\pi\lambda$ $\quad \lambda = +\frac{1}{12}$

$$\Rightarrow \quad \delta = \pi/6.$$

This is the general "F-theory" compactification on a K3. Note that τ is the coupling constant, ~~and~~ $\$ (e^{-\Phi})$, so we have a situation in which the coupling constant ~~moves~~ changes from place to place.

To obtain a model that can be described perturbatively, we would like to go to a special configuration of these 7-branes so that τ will be constant everywhere in space. This is achieved by choosing f^3/g^2 = constant.

$$g = \phi^3 \qquad f = \alpha\phi^2 \qquad \phi(z) = \text{polynomial of degree } 4$$

by rescaling y & x set the coefficient of $z^4 = 1$. Then, $\phi = \prod_{a=1}^{4} (z - z_a)$ (1)

$$\Delta = (4\alpha^3 + 27) \prod_{a=1}^{4} (z - z_a)^6$$

$$j(\tau) = \frac{4 \cdot (24\alpha)^3}{27 + 4\alpha^3}.$$

Thus the 24 7-branes are divided into 4 groups of 6.

τ is constant, but there is an $SL(2,\mathbb{Z})$ monodromy $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ because as z goes around z_i y changes sign $(y, x) \to (-y, x)$

This is the hyperelliptic involution of the torus

[$(\wp'(u), \wp(u)) = (y, x)$, so $y \to -y$, $x \to x \Rightarrow u \to -u$

What is F-theory dual to?

At this special point in the moduli space F-theory is nothing but T-dual of Type-I which is S-dual to Heterotic.

Thus more generally, we conclude that

F-theory on K3 $\Leftrightarrow$ Heterotic on T2

What happens when we move the D-branes away from orientifold planes?

Much more interesting.

A. Sen
hep-th/96051

We have 32 D-branes on T^2 but they are to be identified on to 16 on $T^2/\mathbb{Z}_2$

image

D-brane

Orientifold fixed plane.

$$16 \quad + \quad 4 \text{ orientifold planes.}$$

$\downarrow$ splits into $2 \cdot 4$

$$16 + 8 = 24.$$

quarks

Dyon point

monopole point

4-copies of Seiberg-Witten with SU(2) & $N_f = 4$.

To recapitulate:

We discussed F-theory on an elliptically fibered K3 surface.

Given a K3 surface that has a sphere as a base and a torus as a fiber, we can consider a compactification IIB on the sphere to 8-dimensions, such that there is a nontrivial dilaton-axion background field $\tau = a + ie^{-\phi}$ and $\tau(z)$ is given by simply the modular parameter of the torus as a function of a point z on the base.

fiber

This by-definition is F-theory on K3.

Now, one can read off enhanced gauge symmetries from the singularities of K3.

How do we see that this is dual to Heterotic on T^2.

To do this we consider a special limit of the K3

$$y^2 = x^3 + f(z)x + g(z)$$

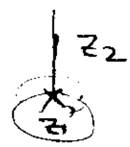
f = degree 8 $\qquad$ g = degree 12

~~consider~~ These are 24 stringy cosmic strings located at the 24 points where the torus degenerates. If they put them together in four groups of six then we get a configuration in which $\tau(z)$ does not vary with z : $\tau(z) = $ constant. ~~We have~~ ~~ϕ~~ ~~singular~~ This is achieved by choosing $g = \phi^3(z)$ $f(z) = \alpha\,\phi^2$, $\alpha =$ constant, $\phi \sim$ polynomial of degree 4.

In this limit we get K3 that is nothing but $T^4/\mathbb{Z}_2$. The base is $T^2/\mathbb{Z}_2$, Fiber T^2, the τ parameter of the fiber is constant but as we go around a fixed point on $\frac{T^2}{\mathbb{Z}_2}$ we ~~get~~ have monodromy $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \in SL(2, \mathbb{Z})$

$= \Omega(-1)^{F_L}$

$(z_1, z_2) \to (-z_1, -z_2)$



$$\frac{\text{IIB on } T_2}{\{1, \Omega(-1)^{F_L} I_1\}}$$

$I_1 : z_1 \to -z_1$

$\Omega(-1)^{F_L} \sim I_2 : z_2 \to -z_2$

This is T-dual to $\frac{\text{IIB on } T_2}{\{1, \Omega\}}$ i.e.

Type-I on T^2

Aspects of Compactifications to $D=6$ with $N=$

The dynamics of string compactifications to $D=6$ with $N=1$ supersymmetry exhibits many novel phenomena such as small instantons, phase transitions in which the number of tensor multiplets change, tensionless strings....
Kachru has discussed some of these in his lectures. These 6d singularities where something interesting happens are related to $N=2$ in $D=4$ singularities upon compactific on T^2.
Orientifolds allow us to understand some of the phases of these theories using perturbative string theory.
Thus, the strange phases of the heterotic strings, that can be understood only nonperturbatively are perturbative in the dual Type-I.

Let us review some aspects of $N=1$ susy in $d=6$. These theories are chiral ~~These~~ and potentially anomalous. Thus considerations of anomaly cancellation on supersymmetry place powerful constraints.

Let us look at the massless representation of $N=1$ in $d=6$.

The little group of a light-ray in ~~SO~~ $d=6$ is $SO(4) = SU(2) \times SU(2)$.

The allowed $N=1$ super multiplets are as follows:

1) The gravity multiplet.

graviton	$(3,3)$	gravitini
self-dual 2-form	$(1,3)$	$2(2,3)$

2) Vector Multiplet

gauge boson	$(2,2)$	gaugino	$2(1,2)$

3) Hyper multiplet

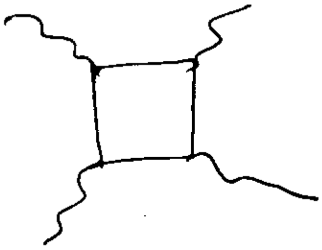
four scalars	$4(1,1)$	fermion	$2(2,1)$

4) Tensor multiplet

anti-self-dual 2-form	$(3,1)$	fermion	$2(2,1)$
scalar	$(1,1)$		

Note that gravitino and gaugino are right-handed wheras the fermions in the other 2- multiplets are left handed.

Gravitational anomalies are possible, (unliken in 4d)



$\sim \; \mathrm{Tr}\, R^4 \qquad (\mathrm{Tr}\, R^2)^2$

coefficient of $\mathrm{Tr}\, R^4$ term must vanish.

$$\boxed{n_H + n_T - n_V = 273}$$

The standard Calabi-Yau compactification to obtain $N=1$ in $d=6$ means we should consider Heterotic string compactified on a K3 surface.

This gives $n_T = 1$, and

$n_H - n_V = 272$. The precise number depe on the what vector bundle we choose, i.e. ~~how we embed~~ to the choice of the connection corresponding to the instantons.

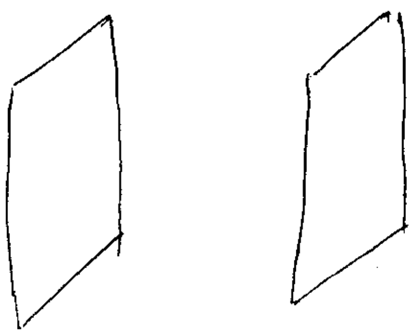
Two kinds of singularities

1) An instanton in $SO(32)$ shrinks to zero size. We can have k such small instantons ~~when~~ coincide one get nonperturb. ~~USp(2k)~~ $USp(2k)$ enhanced gauge symmetry

[~~so~~ $USp(2) = SU(2)$ $\frac{2 \cdot 3}{2} = 3$.]

In heterotic theory, this conclusion was arrived at ~~by~~ from the ADHM constructio ~~of the instanton~~.

2) An instanton in $E_8 \times E_8$ shrinks to zero size. This is harder to understand. Use the M-theory picture of $E_8 \times E_8$ str



10-plane $= R_6 \times K_3$ 10-plane $= R_6 \times K_3$

An E_8-instanton on $K3$ ~~wo~~ is a point on $K3$ but a 5 brane in the remaining noncompact directions

When it shrinks to zero size it can be emitted into the bulk as an M-theory 5-brane. An M-theory 5-brane carries a tensor multiplet. So in this way the number of tensor multiplets can change. Thus we can have theories with more than one tensor multiplet – something that seems impossible in perturbative string theory.

It would be nice to obtain models (due) that give a perturbative description of these phenomena. Some of these are given naturally as orientifolds

I) Consider Type-I theory on a K3.

Take $K3 = T4/\mathbb{Z}_2$

Then this an orientifold of IIB on T4.

The total projection is

$$\left(\frac{1+\Omega}{2}\right)\left(\frac{1+I}{2}\right)$$

I is total reflection I_{6789}

$G = \{1, I, \Omega, \Omega I\}$

1) Fixed points of I on the torus sixteen of these. each gives the twisted sector of Type-IIB.
In $d = 10$ these are really fixed 5-planes

2) Fixed planes of Ω = Orientifold 9-planes
9-planes: So we would need 32 9-bran to cancell off the charge -32 of the orientifold 9-plane.

3) Fixed planes of $I\Omega$ = orientifold 5-plane each located at the fixed point of I. ~~So we basically have~~ The total charge of the orientifolds planes is still comes out to be -32. Now we have 16 orientifold planes each with charge +2. We need 32 D5-branes.

The D-5 branes have the same charge as ~~instanton~~ small instantons.

Thus, the solitonic 5-brane in the Heterotic string with has charge $\int H$

is a Ramond-Ramond charged 5-D-brane because H comes from the RR sector.

a) When 32 9-branes coincides we get

$U(32) \xrightarrow[\text{projection?}]{\Omega} SO(32)$ $\quad \gamma_{\Omega 9}^T = \gamma_{\Omega 9}$

In general $\quad U(k) \xrightarrow{\Omega} SO(k)$.

We put $\gamma_{\Omega 9} = \mathbb{1}$,

and for the vectors,

$\psi_{-\frac{1}{2}}^{i} \, |0 \; ij\rangle \lambda_{ij} \Rightarrow \lambda = -\gamma_{\Omega 9} \lambda^T \gamma_{\Omega 9}$

Demand invariance under Ω $\Rightarrow \lambda = -\lambda^T$ i.e. $SO(k)$

b) When k 5-branes coincide, we again get U(k) group, which needs to be projected under Ω.

Once again we get

$$\lambda = -\gamma_{\Omega 5} \lambda^T \gamma_{\Omega 5}.$$

But one can show (c.f. Gimon-Polchinski) that if $\gamma_{\Omega 9}$ is symmetric, then $\gamma_{\Omega 5}$ must be antisymmetric.

$$\gamma_{\Omega 5}^T = -\gamma_{\Omega 5}.$$

By a unitary change of basis we get $\gamma_{\Omega 5} = J = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

so we need to find a subgroup of $U(K)$ that satifies,

$$\lambda = -J\lambda^T J$$

This gives ~~SO(K)~~ $USp(2K)$ gauge group.

Thus the small instanton has a very simple perturbative description as a D 5-brane in Type-I.

Now, we started with configuration on K3 of 24 instantons in Heterotic. So we can get at most 24 small instantons. In this case, SO(32) will be completely unbroken so the maximal group that we can expect is

$$SO(32) \times USp(48)$$

In the type-I description, we seem to get only 32 5-branes on ~~K3~~ T^4 which are

16 ~~under~~ on $T^4/\mathbb{Z}_2 = K3$.
~~So we seem to have~~

The 5-brane is further identified under $I\Omega$, so we really have only 8 small instantons. The Euler character is still 24. Then what happen to the remaining 16.

Ans: At each fixed point of the orbifold there is a "hidden" instanto

II) How do we describe string vacua with multiple tensor multiplets perturbatively ~ $E_8 \times E_8$ instanton shrinkin to zero size. So one can construct orientifolds with multiple tensors.
Consider IIB compactified on K3.
This gives rise to (0, 2) chiral super symmetry: There are only 2 multiplets now possible

1) $N=2$ Tensor multiplet
$= (N=1)$ Tensor $+ (N=1)$ Hyper

$B^-_{\mu\nu}$ $(3,1) + 5(1,1)$ | $4(2,1)$

Gravity multiplet:

$(3,3) + 5(1,3)$ + gravitini

i.e. $g_{\mu\nu} + 5 B^{+}_{\mu\nu}$.

It's easy to get this spectrum from IIB. Actually, the requirements of cancellation of gravitational anomaly is very restrictive. completely determines the spectrum. 21 Tensor multiplets.

Lets see in IIB on K3.

$B \; B'$ $\frac{1}{0}$

D $22 = 16_a + \underbrace{3_a + 3_s}$ Untwisted sector. $D^{+}_{\mu\nu\alpha\beta}$ self-dual.

0

1 $\uparrow$ from the fixed points twisted sectors $B_{\mu\nu}$ $B'_{\mu\nu}$

B^{+} B^{-} B'^{+} B'^{-}

$$D_{\mu\nu ij} = b^{(\alpha)}_{\mu\nu} \otimes f^{(\alpha)}_{ij}$$

$\in$ harmonic 2-for

This gives: $b^{+(\alpha)}_{\mu\nu}$ $b^{-(\alpha)}_{\mu\nu}$

3   19

$\Rightarrow$ 21 anti-self dual tensors = Tensor multiple

5 self-dual. = Gravity

Now if we consider further projection $\frac{1+\Omega}{2}$ to get Type-I, then under Ω

$D_{\mu\nu\alpha\beta}$ and $B_{\mu\nu}$ are odd

so, all $b^{\alpha\pm}$ and $B^{\pm}$ get projected out and we are left with only

B'^{+} and B'^{-}

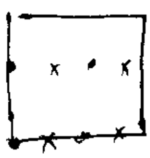
Thus $n = 1$ i.e. a single tensor multiplet. This is to be expected: Heterotic on K3 $\sim$ Type-I on K3.

<u>But</u> Now consider some symmetry S that acts on the cohomology of K3.

<u>Example</u>

$$S: \quad (z_1, z_2) \to (-z_1 + \tfrac{1}{2} \quad -z_2 + \tfrac{1}{2})$$

$$I: \quad (z_1, z_2) \longrightarrow (-z_1, \ -z_2)$$



S basically takes the 16 fixed points of I into each other

so out of the 16_a 2-forms, 8 are odd and 8 are even. So if we consider the ~~operat~~ projection,

$\frac{(1+\Omega S)}{2}$ instead of $\left(\frac{1+\Omega}{2}\right)$ we get 8 more tensors altogether $n = 9$.