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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS  
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Research Workshop on Condensed Matter Physics  
30 June - 22 August 1997

**MINIWORKSHOP ON  
QUANTUM MONTE CARLO SIMULATIONS OF LIQUIDS AND SOLIDS  
30 JUNE - 11 JULY 1997  
and  
CONFERENCE ON  
QUANTUM SOLIDS AND POLARIZED SYSTEMS  
3 - 5 JULY 1997**

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## **"Quantum Solids and Polarised Systems"**

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**These are preliminary lecture notes, intended only for distribution to participants.**

# **Quantum Solids and Polarised Systems**

## **ICTP, Trieste, July 1997**

**Recent experiments on helium films adsorbed on graphite**

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- 1. Heat capacity of  $^3\text{He}$  -  $^4\text{He}$  mixture films**  
2D and layered Fermi fluids
- 2. Superfluidity of atomically layered  $^4\text{He}$  films**  
Observation of sub-monolayer superfluidity
- 3. NMR and heat capacity of 2D  $^3\text{He}$  solid**  
Frustrated 2D magnets and multiple spin exchange

Research in collaboration with Marcio Siquiera, Martin Dann, Bob Ray, Jan Nyéki and Brian Cowan

Supported by the Engineering and Physical Sciences Research Council (United Kingdom)

## 2D FERMI LIQUIDS

$$g(\epsilon) = \text{const.} = \frac{A}{\pi \hbar^2} m^*$$

$$T_F = \frac{\pi \hbar^2}{k_B m^*} \cdot n_2 = \frac{0.505}{(m^*/m)} \cdot n_2 \quad \text{K nm}^{-2}$$

heat capacity  $c = \gamma T$  where  $\gamma = \frac{\pi k_B^2 m^* A}{3 \hbar^2}$

$$= 8.99 \cdot 10^{-2} A \frac{m^*}{m} \frac{\text{J}}{\text{K}}$$

### SYSTEMS

- $^3\text{He}$  on bare graphite - 1<sup>st</sup> or 2<sup>nd</sup> layer fluid

$$m^* = m \left( 1 + \frac{1}{2} F_1^s \right)$$

as  $n_2 \rightarrow 0$   $m^* \rightarrow m$  ;  $\frac{m^*}{m} \sim 6$  at  $5.5 \text{ nm}^{-2}$

(cf bulk liquid at  $p=0$   $\frac{m^*}{m} = 2.80$  , 1 bulk "liquid" layer  $\approx 6.5 \text{ nm}^{-2} \Rightarrow$  low dimensionality  $\rightarrow$  bigger interactions)

ideal gas value  $\rightarrow \frac{\chi}{\chi_0} = \frac{m^*/m}{1 + F_0^s} \quad F_0^s \rightarrow -0.76$

Greywall Phys. Rev. B 41, 1842 (1990) ; Lusher, Saunders and Cowan Phys. Rev. Lett. 67, 2497 (1991) ; Morhard, Bäuerle, Bossy, Bunkov, Fisher and Godfrin, Phys. Rev. 53, 2658 (1996)

- $^3\text{He}$  at the surface of  $^4\text{He}$  (Andreev Sov. Phys. JETP 23, 939 (1966))

$$m^* = M_H \left( 1 + \frac{1}{2} F_1^s \right) \quad (\text{cf in bulk solutions } M_H/m = 2.24)$$

$\hookrightarrow$  hydrodynamic mass

$$\frac{M_H}{m} = 1.45 \pm 0.1 \text{ at surface of bulk} \quad \text{Edwards \& Salam. Prog Low Temp Phys VII A (1975)}$$

# HEAT CAPACITY ANOMALY IN TWO DIMENSIONAL FLUID $^3\text{He}$

NEW LETTERS

Greywall & Busch

2 JULY 1990

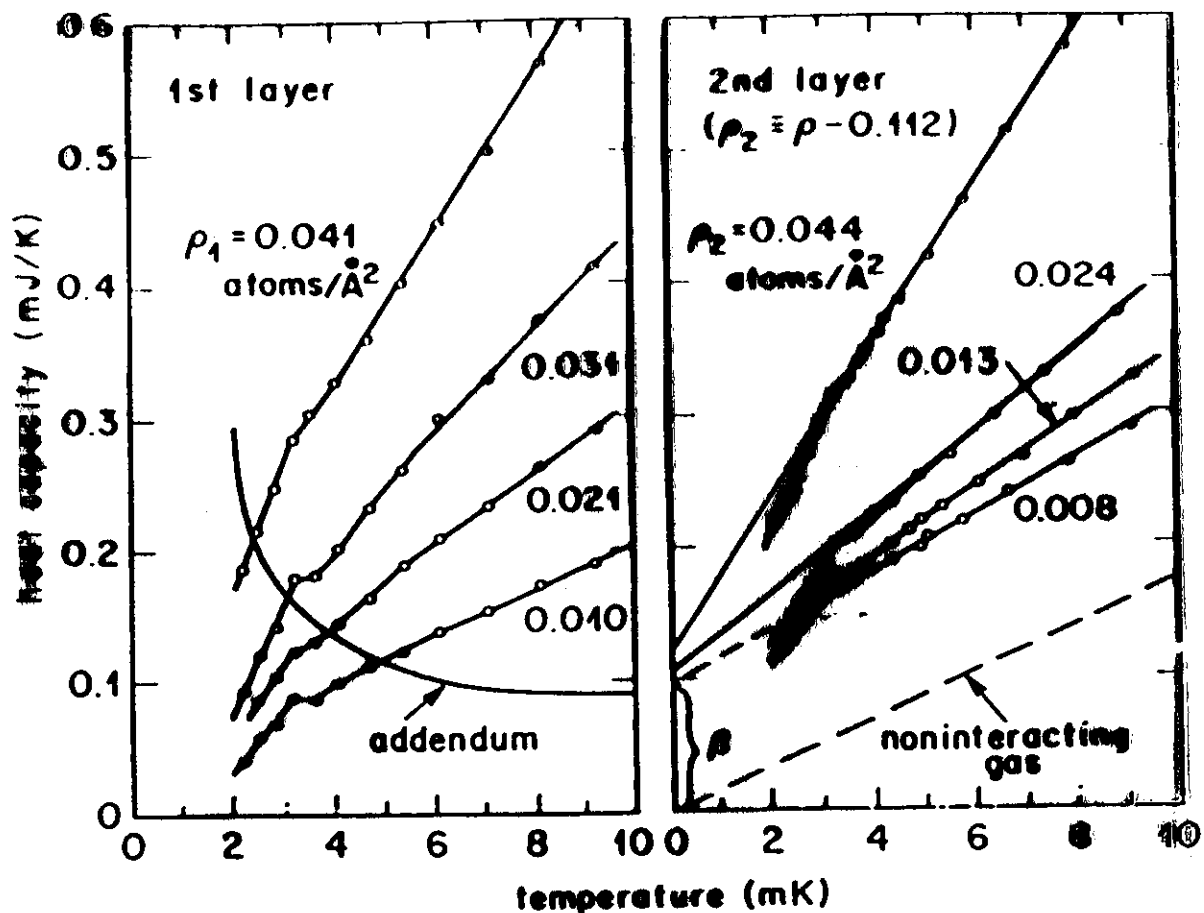


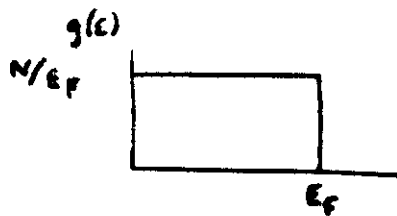
FIG. 2. First- and second-layer fluid heat capacities in the vicinity of a sharp feature at 3.2 mK. Also indicated are the addendum and the heat capacity of an ideal degenerate 2D Fermi gas.

$$c = \beta + \gamma T$$

↑  
believed to arise from heterogeneity

# NUCLEAR MAGNETIC SUSCEPTIBILITY OF TWO DIMENSIONAL FLUID $^3\text{He}$

FERMI GAS



$$\chi = N^2 g(E_F) = \frac{N \mu^2}{k_B T_F}$$

$$T_F = \frac{\pi \hbar^2}{k_B m} \cdot \frac{N}{A}$$

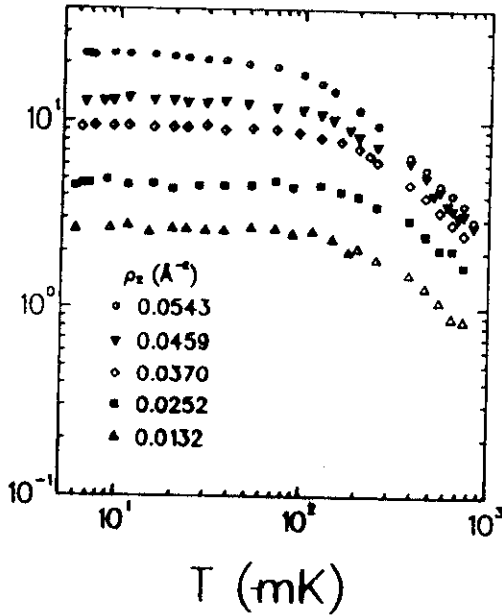
$$\chi_0 = C/T_F \text{ independent of } \rho = N/A$$

FERMI LIQUID

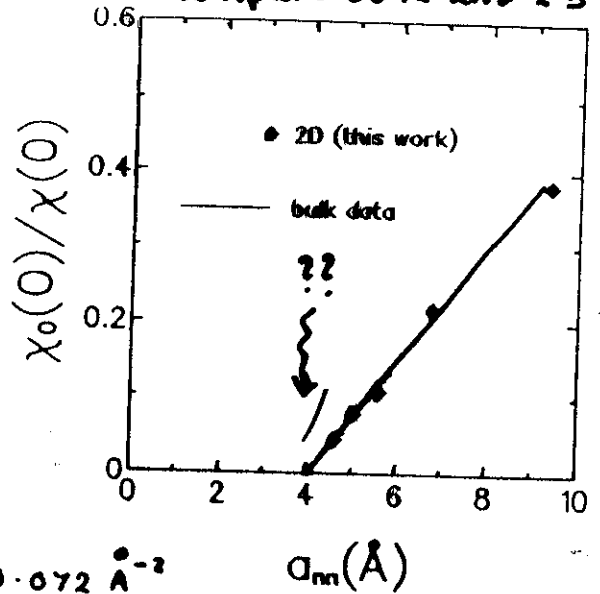
$$\chi/\chi_0 = m^*/m (1 + F_0^a)^{-1}$$

(INTERACTIONS)

$\chi/\chi_0(0)$

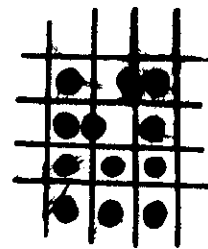
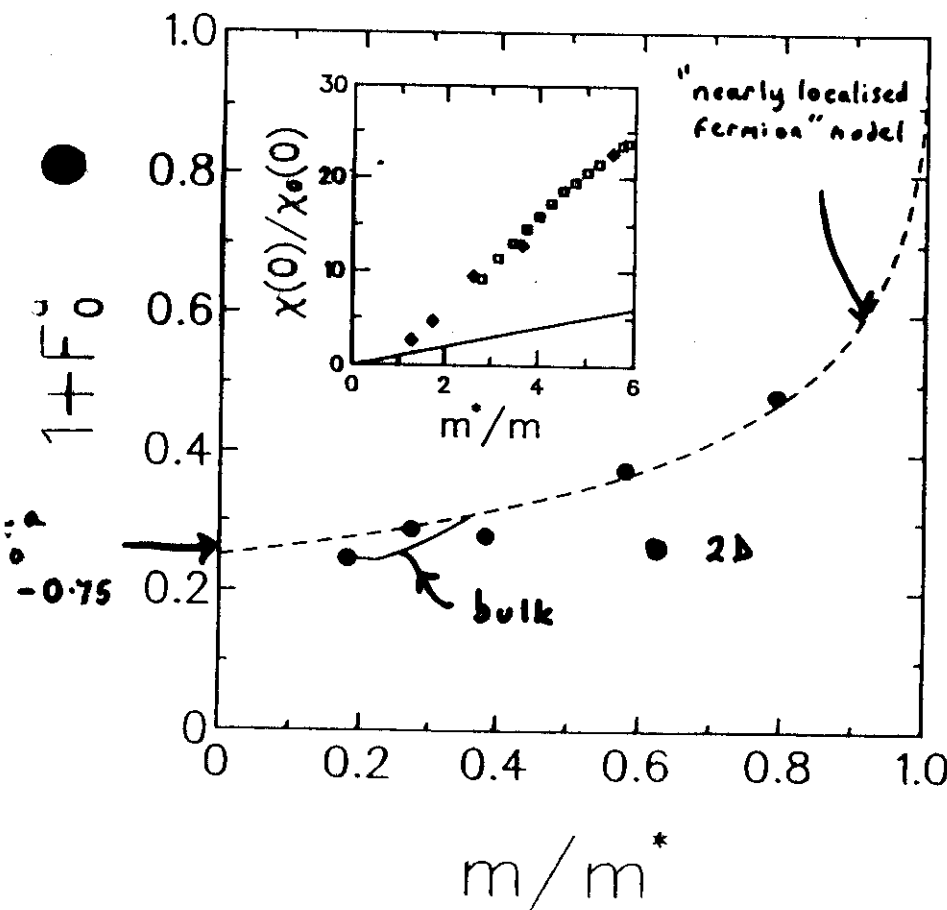


compare bulk and 2D



$$?? \equiv 0.072 \text{ A}^{-2}$$

$1 + F_0^a$



lattice gas model  
repulsive interaction  
 $I = U_c$

$$m^*/m = (1 - I^2)^{-1}$$

$$F_0^a \sim -[1 - (1 + I)^{-2}]$$

note as  $I \rightarrow 1$

$$m^*/m \rightarrow \infty$$

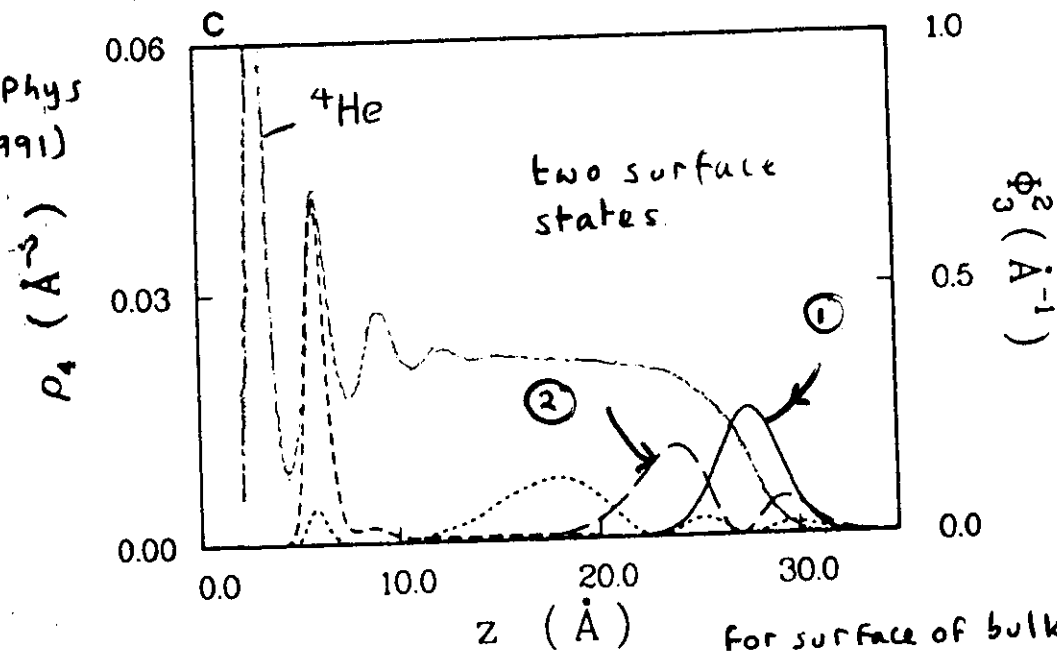
$$F_0^a \rightarrow -0.75$$

# Structure of $^4\text{He}$ films

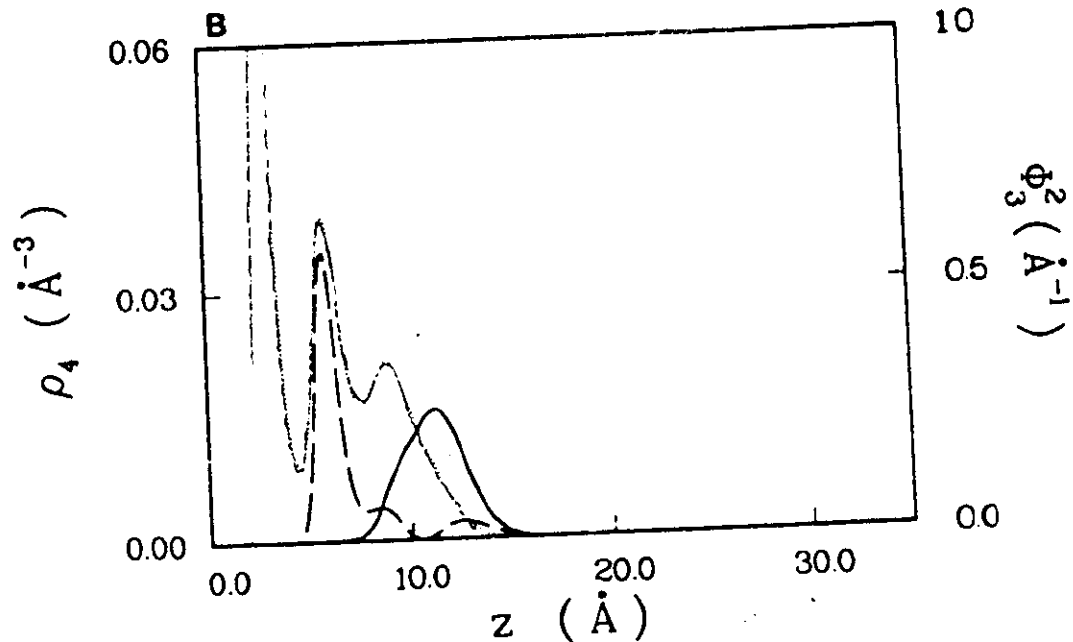
## $^3\text{He}$ surface states on $^4\text{He}$ films

these pictures from  
Pavloff &  
Treiner

1. Low Temp Phys  
83, 331 (1991)



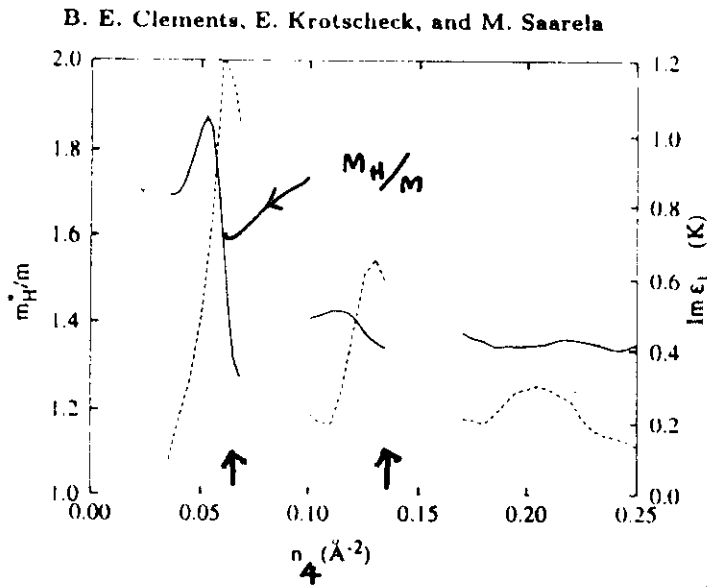
for surface of bulk liquid see  
Dalfovo & Stringari, Physica  
Scripta 38, 204 (1988)



calculations by Krotscheck and co-workers since ~1985 see  
Clements, Krotscheck, Saarela Phys. Rev B 55, 5959 (1997) and refs. therein

# Predicted coverage dependence of hydrodynamic effective mass

Krotscheck, Clements, Saarela



gaps arise from  
layer by layer 2D  
condensation of  $^4\text{He}$   
film

↑ indicates where new layer  
begins to form

## Energetics of surface states

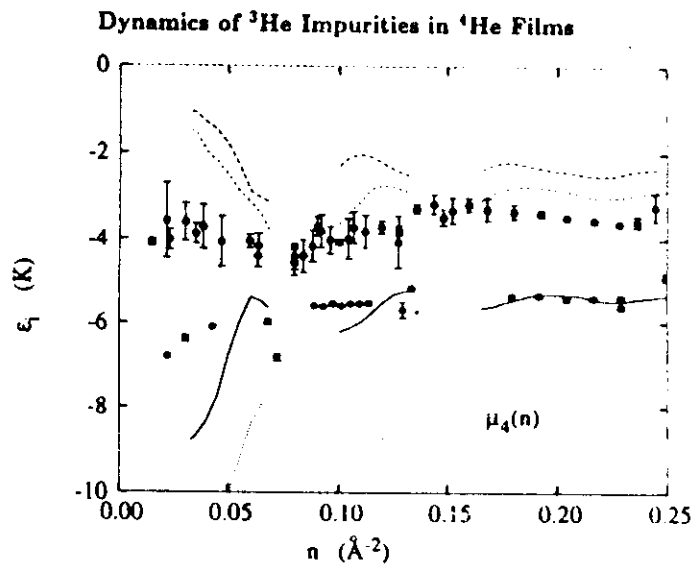
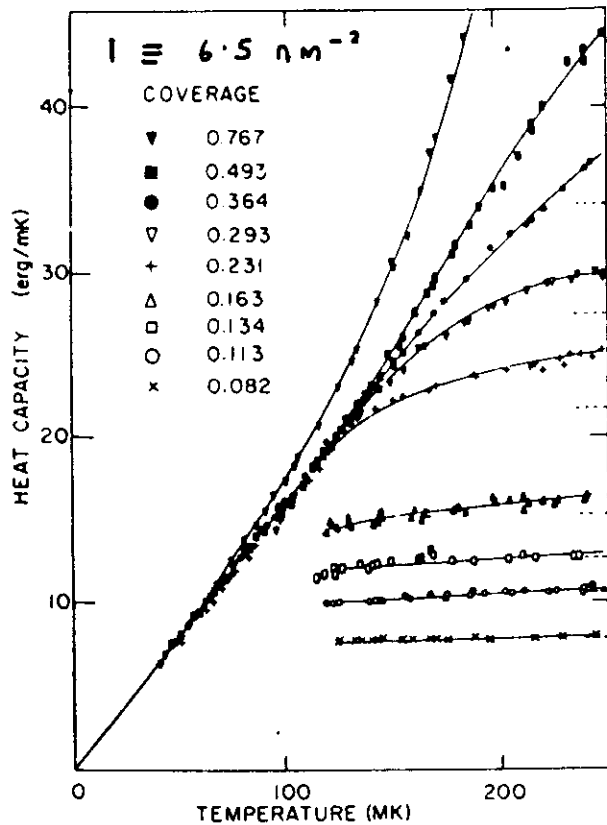


Fig. 1. Our theoretical results for the ground-state energy of  $^3\text{He}$  impurities in  $^4\text{He}$  films (solid line) and the first excited state (short-dashed line) are compared with the data of Ref. 5 for the ground state (solid circles) and the first excited state (open circles) with error bars. Also shown is the first excitation energy calculated within the static approximation (long-dashed line) and the chemical potential of the background (dotted line). The experimental data were shifted horizontally by  $0.25 \text{ \AA}^{-2}$  to account for the first two solid layers.

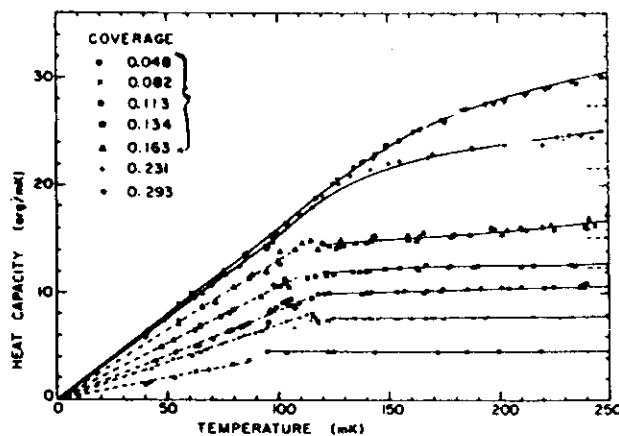
## Early heat capacity data

Gasparini and co-workers (State University of New York, Buffalo).

substrate; nuclepore filter paper



anomalous results at low coverages  $n_3 < 1.3 \text{ nm}^{-2}$   
Puddling?



$$\gamma \propto n_3$$

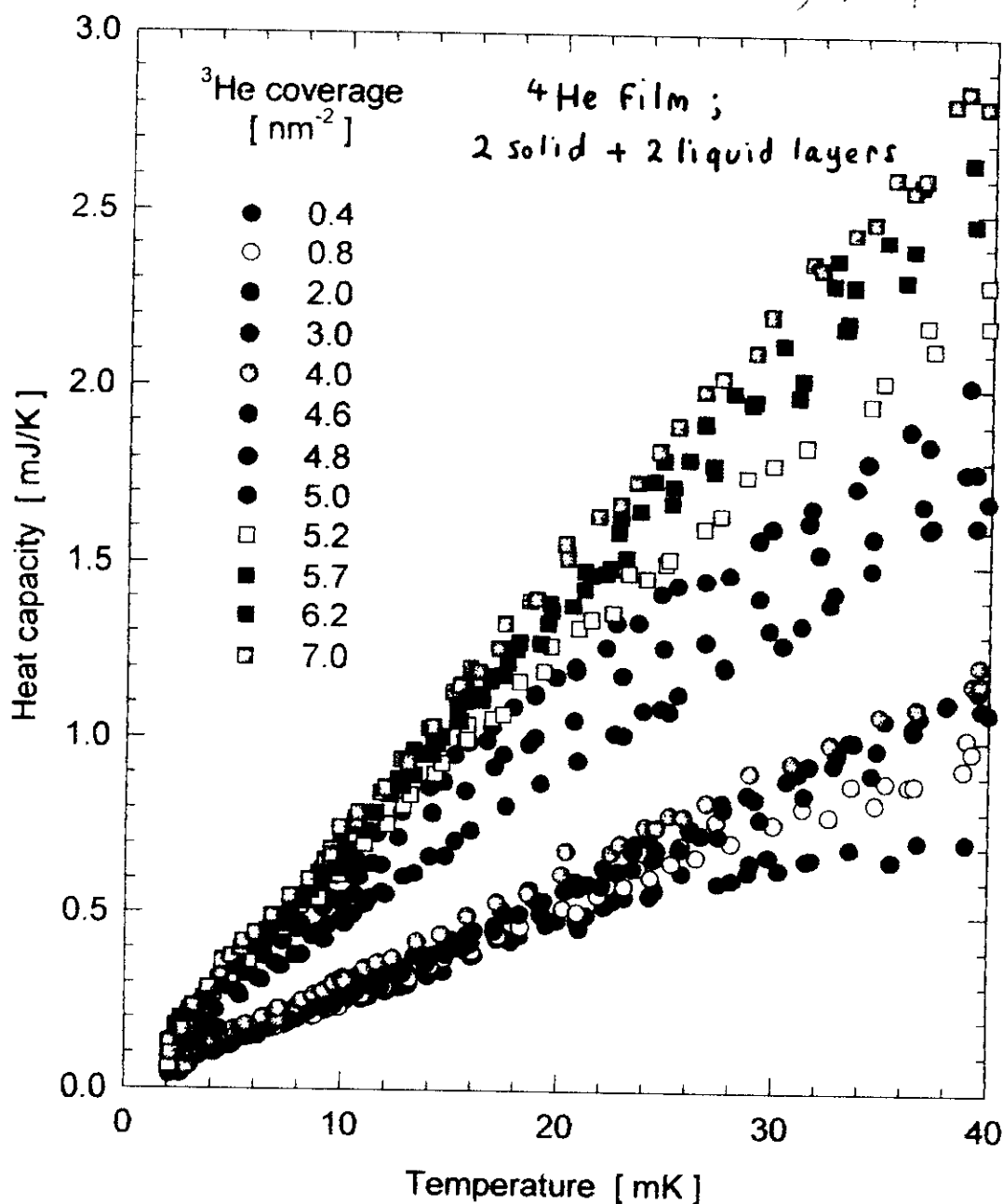
FIG. 1. Heat capacity of  $^3\text{He}$  at various coverages on a  $10\text{-}\text{\AA}$   $^4\text{He}$  film. The solid lines represent a fit of the data under the assumption that the  $^3\text{He}$  is homogeneously spread out over the surface of the film. The dashed lines are drawn to guide the eye.

$^3\text{He} - ^4\text{He}$  mixture films on graphite

(Dann, Nyéki, Cowan, Saunders - submitted to J. Low Temp. Phys. (QFS 97))

Fit data to  $c = \cancel{\beta} + 8T^2 + BT^3$

note ; (i) no  $\beta$  term (ii) for  $n_3 < 4 \text{ nm}^{-2}$  no strong  $n_3$  dependence  
so no 'puddling'



$$\gamma = 8.99 \cdot 10^{-2} \times 182 \times \frac{m^*}{m} \quad (\text{mJ K}^{-2})$$

total surface area  $A = 182 \text{ m}^2$   
-8-

before step  $n_2 < n_c$  **EFFECTIVE MASS**  $m^* = m_H (1 + \frac{1}{2} F_1^S)$

1) before step  $n_3 < 4 \text{ nm}^{-2}$

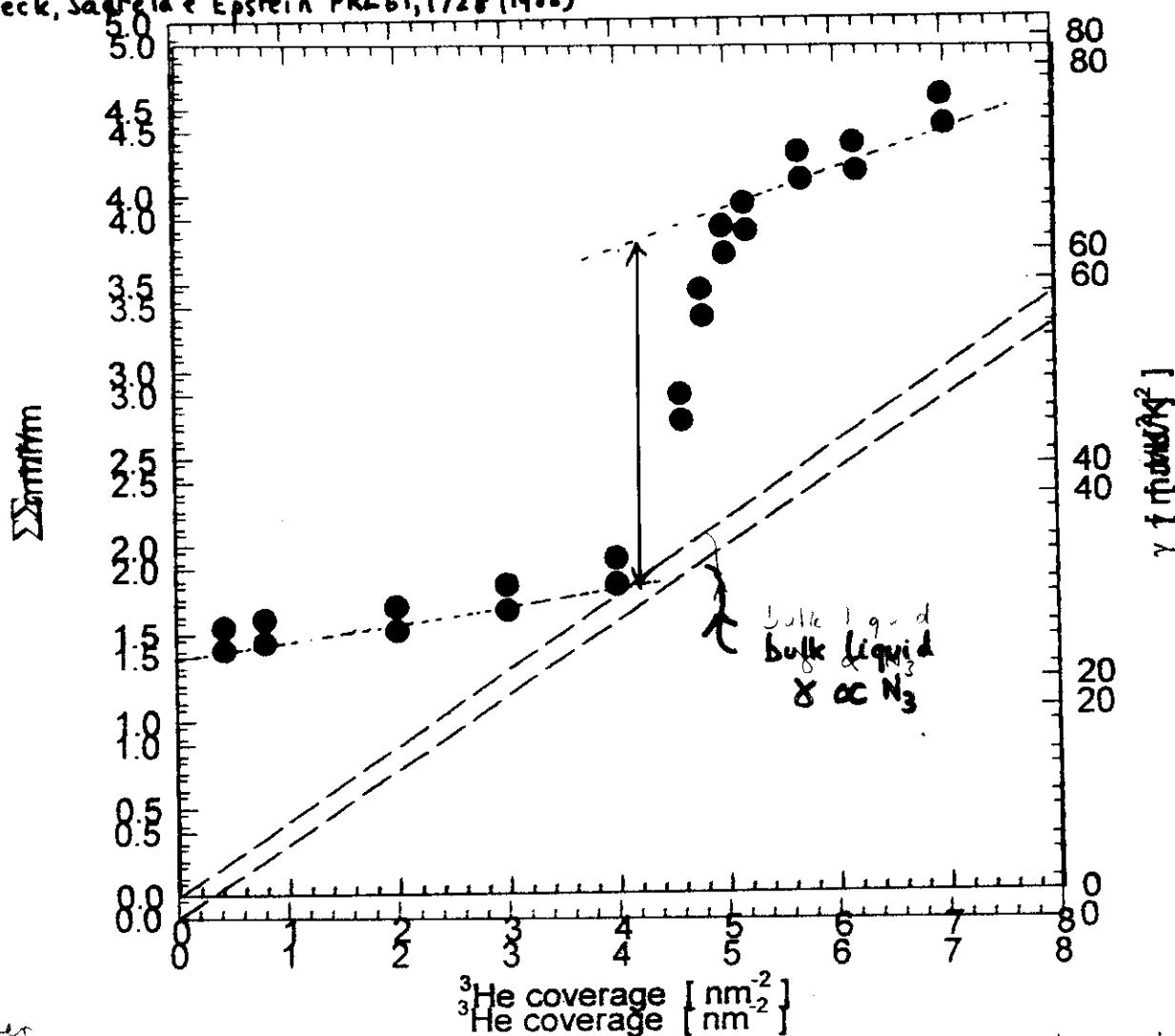
2) Fermi liquid  $m^* = m_H (1 + \frac{1}{2} F_1^S)$

$$\Rightarrow \frac{m_H}{m} = 1.48$$

$$F_1^S = 0.40 \pm 0.04 \text{ at } n_3 = 3.2 \text{ nm}^{-2}$$

in reasonable agreement with Krotscheck, Sagdeev & Epstein PRL 61, 1728 (1988)

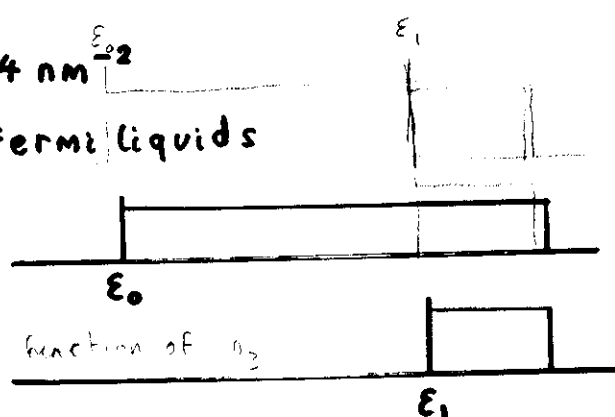
( $\Rightarrow F_0^a \sim -0.4$  using magnetization data)



after

step  $n_3 > 4 \text{ nm}^{-2}$

two 'layered' 2D Fermi liquids



but  $\epsilon_1 - \epsilon_0$  is a function of  $n_3$

height of step

height of step

$$\Rightarrow \frac{m_H}{m} \sim 1.9$$

in 1<sup>st</sup> excited state

# Magnetization measurements

Hallock and co-workers (University of Massachusetts)

Higley, Sprague and Hallock Phys. Rev. Lett. 63, 2570 (1989)

observed magnetization step

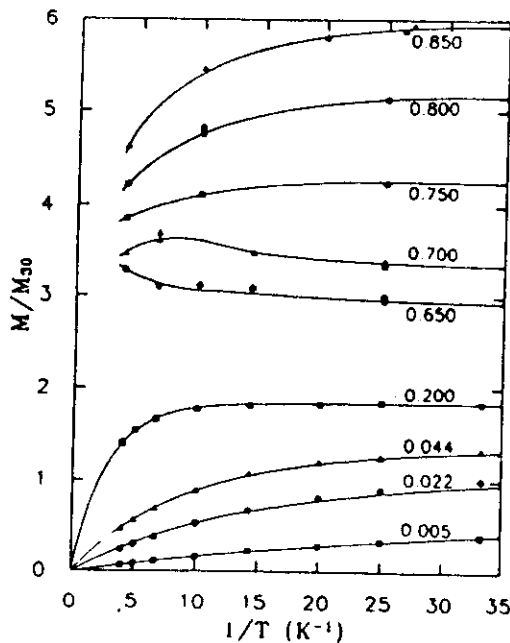


FIG. 1. Magnetization of the sample vs  $1/T$  at fixed  $^4\text{He}$  coverage  $44 \mu\text{mol/m}^2$  for the  $^3\text{He}$  coverages shown at the right. The positive  $dM/dT$  at  $d_3=0.65$  layer precedes the step in magnetization seen in Fig. 2. The curves for  $d_3 \leq 0.2$  layer are 2D Fermi-gas fits, and the curves for  $d_3 \geq 0.65$  layer are a guide to the eye.

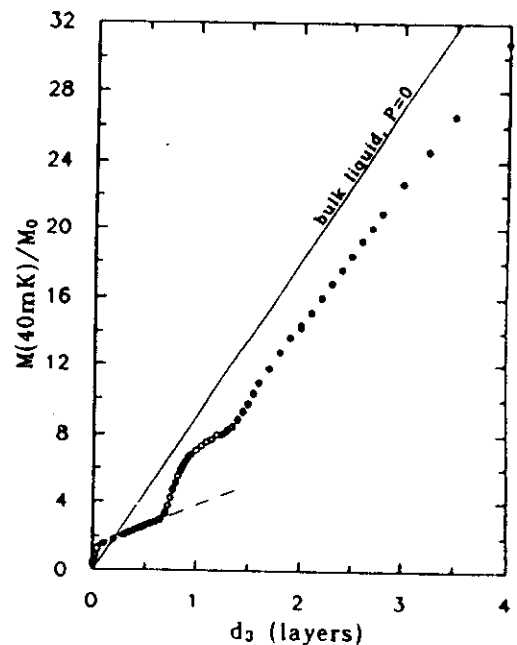


FIG. 2. Magnetization at  $T=40 \text{ mK}$  vs  $d_3$ , showing steps in magnetization at  $d_3 \approx 0.8$  and  $1.5$  layers, and an increase in magnetization with the same slope as for bulk liquid above  $d_3=2$  layers. The dashed line extrapolates low-coverage data to obtain the hydrodynamic mass  $m_h = 1.38m_3$ .

bulk liquid behavior is observed for rather thin films

state of a thicker film

$^3\text{He}$

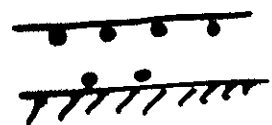
$^4\text{He}$

quantum state effects

see review by Hallock in Prog. Low Temp. Phys. XIV (1985)

## SOME FUTURE PROSPECTS

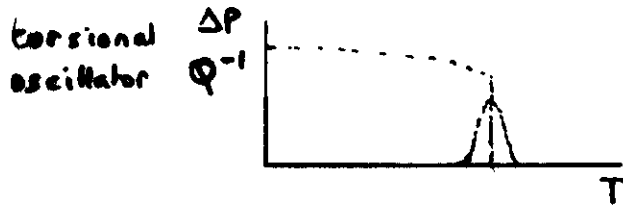
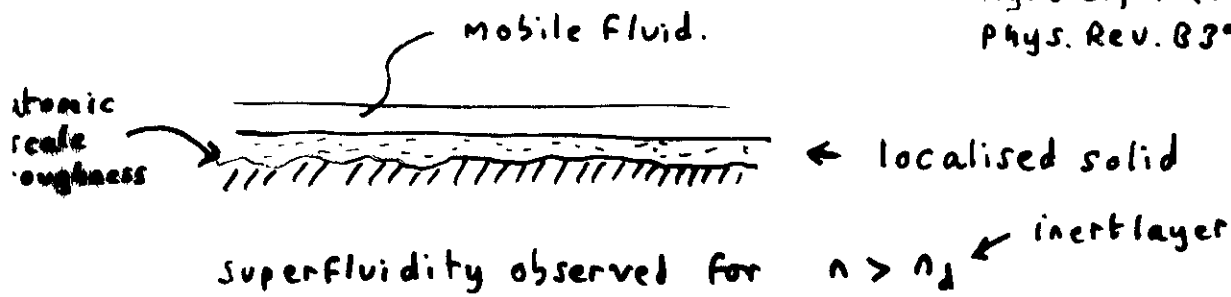
for  $^3\text{He} - ^4\text{He}$  liquid "mixture films" on graphite

- In this system a  $^4\text{He}$  film with an integral number of layers can be prepared with good precision
- to observe the predicted structure of  $M_H$  vs  $A_H$  (2D phase separation) ( $M_H$  gives information about where  $^3\text{He}$  sits in film)
- is there a substrate state? 
- quasiparticle interactions with  $n_q = 2$  (solid) layers and  $n_q = 3$  layers (2 solid + 1 sfl)
- measurements of  $\chi$  and  $c \rightarrow F_0^A + F_1^S$  on same system
- phase transitions — dimerization + superfluidity.

# SUPERFLUIDITY OF THIN $^4\text{He}$ FILMS

## 1. Amorphous substrate

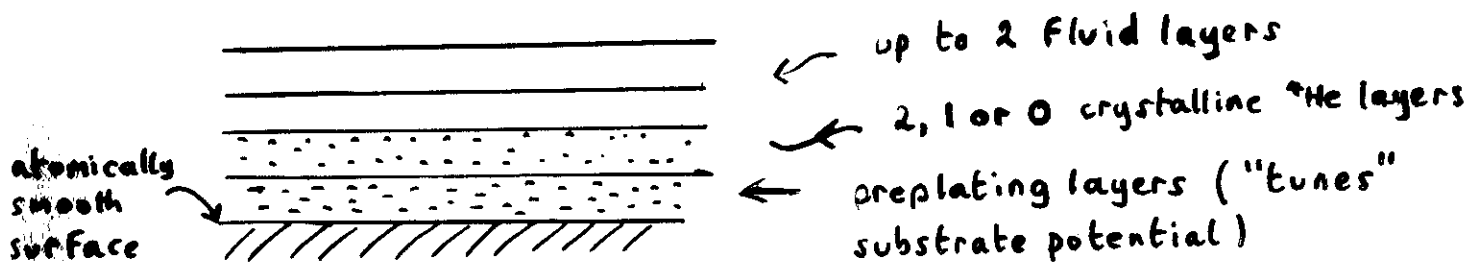
Bishop & Reppy PRL 40, 1727 (1980), Phys. Rev B 22, 5171 (80)  
 Agnollet, McQueeney & Reppy Phys. Rev. B 39, 8934 (1989)



universal Kosterlitz - Thouless relation

$$\frac{\rho_s(T_c)}{T_c} = \frac{2m^2 k_B}{\pi \hbar^2} = 5.254 \text{ nm}^{-2} \text{ K}^{-1}$$

## 2. Atomically flat substrate → atomically layered film



2  $^4\text{He}$  solid layers ← no prelating.

1  $^4\text{He}$  solid layer ← 2 layers HD  
 3 layers HD

no  $^4\text{He}$  solid layer ← 2 layers Ne + 5 layers HD

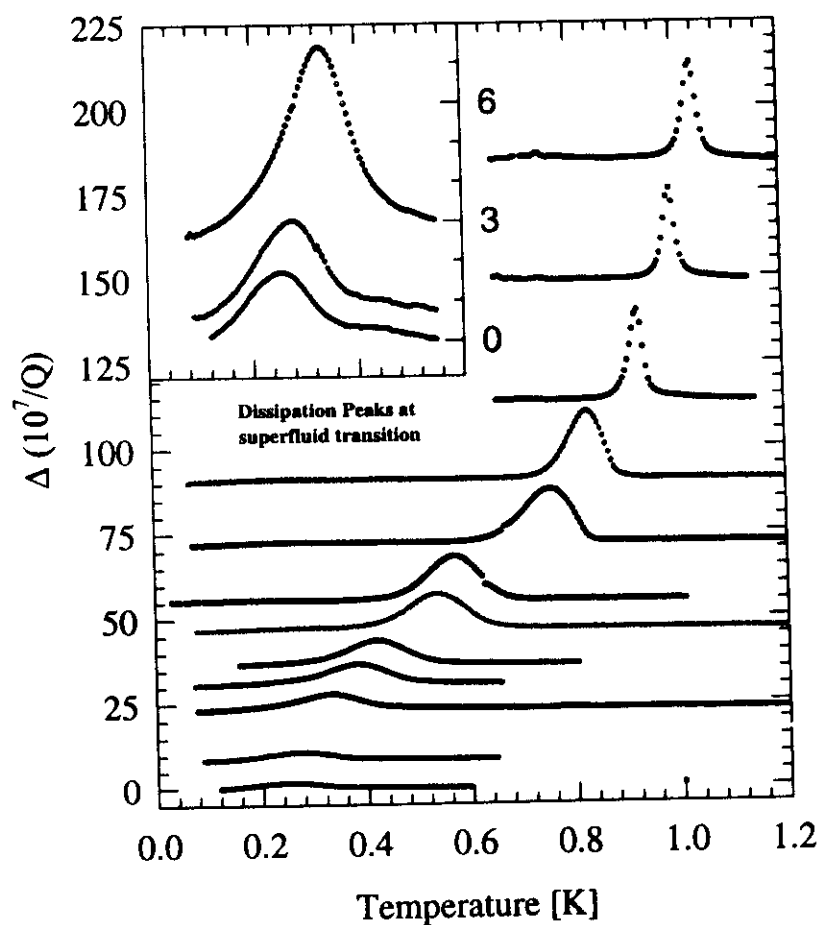
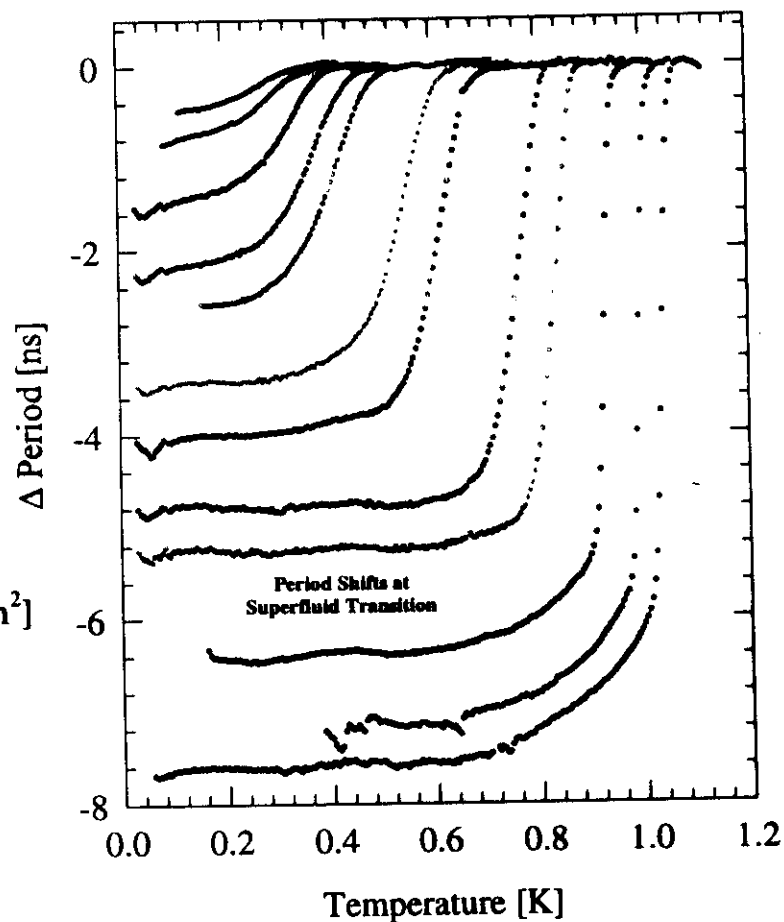
↓  
 SUBMONOLAYER SUPERFLUIDITY |  
 ○

- PIMC calculations of  $^4\text{He}$  on  $\text{H}_2$  Wagner & Ceperley J. Low Temp. Phys. 94, 185 (1994)
- $3^{\text{rd}}$  sound studies by Mochel & co-workers

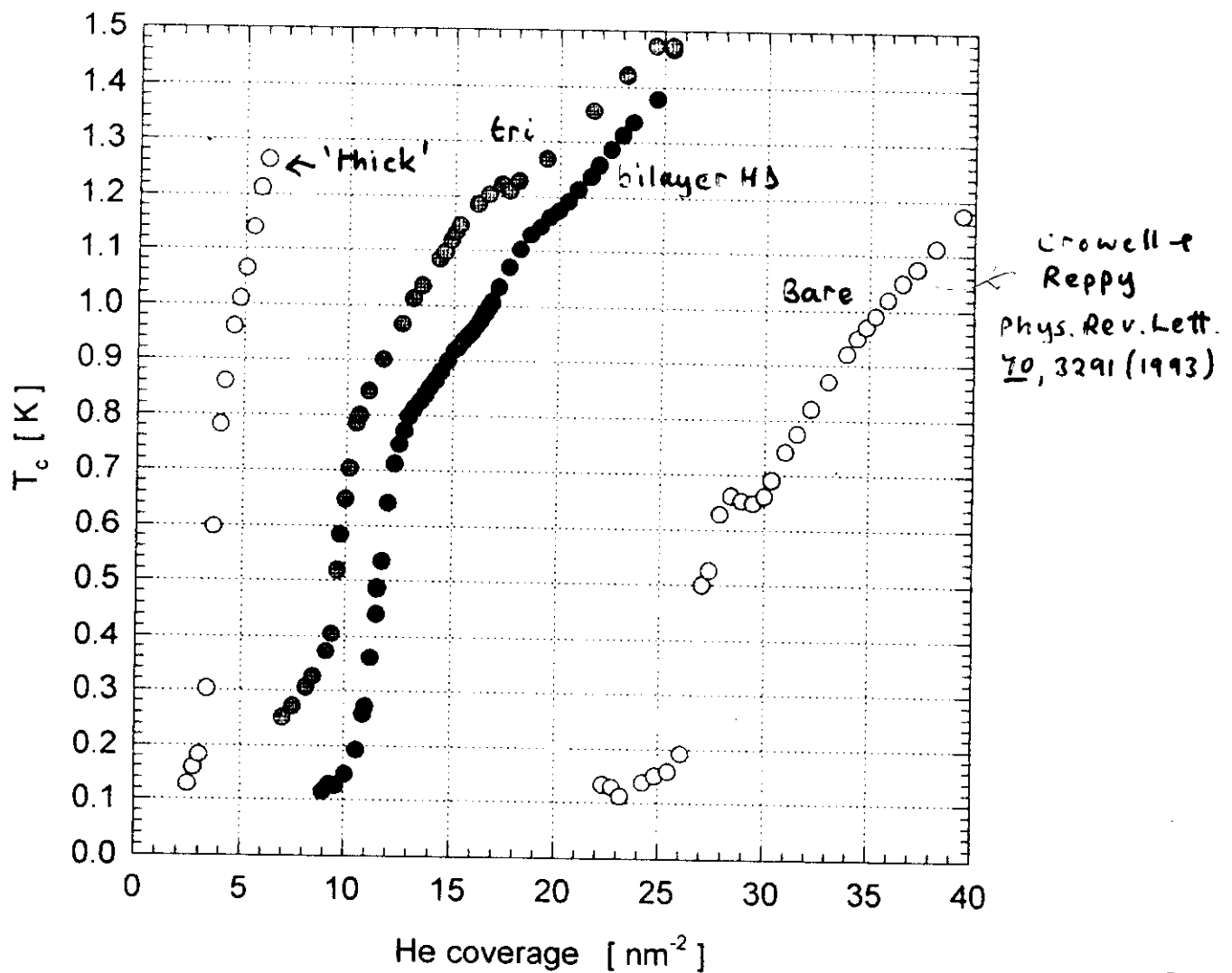
# Trilayer Preplating

Coverage [atoms/nm<sup>2</sup>]

- 7.02
- 7.50
- 8.15
- 8.47
- 9.11
- 9.35
- 9.61
- 9.74
- 10.16
- 10.60
- 11.71
- 12.58
- 13.11



# Temperature of Dissipation Peak as a Function of Coverage For various preplatings

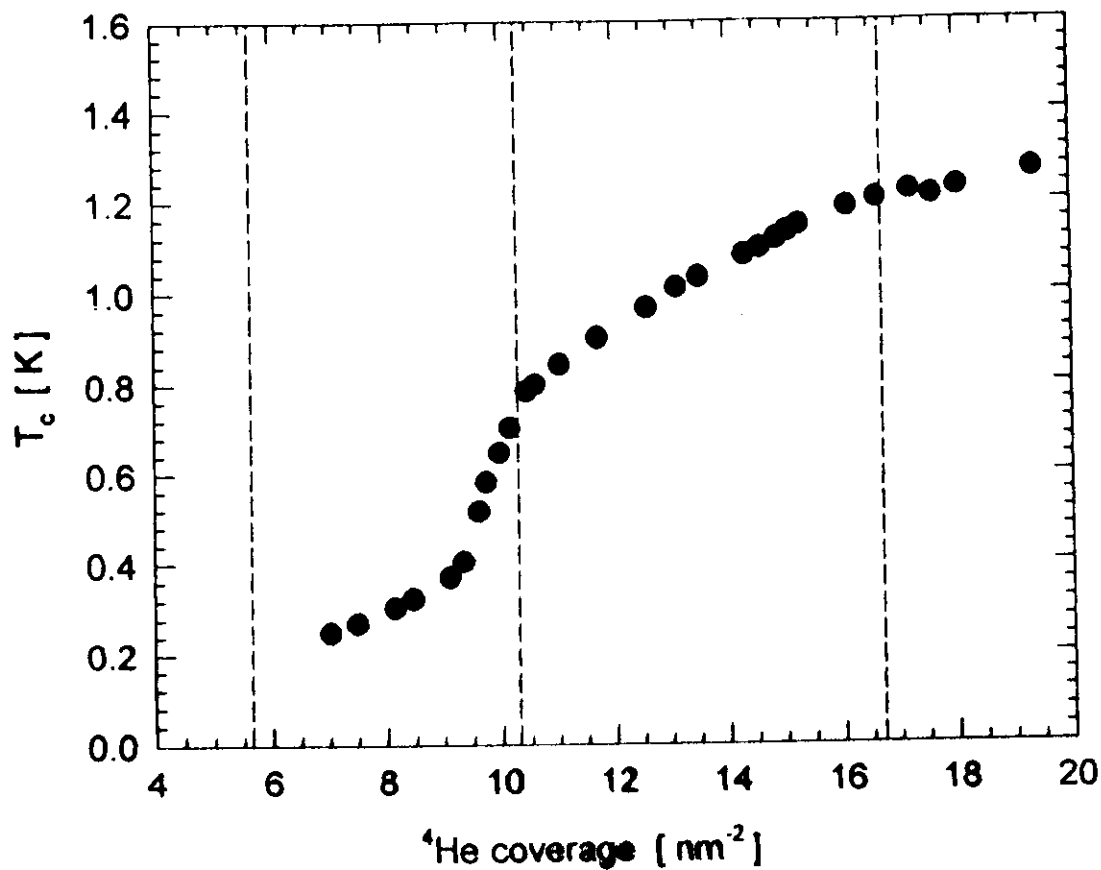


Bare grafoil - no preplating [1]. First two He layers are solid.  
3rd layer completion at  $28 \text{ nm}^{-2}$   
4th layer completion at  $35.6 \text{ nm}^{-2}$

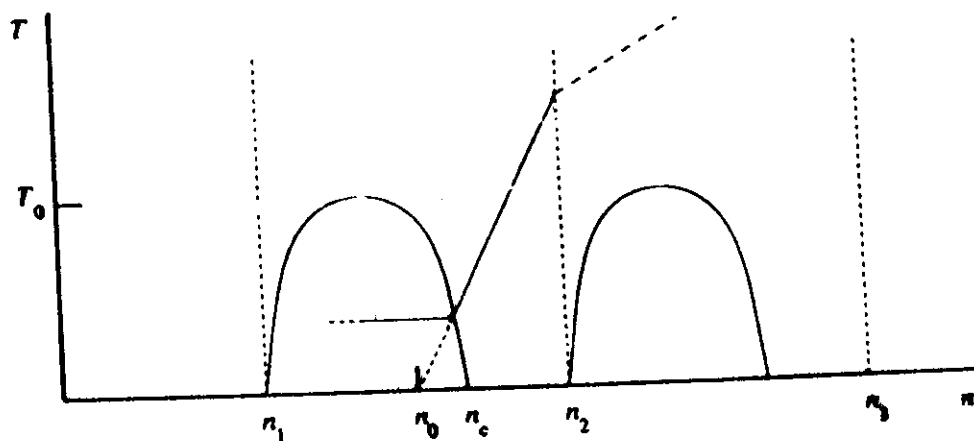
Grafoil preplated by HD trilayer. Total HD density  $27.54 \text{ nm}^{-2}$   
Isothermal compressibility of He film has minima at  
 $5.65 \text{ nm}^{-2}$ ,  $10.3 \text{ nm}^{-2}$ ,  $16.7 \text{ nm}^{-2}$  and  $24.0 \text{ nm}^{-2}$ , respectively

Grafoil preplated by HD bilayer. Total HD coverage  $19.16 \text{ nm}^{-2}$   
Compressibility minima at  
 $7.3 \text{ nm}^{-2}$ ,  $12.5 \text{ nm}^{-2}$ ,  $18.7 \text{ nm}^{-2}$ ,  $25.2 \text{ nm}^{-2}$

# Temperature of the dissipation peak



CLEMENTS, KROTSCHKE, LAUTER, PRL 70 (1993) 1287 - theory  
 EBEL, VILCHES, private communication, heat capacity measurements  
 on  $\text{H}_2$  plated graphite



Cheng, Cole, Saam, Treiner; PRB 46 (1992) p.13957

KT relation 
$$T_c = \frac{M_s(T_c) \pi \hbar^2}{2m k_B} \approx 0.22 \frac{M_s(0) \pi \hbar^2}{2m k_B}$$

## TWO DISTINCT REGIMES

### I single fluid layer

structure 2D gas liquid coexistence region (puddles)  $n < n_c$   
↓  
uniform fluid  $n > n_c$

- coverage dependence of  $T_c$  and  $\Delta F$  suggest that superfluidity is suppressed in the uniform film.  
This suppression is coverage dependent.\*

possible

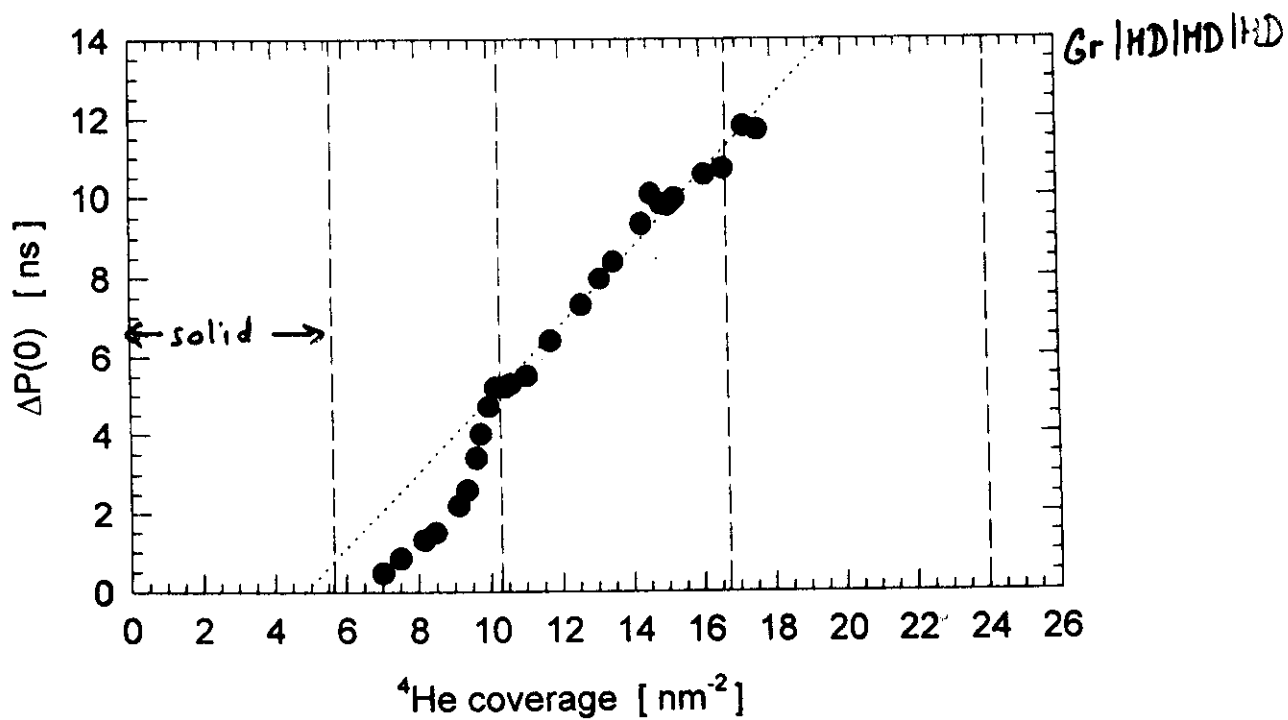
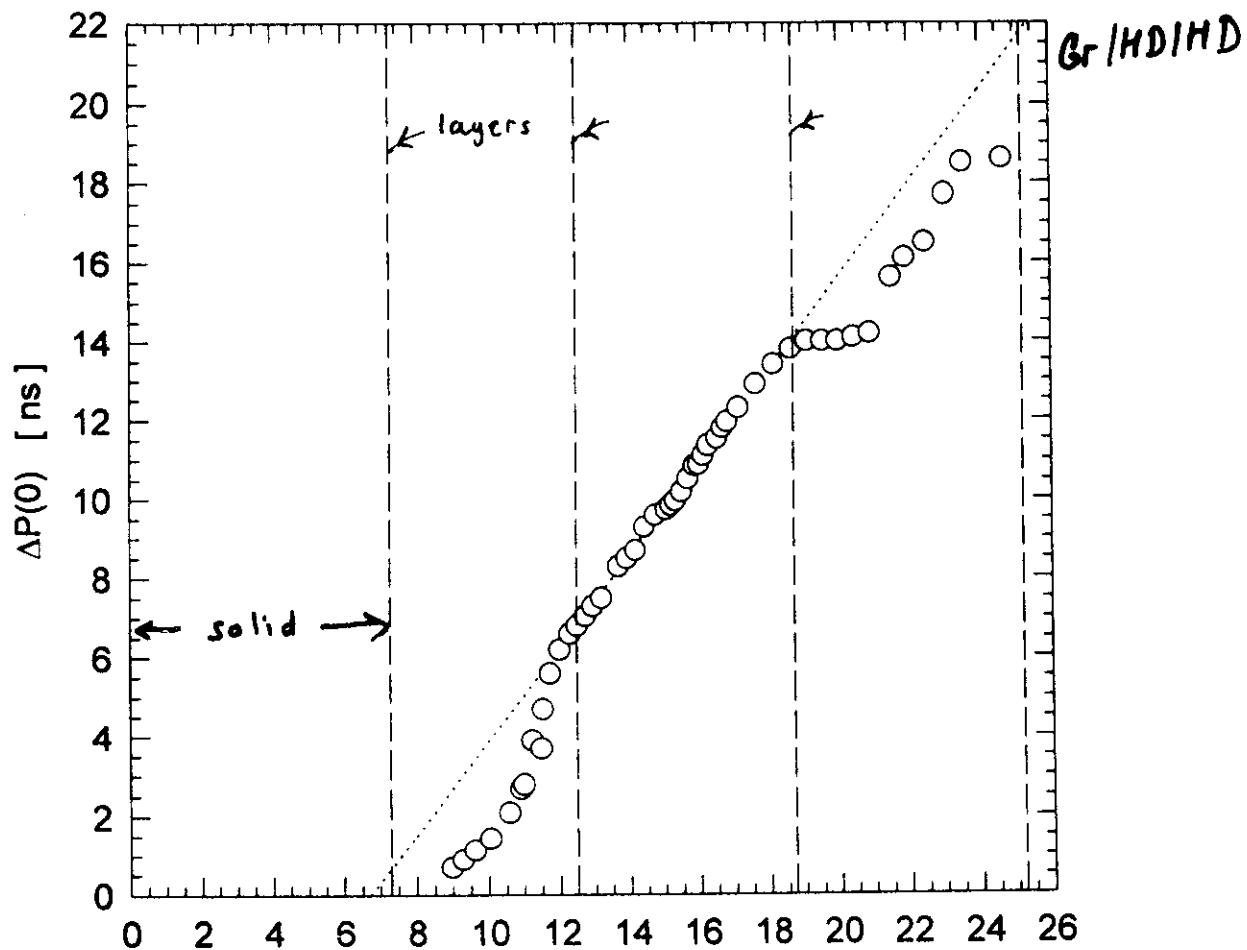
mechanism (Saarela, Clements, Krottscheck, Kuznartsev J. Low Temp. Phys 93, 971 (1993) find that for  $n < 3.7 \text{ nm}^{-2}$  vortex-antivortex pairs are spontaneously created)

\* i.e.  $\exists$  inert layer in a uniform fluid film

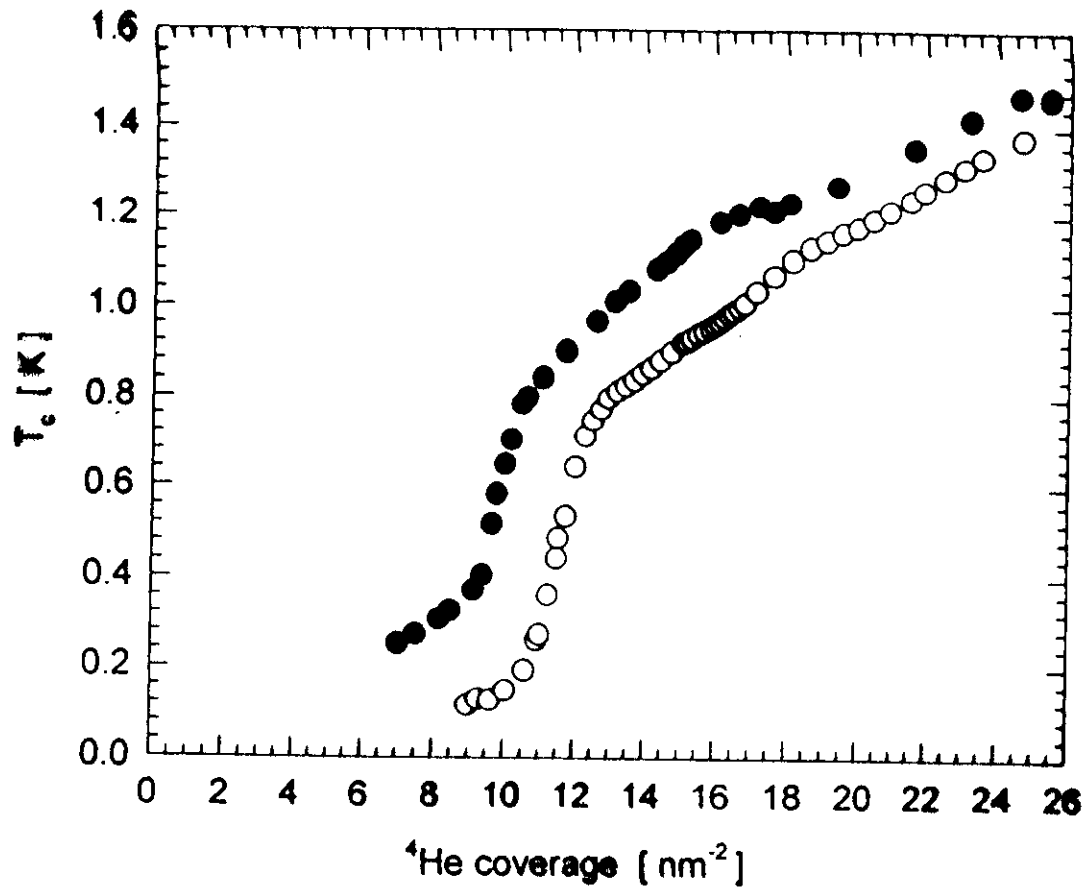
### II two fluid layers

- all of the two fluid layers participates in superfluidity (see period shift data)  
(i.e. "dead" or "inert" layer is just solid layer)
  - $T_c$  depends much more weakly on  $n$  than K-T relation ( $T_c = 0.156 n \text{ K nm}^{-2}$ )  
(similar to mylar data)
- $\Rightarrow$  K-T theory does not apply to bi-layer  
dynamics of vortices in a superfluid bi-layer?

# Low temperature period shift



# Temperature of the dissipation peak



G. AGNOLET, D. F. McQUEENY, AND J. D. REPPY , PRB 39 (1989) 8034

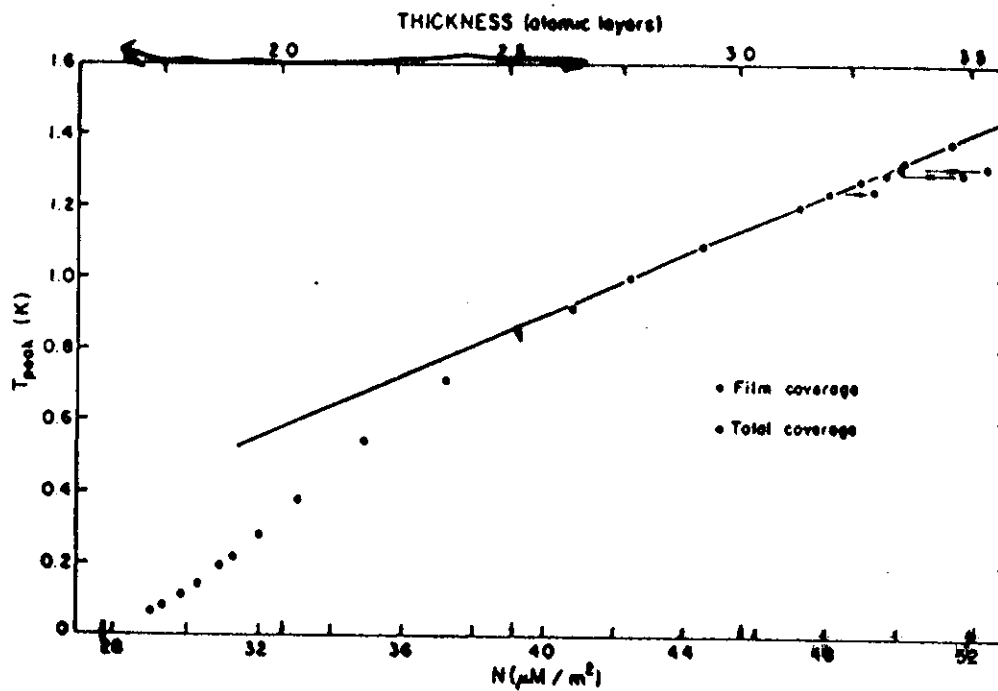


FIG. 14. Variation of  $T_{\text{peak}}$  with film coverage. The line is drawn to emphasize the coverage dependence of the lower coverage data. The higher coverages have been corrected for depletion by the gas. The nominal thickness in atomic layers is shown.

