UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL ATOMIC ENERGY AGENCY
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR.996 - 10

Lecture I

**SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY**

2 June - 4 July 1997

# THE STANDARD MODEL

H. HABER
Department of Physics
University of California, Santa Cruz
Santa Cruz, CA 95064
USA

Please note: These are preliminary notes intended for internal distribution only.

MAIN BUILDING STRADA COSTIERA, 11 TEL. 2240111 TELEFAX 224163 TELEX 460392 ADRIATICO GUEST HOUSE VIA GRIGNANO, 9 TEL.224241 TELEFAX 224531 TELEX 460449
MICROPROCESSOR LAB. VIA BEIRUT, 31 TEL. 2249911 TELEFAX 224600 TELEX 460392 GALILEO GUEST HOUSE VIA BEIRUT, 7 TEL.2240311 TELEFAX 2240310 TELEX 460392

# THE STANDARD MODEL

HOWARD E. HABER
16–19 JUNE 1997
ICTP, TRIESTE

## AN INTRODUCTION IN FOUR LECTURES

1. THE STRUCTURE OF SPONTANEOUSLY BROKEN NON-ABELIAN GAUGE THEORY AND APPLICATION TO $SU(2) \times U(1)$ ELECTROWEAK THEORY

2. FERMIONS AND THEIR ELECTROWEAK INTERACTIONS

3. CONSIDERATIONS BEYOND TREE-LEVEL
   - PRECISION ELECTROWEAK MEASUREMENTS
   - EFFECTIVE POTENTIAL TECHNIQUES

4. HIGGS BOSON PHYSICS

# References


Introduction to Gauge Field Theory,
by D. Bailin and A. Love

Quantum Field Theory
by Lewis H. Ryder

Gauge Theories of Elementary Particle Physics
by Ta-Pei Cheng and Ling-Fong Li

Particle Data Group summaries, Phys. Rev D
July, 1996

- Standard Model of Electroweak Interactions p. 85-93.
- The Cabibbo-Kobayashi-Maskawa Mixing Matrix p. ~~94~~-97.

# Gauge Theory Toolkit

**matter**: spin 0 $\phi_i(x)$
spin ½ $\psi_i(x)$

**symmetry group $G$** (Lie group)

$\mathcal{L}$ is invariant under

$$\phi_i(x) \to U_{ij}\,\phi_j(x) \qquad i,j = 1,2,\ldots,\dim \underset{\substack{\uparrow\\ \text{representation}\\ \text{of } G}}{R}$$

$U$ is a unitary matrix representation of $G$
global: $U$ is independent of $x$
local: $U$ depends on $x$

$$U = e^{-ig\Lambda^a T^a} \simeq 1 - ig\Lambda^a T^a$$

$T^a$: generators of the Lie algebra of $G$

$$\phi_i(x) \longrightarrow \phi_i(x) + \delta\phi_i(x),$$

$$\delta\phi_i(x) = -ig\,\Lambda^a\,T^a_{ij}\,\phi_j(x)$$

$$[T^a, T^b] = i\underset{\substack{\uparrow\\ \text{structure constants of } G.}}{f^{abc}}\,T^c$$

The $T^a$ are hermitian matrix representation of the Lie algebra of $G$.

## Aside: Facts about Lie algebras

$$[T^a, T^b] = i f^{abc} T^c \qquad \text{abstract definition}$$

A Lie algebra is a vector space. The $T^a$ are basis vectors. Arbitrary elements of the vector space are $c^a T^a$ (where $c^a$ are <u>real</u> numbers). Additional structure: the commutator of any two elements of the Lie algebra yield another element of the Lie algebra. In addition, the Jacobi identity is satisfied: $[T_a,[T_b,T_c]] + [T_b,[T_c,T_a]] + [T_c,[T_a,T_b]] = 0$.

choice of basis: $\mathrm{Tr}(T_a T_b) = T_R\,\delta_{ab}$

where $T_R$ depends on the representation. (In this basis, $f^{abc}$ is totally antisymmetric.)

basic building blocks: irreducible representations

- reducible representations are direct sums of irreducible representations

$$T^a = \begin{pmatrix} \square & & & 0 \\ & \square & & \\ & & \ddots & \\ 0 & & & \square \end{pmatrix} \qquad \text{(in some basis)}$$

<u>examples</u>:

| <u>algebra</u> ($G$) | <u>description</u> | <u>dimension of $G$</u> | <u>rank of $G$</u> |
|---|---|---|---|
| $SU(n)$ | $n\times n$ traceless hermitian | $n^2-1$ | $n-1$ |
| $SO(2n)$ | $2n\times 2n$ pure imaginary, antisymmetric | $n(2n-1)$ | $n$ |
| $SO(2n+1)$ | $(2n+1)\times(2n+1)$ pure imaginary, antisymmetric | $n(2n+1)$ | $n$ |

dimension: number of linearly independent generators

rank: maximal number of simultaneously diagonalizable generators

## examples of representations

1. Defining (or fundamental) representation
2. Adjoint representation

$$(T^a)_{bc} = -i f_{abc} \qquad a,b,c = 1,2,\dots,\dim G$$

$$\dim(\text{adjoint}) = \dim G$$

## Standard Normalization

$$\mathrm{Tr}(T_a T_b) = \tfrac{1}{2}\delta_{ab} \quad \text{in the fundamental representation}$$

i.e. $T_F = \frac{1}{2}$

**Casimir invariant:** an operator that commutes with all the generators.

Quadratic Casimir operator

$$T^a_{ik} T^a_{kj} = C_2(R)\,\delta_{ij}$$

eigenvalue of quadratic Casimir operator in representation R.

**Theorem:** $T_R\, d(G) = C_2(R)\, d(R)$

$d(G)$ = dimension of the Lie algebra
$d(R)$ = dimension of matrix representation

example:

$$\mathcal{L} = (\partial_\mu \phi_i^*)(\partial^\mu \phi_i) - V(\phi, \phi^*)$$

where $V$ is a group invariant potential:

$$V(U\phi, (U\phi)^*) = V(\phi, \phi^*)$$

is invariant under global symmetry transformations, but not under local symmetry transformations.

reason: $\partial_\mu \phi \rightarrow \partial_\mu (U\phi) = U \partial_\mu \phi + (\partial_\mu U)\phi$

covariant derivative

$$D^\mu = \partial^\mu + ig A_\mu^a T^a \qquad a = 1, 2, \ldots, \dim G$$

$A_\mu^a(x)$: gauge fields, with transformation laws chosen such that:

$$D^\mu \phi \rightarrow U D^\mu \phi$$

in which case,

$$\mathcal{L} = (D_\mu \phi_i^*)(D^\mu \phi_i) - V(\phi, \phi^*)$$

is invariant under local symmetry transformations.

matrix-valued gauge field: $A_\mu \equiv A_\mu^a T^a$

transformation law:

$$\boxed{A_\mu \longrightarrow U A_\mu U^{-1} - \frac{i}{g} U(\partial_\mu U^{-1})}$$

Using this result, it is easy to show that $D_\mu \phi$ transforms as expected:

$$D_\mu \phi = (\partial_\mu + igA_\mu)\phi$$

$$\longrightarrow [\partial_\mu + ig U A_\mu U^{-1} + U(\partial_\mu U^{-1})] U\phi$$

$$= U D_\mu \phi + [\partial_\mu U + U(\partial_\mu U^{-1}) U]\phi$$

$$= U D_\mu \phi$$

where we have used the fact that

$$\partial_\mu U + U(\partial_\mu U^{-1})U = [(\partial_\mu U)U^{-1} + U(\partial_\mu U^{-1})]U$$

$$= [\partial_\mu (U U^{-1})]U$$

$$= 0$$

since $UU^{-1} = I$.

Infinitesimal version of the transformation law: $[U \simeq 1 - ig T^a \Lambda^a(x)]$

$$\delta A_\mu^a = \underbrace{g f^{abc} \Lambda^b A_\mu^c}_{\text{transformation law for an adjoint field}} + \underbrace{\partial_\mu \Lambda^a}_{\text{inhomogeneous term}}$$

transformation law for an adjoint field: $(T^a)_{bc} = -i f^{abc}$

$$= +i f^{bac}$$

$$= D_\mu^{ab} \Lambda^b$$

## gauge field dynamics

Consider $[D_\mu, D_\nu]$ acting on $\phi$.

$$[D_\mu, D_\nu]\phi = [\partial_\mu + igA_\mu, \partial_\nu + igA_\nu]\phi$$

$$= ig\{\partial_\mu A_\nu + \partial_\nu A_\mu + ig[A_\mu, A_\nu]\}\phi$$

- **definition:** $[D_\mu, D_\nu] \equiv igF_{\mu\nu}$

where $F_{\mu\nu} \equiv F_{\mu\nu}^a T^a$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_b^\mu A_c^\nu$$

- **transformation law:**

$$\phi \to U\phi$$

$$D_\mu\phi \to UD_\mu\phi$$

$$[D_\mu, D_\nu]\phi \longrightarrow U[D_\mu, D_\nu]\phi$$

Hence,

$$F_{\mu\nu}\phi \longrightarrow UF_{\mu\nu}\phi = (UF_{\mu\nu}U^{-1})U\phi$$

i.e.

$$\boxed{F_{\mu\nu} \longrightarrow UF_{\mu\nu}U^{-1}}$$

which is the transformation law for an adjoint field.

Note: for Abelian gauge groups, $UF_{\mu\nu}U^{-1} = UU^{-1}F_{\mu\nu} = F_{\mu\nu}$ so that $F^{\mu\nu}$ is gauge invariant (i.e. neutral under the gauge group). For non-Abelian gauge group, $F_{\mu\nu}$ transforms non-trivially; i.e. it carries non-trivial gauge charge.

A gauge-invariant Lagrangian for the gauge fields:

$$\mathcal{L}_{gauge} = -\frac{1}{4T_R} \mathrm{Tr}(F_{\mu\nu} F^{\mu\nu})$$

Note: under $F_{\mu\nu} \to U F_{\mu\nu} U^{-1}$, $\mathcal{L}_{gauge} \to \mathcal{L}_{gauge}$

Using:

$$\begin{aligned} \mathrm{Tr}(F_{\mu\nu} F^{\mu\nu}) &= F_{\mu\nu}^a F^{\mu\nu b}\, \mathrm{Tr}\, T^a T^b \\ &= T_R\, F_{\mu\nu}^a F^{\mu\nu a} \end{aligned}$$

we end up with:

$$\begin{aligned} \mathcal{L}_{gauge} &= -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} \\ &= -\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c) \\ &\quad \times (\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a} - g f^{ade} A_d^\mu A_e^\nu) \end{aligned}$$

which contains three-point and four-point gauge boson interactions.

As in QED, the quadratic terms are not invertible, so we cannot define a propagator.

$$\begin{aligned} &-\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)(\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a}) \\ &\qquad = \frac{1}{2} A_\mu^a (g^{\mu\nu}\Box - \partial^\mu\partial^\nu) A_\nu^a + \text{total divergence} \end{aligned}$$

But, $(g^{\mu\nu}\Box - \partial^\mu\partial^\nu)\partial_\nu = 0$ means that $g^{\mu\nu}\Box - \partial^\mu\partial^\nu$ is not invertible.

<u>Solution:</u> add gauge-fixing term
add Faddeev-Popov ghosts $(\omega_a, \omega_a^*)$

$$\boxed{\mathcal{L}_{YM} = -\frac{1}{4} F_{\mu\nu}^{a} F^{\mu\nu a} - \frac{1}{2\xi}(\partial_\mu A^\mu_a)^2 + \partial^\mu \omega_a^* D_\mu^{ab}\omega_b}$$

is invariant under BRS extended gauge symmetry:

$$\delta A_\mu^a = \varepsilon D_\mu^{ab}\omega_b$$
$$\delta \omega_a = \frac{1}{2}\varepsilon g f^{abc}\omega_b \omega_c$$
$$\delta \omega_a^* = -\frac{1}{\xi}\varepsilon \partial_\mu A^\mu_a$$

where $\varepsilon$ is an infinitesimal anticommuting parameter and $\omega, \omega^*$ are independent anticommuting scalar fields.

## <u>SUMMARY</u>

If $\mathcal{L}_{matter}$ is invariant under a group $G$ of global transformations, then

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_{matter}(\partial_\mu \to D_\mu)$$

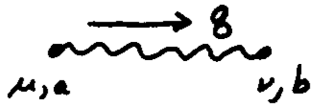
is invariant under a group $G$ of local transformations.
When $D_\mu = \partial_\mu + i g A_\mu^a T^a$ acts on matter fields, we the matrix representation $T^a$ appropriate to the matter fields.

## Are gauge bosons massless?

$$\mathcal{L}_{mass} = \tfrac{1}{2} m^2 A_\mu^a A^{\mu a}$$

violates gauge invariance, so apparently gauge bosons must be massless. Let's "prove" this to all orders in perturbation theory.

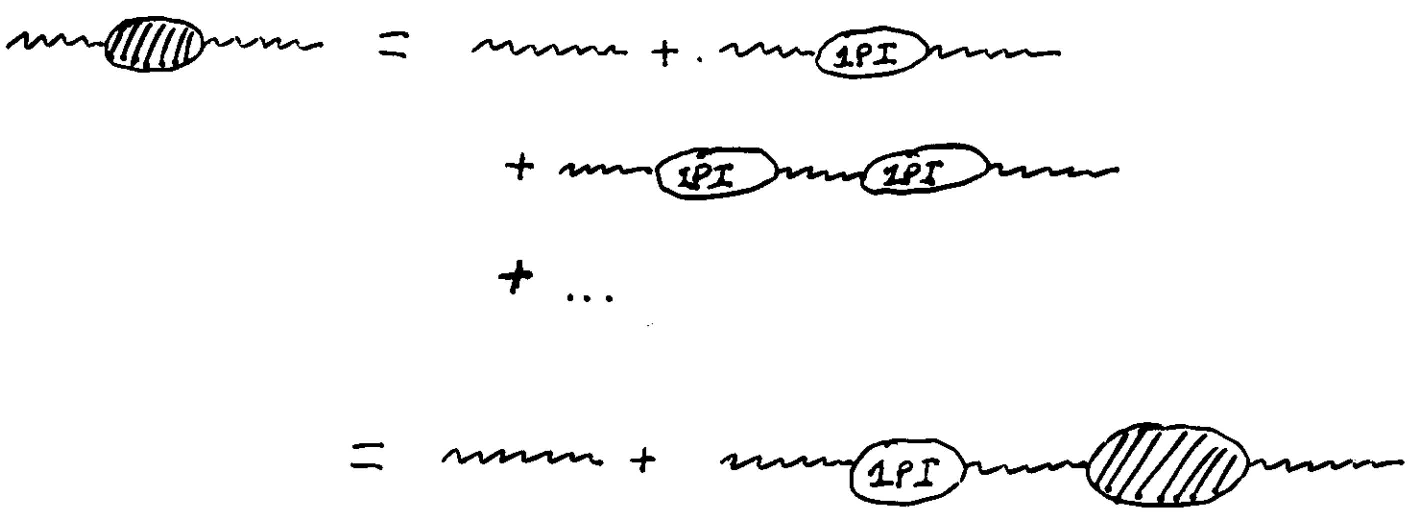
### tree-level propagator



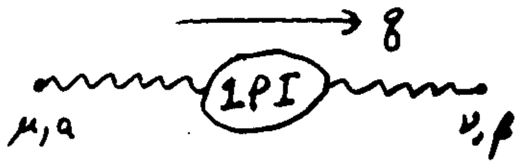

$$\frac{-i\delta^{ab}}{q^2+i\varepsilon}\left(g_{\mu\nu} - (1-\xi)\frac{q_\mu q_\nu}{q^2}\right)$$

$$= \frac{-i\delta^{ab}}{q^2+i\varepsilon}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) - \frac{i\xi\delta^{ab}}{q^2+i\varepsilon}\frac{q_\mu q_\nu}{q^2}$$

To all orders in perturbation theory,




where one-particle irreducible (1PI) graphs cannot be split into two separate graphs by cutting through one internal line.




The Ward identity of the theory (a consequence of gauge invariance) implies that

$$q_\mu \Pi^{\mu\nu} = q_\nu \Pi^{\mu\nu} = 0.$$

$$i\Pi^{\mu\nu}_{ab}(q) = -i(q^2 g^{\mu\nu} - q^\mu q^\nu)\,\Pi(q^2)\,\delta^{ab}$$

 $D^{\mu\nu}$

 $\mathcal{D}^{\mu\nu}$

$$\mathcal{D}^{\mu\nu}(q) = D^{\mu\nu}(q) + D^{\mu\lambda}(q)\, i\Pi_{\lambda\rho}(q)\,\mathcal{D}^{\rho\nu}(q)$$

Multiply on the left by $D^{-1}$ and on the right by $\mathcal{D}^{-1}$, where e.g. $D^{-1}_{\mu\lambda} D^{\lambda\nu} = g_\mu{}^\nu$.

$$\boxed{\mathcal{D}^{-1}_{\mu\nu}(q) = D^{-1}_{\mu\nu}(q) - i\Pi_{\mu\nu}(q)}$$

Decompose into transverse and longitudinal pieces:

$$D_{\mu\nu}(q) = D(q^2)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + D^{(l)}(q^2)\frac{q_\mu q_\nu}{q^2}$$

$$D^{-1}_{\mu\nu}(q) = \frac{1}{D(q^2)}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \frac{1}{D^{(l)}(q^2)}\frac{q_\mu q_\nu}{q^2}$$

etc.

Since $\Pi_{\mu\nu}(q)$ is transverse, we learn that

(a) $\quad \mathcal{D}^{(\ell)}(q^2) = D^{(\ell)}(q^2) = \dfrac{-i\xi}{q^2 + i\epsilon}$

(b) $\quad \dfrac{1}{\mathcal{D}(q^2)} = \dfrac{1}{D(q^2)} + iq^2\,\Pi(q^2)$

Using $D^{-1}(q^2) = iq^2$, we conclude that:

$$\mathcal{D}^{\mu\nu}(q) = \frac{-i}{q^2\left[1+\Pi(q^2)\right]}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) - \frac{i\xi\, q^\mu q^\nu}{q^4}$$

(omitting the $i\epsilon$ pieces for simplicity)

The pole at $q^2 = 0$ is not shifted. <u>Thus, the gauge boson mass remains zero to all orders in perturbation theory</u>.

## The loophole

If $\Pi(q^2)$ develops a pole at $q^2=0$, then the pole of $\mathcal{D}^{\mu\nu}(q)$ shifts away from zero: $\Pi(q^2) \simeq \dfrac{-m_V^2}{q^2}$ as $q^2 \to 0$ implies that $\mathcal{D}(q^2) \simeq \dfrac{-i}{q^2 - m_V^2}$. This requires some non-trivial dynamics that generates a massless intermediate state in $\Pi_{\mu\nu}(q)$:

The Standard Model employs the dynamics of elementary scalar fields in order to generate the Goldstone mode.

<u>Goldstone's Theorem</u>: If the Lagrangian is invariant under a continuous global symmetry group $G$, but the vacuum state of the theory does not respect all $G$-transformations, then the theory exhibits spontaneous symmetry breaking. If the vacuum state respects $H$-transformations, then we say that the group $G$ breaks to a subgroup $H$. The physical spectrum will then contain $n$ massless scalar excitations (called Goldstone bosons) where

$$n \equiv \dim G - \dim H$$

This theorem is proved independent of perturbation theory.

## A tree-level example

Let $\phi_i(x)$ be a set of real scalar fields. The Lagrangian

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi_i)(\partial^\mu \phi_i) - V(\phi_i)$$

is assumed to be invariant under

$$\delta \phi_i(x) = -ig\, T^a_{ij} \Lambda^a \phi_j(x)$$

where the $T^a$ are imaginary antisymmetric matrices.
Since under the symmetry transformation, $\phi_i \to \mathcal{O}_{ij}\phi_j$ (where $\mathcal{O}$ is orthogonal), the kinetic energy term is automatically invariant.

We also assume that $V(\phi)$ is invariant, which implies that:

$$V(\phi+\delta\phi) = V(\phi) \qquad \text{invariance}$$

$$\simeq V(\phi) + \frac{\partial V}{\partial \phi_i}\delta\phi_i \qquad \text{Taylor expansion}$$

Since $\delta\phi_i = -ig\, T^a_{ij}\Lambda^a \phi_j(x)$, it follows that:

$$\boxed{\frac{\partial V}{\partial \phi_i} T^a_{ij}\phi_j = 0}$$

Suppose $V(\phi)$ has a minimum at $\langle\phi_i\rangle = v_i$ which is not invariant under the global symmetry group:

$$\mathcal{O}_{ij} v_j \simeq \left(\delta_{ij} - ig\, T^a_{ij}\Lambda^a\right) v_j \neq v_i$$

i.e. there exists at least one a such that:

$$T^a v \neq 0$$

The symmetry is spontaneously broken. Now, shift the field:

$$\phi_i \equiv \eta_i + v_i$$

and express $\mathcal{L}$ in terms of the $\eta_i$:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta_i)(\partial^\mu \eta_i) - \frac{1}{2} M^2_{ij}\eta_i\eta_j + \dots$$

$$M^2_{ij} \equiv \left(\frac{\partial^2 V}{\partial\phi_i \partial\phi_j}\right)_{\phi_i = v_i}$$

Since $V$ must satisfy: $\frac{\partial V}{\partial \phi_i} T^a_{ij} \phi_j = 0$

we can differentiate with respect to $\phi_k$ and then set all $\phi_i = v_i$.
By definition, $\left(\frac{\partial V}{\partial \phi_i}\right)_{\phi_i = v_i} = 0$, so

$$M^a_{ki} (T^a v)_i = 0$$

Either $T^a v = 0$ or $(T^a v)_i$ is an eigenvector of $M^2$ with zero eigenvalue.

$$G^a = \eta_i T^a_{ij} v_j$$

is precisely the linear combination of scalar fields whose mass is zero. These are the Goldstone bosons, and there are $\dim(G) - \dim(H)$ independent Goldstone modes, where $H$ is the residual symmetry group (corresponding to the maximal number of linearly independent elements of the Lie algebra that annihilate the vacuum).

# Spontaneously broken non-abelian gauge theory – the recipe

Consider a locally gauge invariant Yang-Mills theory coupled to some representation (perhaps reducible) of scalar fields. Without loss of generality, we can assume that the scalar fields are real.

For example, a theory of one complex scalar $\phi(x) = \frac{1}{\sqrt{2}}(\phi_1(x) + i\phi_2(x))$ is equivalent to a theory of two real scalars $\phi_1(x)$, $\phi_2(x)$.

Having chosen to work in a "real representation", the generators that act on these scalars must be pure imaginary antisymmetric matrices. That is, $iT^a$ are real and antisymmetric.

If the gauge group is a product

$$G = G_1 \times G_2 \times \ldots$$

where the $G_i$ are either simple Lie groups or $U(1)$, then the covariant derivative acting on the scalar fields takes the form:

$$(D_\mu)_{ij} = \delta_{ij}\partial_\mu + i g^a T^a_{ij} A^a_\mu$$

There is an implicit sum over a. Here $g^a$ means take the gauge coupling appropriate to which $G_i$ the generator $T^a$ belongs.

<u>Notation:</u> $L^a \equiv i g^a T^a$ (no sum over a)

Then, $D_\mu = \partial_\mu + L^a A^a_\mu$

The recipe – Step 3

- Determine the most general scalar potential that is invariant under $G$-transformations (consistent with other global discrete and/or continuous symmetries if imposed).

- Determine a plausable minimum at $\phi_i = v_i$. Then shift:

$$\phi_i = \eta_i + v_i$$

such that $\langle \eta_i \rangle = 0$.

- Check that the desired minimum is an actual local minimum.
  e.g. all squared-masses of physical Higgs scalars are non-negative.

- Verify that the minimum corresponds to some finite range of the parameters of the scalar potential.

The scalar part of the Lagrangian is:

$$\mathcal{L} = \frac{1}{2}(D_\mu \phi_i)(D^\mu \phi_i) - V(\phi)$$

Inserting $D_\mu = \partial_\mu + L^a A^a_\mu$, study the terms quadratic and linear in $A^a_\mu$. When we shift the scalar field: $\phi_i = \eta_i + v_i$, we find:

$$\mathcal{L}_{mass} = \frac{1}{2} M^2_{ab} A^a_\mu A^{\mu b}$$

with

$$M^2_{ab} = (L_a v, L_b v)$$

notation: $(x,y) \equiv \sum_i x_i y_i$

If $L_a v \neq 0$ for at least one $a$, then the symmetry is broken and the gauge bosons acquire mass. Diagonalize the gauge boson mass matrix:

$$\mathcal{O} M^2 \mathcal{O}^T = \mathrm{diag}(0, 0, \ldots, 0, m_1^2, m_2^2, \ldots)$$

If we define:

$$\boxed{\tilde{L}_a \equiv \mathcal{O}_{ab} L_b}$$

then:

$$(\mathcal{O} M^2 \mathcal{O}^T)_{ab} = (\tilde{L}_a v, \tilde{L}_b v) = m_a^2 \delta_{ab}$$

$m_a^2 = 0$, $\tilde{L}_a v = 0$ $\quad a = 1, \ldots, M$ $\quad$ unbroken generators

$m_a^2 \neq 0$, $\tilde{L}_a v \neq 0$ $\quad a = M+1, \ldots, N$ $\quad$ broken generators

$M = \dim H$
$N = \dim G$

The number of massive gauge bosons is equal to $\dim(G) - \dim(H)$.

## The recipe – Step 2

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi_i + L^a_{ij}A^a_\mu\phi_j)(\partial^\mu\phi_i + L^b_{ik}A^b_\mu\phi_k) - V(\phi)$$

$$= \frac{1}{2}(\partial_\mu\phi_i)(\partial^\mu\phi_i) + \frac{1}{2}M^2_{ab}A^a_\mu A^{\mu b}$$

$$+ \frac{1}{2}(L^a A^a_\mu v, \partial^\mu\eta) + \frac{1}{2}(\partial_\mu\eta, L^a A^a_\mu v) + \ldots$$

Define: $$\boxed{\tilde{A}^a_\mu \equiv \theta_{ab}A^b_\mu}$$

so that $$\tilde{L}^a\tilde{A}^a_\mu = \underbrace{\theta_{ac}\theta_{ab}}_{\delta_{cb}} L^c A^b_\mu = L^c A^c_\mu$$

Then,

$$\frac{1}{2}(L^a A^a_\mu v, \partial^\mu\eta) + h.c. = \frac{1}{2}\tilde{A}^a_\mu\partial^\mu(\tilde{L}^a v, \eta) + h.c.$$

which vanishes unless $\tilde{L}^a v \neq 0$.

Define: $$\boxed{G^a = \frac{1}{m_a}(\tilde{L}^a v, \eta)}$$

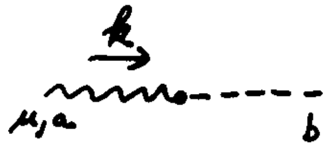
where $m_a$ is the corresponding gauge boson mass.

We recognize $G^a$ as the Goldstone boson in the globally symmetric theory without gauge bosons.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi_i)(\partial^\mu\phi_i) + \frac{1}{2}M^2_{ab}A^a_\mu A^{\mu b} + m_a\tilde{A}^a_\mu\partial^\mu G^a + \ldots$$

The coupling $m_a A^a_\mu \partial^\mu G^a$ explains the mass generation mechanism.

Feynman rule:



$m_a k^\mu \delta^{ab}$

Then, we evaluate the contribution to $i\Pi^{\mu\nu}(k)$:

$$i\Pi^{\mu\nu}(k) = m_a^2 k^\mu(-k^\nu)\frac{i}{k^2} + \ldots$$

$$= -i m_a^2 \frac{k^\mu k^\nu}{k^2} + \ldots$$

This is the only source for a pole at $k^2=0$ at this order in perturbation theory. Gauge invariance ensures that

$$i\Pi^{\mu\nu}(k) = i(k^\mu k^\nu - k^2 g^{\mu\nu})\,\Pi(k^2)$$

so that:

$$\Pi(k^2) \simeq \frac{-m_a^2}{k^2}$$

and

$$D(k^2) = \frac{i}{k^2[1+\Pi(k^2)]} = \frac{i}{k^2 - m_a^2}$$

We say that the gauge boson "eats" or absorbs the corresponding Goldstone boson and thereby acquires mass via the <u>Higgs mechanism</u>.

To show this interpretation is correct, we demonstrate that the Goldstone bosons are gauge artifacts that can be removed by a gauge transformation.

From $\delta\phi_i = -ig T^a_{ij} \Lambda^a \phi_j = -L^a_{ij}\Lambda^a\phi_j$

and using $\phi_i = \eta_i + v_i$ with $\delta v_i = 0$,

$$\begin{aligned}\delta\eta_i &= -L^a\Lambda^a(\eta + v)\\ &= -(\Lambda\eta + \Lambda v)\end{aligned}$$

where $\Lambda = L^a\Lambda^a = \tilde{L}^a\tilde{\Lambda}^a$ if we define $\tilde{\Lambda}_a = \mathcal{O}_{ab}\Lambda_b$.

Then,

$$\begin{aligned}\delta G^a &= \frac{1}{m_a}(\tilde{L}^a v, \delta\eta) = \frac{-1}{m_a}(\tilde{L}^a v, \Lambda\eta + \Lambda v)\\ &= -\frac{1}{m_a}(\tilde{L}^a v, \Lambda\eta) - \frac{1}{m_a}\underbrace{(\tilde{L}^a v, \tilde{L}^b v)}_{m_a^2\delta^{ab}}\tilde{\Lambda}_b\\ &= -\frac{1}{m_a}(\tilde{L}^a v, \Lambda\eta) - m_a\tilde{\Lambda}_a\end{aligned}$$

inhomogeneous term independent of $\eta$.

Thus, I can choose $\tilde{\Lambda}_a$ such that $G^a = 0$. This is called the <u>unitary gauge</u>.

# $R_\xi$ - gauge

Instead of fixing the gauge by setting $G^a = 0$, it is sometimes more convenient to add a gauge-fixing term to the Lagrangian. The $R_\xi$ gauge corresponds to adding

$$\mathcal{L}_{GF} = -\frac{1}{2\xi}\left(\partial^\mu \tilde{A}^a_\mu - \xi \eta_i (\tilde{L}^a v)_i\right)^2$$

$$= -\frac{1}{2\xi}(\partial^\mu \tilde{A}^a_\mu)^2 + \frac{1}{2}\left((\tilde{L}^a \partial^\mu \tilde{A}^a_\mu) v, \eta\right)$$

$$+ \frac{1}{2}(\eta, \tilde{L}^a(\partial^\mu \tilde{A}^a_\mu) v) - \xi\left[\eta_i (\tilde{L}^a v)_i\right]^2$$

$$= -\frac{1}{2\xi}(\partial^\mu \tilde{A}^a_\mu)^2 + \underbrace{m_a G^a(\partial^\mu \tilde{A}^a_\mu)} - \frac{\xi m_a^2}{2} G^a G^a$$

This piece combines with $m_a \tilde{A}^a_\mu \partial^\mu G$ to yield

$$m_a\left[G^a \partial^\mu \tilde{A}^a_\mu + \tilde{A}^a_\mu \partial^\mu G^a\right]$$

$$= m_a \partial^\mu (G^a \tilde{A}^a_\mu)$$

which is a total divergence which can be dropped from the Lagrangian

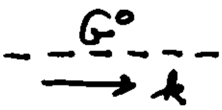
In the $R_\xi$ gauge, we find the following propagators:


$$\frac{i}{k^2+i\varepsilon}\left[-g_{\mu\nu}+(1-\xi_\gamma)\frac{k_\mu k_\nu}{k^2}\right]$$


$(V=W,Z)$

$$\frac{i}{k^2-m_V^2+i\varepsilon}\left[-g_{\mu\nu}+(1-\xi)\frac{k_\mu k_\nu}{k^2-\xi m_V^2}\right]$$


$$\frac{i}{k^2-\xi m_W^2+i\varepsilon}$$




$$\frac{i}{k^2-\xi m_Z^2+i\varepsilon}$$

Note: in principle $\xi_\gamma \neq \xi$ is possible since for $\tilde{L}^a v = 0$ we are free to choose an arbitrary $\xi$-parameter.

$\xi \to \infty$ limit returns you to the unitary gauge.

In addition, one must now include all interaction terms involving $G^\pm$ and/or $G^0$. Finally, one must also include Faddeev-Popov ghosts:

$$\mathcal{L}_{FP} = \partial^\mu \omega_a^* \underbrace{D_\mu^{ab}\omega_b} - \xi\omega_a^* M_{ab}^2 \omega_b - \xi g^2 \omega_a^* \omega_b (\eta, T^a T^b v)$$

gauge boson squared-mass matrix

$$D_\mu^{ab} = \delta^{ab}\partial_\mu + g f^{abc} A_\mu^c$$

# The recipe - Step 3

The original model consisted of:

gauge boson degrees of freedom: $2(\dim G)$

scalar degrees of freedom: $\dim R$

After the Higgs mechanism,

gauge boson degrees of freedom:

massless: $2(\dim H)$

massive: $3(\dim G - \dim H)$

scalar degrees of freedom

Goldstone: removed

physical Higgs bosons: $\dim R - (\dim G - \dim H)$

total: $2(\dim G) + \dim R$

Let's check that the physical Higgs bosons cannot be removed by a gauge transformation. We divide the scalars into ~~the~~ two classes:

(i) $G^a = \frac{1}{m_a}(\tilde{L}^a v)_j \eta_j$ $\quad a = M+1, \ldots, N$ $\quad$ Goldstone bosons

(ii) $\tilde{H}_k = c_j^{(k)} \eta_j$ $\quad$ physical Higgs bosons

where $\sum_j c_j^{(k)} (\tilde{L}^a v)_j = 0$ $\quad$ [orthogonality]

Note that under a gauge transformation,

$$\delta\left(c_j^{(k)}\eta_j\right) = c_j^{(k)}\delta\eta_j$$
$$= -c_j^{(k)}(\Lambda\eta + \Lambda v)_j \longleftarrow \quad c_j(\Lambda v)_j = c_j\tilde{\Lambda}^a(\tilde{L}^a v)_j$$
$$= 0$$

by orthogonality.

$$= -c_j^{(k)}(\Lambda\eta)_j$$

which is homogeneous, so that $\eta_j$ cannot be removed by a gauge transformation

## Physical Higgs mass matrix

$\{G^a, \tilde{H}_k\}$ provide a basis for the scalars

$$M^2_{ij} = \left(\frac{\partial^2 V}{\partial\varphi_i\,\partial\varphi_j}\right)_{\phi_i = v}$$

satisfies $(M^2)_{ij}(\tilde{L}^a v)_j = 0$. Thus, the quadratic terms in the scalar part of the Lagrangian is:

$$\mathcal{L}_{\text{scalar mass}} = -\tfrac{1}{2}(\mathcal{M}^2)_{k\ell}\,\tilde{H}_k\tilde{H}_\ell$$

Diagonalizing $\mathcal{M}^2$ yields the Higgs boson eigenstates and their respective squared-masses.

# $SU(2)_L \times U(1)_Y$ model of electroweak interactions

## gauge sector

| group | field | coupling | generator (in fundamental representation) |
|---|---|---|---|
| $SU(2)_L$ | $W_\mu^a$ | $g$ | $\frac{1}{2}\sigma^a$ |
| $U(1)_Y$ | $B$ | $g'$ | $\frac{1}{2}Y$ |

commutation relations

$$\left[\tfrac{1}{2}\sigma^a, \tfrac{1}{2}\sigma^b\right] = i\varepsilon^{abc}\left(\tfrac{1}{2}\sigma^c\right) \qquad a,b,c = 1,\dots,3$$

$$\left[\tfrac{1}{2}\sigma^a, \tfrac{1}{2}Y\right] = \left[\tfrac{1}{2}Y, \tfrac{1}{2}Y\right] = 0$$

## scalar sector

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \longrightarrow \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}$$

real representation.

Introduce a complex doublet of Higgs scalars

$$D_\mu \Phi_i = \partial_\mu \Phi_i + L^a_{ij} W^a_\mu \Phi_j + L^4_{ij} B_\mu \Phi_j$$

$$L^a_{ij} = \tfrac{1}{2} i g \sigma^a_{ij}$$

$$L^4_{ij} = \tfrac{1}{2} i g' Y$$

To work in the real representation, we need explicit forms for the generators:

$$L^1 = \frac{g}{2}\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \qquad L^2 = \frac{g}{2}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$$L^3 = \frac{g}{2}\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \qquad L^4 = \frac{g'}{2}\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

<u>examples:</u>

$$L^1\Phi = \frac{ig}{2\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} = \frac{g}{2\sqrt{2}}\begin{pmatrix} -\phi_4 + i\phi_3 \\ -\phi_2 + i\phi_1 \end{pmatrix}$$

which means that

$$L^1\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix} = \frac{g}{2}\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Similarly,

$$L^4\Phi = \frac{ig'}{g} Y \Phi = \frac{ig'}{2}\Phi = \frac{g'}{2}\begin{pmatrix} -\phi_2 + i\phi_1 \\ -\phi_4 + i\phi_3 \end{pmatrix}$$

etc.

<u>Note</u>: the $L^a$ are real antisymmetric matrices as expected.

## Step 0: Determine the scalar vacuum expectation values.

most general quartic (hence, renormalizable) scalar potential that is invariant under $SU(2)_L \times U(1)_Y$ transformations:

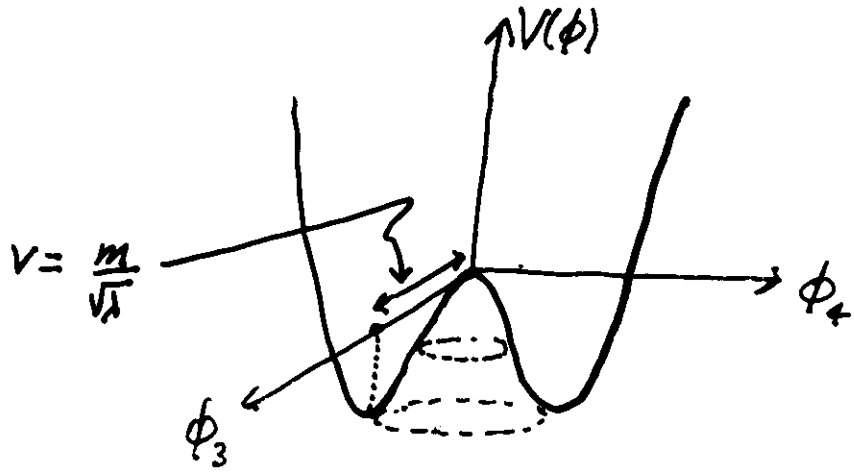
$$V(\Phi) = -m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$
$$= -\tfrac{1}{2} m^2 (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) + \tfrac{1}{4} \lambda (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)^2$$

$$\frac{\partial V}{\partial \phi_i} = 0 = m^2 \phi_i + \lambda \phi_i (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = 0$$

solutions:

(a) symmetric $\qquad \phi_i = 0$

(b) asymmetric $\qquad \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = \dfrac{m^2}{\lambda}$




Convention: $\quad v = \langle \phi_3 \rangle, \quad \langle \phi_1 \rangle = \langle \phi_2 \rangle = \langle \phi_4 \rangle = 0$

i.e. $\quad \vec{v} = \begin{pmatrix} 0 \\ 0 \\ v \\ 0 \end{pmatrix}$

**Step 1**: Gauge boson mass matrix and determination of the $\tilde{L}^a$.

$$M^2_{ab} = (L^a v, L^b v) \qquad v = \begin{pmatrix} 0 \\ 0 \\ v \\ 0 \end{pmatrix}$$

$$M^2_{ab} = \begin{pmatrix} \frac{1}{4}g^2v^2 & 0 & 0 & 0 \\ 0 & \frac{1}{4}g^2v^2 & 0 & 0 \\ 0 & 0 & \frac{1}{4}g^2v^2 & -\frac{1}{4}gg'v^2 \\ 0 & 0 & -\frac{1}{4}gg'v^2 & \frac{1}{4}g'^2v^2 \end{pmatrix}$$

Diagonalization of $\frac{1}{4}v^2\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}$ can be done by inspection.

| eigenvector | eigenvalue | state |
|---|---|---|
| $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ | $m_W^2 = \frac{1}{4}v^2g^2$ | $W^1$ |
| $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ | $m_W^2 = \frac{1}{4}v^2g^2$ | $W^2$ |
| $\frac{1}{\sqrt{g^2+g'^2}}\begin{pmatrix} 0 \\ 0 \\ g' \\ g \end{pmatrix}$ | $m_\gamma^2 = 0$ | $\gamma$ |
| $\frac{1}{\sqrt{g^2+g'^2}}\begin{pmatrix} 0 \\ 0 \\ g \\ -g' \end{pmatrix}$ | $m_Z^2 = \frac{1}{4}v^2(g^2+g'^2)$ | $Z$ |

Orthogonal diagonalizing matrix (lower 2×2 block)

$$\mathcal{O} = \frac{1}{\sqrt{g^2+g'^2}} \begin{pmatrix} g & -g' \\ g' & g \end{pmatrix} \equiv \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix}$$

where $\sin\theta_W = \dfrac{g'}{\sqrt{g^2+g'^2}}$

## W-Z mass relation

(depends critically on choice of scalar representation)

$$m_W^2 = \tfrac{1}{4} g^2 v^2$$
$$m_Z^2 = \tfrac{1}{4}(g^2+g'^2)v^2$$

$$\boxed{v = 246\,\mathrm{GeV}}$$

$$\Rightarrow \boxed{\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1}$$

The $\tilde{L}_a$ are given by:

$$\tilde{L}_1 = L_1$$
$$\tilde{L}_2 = L_2$$
$$\tilde{L}_3 = L_3 \cos\theta_W - L_4 \sin\theta_W$$
$$\tilde{L}_4 = L_3 \sin\theta_W + L_4 \cos\theta_W = \frac{gg'}{\sqrt{g^2+g'^2}} \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \equiv ieQ$$

$$\boxed{e = \frac{gg'}{\sqrt{g^2+g'^2}} = g\sin\theta_W = g'\cos\theta_W}$$

Note that $L_a v \neq 0 \quad a=1,2,3$

$\tilde{L}_4 v = 0$

Thus, we identify $\tilde{L}_4$ as proportional to the electric charge operator $Q$. So, $SU(2)_L \times U(1)_Y$ has broken down to $U(1)_{EM}$.

To verify this interpretation, examine the covariant derivative:

$$D_\mu = \partial_\mu + L^a W_\mu^a + L^4 B_\mu$$
$$= \partial_\mu + \tilde{L}^1 W_\mu^1 + \tilde{L}^2 W_\mu^2 + \tilde{L}_3 Z_\mu + \tilde{L}_4 A_\mu$$

$A_\mu = W_\mu^3 \sin\theta_W + B_\mu \cos\theta_W$ (massless photon)

$Z_\mu = W_\mu^3 \cos\theta_W - B_\mu \sin\theta_W$

which confirms the identification of $\tilde{L}_4 = ieQ$.

<u>Diagonal" generators</u> ["diagonal": a basis exists where these generators are diagonal]

$SU(2) \times U(1)$ has two diagonal generators: $T^3$ and $\frac{Y}{2}$.

Since $L^3 = igT^3$ and $L^4 = ig'\frac{Y}{2}$, it is easy to work out:

$$\tilde{L}_3 = \frac{ig}{\cos\theta_W}\left[T^3 - Q\sin^2\theta_W\right]$$

$$\tilde{L}_4 = ie\left[T^3 + \frac{Y}{2}\right] = ieQ$$

i.e. $\boxed{Q = T^3 + \frac{Y}{2}}$

## Charged $W^\pm$ eigenstates

$W^a$ are $SU(2)$-adjoint gauge fields with $Y=0$. The charge operator acting on $W^a$ is $Q=T^3$.

$$(T^a)_{bc} = -i\varepsilon_{abc} \qquad \text{for } SU(2)\text{-adjoint representation}$$

So,

$$Q = T^3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In particular,

$$Q \begin{pmatrix} W_\mu^1 \\ W_\mu^2 \\ W_\mu^3 \end{pmatrix} = \begin{pmatrix} -iW_\mu^2 \\ iW_\mu^1 \\ 0 \end{pmatrix}$$

If we define:

$$\boxed{W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right)}$$

then $QW_\mu^\pm = \pm W_\mu^\pm$
$QW_\mu^3 = 0$

**Summary:**

$$\boxed{\begin{aligned} D^\mu = \partial^\mu &+ \frac{ig}{\sqrt{2}}\left(T^+ W_\mu^+ + T^- W_\mu^-\right) \\ &+ \frac{ig}{\cos\theta_W}\left(T^3 - Q\sin^2\theta_W\right) Z_\mu + ieQA_\mu \end{aligned}}$$

where $T^\pm = T^1 \pm iT^2$.

## Step 2: Identifying the Goldstone bosons and physical Higgs scalars

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 + v \\ \eta_4 \end{pmatrix}$$

$$G^a = \left(\frac{\tilde{L}^a v}{m_a}\right)_i \eta_i$$

i.e. $G^1 = \eta_1$ $\qquad G^{\pm} = \frac{1}{\sqrt{2}}(G_1 \pm i G_2)$

$G^2 = \eta_2$ $\qquad G^0 = -G^3$

$G^3 = -\eta_4$

One scalar degree of freedom left: the Higgs boson

$$H^0 = \eta_3$$

## Step 3: The physical Higgs mass

Go to the unitary gauge by setting $G^a = 0$. Then,

$$V(H) = -\frac{m^2}{2}(H+v)^2 + \frac{\lambda}{4}(H+v)^4 \qquad \text{where } v^2 = \frac{m^2}{\lambda}$$

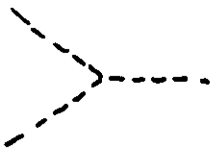
$$= \text{constant} + \frac{1}{2}m_H^2 H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4$$

where $\boxed{m_H^2 = 2\lambda v^2}$

# Unitary gauge Feynman rules

## ① Scalar interactions

Feynman rules are read off $i\mathcal{L} = -iV + \ldots$



$$-(3!)i\lambda v = \frac{-3ig\,m_H^2}{2m_W}$$

$$-(4!)\frac{i\lambda}{4} = \frac{-3ig^2 m_H^2}{4m_W^2}$$

## ② Scalar-vector interactions

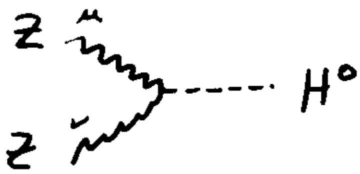
Obtained from $\frac{1}{2}(D_\mu\phi_i)(D^\mu\phi_i)$, with $\phi_i = \begin{pmatrix} 0 \\ 0 \\ v+H \\ 0 \end{pmatrix}$

$$D_\mu \begin{pmatrix} 0 \\ 0 \\ v+H \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{ig}{2\sqrt{2}}(W_\mu^+ - W_\mu^-)(v+H) \\ \frac{g}{2\sqrt{2}}(W_\mu^+ + W_\mu^-)(v+H) \\ \partial_\mu H \\ \frac{-g}{2\cos\theta_W} Z_\mu (v+H) \end{pmatrix}$$

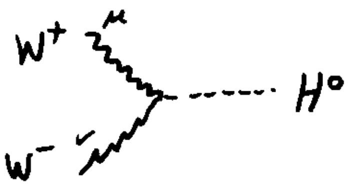
$$\frac{1}{2}(D_\mu \Phi_i)(D^\mu \Phi_i) = \frac{1}{2}(\partial_\mu H)^2 + \frac{g^2}{8\cos^2\theta_W}(v^2 + 2vH + H^2) Z_\mu Z^\mu$$

$$+ \frac{1}{4} g^2 (v^2 + 2vH + H^2) W_\mu^+ W^{\mu -}$$

which yields $m_W^2 = m_Z^2 \cos^2\theta_W = \frac{1}{4} g^2 v^2$ and:




$$\frac{i g m_Z}{\cos\theta_W} g_{\mu\nu}$$




$$i g m_W g_{\mu\nu}$$


$$\frac{i g^2}{2\cos^2\theta_W} g_{\mu\nu}$$

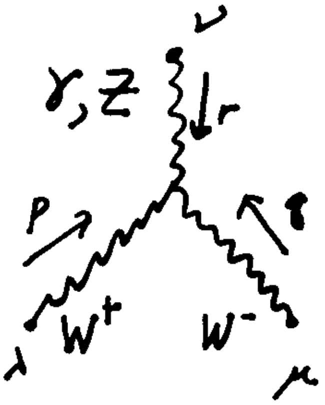

$$\frac{i g^2}{2} g_{\mu\nu}$$

## ③ Vector boson self-interactions

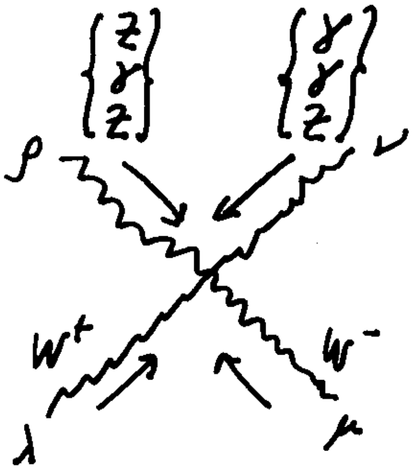
Obtained from

$$\mathcal{L}_{gauge} = -\frac{1}{4} W_{\mu\nu}^a W^{\mu\nu a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

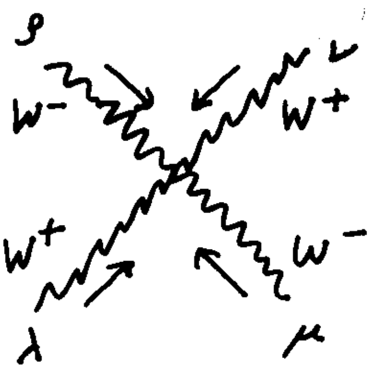
$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g\,\varepsilon^{abc} W_\mu^b W_\nu^c$$




$$-i\begin{Bmatrix} e \\ g\cos\theta_W\end{Bmatrix}\left[(r-q)_\lambda g_{\mu\nu} + (q-p)_\nu g_{\lambda\mu} + (p-r)_\mu g_{\nu\lambda}\right]$$




$$-i\begin{Bmatrix} eg\cos\theta_W \\ e^2 \\ g^2\cos^2\theta_W\end{Bmatrix}\left[2g_{\nu\rho}g_{\mu\lambda} - g_{\mu\rho}g_{\nu\lambda} - g_{\mu\nu}g_{\lambda\rho}\right]$$




$$ig^2\left(2g_{\lambda\nu}g_{\mu\rho} - g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho}\right)$$