UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL ATOMIC ENERGY AGENCY
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR.998a - 18

Research Workshop on Condensed Matter Physics
30 June - 22 August 1997
MINIWORKSHOP ON
QUANTUM MONTE CARLO SIMULATIONS OF LIQUIDS AND SOLIDS
30 JUNE - 11 JULY 1997
and
CONFERENCE ON
QUANTUM SOLIDS AND POLARIZED SYSTEMS
3 - 5 JULY 1997

"Crystallization and melting of spin-polarized 3He"

A.F. ANDREEV
Russian Academy of Sciences
P.L. Kapitza Institute for Physical Problems
Kosygin Street 2
117334 GSP-1 Moscow
RUSSIAN FEDERATION

Crystallization and Melting of Spin-Polarized ^3He.

(Trieste, 1997)

1. Nondissipative crystallization of helium quantum crystals.
2. Crystallization waves in ^4He and in nonpolarized ^3He.
3. Spin conservation in crystallization and melting processes:
   a) High-temperature region.
      Nonequilibrium spin-densities.

   b) Low-temperature region.
      Spin-supercurrents.
      Nondissipative decay of spin-polarization
4. Magnetic crystallization waves in spin-polarized ^3He.

NEW QUANTUM STATES IN HELIUM CRYSTALS

KEY POINT: QUANTUM (TUNNELING) DELOCALIZATION OF PARTICLES, IMPURITIES, AND POINT DEFECTS OF **ALL** TYPES.

IMPURITY QUASIPARTICLES,
VACANCY QUASIPARTICLES, ETC.

GENERAL PROCEDURE OF CONSTRUCTING NEW QUANTUM STATES:
(A.A. and Lifshitz, 1969)

CLASSICAL CRYSTAL WITH POINT DEFECTS OF SOME PARTICULAR TYPE (DISORDERED SYSTEM)	DELOCALIZATION → OF DEFECTS	CORRESPONDING NEW QUANTUM STATE OF IDEAL CRYSTAL

EXAMPLES:

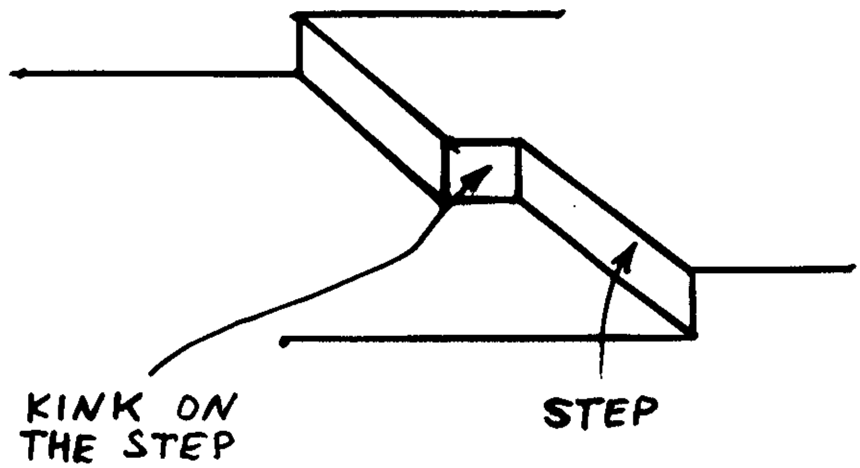
^{3}HE IMPURITIES → QUANTUM DIFFUSSION

VACANCIES → SUPERFLUID CRYSTALS

OUR PRESENT PROBLEM:

SURFACE DEFECTS.

SURFACE DEFECTS



QUANTUM DELOCALIZATION OF KINKS AND STEPS:

Stationary states with nonzero flux of matter through the interface,

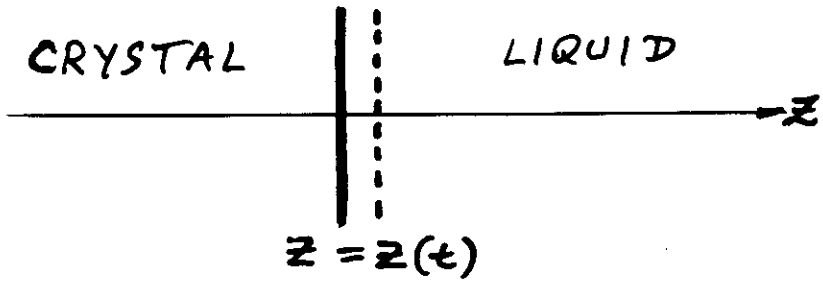
NONDISSIPATIVE CRYSTAL GROWTH OF MELTING.

(AA. and Parshin, 1978)

PROBLEM:

CONSERVATION LAWS!

DIFFICULTIES WITH CONSERVATION LAWS:



- MASS CONSERVATION

  Densities ρ_ℓ and ρ_c of the liquid and of the crystal are not equal: $\rho_c > \rho_\ell$.

  RESULT:

  Nonzero velocity $\vec{v}_\ell$ in the liquid:

  $$\rho_\ell v_{\ell z} = -(\rho_c - \rho_\ell)\dot{z}.$$

- ENERGY CONSERVATION

  If the entropy densities σ_ℓ and σ_c are not equal, conditions of mass conservation and of energy conservation are not the same.

  RESULT:

  Temperature T is not uniform:

  DISSIPATION.

  POSSIBILITY:

  $T \to 0$, so $\sigma_c \simeq \sigma_\ell \simeq 0$ and energy conservation $\equiv$ mass conservation.

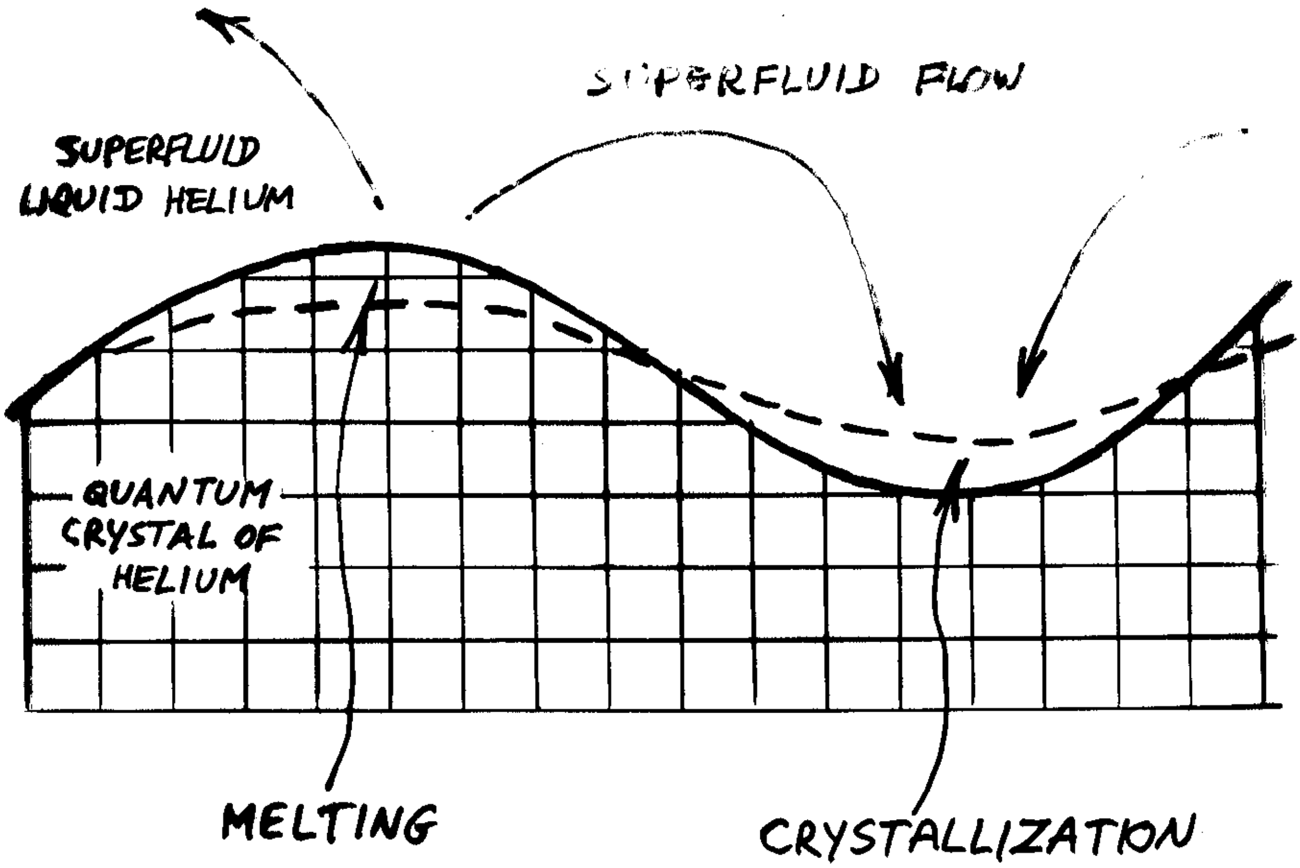
NONDISSIPATIVE CRYSTALLIZATION

for ^4He at $T < 1K$

for ^3He at $T < 1mK$

liquid is superflui in both cases

CRYSTALLIZATION WAVES

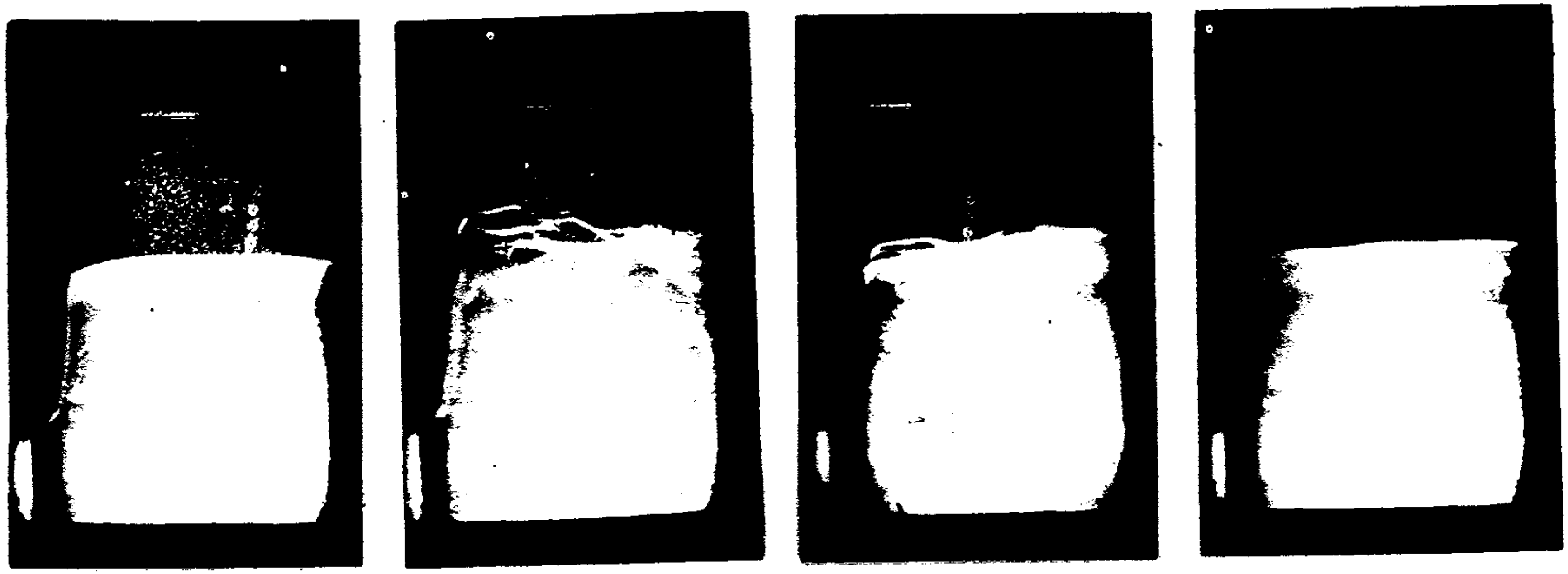


POTENTIAL ENERGY = SURFACE ENERGY OF CURVED INTERFACE

KINETIC ENERGY = ENERGY OF SUPERFLUID FLOW

AND NO DISSIPATION

MOTION PICTURE OF EXCITATION AND PROPAGATION OF CRYSTALLIZATION WAVES



(Keshishev, Parshin, Babkin - 1981)

SPIN CONSERVATION (APPROXIMATE) IN SPIN-POLARIZED ^{3}HE

Equilibrium spin-densities : $S_\ell = \frac{\chi_\ell}{\gamma} H$, $S_c = \frac{\chi_c}{\gamma} H$.

where χ_ℓ, χ_c are magnetic susceptibilities,
$\gamma = 2\mu/\hbar$ is gyromagnetic ratio.

$\chi_c \gg \chi_\ell$, so $S_c \gg S_\ell$.

Spin conservation is provided by very different ways depending on whether $T \gtrsim T_N, T_c$ or $T \ll T_N, T_c$

where T_N is Neel temperature in the crystal
T_c is the superfluid transition temperature in the liquid.

I HIGH-TEMPERATURE REGION:

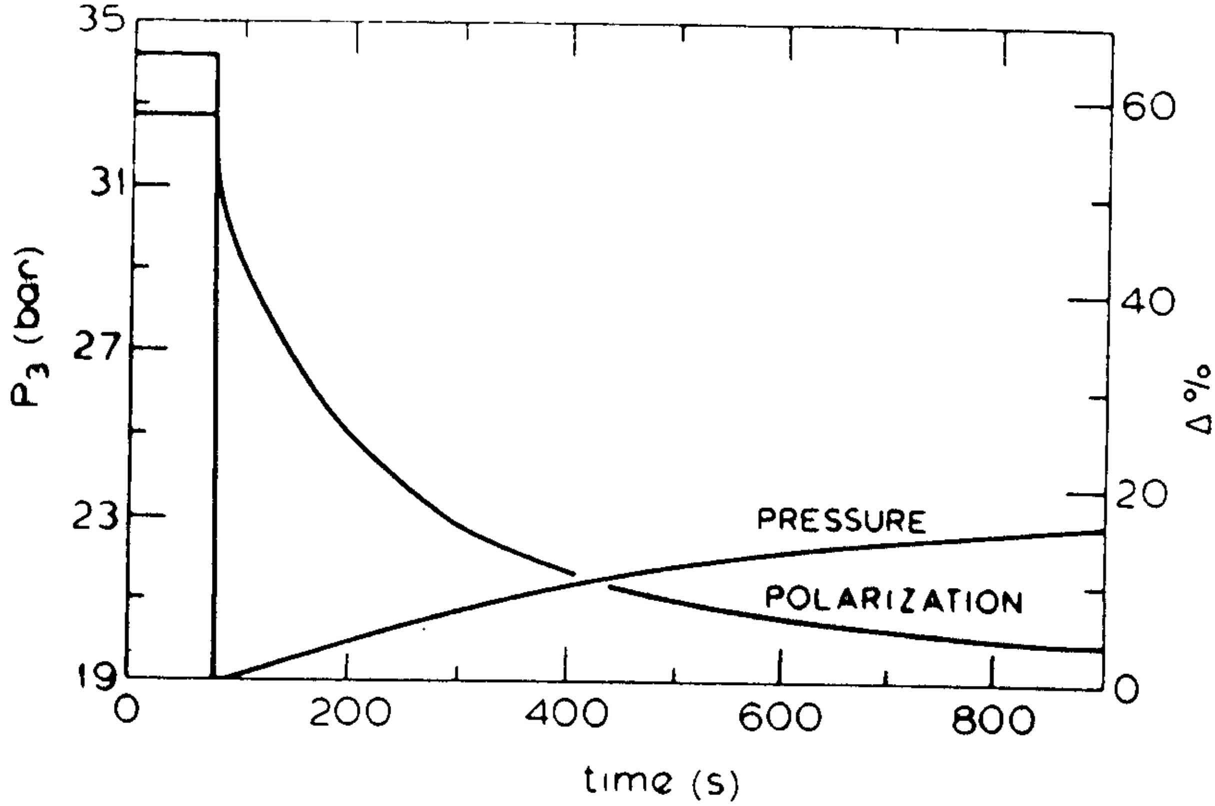
LIQUID WHICH APPEARS UPON MELTING OF SPIN-POLARIZED CRYSTAL HAS A HIGH NONEQUILIBRIUM SPIN DENSITY (Castaing and Nozières, 1979)

REASONS:

spin-relaxation time $T_1 \sim 10^5$ s due to weak dipole-dipole coupling of individual spins and long mean free time of Landau quasiparticles.

MELTING OF SPIN-POLARIZED CRYSTALS

IN HIGH TEMPERATURE REGION IS AN EFFICIENT METHOD TO OBTAIN THE HIGHLY POLARIZED (50%) METASTABLE (1000 s) LIQUID ^{3}HE ($H_{eff} \sim 100T$).



A rapid-melting experiment to produce highly polarized quid ^{3}He: P_3 and Δ vs. time.

(Vermeulen, Wiegers, Kranenburg, Jochemsen, and Frossati, 1987)

II LOW TEMPERATURE REGION

There are neither quasiparticles nor individual spins.

Ordered phases are described by hydrodynamic Eqs.

(SIMPLEST CASE: magnetic field $H \parallel \hat{z}$, liquid $= {}^3He$-B
crystal = antiferromagnetic u2d2 struct.

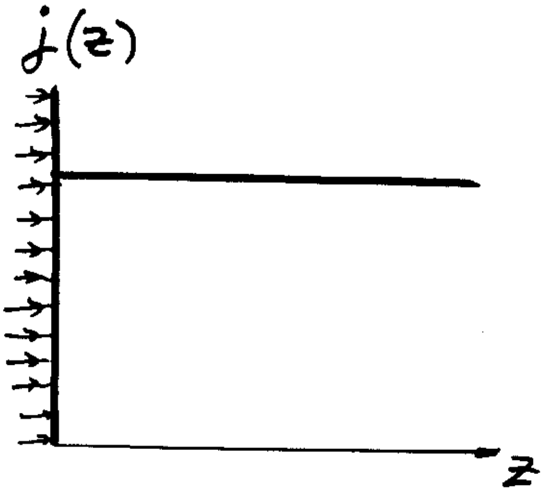
ORBITAL HYDRODYNAMICS OF SUPERFLUID ${}^3He$-B	SPIN HYDRODYNAMICS OF BOTH ${}^3He$-B AND SOLID u2d2
HYDRODYNAMIC Eqs.	
$\ddot{\varphi} - c^2 \Delta \varphi = 0$, c is sound velocity, $\varphi(\vec{r},t)$ is the phase of the order parameter,	$\ddot{\theta} - u^2 \frac{\partial^2 \theta}{\partial z^2} + \Omega^2 \theta = 0$, u is spin wave velocity, $\theta(\vec{r},t)$ is the spin rotation angle around $\hat{z}$, $\Omega \sim 10^5$ Hz is the longitudinal NMR frequency in ${}^3He$-B or AFMR frequency in the crystal.
MASS AND SPIN CURRENTS	
$\vec{j} = \rho_e \vec{v}_s = \frac{\hbar \rho_e}{2m} \nabla \varphi$	$j \equiv j_{zz} = -\frac{\chi u^2}{\gamma^2} \frac{\partial \theta}{\partial z}$
MASS AND SPIN DENSITIES	
$\rho_e = \rho_{e0} - \frac{\hbar \rho_{e0}}{2mc^2} \dot{\varphi}$ (ρ_{e0} is the equilibrium value).	$S = \frac{\chi H}{\gamma} + \frac{\chi}{\gamma^2} \dot{\theta}$

Spin conservation in low temperature region should be provided by a mechanism similar to that for mass conservation:

INSTEAD OF NONEQUILIBRIUM SPIN DENSITIES, SPIN SUPERCURRENTS ARISE BOTH IN THE LIQUID AND IN THE CRYSTAL

MASS CONSERVATION

stationary mass current into the bulk phase $j(0)$

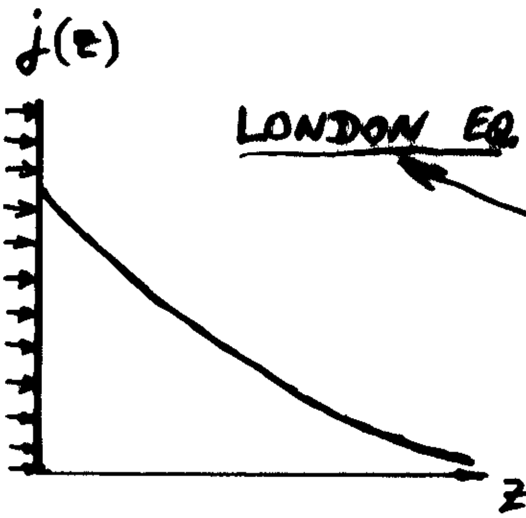


$$\cancel{\ddot{\varphi}} - c^2 \frac{\partial^2 \varphi}{\partial z^2} = 0,$$

$$j(z) = \frac{\hbar \rho_{s0}}{2m} \frac{\partial \varphi}{\partial z} = j(0)$$

SPIN (NON)CONSERVATION

stationary spin current into the bulk phase $j(0)$



LONDON EQ.

$$\cancel{\ddot{\theta}} - u^2 \frac{\partial^2 \theta}{\partial z^2} + \Omega^2 \theta = 0,$$

$$\ell^2 \frac{\partial^2 \theta}{\partial z^2} - \theta = 0,$$

$$(\ell = u/\Omega)$$

$$j(z) = j(0) e^{-z/\ell}.$$

$$l_e \sim 10^{-3}\,\text{cm}, \quad l_c \sim 10^{-5} - 10^{-6}\,\text{cm}.$$

DECAY OF SPIN CURRENT = VIOLATION OF SPIN CONSERVATION

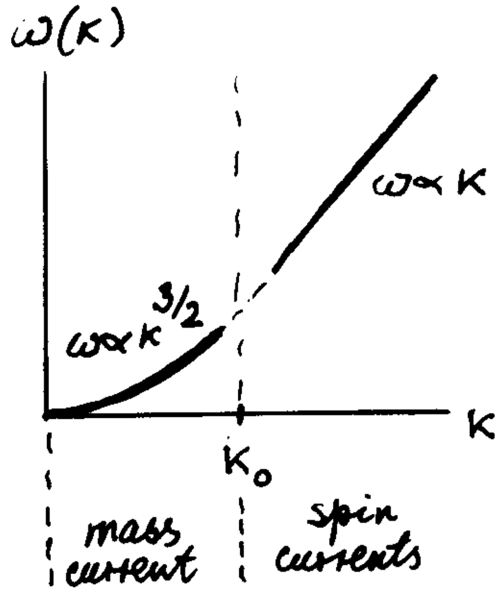
THIS IS NOT A SPIN RELAXATION!

LIKE IN SUPERCONDUCTORS, IT IS A NONDISSIPATIVE PHENOMENON.

CRYSTALLIZATION WAVES IN SPIN-POLARIZED ^{3}HE

kinetic energy of crystallization wave = kinetic energy of mass current in liquid + kinetic energy of spin currents in both liquid and crystal

SPECTRUM OF THE WAVES



$$K_0 \sim \frac{1}{l_e}\left(\frac{H_0}{H}\;\frac{\Delta\rho}{\rho}\right)^2$$

where $\Delta\rho = \rho_c - \rho_e \ll \rho$,

$\rho \simeq \rho_c \simeq \rho_e$,

H_0 is an exchange field:

$\mu H_0 \sim T_N$.

QUALITY FACTOR Q

Attenuation of the waves is mainly due to the interaction of the moving interface with thermal spin waves in the crystal:

$$Q^{-1} \simeq 10^2 (T/T_N)^4 \cdot \text{(most optimistic!)}$$

$T < 0.3\,T_N$ waves could be weakly damped.

EXPERIMENTS by Nomura, Kensley, Matsushita, and Mizusaki (1994):

The melting rate was too fast to be measured quantitatively.

The crystal melts faster at lower temperature

LT 21

Proceedings of the 21st International Conference on Low Temperature Physics
Prague, August 8-14, 1996

Part S6 – Plenary and invited papers

Crystallization and Melting of Spin-Polarized ^{3}He

Alexander F. Andreev

Kapitza Institute for Physical Problems, Russian Academy of Sciences,
Kosygin st. 2, 117334 Moscow, Russia

The melting and growth of ^{3}He crystals, spin-polarized by an external magnetic field, are different in nature depending on whether the temperature is higher or lower than the characteristic ordering temperatures in the crystal (the Neel temperature T_N) and in the liquid (the superfluid transition temperature T_c). In the high-temperature region ($T \geq T_N, T_c$) the liquid which appears upon melting has a high nonequilibrium spin density. In the low-temperature region ($T \ll T_N, T_c$) the melting and growth are accompanied by spin supercurrents both in the liquid and in the crystal in addition to mass supercurrents in the liquid. The crystallization waves at the liquid-solid interface should exist in the low-temperature region. With increasing magnetic field the waves change in nature, because the spin currents begin to play a dominant role. The wave spectrum becomes linear with a velocity inversely proportional to the magnetic field. The attenuation of the waves at low enough temperatures is mainly due to the interaction of the moving crystal-liquid interface with thermal spin waves in the crystal. The waves could be weakly damped at temperatures below a few hundreds microkelvins.

1. INTRODUCTION

Quantum nondissipative nature of crystallization and melting of ^{4}He crystals at low temperatures manifests itself directly in the existence of crystallization waves [1, 2] at the interface between solid and superfluid liquid phases. We would naturally expect similar phenomena also in ^{3}He at low enough temperatures. The criterion is that the melting and growth of crystals are quantum collective processes and so all individual degrees of freedom are frozen. This criterion is satisfied in ^{3}He at temperatures below the temperature T_c of the superfluid transition in the liquid phase and below the Neel temperature T_N of the antiferromagnetic transition in the solid phase.

The most interesting features of the growth and melting of ^{3}He crystals are revealed when the crystals are spin-polarized by an external magnetic field H. In this case, the solid and liquid phases are characterized by different values of not only the mass density but also another (approximately) conserved quantity: the spin density. In contrast with the relative difference between the mass densities, the relative difference between the equilibrium spin densities is by no means small in a magnetic field. The magnetic susceptibility of the crystal greatly exceeds the magnetic susceptibility of the liquid. The conservation of the total spin of the system during the melting or growth process is provided by very different ways depending on whether the temperature is higher or lower than the characteristic ordering temperatures T_c and T_N.

2. HIGH TEMPERATURE REGION (NONEQUILIBRIUM POLARIZATION)

In the high-temperature region ($T \geq T_N, T_c$), the liquid which appears upon the melting of the spin-polarized crystal has a high nonequilibrium spin density [3–6]. This is because of the extremely large value of the spin-relaxation time T_1. In the normal liquid ^{3}He, spin-relaxation takes place via the dipole-dipole coupling of individual spins of Landau quasiparticles during their scattering from one another. Being inversely proportional to the square of the weak dipole coupling constant and proportional to the long mean free time of quasiparticles, T_1 reaches values of order 10^5s at 1-10mK [7].

As a result, the melting of spin-polarized helium crystals in the high-temperature region is a powerful method to obtain the highly polarized metastable liquid. Using a Pomeranchuk cell at T<100mK one can obtain [6] liquid ^{3}He with a polarization of at least 50%.

3. LOW TEMPERATURE REGION (SPIN SUPERCURRENTS)

In the low-temperature region ($T \ll T_N, T_c$), both phases are completely ordered. There are neither quasiparticles nor individual spins. Properties of the ordered ground states are described by hydrodynamic equations for both spin and orbital degrees of freedom. Below we consider the simplest case to demonstrate main features of the phenomenon which are in fact the same in the general case where hydrodynamic equations are more complicated.

Let us assume that the magnetic field $\boldsymbol{H}$, directed along the normal to the crystal-liquid interface, $\hat{z}$, satisfies the condition $H \ll H_0$ where $\mu H_0 \sim T_N$, and μ is the magnetic moment of the ^{3}He nucleus. Under these conditions the solid phase has an antiferromagnetic u2d2 structure, while the liquid phase is ^{3}He-B. Let us assume, however, that the magnetic field is strong in comparison with the characteristic fields at which the anisotropy vector $\boldsymbol{n}$ of ^{3}He-B aligns parallel to the field, i.e. along $\hat{z}$, and at which the unit antiferromagnetic vector $\boldsymbol{l}$ of the solid phase runs perpendicular to $\boldsymbol{H}$. Under all conditions formulated above the spin hydrodynamic equations both for the liquid and for the crystal can be expressed in terms of the angle θ of the spin rotation around $\hat{z}$ [8–10]. The spin densities (the densities of the z-component of the spin) in the crystal $S_z \equiv S = S_c$ and in the liquid $S = S_l$ are

$$S_c = \frac{\chi_c}{\gamma} H + \frac{\chi_c}{\gamma^2}\dot{\theta}\,, \qquad S_l = \frac{\chi_l}{\gamma} H + \frac{\chi_l}{\gamma^2}\dot{\theta}\,, \qquad (1)$$

where χ_c is the magnetic susceptibility of the crystal in the direction perpendicular to $\boldsymbol{l}$, χ_l is the magnetic susceptibility of ^{3}He-B, $\chi_l \ll \chi_c$, and $\gamma = 2\mu/\hbar$ is the gyromagnetic ratio. The spin hydrodynamic equations for the liquid ($z > 0$) and for the crystal ($z < 0$) are

$$\ddot{\theta} - u_c^2 \frac{\partial^2 \theta}{\partial z^2} + \Omega_c^2 \theta = 0, \qquad \ddot{\theta} - u_l^2 \frac{\partial^2 \theta}{\partial z^2} + \Omega_l^2 \theta = 0, \quad (2)$$

where u_c and u_l are the velocities of the corresponding spin waves in the solid and liquid phases, $\Omega_l \sim 10^5$Hz is the frequency of the longitudinal NMR in liquid ^{3}He-B, $\Omega_c \sim 10^5$Hz is the frequency of the uniform oscillations of the vector $\boldsymbol{l}$ in the plane perpendicular to the magnetic field. This frequency depends on the angle ψ, between the normal $\hat{z}$ and the direction of the edge of the cubic cell of the crystal, along which there is a uudd alternation of spins: $\Omega_c^2 = \Omega_{c0}^2 \sin^2\psi$, where Ω_{c0} is a constant [9]. We assume that the characteristic length of the problem (in the plane $\hat{x}\hat{y}$) under consideration is large in comparison with the dipole length $l_D = u_l/\Omega_l \sim 10^{-3}$cm. Since the corresponding dipole length in the crystal, $l_c = u_c/\Omega_c \sim 10^{-5} - 10^{-6}$cm, satisfies the condition $l_c \ll l_D$, the spin rotation angle θ depends in Eq.(2) on the coordinate z only.

The superfluid motion in ^{3}He-B is determined by the usual equation of the orbital hydrodynamics

$$\ddot{\varphi} - c^2 \Delta\varphi = 0 \qquad (3)$$

where φ is the phase of the order parameter, c is the velocity of sound. The superfluid velocity is $\boldsymbol{v} = (\hbar/2m)\nabla\varphi$ where m is the ^{3}He particle mass. The density ρ_l of the liquid is determined by

$$\rho_l = \rho_{l0} - \frac{\hbar\rho_{l0}}{2mc^2}\dot{\varphi} \qquad (4)$$

where ρ_{l0} is the equilibrium value of the density.

Equations (1),(2) are similar to the hydrodynamic equations (3),(4), the spin-rotation angle θ playing the role of the phase φ. So in low-temperature region, the conservation of spin should be provided by a mechanism similar to that for the conservation of mass. Namely, instead of nonequilibrium spin densities, spin supercurrents should arise both in the liquid and in the crystal during the growth of spin-polarized crystals. The spin supercurrents (the flux densities of the z-component of the spin along the $\hat{z}$ direction) in the liquid, j_l, and in the crystal, j_c, are determined by formulas:

$$j_l = -\frac{\chi_l u_l^2}{\gamma^2}\frac{\partial\theta}{\partial z}\,, \qquad j_c = -\frac{\chi_c u_c^2}{\gamma^2}\frac{\partial\theta}{\partial z}\,, \qquad (5)$$

which are analogous to the expression for the superfluid mass current:

$$\mathrm{j} = \rho_l \mathrm{v} = \frac{\hbar\rho_l}{2m}\nabla\varphi\,. \qquad (6)$$

There is, however, a difference between spin and mass supercurrents connected with the violation of the conservation of spin due to the weak dipole-dipole interaction described by the last terms in both equations (2). To see the difference, let us assume that the characteristic frequency of the problem under consideration is low in comparison with Ω_l and Ω_c. We can then ignore the first terms in both equations (2). We obtain the equation

$$l^2 \frac{\partial^2\theta}{\partial z^2} - \theta = 0\,, \qquad (7)$$

where $l = l_D$ in the liquid and $l = l_c$ in the crystal. Eq.(7) coincides with the London equation for superconductors, the dipole lengths, l_D and l_c, playing the role of the penetration depths. In our case, Eq.(7) describes the decay of the spin currents (given by Eq.(5)) and the nonequilibrium spin densities (given by the second terms in both equations(1)), as functions of the distance from surfaces of the bulk ordered phases. It is important to note that this decay, like in superconductors, is not a (spin) relaxation. It is a completely nondissipative phenomenon.

4. CRYSTALLIZATION WAVES IN ZERO MAGNETIC FIELD

In zero magnetic field, crystallization waves in ^{3}He should be completely analogous to the case of ^{4}He. The waves are oscillations of the positions of the interface caused by periodic processes of melting and crystallization. To calculate the spectrum of the waves and attenuation we may use the results obtained for ^{4}He (see Ref.[11]). Let the initially flat crystal-liquid interface, $z = 0$, experience a displacement

$$z = \zeta(x,t) = \zeta_0(t)\, e^{ikx} \tag{8}$$

in the direction normal to the interface. The corresponding change E_{pot} in the surface energy

$$\int \alpha(\mathrm{N})\, dS \tag{9}$$

plays the role of the potential energy of the crystallization wave. Here $\alpha(N)$ is the surface energy as a function of the direction N of the normal to the surface, dS is the surface element. For small ζ we can assume $N_x = \sin\phi \approx \partial\zeta/\partial x$, $N_y = 0$, $N_z = \cos\phi \approx 1$,

$$\begin{aligned} dS &= dx\, dy \left[1 + \frac{1}{2}\left(\frac{\partial\zeta}{\partial x}\right)^2\right], \\ \alpha(\phi) &= \alpha(0) + \frac{\partial\alpha}{\partial\phi}\frac{\partial\zeta}{\partial x} + \frac{1}{2}\frac{\partial^2\alpha}{\partial\phi^2}\left(\frac{\partial\zeta}{\partial x}\right)^2. \end{aligned} \tag{10}$$

So, we obtain

$$E_{pot} = \frac{S}{4}\,\kappa(k)\,|\zeta|^2, \tag{11}$$

where $\kappa(k) = \tilde{\alpha}k^2$ is the effective stiffness of the crystallization wave, S is the total surface area, $\tilde{\alpha} = \alpha + (\partial^2\alpha/\partial\phi^2)$ is the surface stiffness of the interface.

Since the densities of the crystal ρ_c and the liquid ρ_l differ from each other, the nonzero velocity of the interface $\dot{\zeta}$ leads to the superfluid motion in the liquid. The superfluid motion is determined by Eq.(3). The boundary conditions on this equation are that $\varphi(z)$ is finite as $z \to \infty$ and the mass is conserved at $z = 0$:

$$\rho_l v_z = -(\rho_c - \rho_l)\,\dot{\zeta}\,. \tag{12}$$

The velocity of crystallization waves is much smaller than the velocity of sound. We can thus ignore the first term in Eq.(3). Solving the Laplace equation $\Delta\varphi = 0$ under the boundary conditions formulated above, we obtain

$$\varphi = \frac{2m\Delta\rho}{\hbar\rho k}\, e^{-kz}\dot{\zeta} \tag{13}$$

where $\Delta\rho = \rho_c - \rho_l$. Since the condition $\Delta\rho \ll \rho$ holds, we will use the letter ρ without subscript, to denote the common value of the density: $\rho \approx \rho_c \approx \rho_l$. The total kinetic energy E_{kin} of the surface is

$$E_{kin} = \int dx\, dy \int_0^\infty dz\, \frac{\rho}{4}\,|\mathrm{v}|^2 = \frac{S}{4}\, M_m(k)\,|\dot{\zeta}|^2, \tag{14}$$

where $M_m(k) = (\Delta\rho)^2/\rho k$ is the effective mass of the wave.

At finite temperatures T the motion of the interface is accompanied by the energy dissipation caused by the interaction with thermal excitations both in the liquid and in the crystal. Assuming for simplicity that for all branches of thermal excitations the energy spectrum $\varepsilon = \varepsilon(p)$ ($p = |\boldsymbol{p}|$, $\boldsymbol{p}$ is the momentum) is isotropic and the reflection from the stationary interface is total and specular, we can express (see Refs.[12, 13]) the energy $\dot{E}$ which is dissipated per unit time in terms of the energy spectrum of the excitations:

$$\dot{E} = \frac{S}{2}\, f(k)\,|\dot{\zeta}|^2 \tag{15}$$

where

$$f(k) = \frac{1}{2\pi^2\hbar^3}\sum \int_0^\infty p^3 n_0(\varepsilon)\, dp \tag{16}$$

is the effective friction coefficient, $n_0(\varepsilon) = (e^{\varepsilon/T} - 1)^{-1}$ is the equilibrium distribution function, the sum is taken over all branches of thermal excitations. For thermal excitations with a linear energy spectrum $\varepsilon = up$, the friction coefficient

$$f = \frac{\pi^2}{30\hbar^3}\left(\frac{T}{u}\right)^4 \tag{17}$$

is inversely proportional to the fourth power of the velocity u of the excitations.

To find the spectrum $\omega = \omega(k)$ of the crystallization waves and their attenuation, we note that Eqs.(11),(14) and (15) coincide with similar equations for an oscillator with a fundamental frequency determined by

$$\omega^2(k) = \frac{\kappa(k)}{M_m(k)} = \frac{\tilde{\alpha}\rho}{(\Delta\rho)^2}\, k^3 \qquad (18)$$

and a quality factor Q determined by

$$Q^{-1} = \frac{\mathrm{Im}\,\omega}{\omega} = \frac{1}{2}\,\frac{f(k)}{\sqrt{M_m(k)\,\kappa(k)}}. \qquad (19)$$

5. CRYSTALLIZATION WAVES IN SPIN-POLARIZED ^{3}HE

In an external magnetic field the motion of the interface is accompanied in the low-temperature region by spin supercurrents in addition to mass flows. The boundary conditions on Eq.(7) are that $\theta(z)$ is finite as $z \to \pm\infty$, $\theta(z)$ is continuous at $z = 0$, and the z-component of the spin is conserved at $z = 0$:

$$j_l - j_c = -(S_c - S_l)\dot{\zeta} = -\frac{\chi_c}{\gamma}\, H\dot{\zeta}. \qquad (20)$$

Here we have used the condition $\chi_c \gg \chi_l$ and have neglected terms with the nonequilibrium parts of spin densities as second order terms in ζ. Solving Eq.(7) under the formulated boundary conditions we obtain

$$\theta(z) = \frac{-\gamma H \chi_c}{\chi_c u_c \Omega_c + \chi_l u_l \Omega_l}\,\dot{\zeta} \times \begin{cases} \exp\left(\dfrac{-z}{l_D}\right) & \text{if } z>0, \\ \exp\left(\dfrac{z}{l_c}\right) & \text{if } z<0. \end{cases} \qquad (21)$$

In contrast with the mass current in Eq.(13) which penetrates into the liquid on distances of order $1/k \to \infty$ as $k \to 0$, the spin currents defined by Eq.(5) and Eq.(21), in each phase are nonzero only in narrow regions with thicknesses l_D and l_c near the interface.

The kinetic energy of the crystallization wave is generally the sum of the kinetic energies of the mass flow and the spin currents. The energy density of the spin current in the crystal is given by

$$E_c = \frac{\chi_c}{2\gamma^2}\left[u_c^2\left(\frac{\partial\theta}{\partial z}\right)^2 + \Omega_c^2\theta^2\right]. \qquad (22)$$

The energy density in the liquid, E_l, is found from Eq.(22) by replacing χ_c by χ_l, u_c by u_l, and Ω_c by Ω_l. Substituting Eq.(21) into Eq.(22), and integrating the energy density over z, we find the following expression for the spin part of the total kinetic energy of the crystallization wave:

$$E_{kin} = \frac{S}{4}\, M_s\, |\dot{\zeta}|^2, \qquad (23)$$

where

$$M_s = \frac{\chi_c^2 H^2}{\chi_c u_c \Omega_c + \chi_l u_l \Omega_l} \qquad (24)$$

is the spin part of the effective mass of the wave. The mass part of the kinetic energy is determined by Eq.(14) while the potential energy is given by Eq.(11).

The frequency of the crystallization wave is determined by

$$\omega^2(k) = \frac{\tilde{\alpha}k^2}{M(k)}, \qquad (25)$$

where $M(k) = M_m + M_s$. Since the relative difference between the spin densities of the crystal and the liquid is not small in contrast with the difference between the mass densities, the spin part of the effective mass begin to play a decisive role in comparatively weak fields, as we will see below. The general expression for the mass $M(k)$ can be written in the form

$$M(k) = \rho d\left[\left(\frac{H}{H_0}\right)^2 + \left(\frac{\Delta\rho}{\rho}\right)^2 \frac{1}{kd}\right], \qquad (26)$$

where we have introduced a characteristic length

$$d = \frac{\chi_l u_l^2}{\chi_c u_c \Omega_c + \chi_l u_l \Omega_l} \qquad (27)$$

and a characteristic field $H_0 = (u_l/\chi_c)(\rho\chi_l)^{1/2}$. The length d is smaller than the dipole length in the liquid, l_D, by a factor of only several units. The field H_0 is of the order of the exchange field: $\mu H_0 \sim T_N$.

We assume that the magnetic field satisfies the condition $H \gg H_0\Delta\rho/\rho$, i.e., that the parameter $\epsilon = H_0\Delta\rho/(\rho H)$ is small in comparison with one. Spin currents then dominate the mass in Eq.(26) over a broad range of wavelengths satisfying the inequality $kd \gg \epsilon^2$. The spectrum of the crystallization waves is linear, $\omega = sk$, with a wave velocity

$$s = \sqrt{\frac{\tilde{\alpha}}{\rho d}}\,\frac{H_0}{H}. \qquad (28)$$

Only under the condition $kd \ll \epsilon^2$ we can ignore the contribution of spin currents, and only under this condition does the usual law (18) hold.

In the region of weak fields, $H \ll H_0\Delta\rho/\rho$, the contribution of spin currents is negligible for all k satisfying the condition $kd \leq 1$.

6. ATTENUATION OF THE WAVES

Since the energy $\hbar\Omega_{c,l} \sim 10^{-5}$K is very small, we will assume that $T \gg \hbar\Omega_{c,l}$. Under this condition, energy gaps of order $\hbar\Omega_{c,l}$ are negligible in thermal excitations spectra. Among all branches of gapless thermal excitations both in the liquid and in the crystal, the spin waves in the crystal are characterized by a minimum velocity $u \sim u_c$. According to Eq.(17) they give the main contribution to the attenuation of the crystallization waves at low enough temperatures. Since $T_N \sim \hbar u_c/a$, where a is the crystal lattice period, we have from Eq.(17):

$$f \sim \hbar \left(\frac{T}{aT_N}\right)^4 . \tag{29}$$

The quality factor of the crystallization waves is found from Eq.(19) by replacing M_m by M. Using Eq.(19) and Eq.(26), we obtain the following expression for Q^{-1} which is valid, in order of magnitude, for $H \leq H_0$ and $kd \leq 1$ in both regions of high, $H \gg H_0\Delta\rho/\rho$, and weak, $H \ll H_0\Delta\rho/\rho$, fields:

$$Q^{-1} \sim 10^2 \left(\frac{T}{T_N}\right)^4 \left[\frac{H}{H_0}\, kd + \frac{\Delta\rho}{\rho}\sqrt{kd}\right]^{-1} . \tag{30}$$

At given T this value of Q^{-1} is minimum in our region of k and H at $kd \sim 1$ and $H \sim H_0$. Thus, we see from Eq.(30) that the crystallization waves with $kd \leq 1$ could exist in ^{3}He as weakly damped waves at temperatures below a few tenths of T_N.

The growth and melting of u2d2 solid ^{3}He in superfluid ^{3}He-B have been studied by Nomura et al.[14] for temperatures between 0.4 and 0.9 T_N. The melting rate was too fast to be measured quantitatively, but it was noticed that the temperature dependence of the melting rate was rather strong and the solid melts faster at lower temperatures.

REFERENCES

[1] A.F.Andreev and A.Y.Parshin, Sov. Phys. JETP 48 (1978) 763.

[2] K.O.Keshishev, A.Y.Parshin and A.Babkin, JETP Lett. 30 (1979) 56.

[3] B.Castaing and P.Nozieres, J. Phys. (Paris) 40 (1979) 257.

[4] M.Chapelier, G.Frossati and F.B.Rasmussen, Phys. Rev. Lett. 42 (1979) 904.

[5] G.Schumacher, D.Thoulouze, B.Castaing, Y.Chabre, P.Segransan and J.Joffrin, J. Phys. Lett. (Paris) 40 (1979) 143.

[6] G.A.Vermeulen, S.A.J.Wiegers, C.C.Kranenburg, R.Jochemsen, and G.Frossati, Can. J. Phys. 65 (1987) 1560.

[7] D.Vollhardt and P.Wolfle, The Superfluid Phases of Helium 3 (Taylor and Francis, London, 1990) p.394.

[8] W.F.Brinkman and M.C.Cross, in: Progress in Low Temperature Physics, ed. D.F.Brewer, Vol.7A (North-Holland, Amsterdam, 1978) p.106.

[9] D.D.Osheroff, M.C.Cross and D.S.Fisher, Phys. Rev. Lett. 44 (1980) 792.

[10] A.F.Andreev and V.I.Marchenko, Sov. Phys. Usp. 23 (1980) 21.

[11] A.F.Andreev, in: Progress in Low Temperature Physics, ed. D.F.Brewer, Vol.8 (North-Holland, Amsterdam, 1982) p.67.

[12] A.F.Andreev and V.G.Knizhnik, Sov. Phys. JETP 56 (1982) 226.

[13] R.M.Bowley and D.O.Edwards, J. Phys. (Paris) 44 (1983) 723.

[14] R.Nomura, H.H.Hensley, T.Matsushita, and T.Mizusaki, JLTP 94 (1994) 377.