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SMR.998a - 11

Research Workshop on Condensed Matter Physics
30 June - 22 August 1997
MINIWORKSHOP ON
QUANTUM MONTE CARLO SIMULATIONS OF LIQUIDS AND SOLIDS
30 JUNE - 11 JULY 1997
and
CONFERENCE ON
QUANTUM SOLIDS AND POLARIZED SYSTEMS
3 - 5 JULY 1997

"Spin diffusion in polarized fermi gases and solids"

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These are preliminary lecture notes, intended only for distribution to participants.

Spin Diffusion in Polarized Fermi Gases and Solids

W. J. Mullin, University of Massachusetts

Coworkers: Don Candela, Jong Jeon, Brian Cowan, Robert Ragan, Erik Nelson

•Why study polarized gases?

Example of polarization effects in transport:

✓Thermal conductivity

$\kappa = \rho\, v\, \lambda\, c_V$, mean-free path $\lambda \sim 1/\sigma$

low T: $\sigma_{+-} >> \sigma_{++}$ by Pauli principle

experiment: Leduc et al, ENS

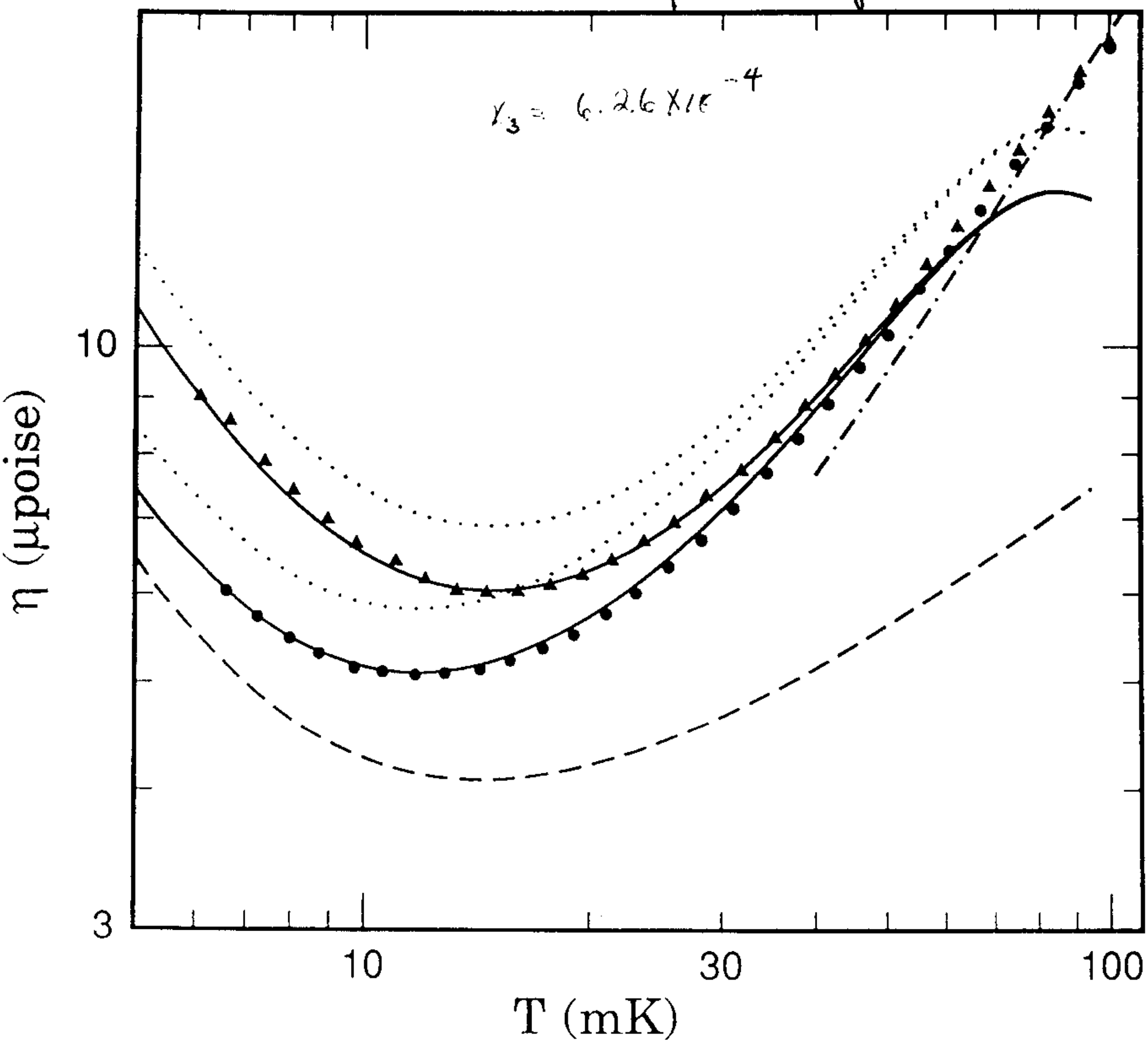
✓Viscosity similar enhancement by polarization

experiments: Owers-Bradley et al, Nottingham
Candela et al, UMass

So... polarized systems provide a probe of quantum mechanics

Candela et al

Dilute Solutions of ^{3}He in liquid ^{4}He



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History

1876: Boltzmann transport equation

$$\frac{\partial n_{p_1}(\mathbf{r},t)}{\partial t} + \frac{\partial n_{p_1}(\mathbf{r},t)}{\partial \mathbf{r}} \mathbf{v} - \frac{\partial n_{p_1}(\mathbf{r},t)}{\partial \mathbf{p}} \frac{\partial \phi}{\partial \mathbf{r}} =$$

$$= 4\pi \sum_{p_2,p_3,p_4} \delta(E_1+E_2-E_3-E_4)\, |\langle \mathbf{p}_1\mathbf{p}_2|T|\mathbf{p}_3\mathbf{p}_4\rangle|^2$$

$$(n_{\mathbf{p}_3}n_{\mathbf{p}_4} - n_{\mathbf{p}_1}n_{\mathbf{p}_2}) \approx -\frac{\delta n_{p_1}}{\tau}$$

1956 Landau Equation for Fermi Liquids:

$$\frac{\partial n_{\mathbf{p}_1}(\mathbf{r},t)}{\partial t} + \frac{\partial n_{\mathbf{p}_1}(\mathbf{r},t)}{\partial \mathbf{r}} \frac{\partial \varepsilon_{\mathbf{p}_1\sigma}}{\partial \mathbf{p}} - \frac{\partial n_{\mathbf{p}_1}(\mathbf{r},t)}{\partial \mathbf{p}} \frac{\partial \varepsilon_{\mathbf{p}_1\sigma}}{\partial \mathbf{r}} =$$

Mean-field terms

$$= 4\pi \sum_{p_2,p_3,p_4} \delta(\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4)\Big\{ \sum_{\sigma\neq\sigma'} |\langle \mathbf{p}_1\mathbf{p}_2|T|\mathbf{p}_3\mathbf{p}_4\rangle|^2$$

final state factors

↑↓ scatt.

$$[n_{\mathbf{p}_{3\sigma}}n_{\mathbf{p}_{4\sigma'}}(1-n_{\mathbf{p}_{1\sigma}})(1-n_{\mathbf{p}_{2\sigma'}}) - n_{\mathbf{p}_{1\sigma}}n_{\mathbf{p}_{2\sigma'}}(1-n_{\mathbf{p}_{3\sigma}})(1-n_{\mathbf{p}_{4\sigma'}})] +$$

$$+[\sum_{\sigma} |\langle \mathbf{p}_1\mathbf{p}_2|T|\mathbf{p}_3\mathbf{p}_4\rangle|^2 - \langle \mathbf{p}_1\mathbf{p}_2|T|\mathbf{p}_3\mathbf{p}_4\rangle\langle \mathbf{p}_1\mathbf{p}_2|T|\mathbf{p}_4\mathbf{p}_3\rangle]$$

↑↑ or ↓↓ scattering

$$[n_{\mathbf{p}_{3\sigma}}n_{\mathbf{p}_{4\sigma}}(1-n_{\mathbf{p}_{1\sigma}})(1-n_{\mathbf{p}_{2\sigma}}) - n_{\mathbf{p}_{1\sigma}}n_{\mathbf{p}_{2\sigma}}(1-n_{\mathbf{p}_{3\sigma}})(1-n_{\mathbf{p}_{4\sigma}})]\Big\}$$

1958 Silin Equation

Generalized Landau eqn for arbitrary spin.
$\underline{\underline{n}}_{\mathbf{p}}(\mathbf{r},t)$ is a 2x2 matrix distribution function.

$$\frac{\partial \underline{\underline{n}}_{\mathbf{p}}}{\partial t} + \frac{1}{2}\left[\frac{\partial \underline{\underline{n}}_{\mathbf{p}}}{\partial \mathbf{r}}\frac{\partial \underline{\underline{\varepsilon}}_{\mathbf{p}}}{\partial \mathbf{p}} + \frac{\partial \underline{\underline{\varepsilon}}_{\mathbf{p}}}{\partial \mathbf{p}}\frac{\partial \underline{\underline{n}}_{\mathbf{p}}}{\partial \mathbf{r}}\right] - \frac{1}{2}\left[\frac{\partial \underline{\underline{n}}_{\mathbf{p}}}{\partial \mathbf{p}}\frac{\partial \underline{\underline{\varepsilon}}_{\mathbf{p}}}{\partial \mathbf{r}} + \frac{\partial \underline{\underline{\varepsilon}}_{\mathbf{p}}}{\partial \mathbf{r}}\frac{\partial \underline{\underline{n}}_{\mathbf{p}}}{\partial \mathbf{p}}\right]$$

$$+ \frac{i}{\hbar}\left[\underline{\underline{\varepsilon}}_{\mathbf{p}}, \underline{\underline{n}}_{\mathbf{p}}\right] = \mathbf{I}_{\mathbf{p}}$$ ⟷ collision integral not specified

"identical particle spin rotation" $\vec{\sigma}_p \times \vec{h}_p$

Predicted transverse spin waves. Not detected until 1983!

1970 Leggett equations

Derived from Silin equation — spin rotation parameter

$$\frac{J_i(\mathbf{M})}{\partial t} + \frac{\alpha}{m}\frac{\partial \mathbf{M}}{\partial r_i} + \frac{\mu}{\tau}\mathbf{J}_i(\mathbf{M})\mathbf{x}\mathbf{M} - \gamma \mathbf{J}_i(\mathbf{M})\mathbf{x}\mathbf{B} = -\frac{J_i(\mathbf{M})}{\tau}$$

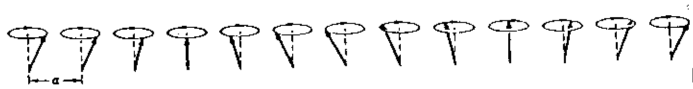
Transverse effects $M_+ = M_x + iM_y$

$$\frac{\partial M_+}{\partial t} = \frac{D_0(1+i\mu M_z)}{1+\mu^2 M^2}\nabla^2 M_+$$

Diffusion eqn with complex D

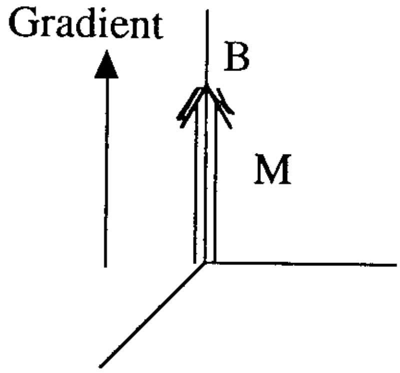
Predictions:
- Transverse spin waves (Silin's prediction)

$$\omega = \omega_L + \frac{D_0 \mu M}{1+\mu^2 M^2}\left[1 - \frac{i}{\mu M}\right]k^2$$

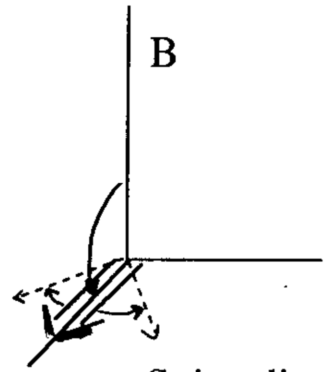


- Leggett- Rice effect (Anomalous spin diffusion).
- Non-linear effects: Levy, Ragan, Nunes

Transverse spin diffusion (spin echo experiment):

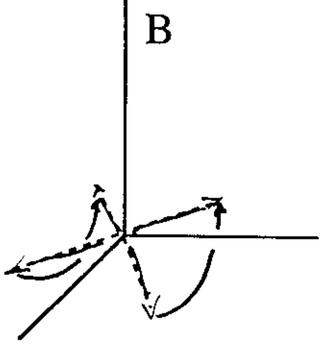


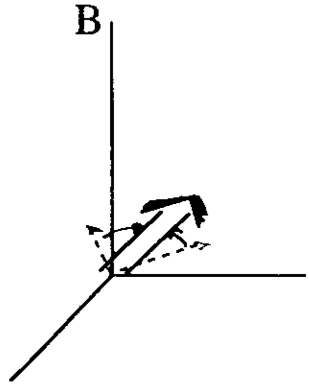
90^0 (or less) pulse



Spins disperse
in gradient

180 pulse

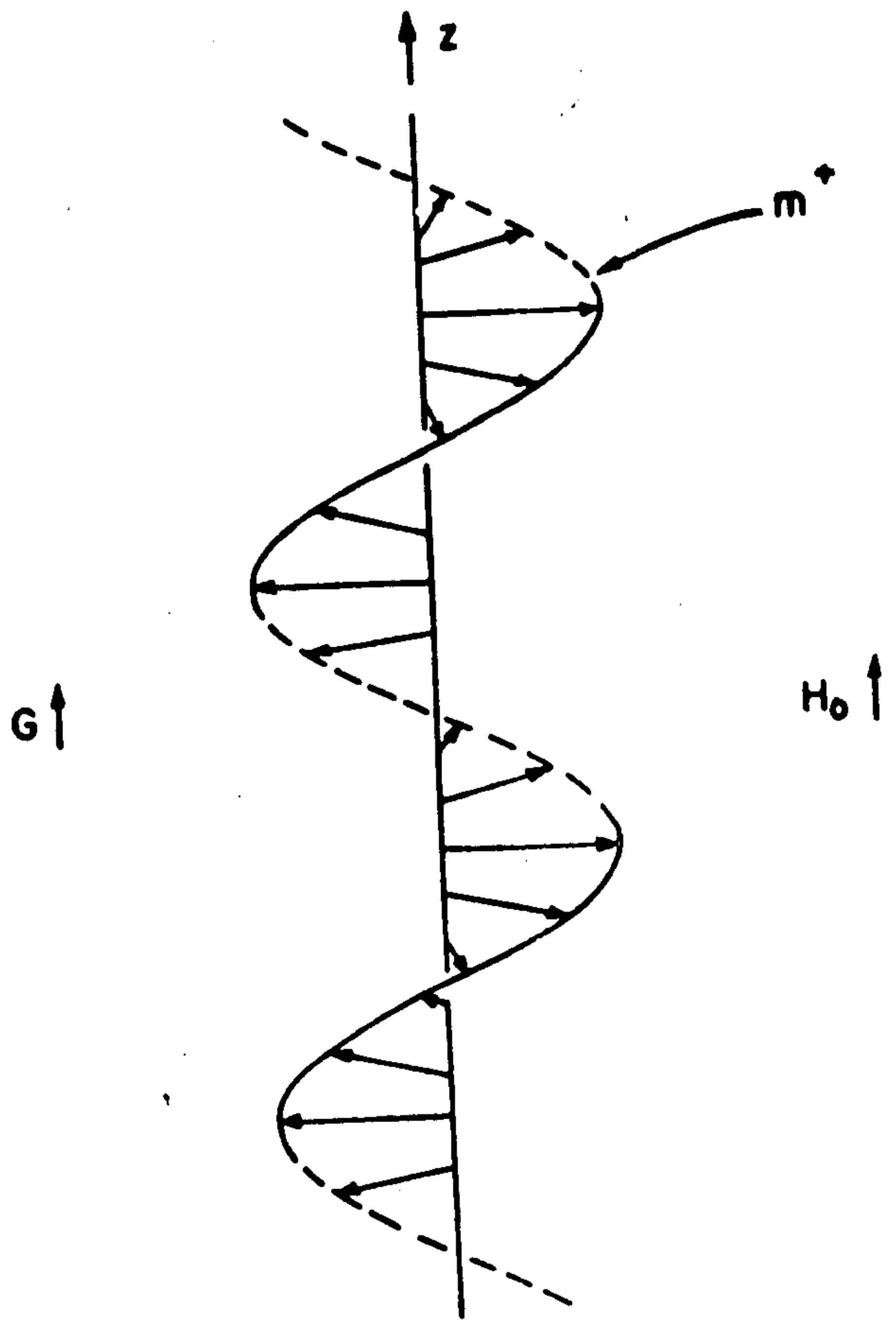




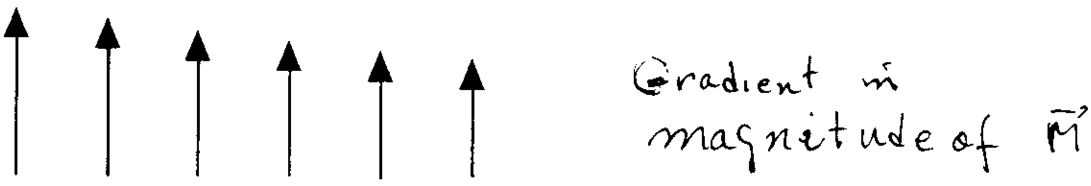
"time reversed"
spins regroup in
echo

However, if spins have diffused along z axis they will have lost phase memory (now in wrong gradient) and the echo height h will be diminished.

h(t) gives transverse diffusion constant.

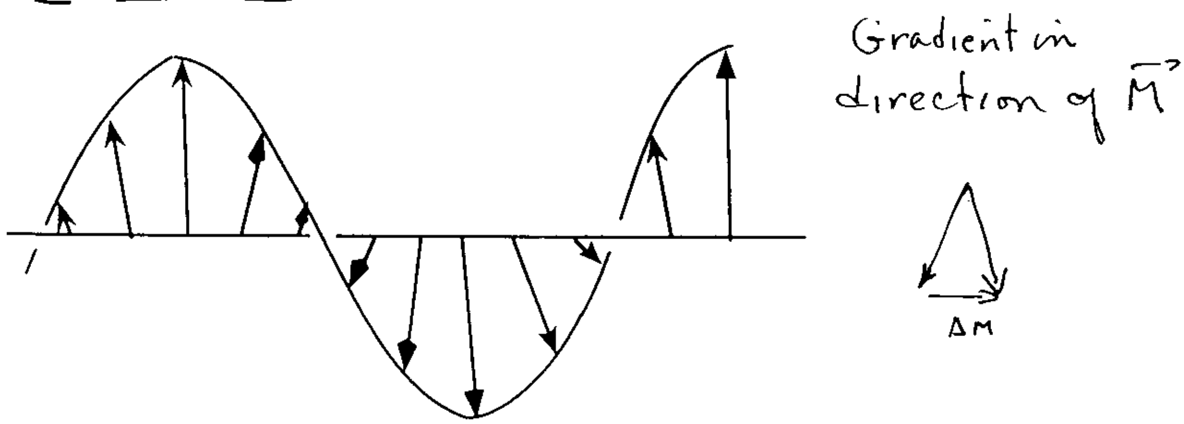

z
m+
G
H0

Longitudinal Spin diffusion:



Measures $D_{\parallel}$
Spin currents along M (in spin space)

Transverse spin diffusion :



Measures $D_{\perp}$
Spin currents perpendicular to M.

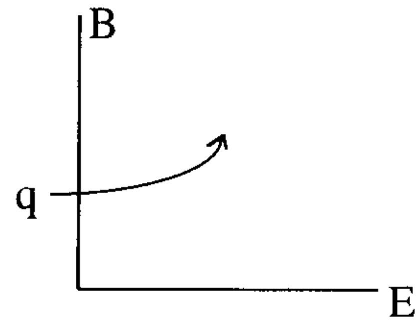
Before 1990 all experiments measured $D_{\perp}$ but all theories computed $D_{\parallel}$ (and they were the same!)

Leggett-Rice effect in transverse spin diffusion.

Hall effect analogy:

$$\frac{\partial \mathbf{v}}{\partial t} = \frac{q}{m}\mathbf{E} + \frac{q}{m}\mathbf{v}\mathbf{x}\mathbf{B} - \frac{\mathbf{v}}{\tau}$$

↑ drag term



$$\sigma = \frac{\sigma_0}{1+\mu^2 B^2}$$

Effective σ is reduced

$$\sigma_0 = q^2 \frac{n\tau}{m}$$

$$\mu = \frac{q\tau}{m}$$

1982 Laloë and Lhuillier generalized the Boltzmann equation to arbitrary polarization direction.

$n_{p\sigma}$ becomes a 2x2 matrix $\rho(\mathbf{p})$.

Boltzmann statistics, but quantum collisions

2 x 2 matrix

$$\frac{\partial}{\partial t}\rho_S(\mathbf{r},\mathbf{p}) + \frac{1}{m}\mathbf{p}.\nabla_{\mathbf{r}}\,\rho_S(\mathbf{r},\mathbf{p})$$

$$= \int d^3q' \;\; v_r \int d^2\hat{q} \left\{ \sigma_{\mathbf{k}}(\theta)\left[f(\mathbf{p}_2')\,\rho_S(\mathbf{p}_1') - f(\mathbf{p}_2)\,\rho_S(\mathbf{p}_1)\right] + \right.$$

$$+ \frac{\varepsilon}{2}\,\sigma_{\mathbf{k}}^{ex.}(\theta)\left[[\rho_S(\mathbf{p}_1'),\rho_S(\mathbf{p}_2')]_+ - [\rho_S(\mathbf{p}_1),\rho_S(\mathbf{p}_2)]_+\right]$$

$$- i\frac{\varepsilon}{2}\int d^3q\, v_r\, \tau_{fwd.}^{ex.}(k)\,[\rho_S(\mathbf{p}),\rho_S(\mathbf{p}-\mathbf{q})]$$

$$\left. + i\frac{\varepsilon}{2}\,\tau_{\mathbf{k}}^{ex.}(\theta)\,[\rho_S(\mathbf{p}_1'),\rho_S(\mathbf{p}_2')] \right\}$$

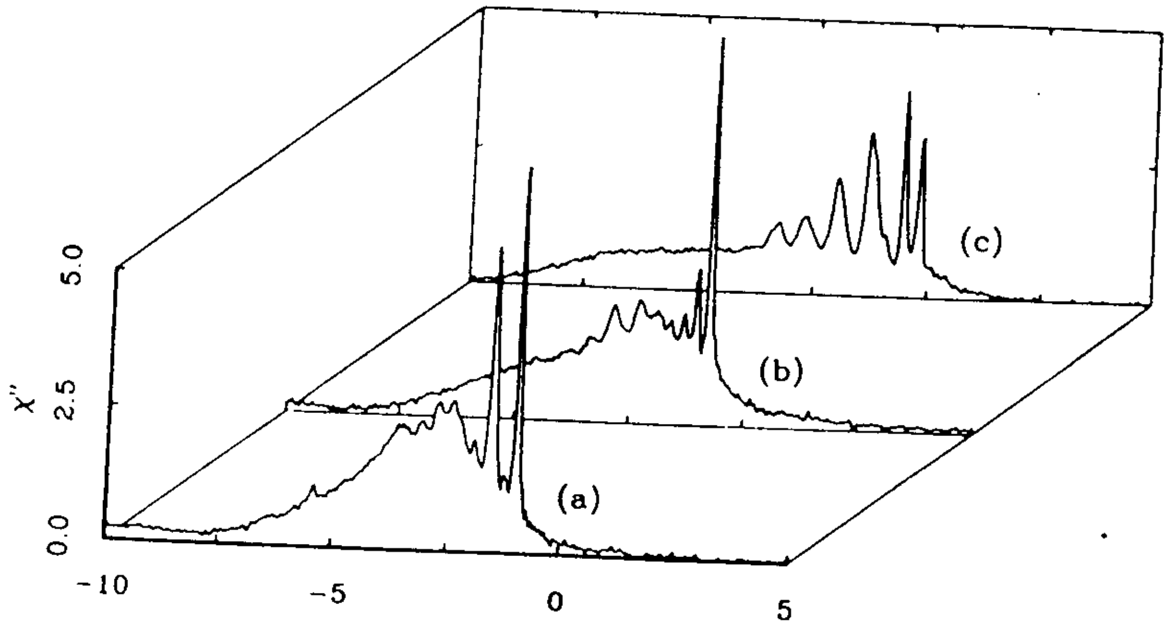
Spin Rotation terms

Results: Leggett equations in Boltzmann gas (Bose or Fermi)!
spin waves, Leggett-Rice effect.

Bashkin, Bashkin and Meyerovich discovered equivalent results.

Experiments: Spin waves seen at Cornell (H↓), ENS (^{3}He gas), Julich (degenerate solutions), Ohio State (pure liq ^{3}He); L-R effect (UMass).

Johnson et al 1984 Cornell



NMR spectra showing the variation of the spectrum with the field gradient. The magnitude of $\nabla\delta H_0(\bar{r})$ was about 0.4 G/cm between traces (a) and (b), and twice that much between (b) ... itude and direction of $\nabla\delta H_0$ were not accurately known. The time between successive spectr spectra were taken at $T = 245$ mK, $n = 3.2\times10^{16}$ atoms/cm^3, and $P \sim -1$ using tipping pulses (

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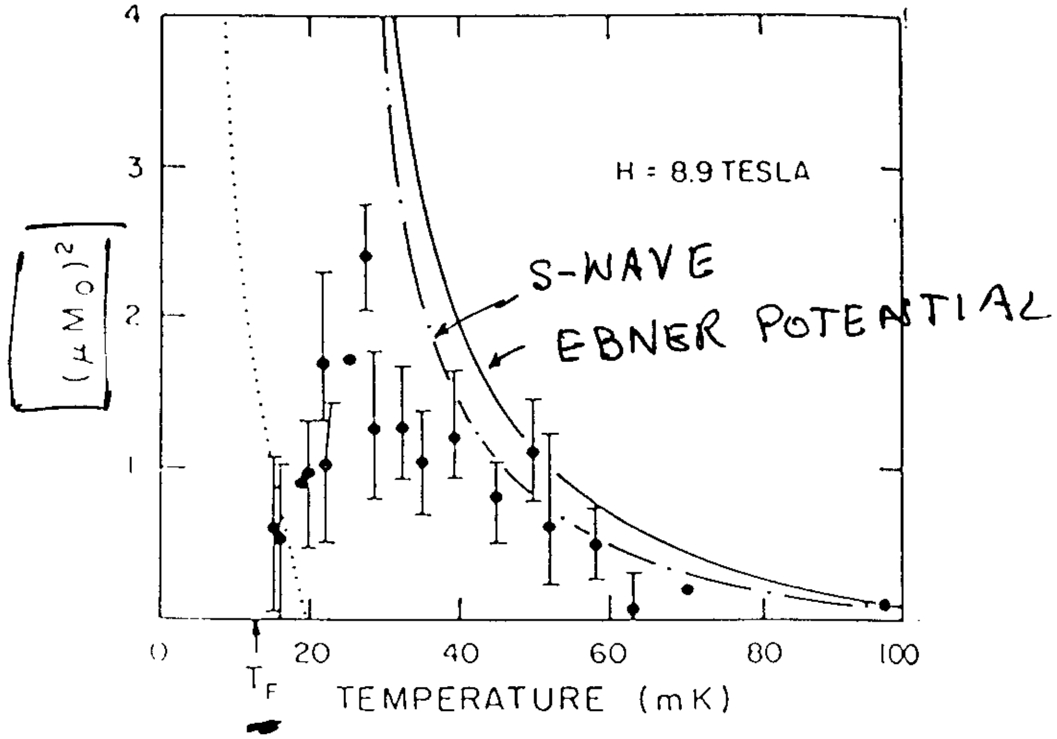
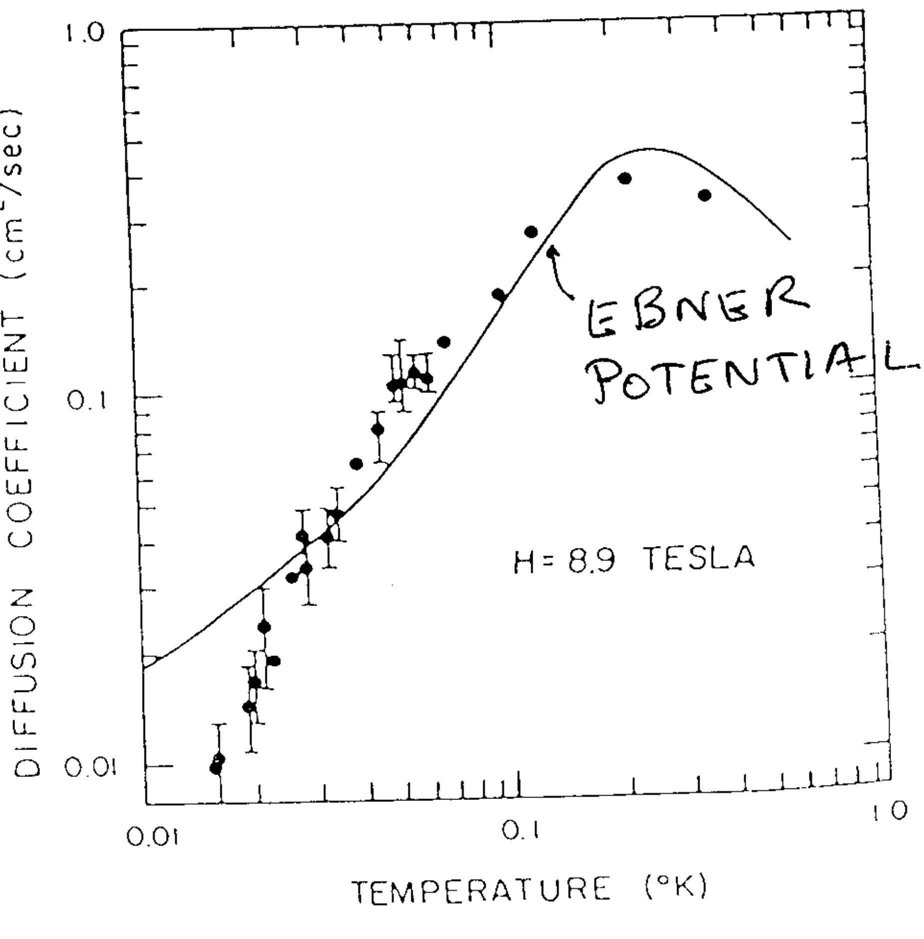
GULLY, MULLIN 1984





Calculations of longitudinal transport properties for polarized systems:

$T >> T_F$

Lhuillier and Laloë
Bashkin and Meyerovich

$T << T_F$:

Bashkin and Meyerovich: κ, η, D, etc (variational)

Miyake and Mullin: κ, η, D, etc (exact)

All T:

Jeon and Mullin: Reduced collision integral to a two-fold integral
Calculated D *at all T* (variational)

1988...Generalization of the collision integral to arbitrary polarization:

Meyerovich (1985): Suggested transverse spin diffusion relaxation time $\tau_\perp$ might not be the same as longitudinal relaxation time $\tau_{||}$.

"We use the following usual τ-representation for the spin-conserving exchange collisions integral:

$$\langle \mathrm{Sp}\, \hat{\sigma} \hat{I} \rangle = 0 \,,$$

$$\langle v_i\, \mathrm{Sp}\, \hat{\sigma} \hat{I} \rangle = -\tau_\perp^{-1} J_i - (\tau_\parallel^{-1} - \tau_\perp^{-1}) e (e \cdot J_i) \,, \qquad (3)$$

where $\tau_{\perp,\parallel}$ are the transverse and longitudinal relaxation times"

Jeon and Mullin (1988,92). {Kadanoff-Baym / Broecker-Dufty} — Left side ≈ Landau-Silin eqn.

$$\frac{\partial}{\partial t} \underline{n}_{\mathbf{p}}(\mathbf{r}, t) - \frac{1}{i\hbar} [\underline{\varepsilon}_{\mathbf{p}}(\mathbf{r}), \underline{n}_{\mathbf{p}}(\mathbf{r})]_- - \tfrac{1}{2}[\nabla_{\mathbf{r}} \underline{\varepsilon}_{\mathbf{p}}(\mathbf{r}), \nabla_{\mathbf{p}} \underline{n}_{\mathbf{p}}(\mathbf{r})]_+ + \tfrac{1}{2}[\nabla_{\mathbf{p}} \underline{\varepsilon}_{\mathbf{p}}(\mathbf{r}), \nabla_{\mathbf{r}} \underline{n}_{\mathbf{p}}(\mathbf{r})]_+$$

$$= \frac{\pi}{\hbar} \int d\mathbf{p}_2 \int d\mathbf{p}_3 \int d\mathbf{p}_4\, \delta(E_{12} - E_{34}) \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4)$$

$$\times \{ |(\mathbf{p}_{12}|T_{12}|\mathbf{p}_{34})|^2 [[\underline{n}_{\mathbf{p}_3}, \tilde{\underline{n}}_{\mathbf{p}_1}]_+ \mathrm{tr}_s(\tilde{\underline{n}}_{\mathbf{p}_2} \underline{n}_{\mathbf{p}_4})$$

$$- [\tilde{\underline{n}}_{\mathbf{p}_3}, \underline{n}_{\mathbf{p}_1}]_+ \mathrm{tr}_s(\underline{n}_{\mathbf{p}_2} \tilde{\underline{n}}_{\mathbf{p}_4})]$$

$$+ \eta\, \mathrm{Re}[(\mathbf{p}_{12}|T_{12}|\mathbf{p}_{34})(\mathbf{p}_{34}|T_{12}^\dagger|-\mathbf{p}_{12})][[\underline{n}_{\mathbf{p}_3} \tilde{\underline{n}}_{\mathbf{p}_2} \underline{n}_{\mathbf{p}_4}, \tilde{\underline{n}}_{\mathbf{p}_1}]_+ - [\tilde{\underline{n}}_{\mathbf{p}_3} \underline{n}_{\mathbf{p}_2} \tilde{\underline{n}}_{\mathbf{p}_4}, \underline{n}_{\mathbf{p}_1}]_+]$$

$$+ i\eta\, \mathrm{Im}[(\mathbf{p}_{12}|T_{12}|\mathbf{p}_{34})(\mathbf{p}_{34}|T_{12}^\dagger|-\mathbf{p}_{12})]$$

$$\times \{[\underline{n}_{\mathbf{p}_3} \tilde{\underline{n}}_{\mathbf{p}_2} \underline{n}_{\mathbf{p}_4}, \tilde{\underline{n}}_{\mathbf{p}_1}]_+ + [\tilde{\underline{n}}_{\mathbf{p}_3} \underline{n}_{\mathbf{p}_2} \tilde{\underline{n}}_{\mathbf{p}_4}, \underline{n}_{\mathbf{p}_1}]_+ - 2[\underline{n}_{\mathbf{p}_3} \underline{n}_{\mathbf{p}_2} \underline{n}_{\mathbf{p}_4}, \underline{n}_{\mathbf{p}_1}]_+ \} \} \qquad (2.32)$$

← new collision integral

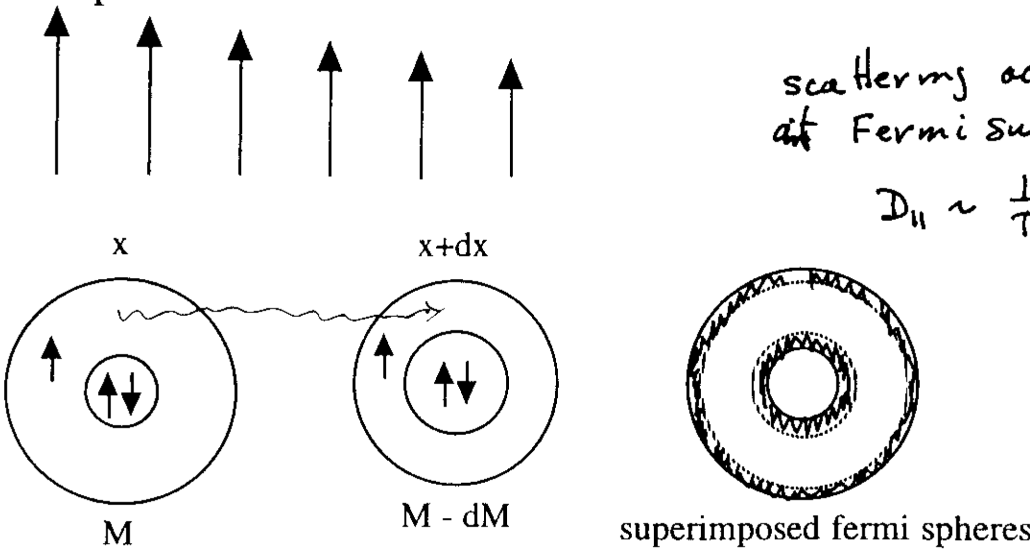
(Did for the degenerate transport equation what Lhuillier-Laloë did for the Boltzmann equation)

Valid for dilute systems only.

Independent calculations by Meyerovich and Musealian (92) ←
Golosov and Ruckenstein (95) ←

Calculations: $D_\perp \neq D_{||}$

Longitudinal spin diffusion:

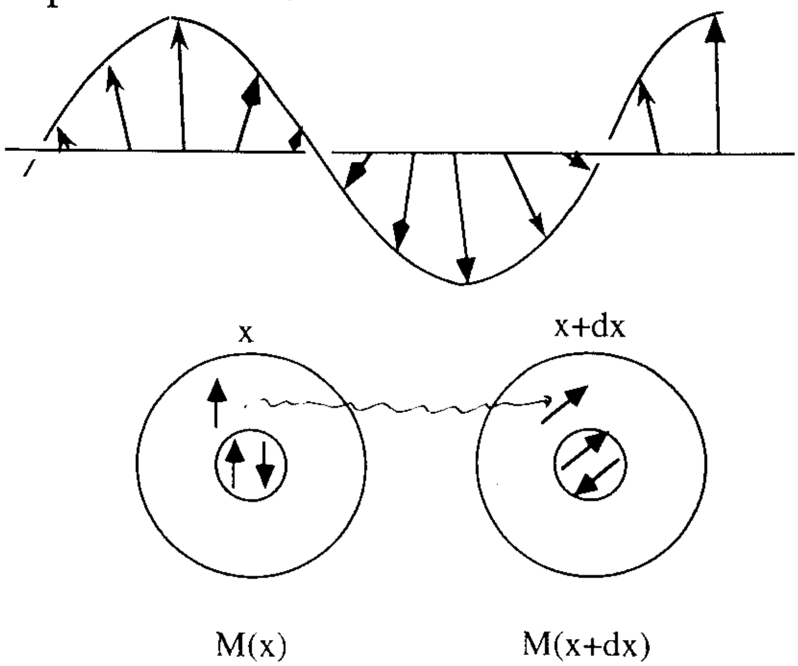


scattering occurs at Fermi surfaces only

$D_{||} \sim \frac{1}{T^2}$ for $T<T_F$

===============

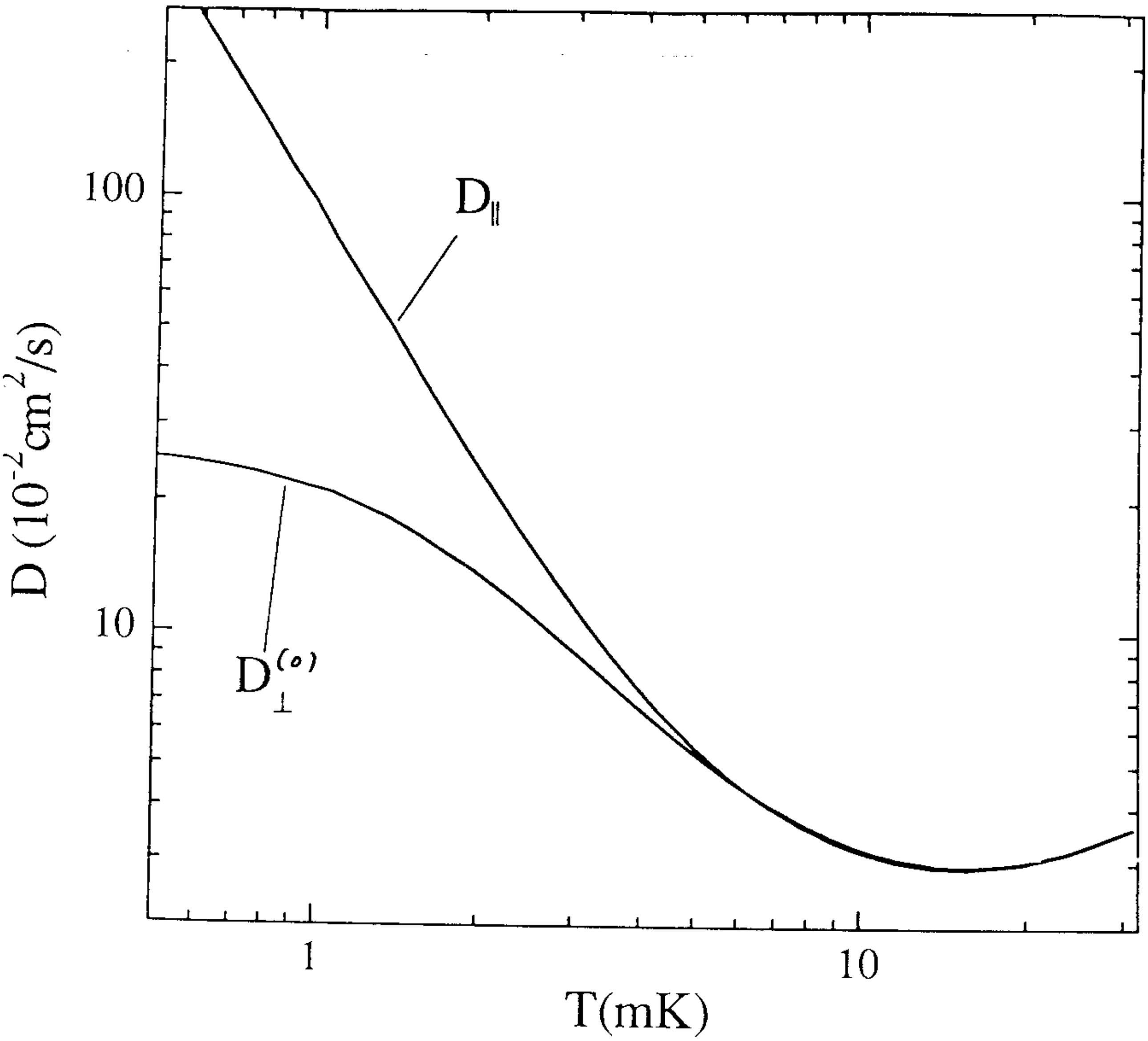
Transverse spin diffusion:



Scattering everywhere between Fermi surfaces

$D_\perp$ independent of T

$T \ll T_F$



$$D_{\perp}^{(\text{Leggett})} = \frac{D_{\perp}^{(0)}}{1 + (\mu M)^2}$$

3

Comparison with experiment:

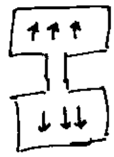
Dilute solutions of ^{3}He in liquid ^{4}He $x_3 \sim 10^{-4}$

Theory applies to dilute gas

Experiments:

- Candela, McAllaster, Wei, and Vermeulen (UMass):
  Spin wave determination of $D_\perp/\mu M$ and μM

- Nunes, Jin, Putnam, and Lee (Cornell):
  Longitudinal spin diffusion, $D_{||}$

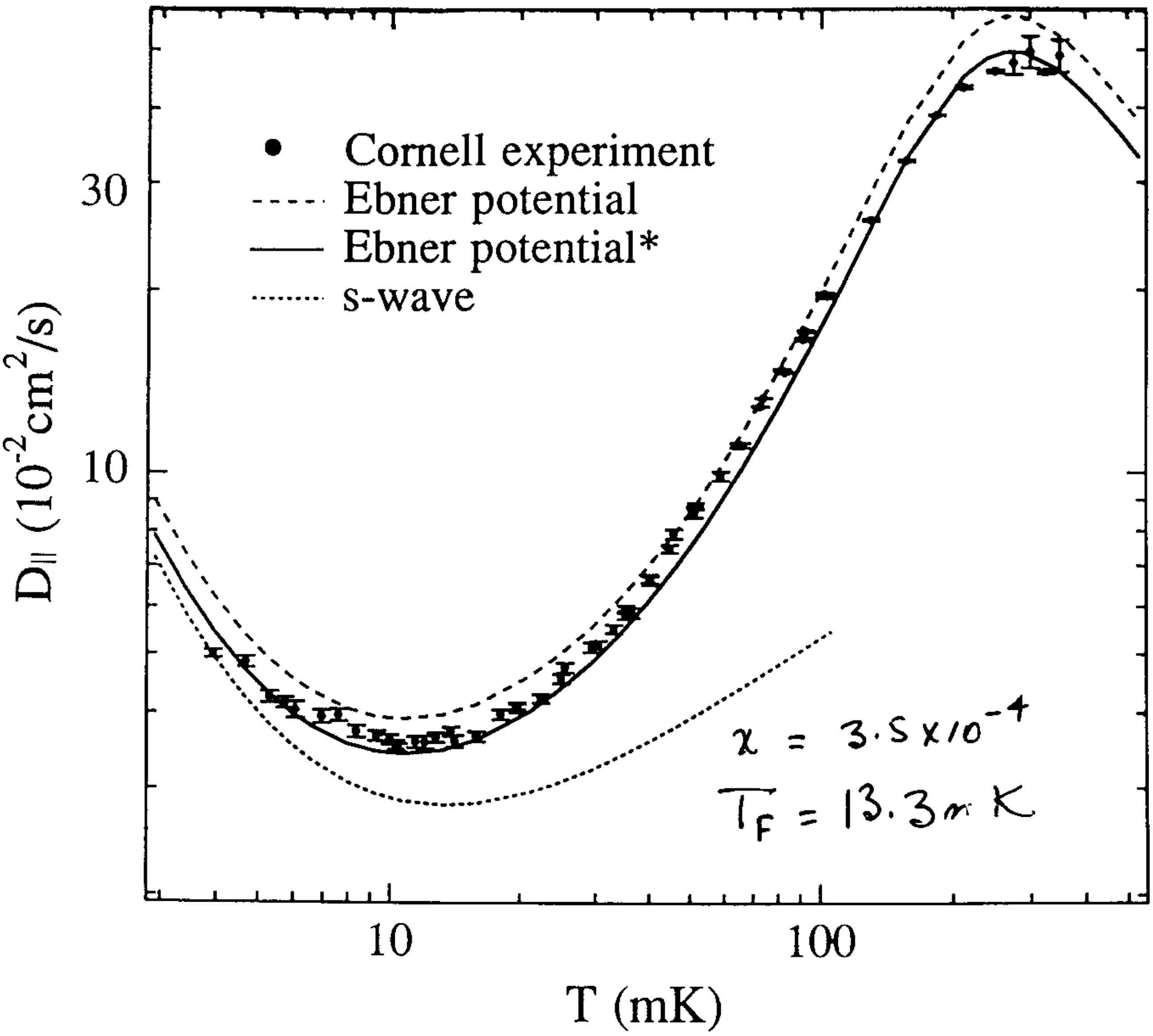


(One potential parameter is adjusted slightly (7%) to fit all data simultaneously.)

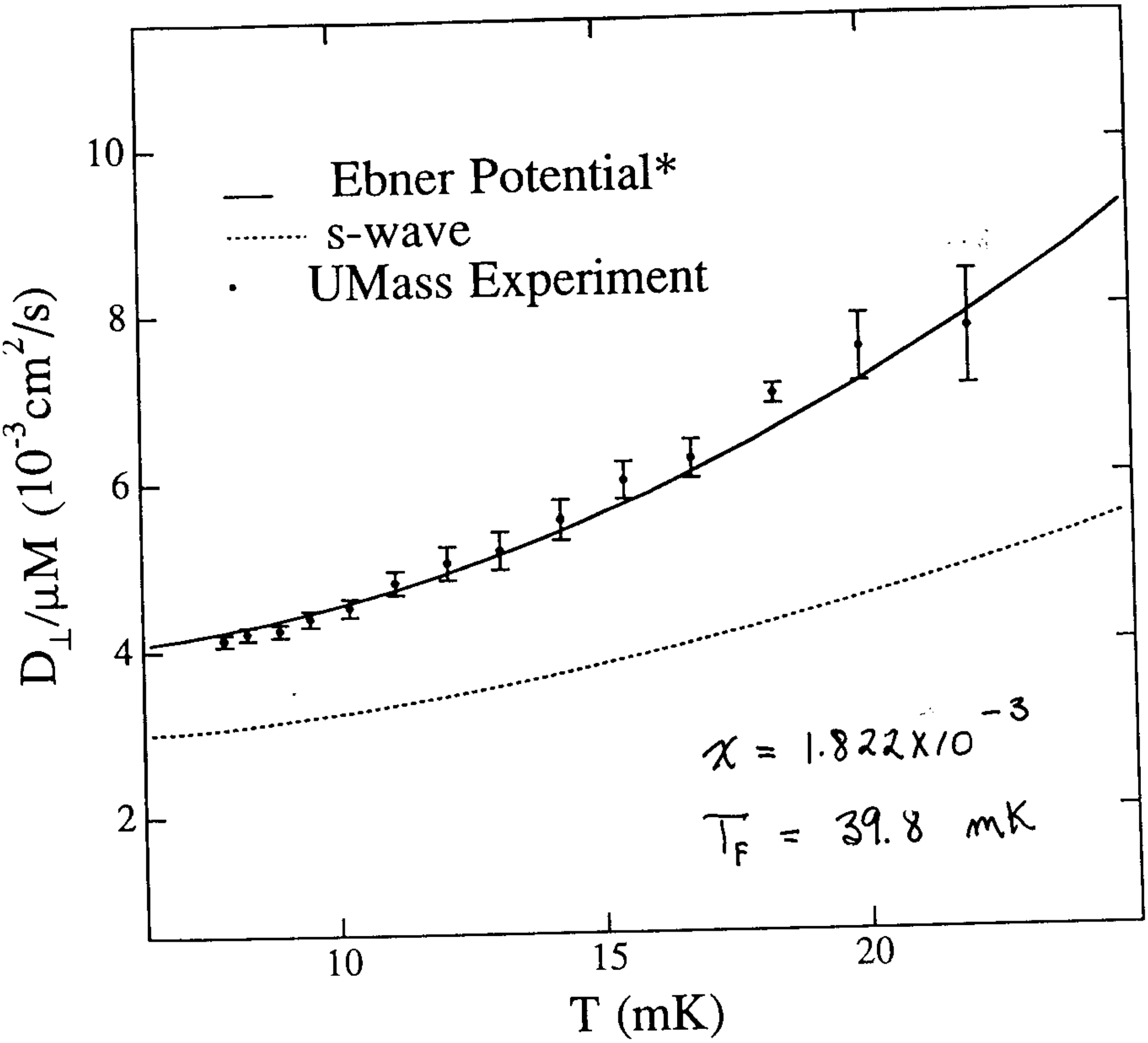
Polarization not large enough or T low enough to see $D_\perp \neq D_{||}$.

- Experiments on pure ^{3}He (Candela et al)

- Experiments on solutions of higher concentrations (Owers-Bradley). 6.4% concentration



Cornell experiment
Ebner potential
Ebner potential*
s-wave
$D_{\parallel}$ (10^{-2}cm^2/s)
30
10
10
100
T (mK)
χ = 3.5 x 10^-4
T_F = 13.3 mK


Ebner Potential*
s-wave
UMass Experiment
$\chi = 1.822 \times 10^{-3}$
$T_F = 39.8$ mK
$D_\perp/\mu M$ (10^{-3}cm^2/s)
T (mK)
10
8
6
4
2
10
15
20

11C

Candela et a   UMASS

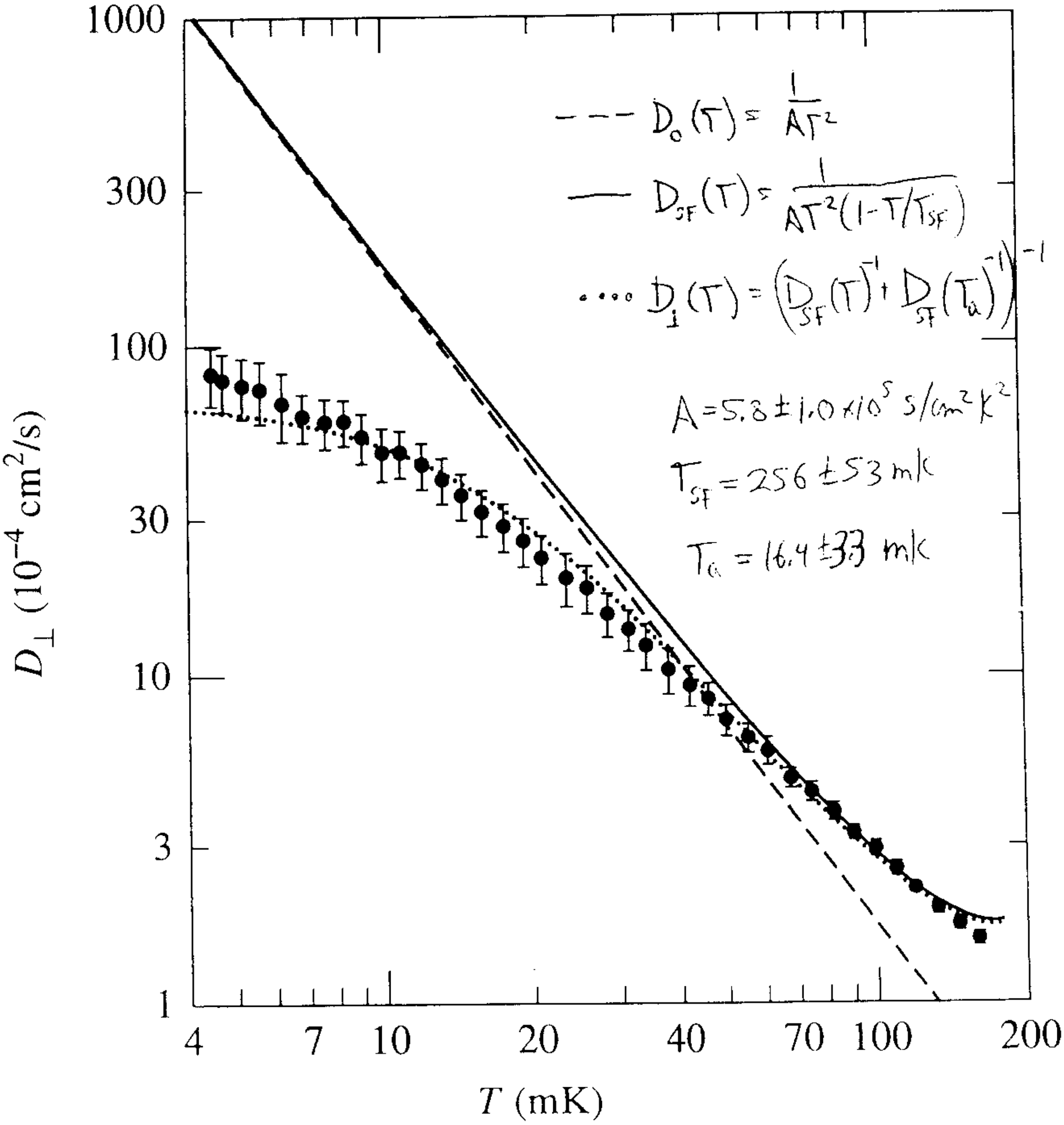


FIG. 1. Experimental results for the transverse spin diffusion coefficient $D_\perp$ in ^{3}He liquid at zero pressure, in a magnetic field $H = 8$ T (points with error bars). The dashed curve shows a simple T^{-2} temperature dependence, while the solid curve contains a spin-fluctuation term. Earlier experiments have shown that $D_\perp$ at low field follows the solid curve for $T = 1 - 100$ mK. Our high-field data fall well below this curve for $T < 20$ mK. Dotted curve is fit of our data to a simple phenomenological model.

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OWERS-BRADLEY et al

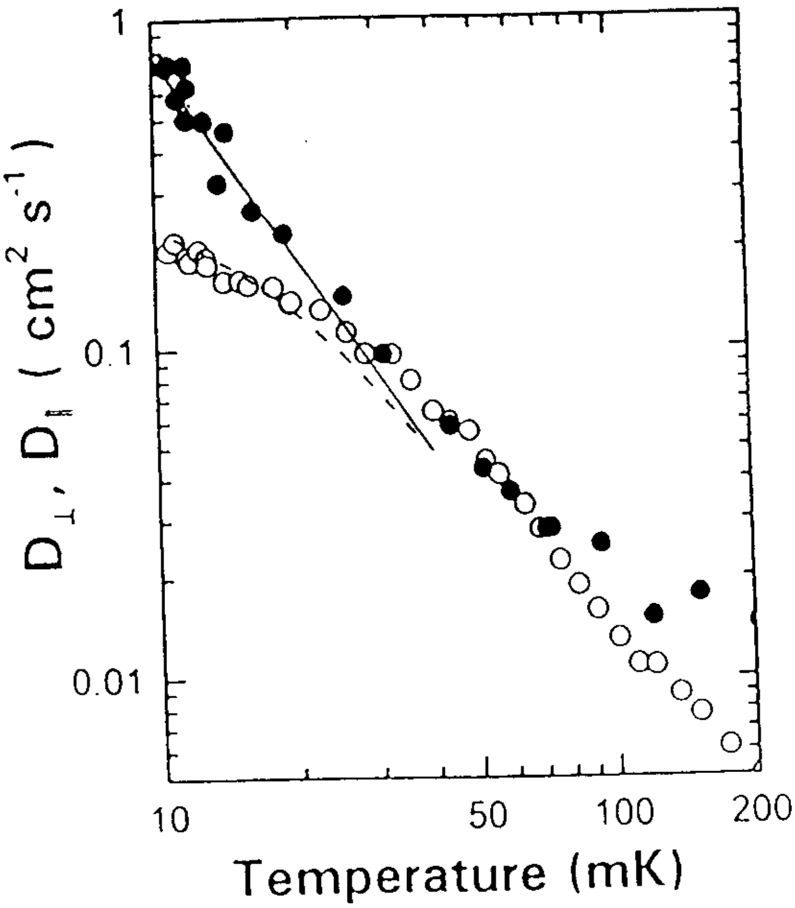


FIG. 2. The longitudinal (●) and transverse (○) spin diffusion coefficients in a saturated solution of ^3He in ^4He in 8.8 T. The solid line shows the T^{-2} dependence of $D_{\|}$ in the degenerate regime. The fit to $D_{\perp}$ (dashed line) yields an anisotropy temperature of (17 ± 2) mK. At temperatures higher than ~ 30 mK the liquid is no longer in the degenerate regime which leads to the deviation from the T^{-2} behavior (see text).

Spin Diffusion in Solid ^{3}He:

$$H_X = -\sum_{i<j} J_{ij}\, \sigma_i \bullet \sigma_j$$

(overly simplified for bcc phase, but okay for hcp)

Fact: Low polarization spin diffusion measured in 1960's. $D \sim Ja^2/\hbar$

Theory: spin waves will propagate at high polarization in paramagnetic phase. (These are not the spin waves seen in ordered phase.)

$$\omega = \omega_0 + J\, a^2 k^2$$

In between - Complex spin diffusion constant??
Leggett-Rice effect???
Nonlinear effects as in liquid??

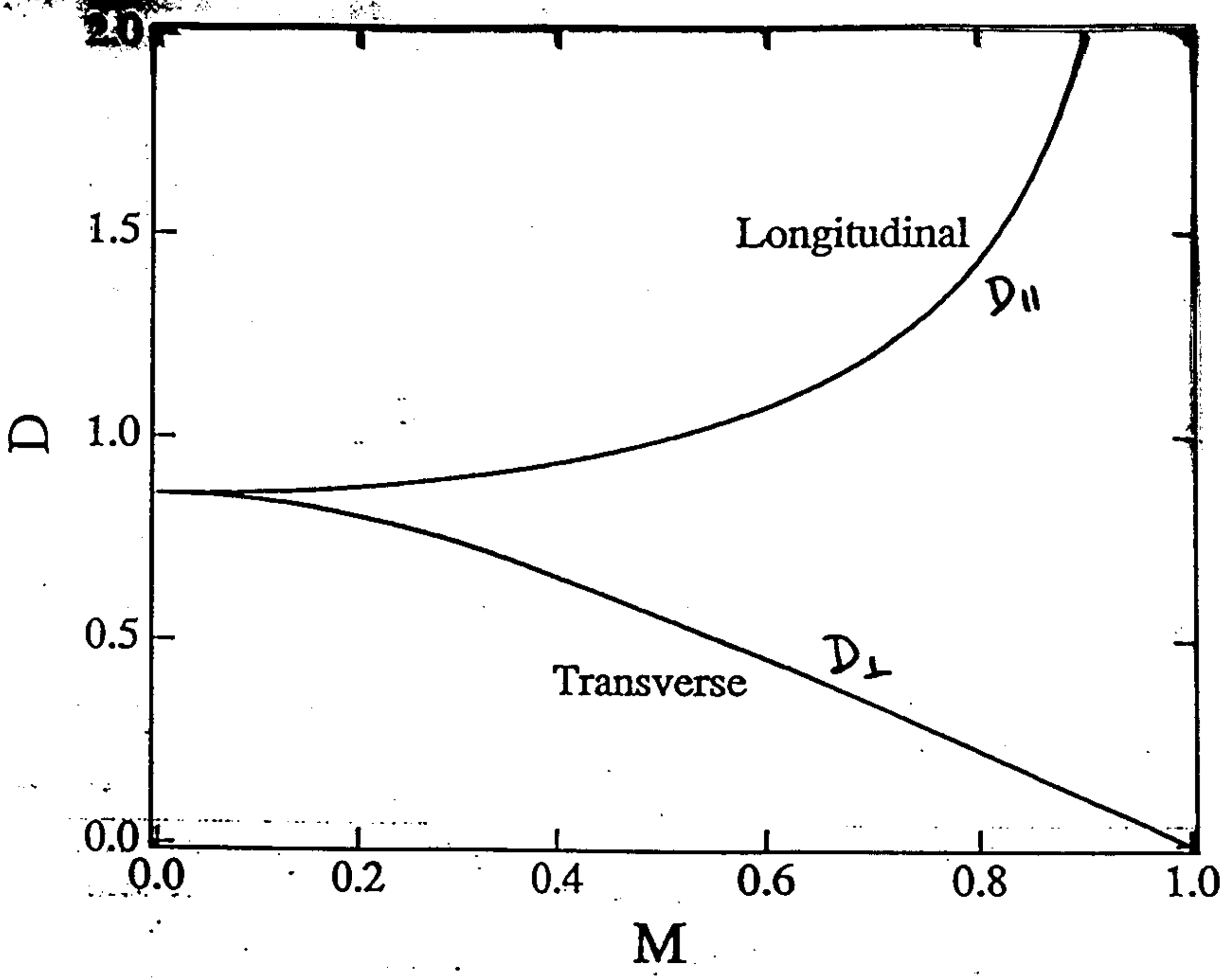
Kubo theory of Transport:

$$D_\perp q^2 \sim \int_0^\infty dt \sum_{i,j} q \bullet R_i\, q \bullet R_j\, \mathrm{Tr}\left\{\rho_0 I_-^i [H_x,[H_x,I_+^j(t)]]\right\}$$

Similar form for $D_\parallel$

Procedure: Expand correlation function in moments, fit first few to appropriate functional form, do integral. Result is $D_\perp$ in terms of moments of the correlation function.

Get spin wave frequency and diffusion constant.



$$D_\perp = \frac{D_0}{1+(\mu M)^2} + \frac{i\mu M D_0}{1+(\mu M)^2}$$

At large M, $D_\perp = \dfrac{D_0}{1+(\mu M)^2} \sim 1\text{-}M$ as seen from graph $\qquad \mu M \sim \dfrac{M}{1\text{-}M}$ from spin wave freq.

So $D_0 \sim \dfrac{1}{1\text{-}M}$ diverges much like $D_\| \sim \dfrac{1}{\sqrt{1\text{-}M}}$

Bibliography for spin-diffusion anisotropy

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70 A. J. Leggett, "Spin diffusion and spin echoes in liquid ^{3}He at low temperature", J. Phys. C **3**, 448 (1970).

72 L. R. Corruccini, D. O. Osheroff, D. M. Lee, and R. C. Richardson, "Spin Wave Phenomena in Liquid ^{3}He Systems", J. Low Temp. Phys. **8**, 229 (1972).

Spin echoes on ^{3}He and 6.4%, 8.3% ^{3}He-^{4}He, p=0-27bar, 7.9-36MHz, $T \geq 4.9$mK. Saw Leggett-Rice effect with μM up to 4.

83 J. R. Owers-Bradley, H. Chocholacs, R. M. Mueller, Ch. Buchal, M. Kubota, and F. Pobell, "Spin Waves in Liquid ^{3}He-^{4}He Mixtures", Phys. Rev. Lett **51**, 2120 (1983).

1MHz CW NMR on 5% ^{3}He-^{4}He (0-20bar) and 9.5% ^{3}He-^{4}He (10bar), T≥0.3mK. In open coil, say shift and broadening of line rather than sharp spin wave resonances.

84 B. R. Johnson, J. S. Denker, N. Bigelow, L. P. Lévy, J. H. Freed, and D. M. Lee, "Observation of Nuclear Spin Waves in Spin-Polarized Atomic Hydrogen Gas", Phys. Rev. Lett **52**, 1508 (1984).

Pulsed NMR on H↓ in 7.7T at 245mK, μM=6.

84 W. J. Gully and W. J. Mullin, "Observation of Spin Rotation Effects in Polarized ^{3}He-^{4}He Mixtures", Phys. Rev. Lett. **52**, 1810 (1984).

Spin echoes on 3.7×10^{-4} ^{3}He-^{4}He in 8.9T $T \geq 17$mK, anomalous max of μM at 28mK.

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CW NMR on polarized ^{3}He gas in 4G $T \geq 2$K, $\mu \leq 0.7$.

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equals the T=0 value of $D_\perp$) and found their result to be $T_a = \gamma\, \hbar\mathrm{bar}\, H_0 / 2\pi k_B$ = 2.0mK for ^{3}He in 8T (my notebook o66 1/29/93).

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96 V. V. Dmitriev, "Coherent Spin Precession in Normal Fermi Liquids", Czechoslovak J. Phys. **46**, Suppl. S6, 3011 (1996) (invited paper for LT-21 conference, Prague, 8/96).

Reviews theory of the "coherently precessing domain" structure based on Leggett's spin-transport equation, as well as recent experiments in ^{3}He-^{4}He and normal ^{3}He. Suggests CPD may be used to observe spin-relaxation anisotropy.

97 I. A. Fomin and G. A. Vermeulen, "The Effect of the Demagnetizing Field on Coherent Spin Precession in a Normal Fermi Liquid", J. Low Temp. Phys. **106**, 133 (1997).

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97 A. E. Meyerovich and A. Stepaniants, "Zero-Temperature Relaxation in Spin-Polarized Fermi Liquids", J. Low Temp. Phys. **106**, 653 (1997), also shorter A. E. Meyerovich and A. Stepaniants, "Dipole effects in spin dynamics of spin-polarized quantum systems", Czechoslovak J. Phys. **46**, Suppl. S1, 203 (1996) (LT-21 conference, Prague, 8/96).

Theorizes about effect of spin-diffusion anisotropy on (a) zero-sound attenuation, and (b) Castaing instability in transverse spin dynamics (says latter is governed by $\tau_{\parallel}$, not $\tau_{\perp}$).

97 I. A. Fomin, "Spin Dynamics of a Spin-Polarized Fermi Liquid", preprint dated 4/15/95 submitted to Pis'ma ZheTPh.

Claims on theoretical grounds that zero-temperature limit of $D_{\perp}$ should not be finite, in disagreement with Meyerovich and Mullin.