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3 - 5 JULY 1997

"Magnetism of 3He on graphite: From ferromagnetism to spin liquid"

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These are preliminary lecture notes, intended only for distribution to participants.

Magnetism of ^{3}He on graphite: From ferromagnetism to spin liquid

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Objective: understanding the competing effects of many-body exchange processes in the 2D solid ^{3}He.

- Multi spin exchange theory

- Classical phase diagram,

  PIMC & experiments

- Exact diagonalizations: **A spin liquid phase**

Multi spin exchange

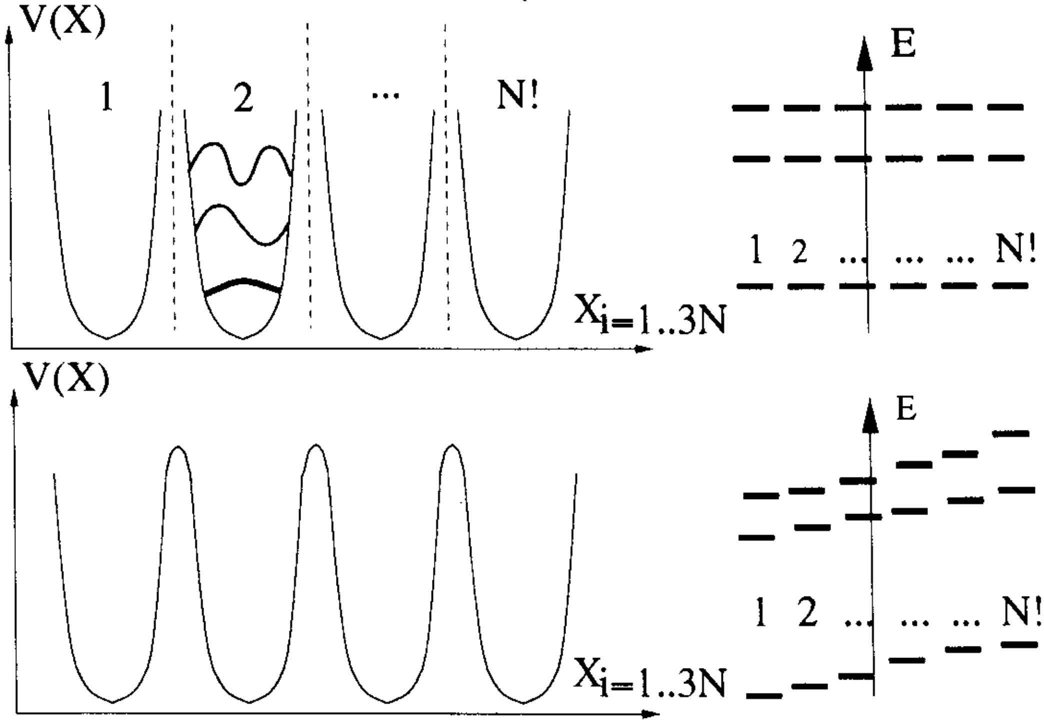
Dirac and Heisenberg (1926), Thouless (1965)

= Effective description of interactions + Pauli principle for quasi localized fermions.

N distinguishable interacting particles:

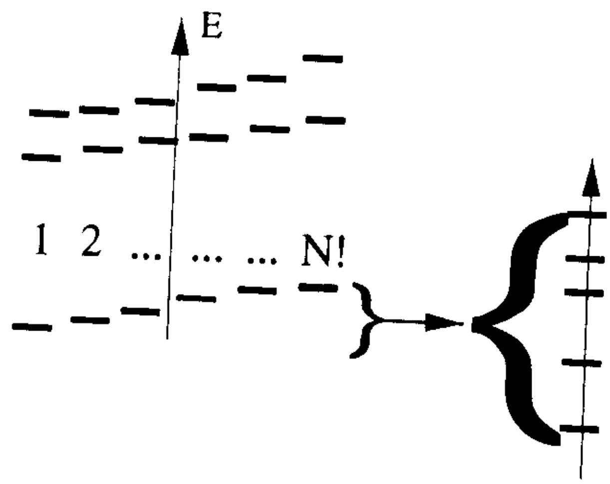
$$H = \sum_{i=1}^{N} \frac{\mathbf{p}_i^2}{2m} + E_1(\mathbf{x}_i) + \sum_{i<j} E_2\left(\mathbf{x}_i, \mathbf{x}_j\right)$$

⇔ 1 particle in a 3N-dimensional space + potential V with N! equivalent cavities

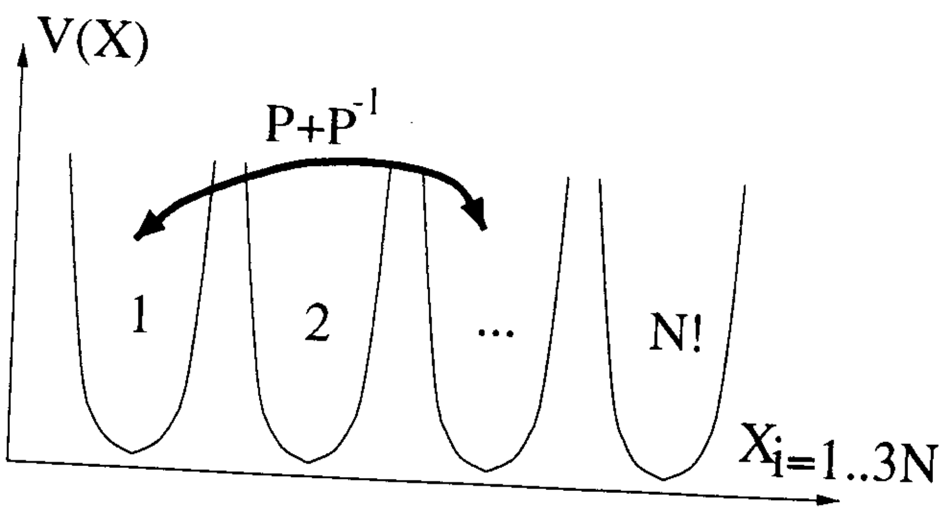


N!-fold degeneracy lifted by tunneling processes

Permutation = Hopping in phase space



$$H \longrightarrow H_{eff} = - \sum_{P \in \sigma_N} J_P \left(P + P^{-1} \right)$$



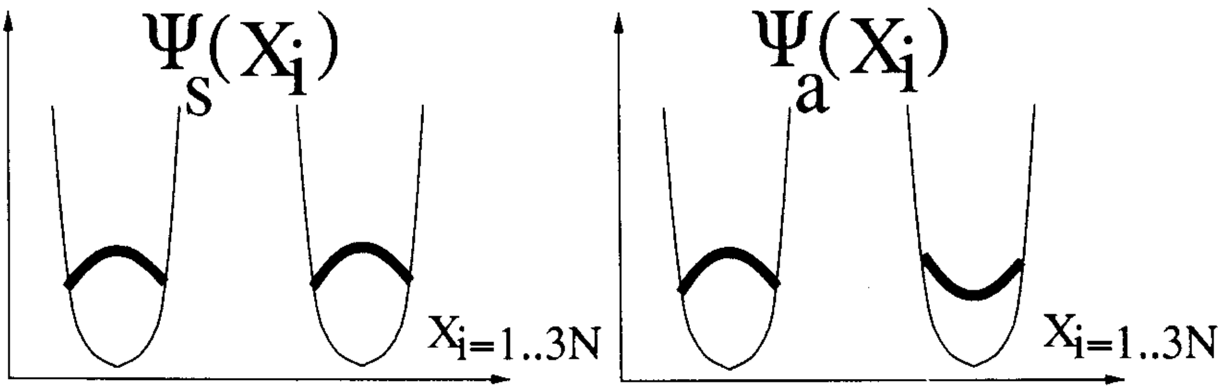
$J_P > 0$

Since tunneling events $\sim$ rare+independent
$\Rightarrow$ We focus on one **single** event.

- Consider a single term $-J_P\left(P + P^{-1}\right)$ when the system starts from the 1^{st} cavity.

$$\{|1\rangle\,, P\,|1\rangle\}$$

- 2 eigenstates in the restricted Hilbert space:



$$\begin{aligned} |\Psi_s\rangle = |1\rangle + P\,|1\rangle \quad E_s &= -J_P \\ |\Psi_a\rangle = |1\rangle - P\,|1\rangle \quad E_a &= J_P \end{aligned}$$

- $|\Psi_s\rangle$ (nodeless) has the lowest energy

$$\Rightarrow J_P > 0$$

Pauli principle

$|\Psi\rangle$ is a fermion wave function $\Leftrightarrow$

$$\forall P \quad P_{spin} P_{space} |\Psi\rangle = (-1)^P |\Psi\rangle$$

$$\begin{aligned} H_{eff} |\Psi\rangle &= - \sum_{P \in \sigma_N} J_P \left(P_{space} + P^{-1}_{space} \right) |\Psi\rangle \\ &= - \sum_{P \in \sigma_N} (-1)^P J_P \left(P^{-1}_{spin} + P_{spin} \right) |\Psi\rangle \end{aligned}$$

$$\Rightarrow H^{spin}_{eff} = - \sum_{P \in \sigma_N} (-1)^P J_P \left(P^{-1}_{spin} + P_{spin} \right)$$

From now, $P_{spin} \longrightarrow P$

$$\begin{aligned} H^{spin}_{eff} &= J_2 \sum P_{1,2} - J_3 \sum \left(P_{1\ldots3} + P^{-1} \right) \\ &+ J_4 \sum \left(P_{1\ldots4} + P^{-1} \right) - J_5 \sum \left(P_{1\ldots5} + P^{-} \right. \\ &+ \cdots \end{aligned}$$

^3He on graphite: 2^{nd} layer

= spins 1/2 on a triangular lattice.

Heisenberg hamiltonian = 2-spins exchange:

$$\mathbf{S}_i\mathbf{S}_j = \frac{1}{2}\left(P_{i,j} - \frac{1}{2}\right)$$

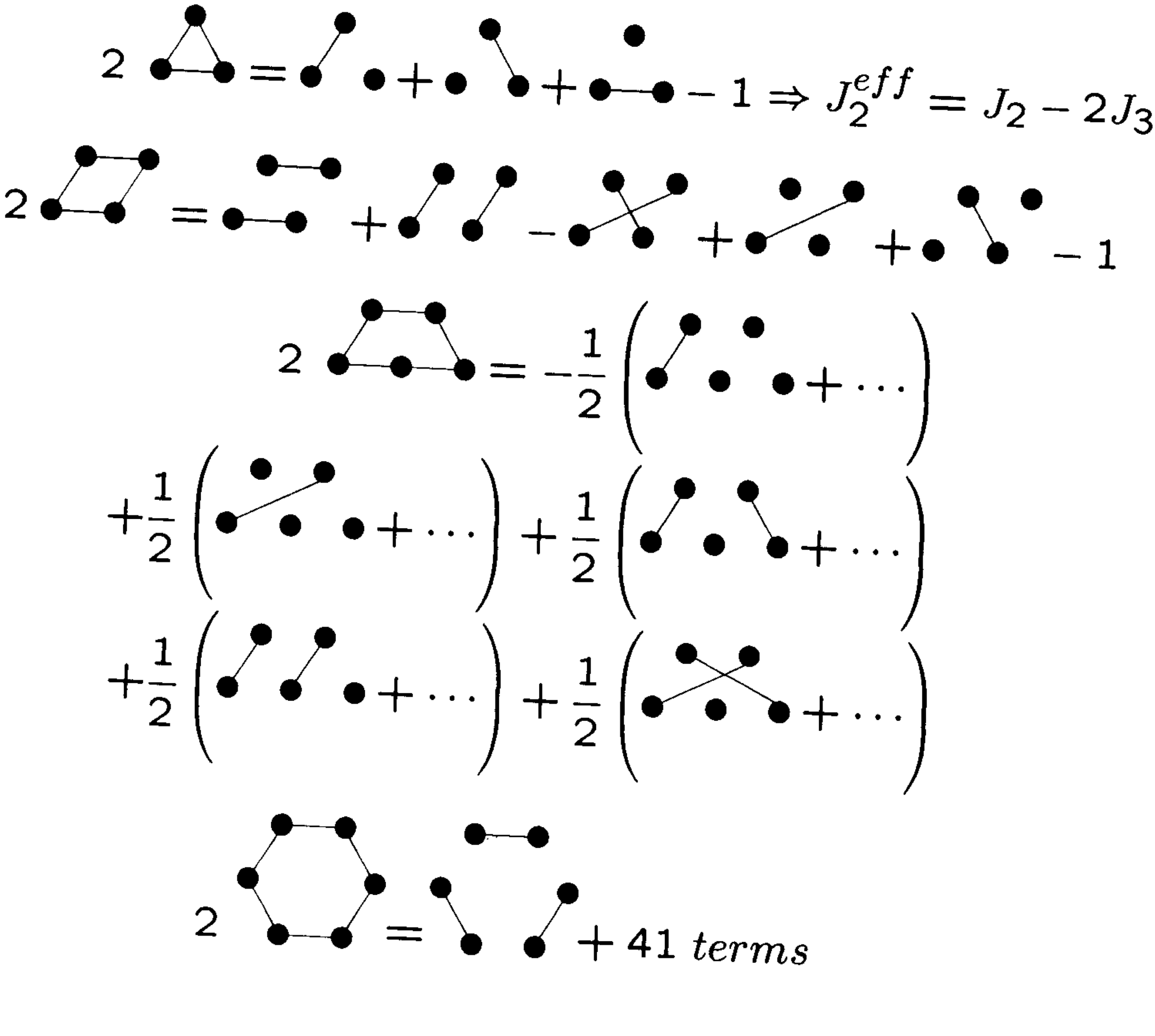
But experimental $C_V(T)$ and $\chi(T)$ data are incompatible with a pure Heisenberg hamiltonian.

Path **I**ntegral **M**onte **C**arlo calculations of J_n:

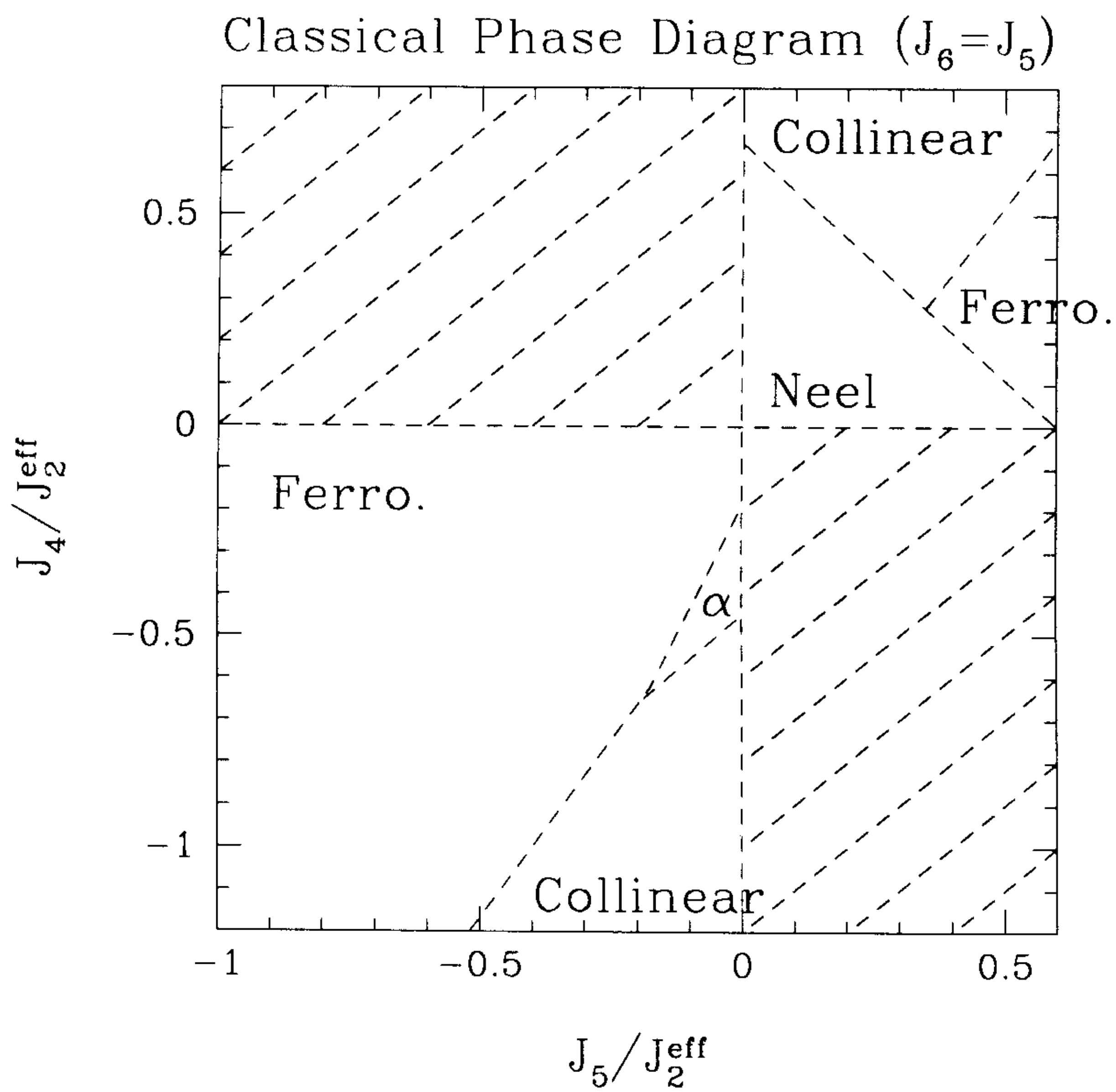
3N $J_2 P_{1,2}$	$J_2 = 10.9 \pm 0.6 mK$
2N $J_3\,(P_{1\cdots 3} + P_{3\cdots 1})$	$J_3 = 8.3 \pm 0.5 mK$
3N $J_4\,(P_{1\cdots 4} + P_{4\cdots 1})$	$J_4 = 5.8 \pm 0.4 mK$
6N $J_5\,(P_{1\cdots 5} + P_{5\cdots 1})$	$J_5 = 2.4 \pm 0.3 mK$
N $J_6\,(P_{1\cdots 6} + P_{6\cdots 1})$	$J_6 = 2.6 \pm 0.5 mK$

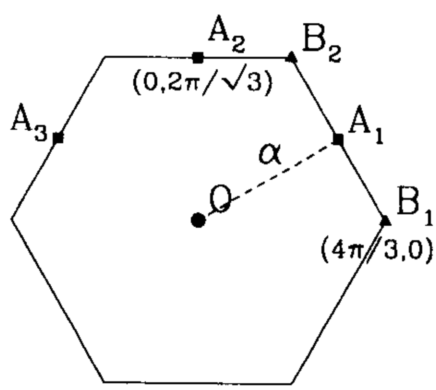
($\rho = 0.179$ atom/$\AA^{-2}$, Bernu and Ceperley)

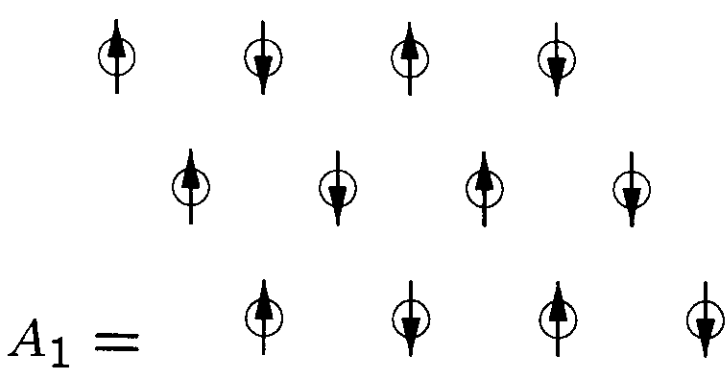
From permutations to classical spins

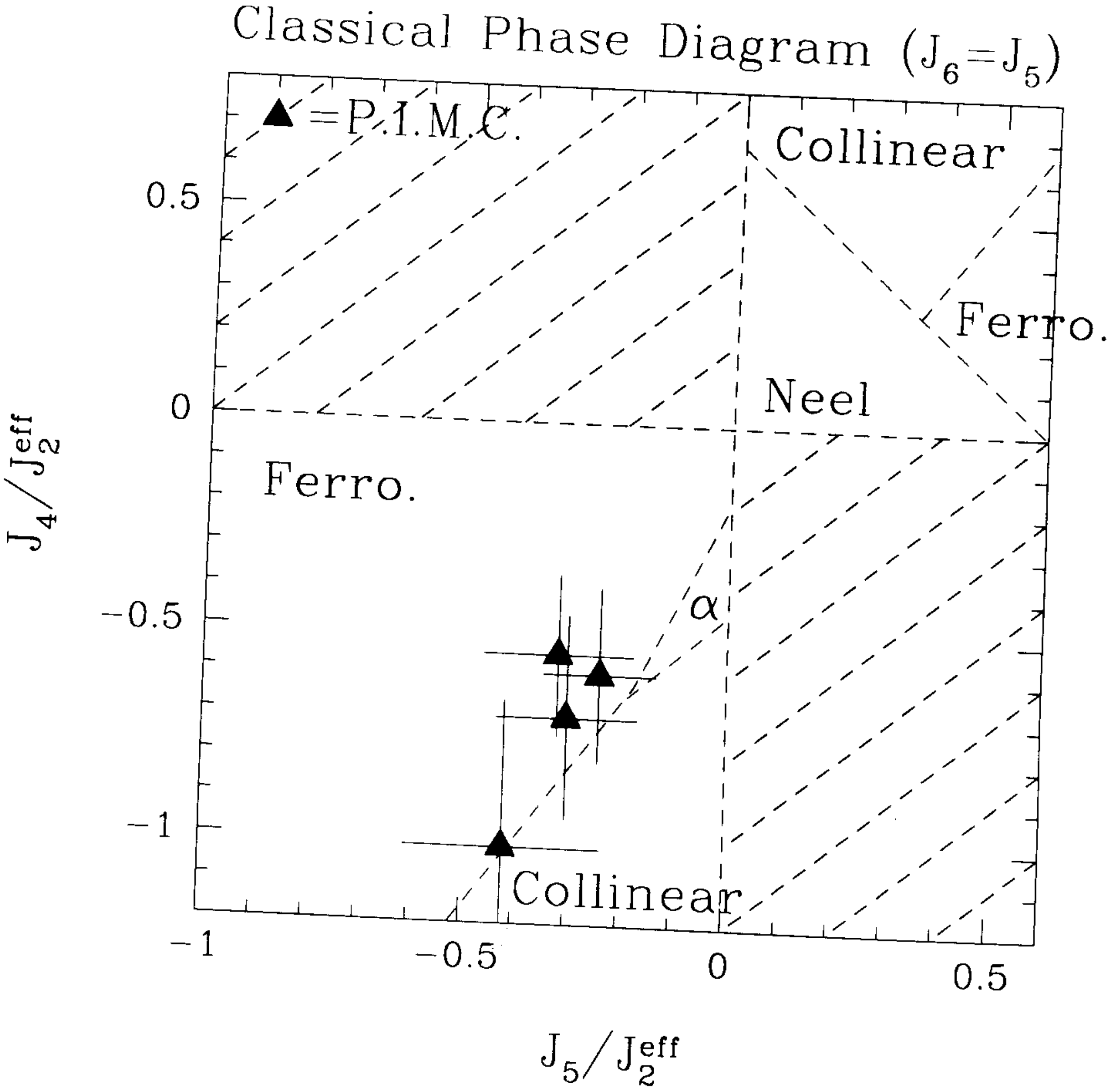


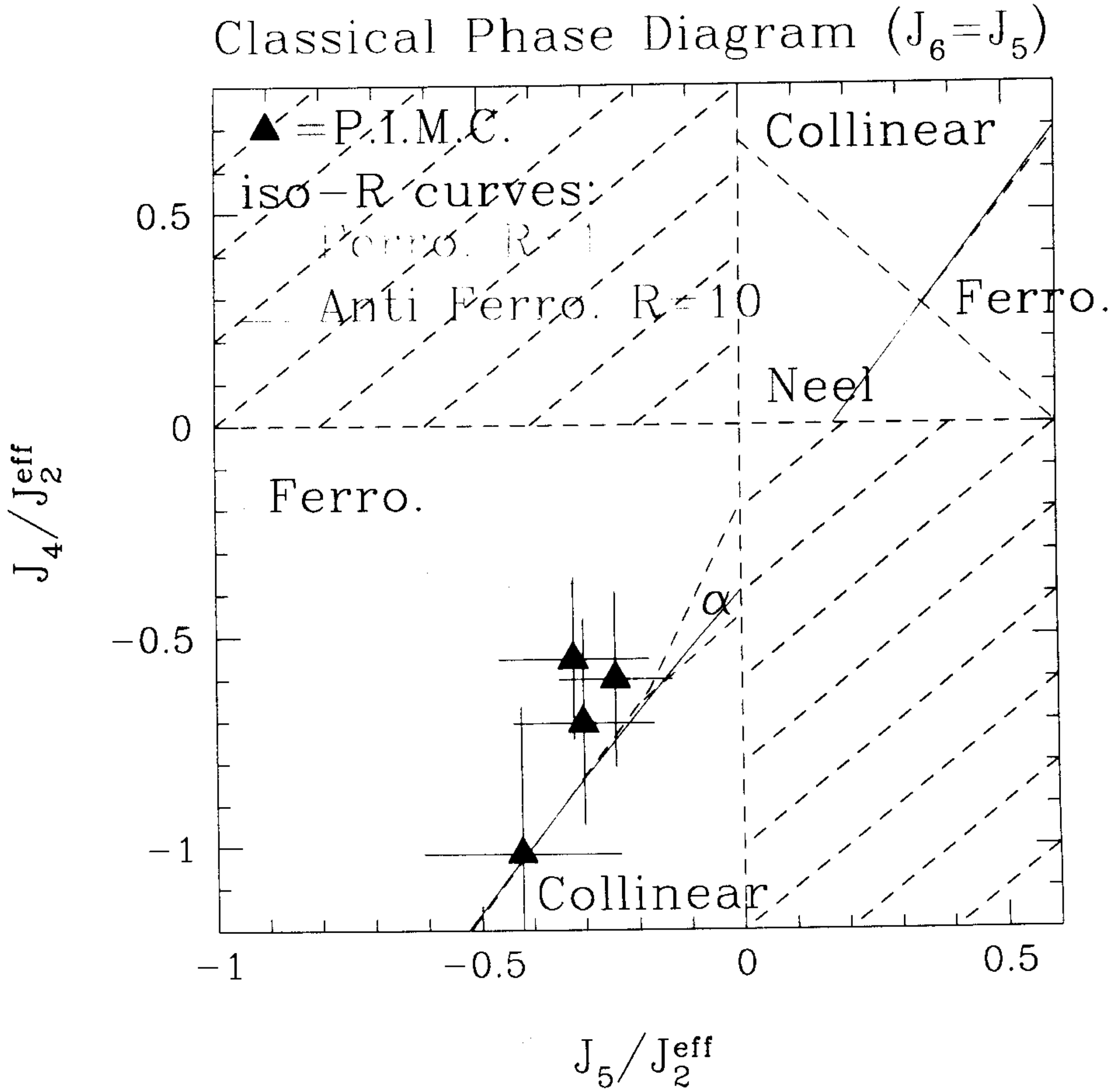
$$H\left[\cdots P_n \cdots\right] \longrightarrow H\left[\cdots \ \left(\mathbf{S}_i \cdot \mathbf{S}_{j\neq i}\right) \cdots\right]$$


Classical Phase Diagram (J_6=J_5)
Collinear
Ferro.
Neel
Ferro.
α
Collinear
0.5
0
−0.5
−1
−1
−0.5
0
0.5
J_4/J_2^{eff}
J_5/J_2^{eff}


A_2
B_2
$(0,2\pi/\sqrt{3})$
A_3
A_1
α
O
B_1
$(4\pi/3,0)$


$A_1 =$


Classical Phase Diagram (J_6=J_5)
▲=P.I.M.C.
Collinear
Ferro.
Neel
Ferro.
α
Collinear
0.5
0
−0.5
−1
−1
−0.5
0
0.5
J_4/J_2^{eff}
J_5/J_2^{eff}



$$C_V = \frac{9N}{4}\left(\frac{J_{C_V}}{T}\right)^2 + O(1/T^3)$$

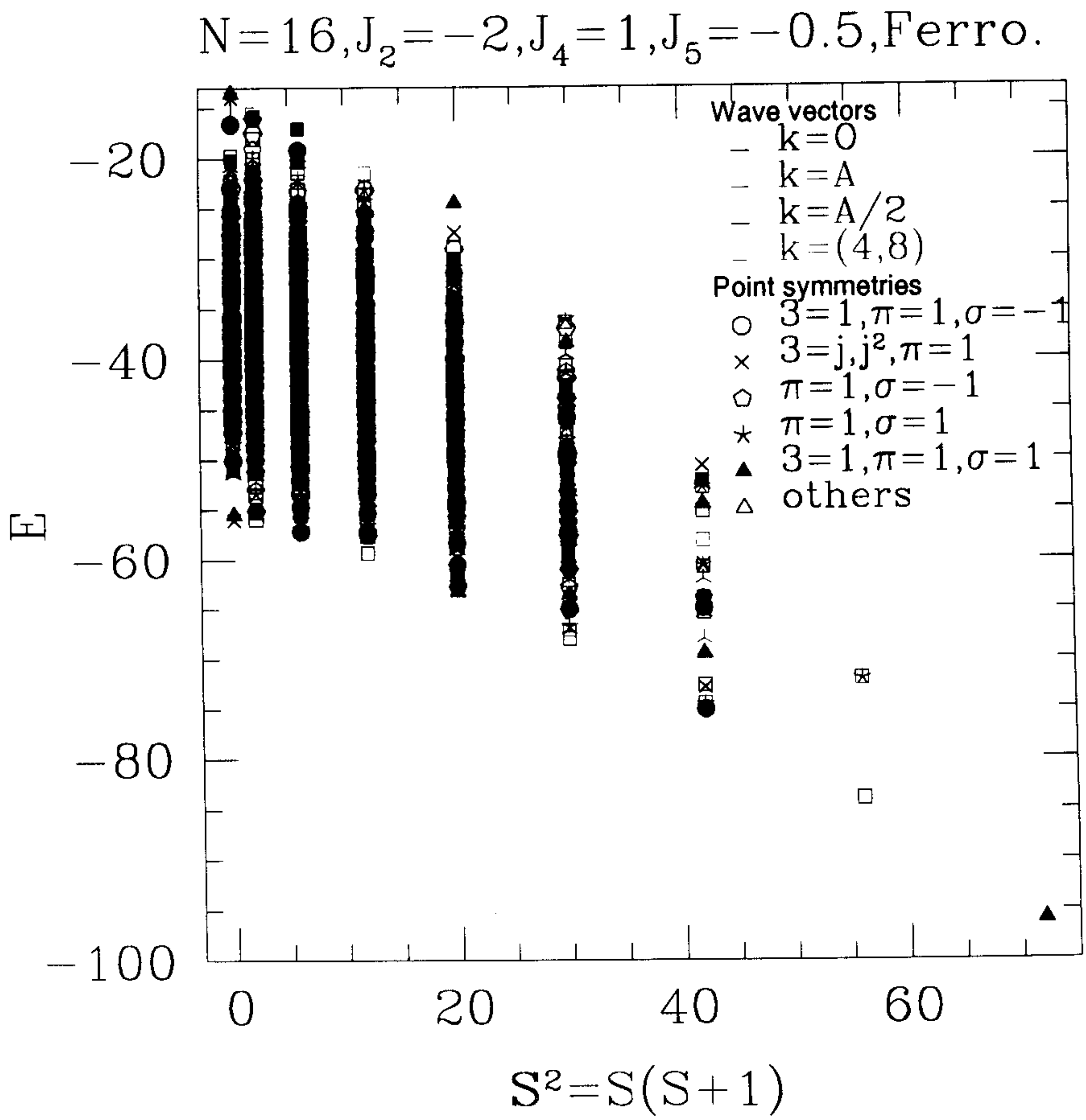
$$\frac{4\chi}{N} = \frac{1}{T} + \frac{3J_\chi}{T^2} + O(1/T^3) \;\; R := \left(\frac{J_{C_V}}{J_\chi}\right)^2$$

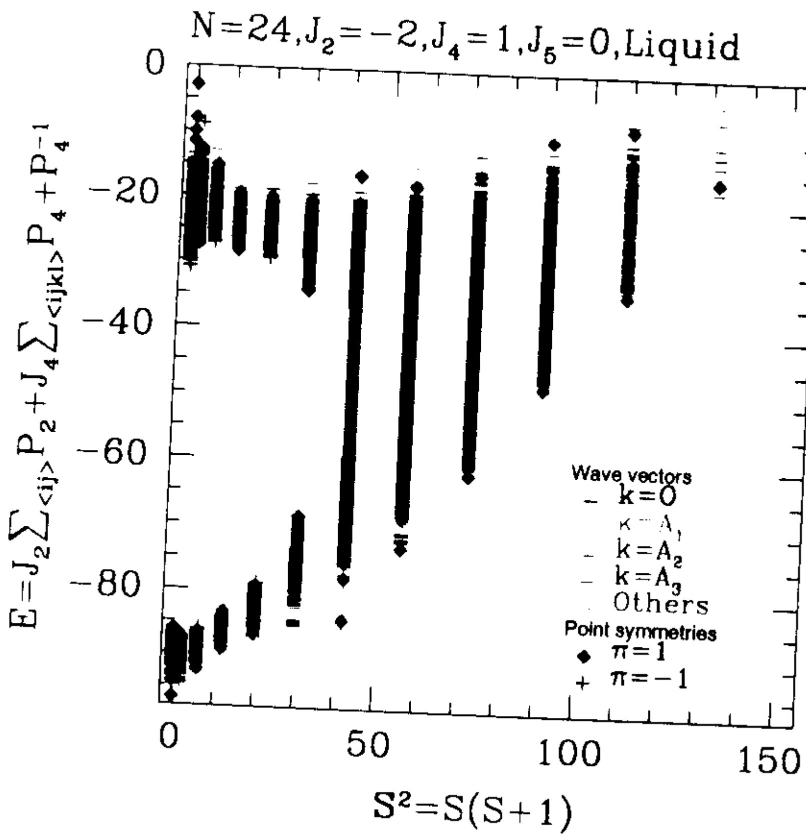
Exact diagonalizations: spin liquid phase

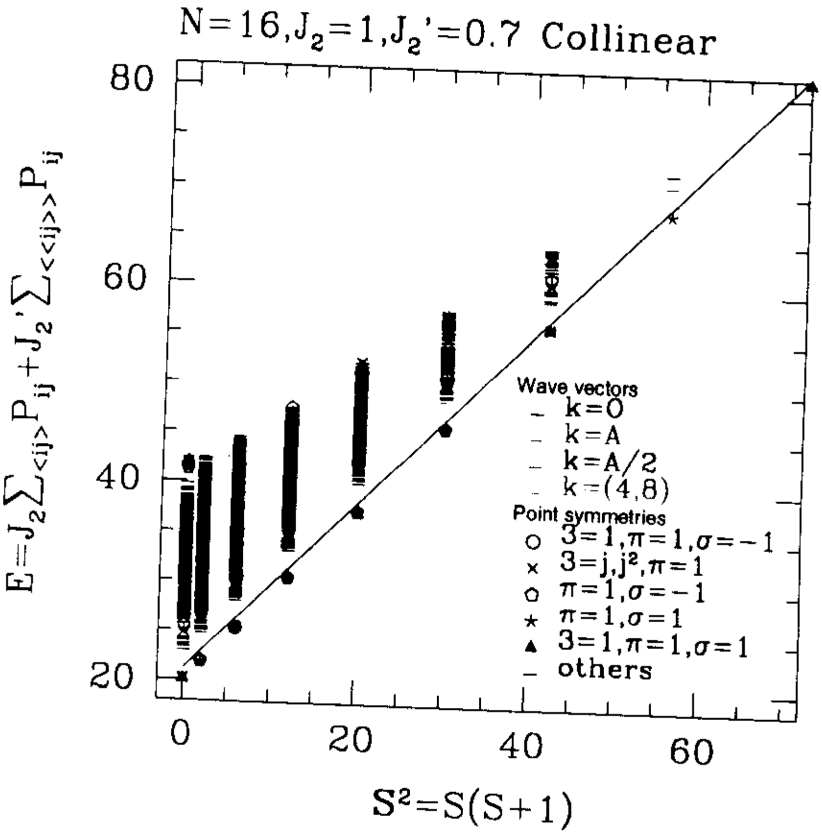
Performed for N=12, 16, 18, 20, 24 and 28

- Ferromagnetic region $\longrightarrow$OK
- Collinear region $\longrightarrow$ Classical long range order destroyed by quantum fluctuations =spin liquid phase

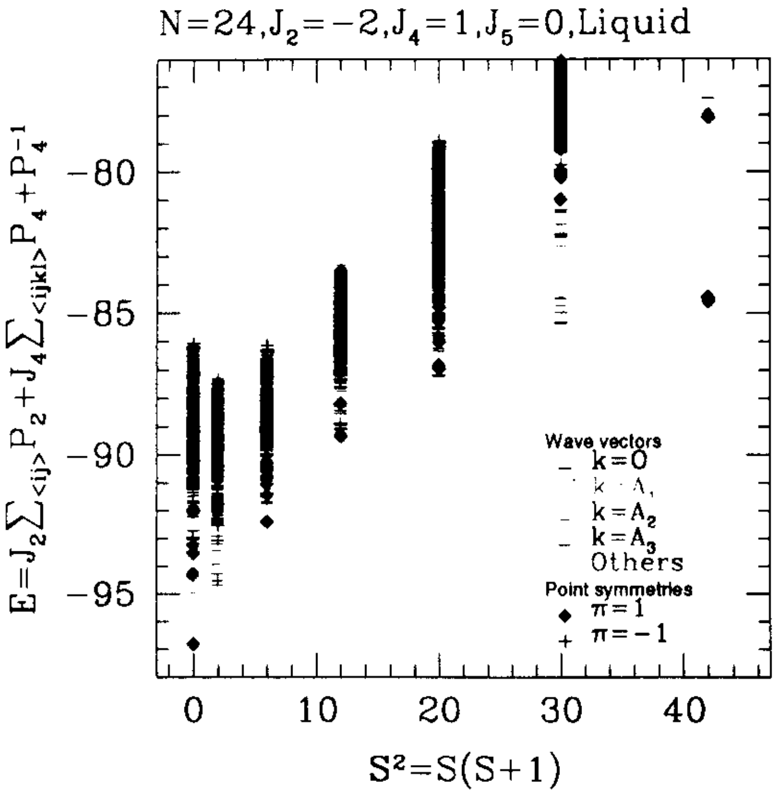
Ferromagnetism: ground state $S = \frac{N}{2}$

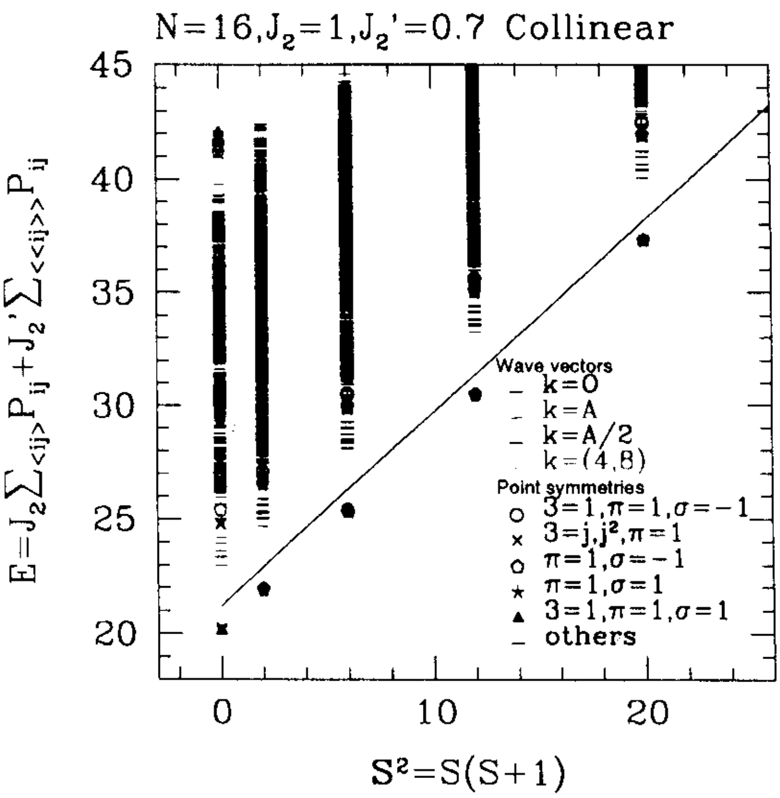



N=24,J_2=−2,J_4=1,J_5=0,Liquid
E=J_2∑_<ij> P_2+J_4∑_<ijkl> P_4+P_4^-1
S²=S(S+1)
Wave vectors
k=0
k=A_2
k=A_3
Others
Point symmetries
π=1
π=−1

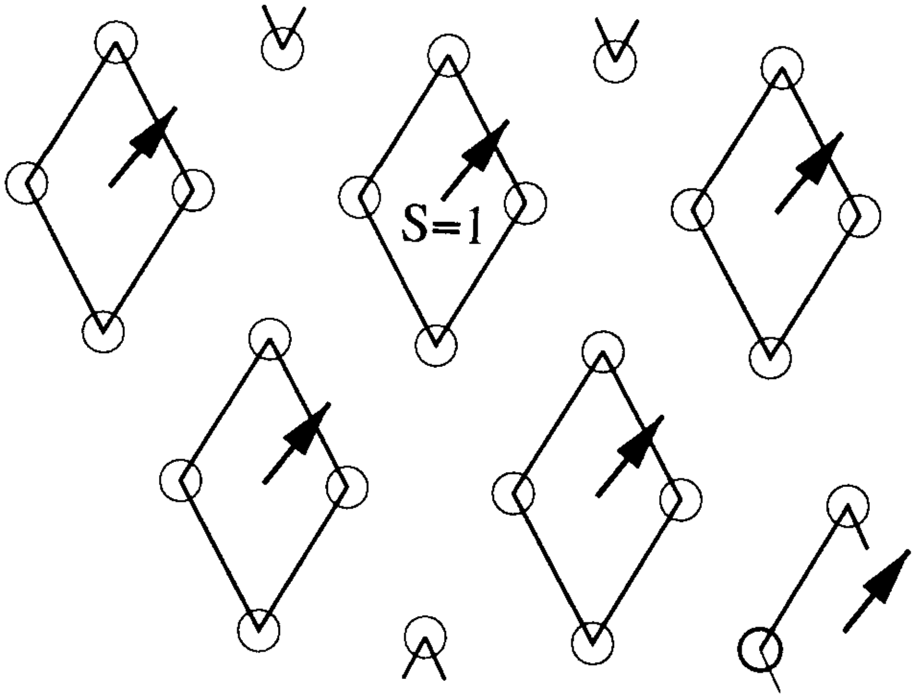

N=16,J_2=1,J_2'=0.7 Collinear
E=J_2∑_<ij> P_ij+J_2'∑_<<ij>> P_ij
S²=S(S+1)
Wave vectors
k=0
k=A
k=A/2
k=(4,8)
Point symmetries
3=1,π=1,σ=−1
3=j,j²,π=1
π=1,σ=−1
π=1,σ=1
3=1,π=1,σ=1
others

No tower of states:

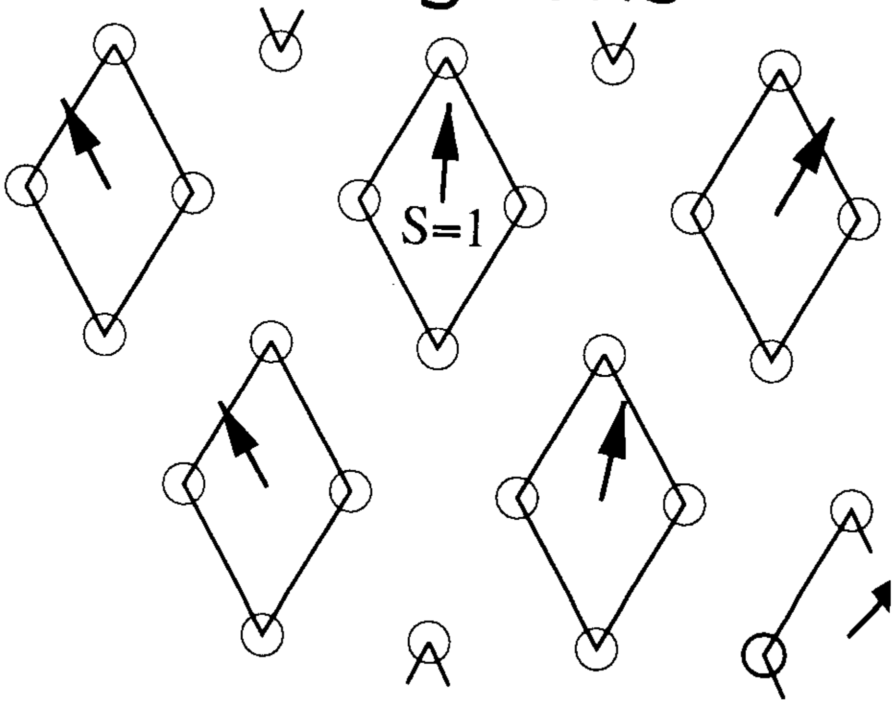




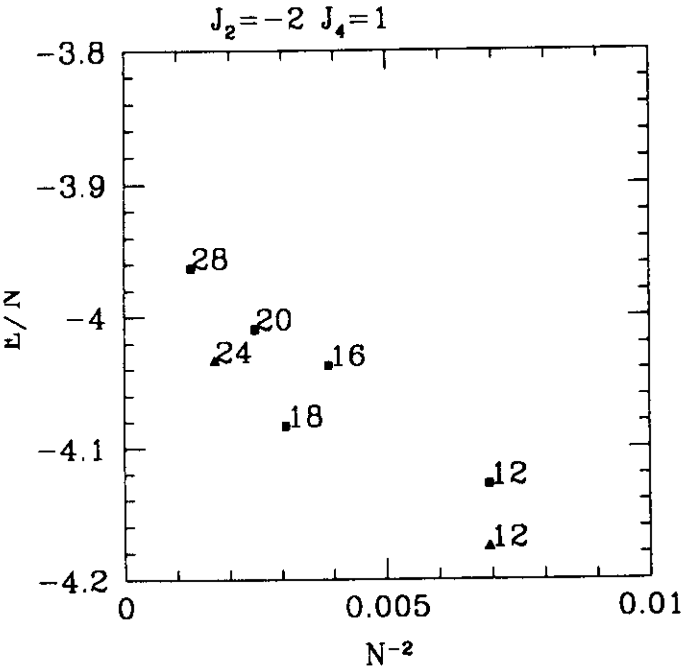
4-spins plaquettes

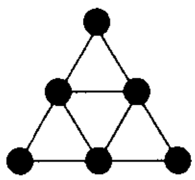


Magnons



Ground state: 6-spins arrangements ?

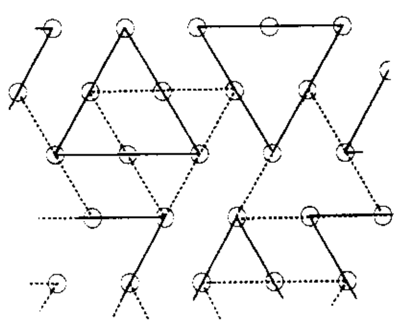



=The only $N \leq 8$ pattern with:

⇒Toy-model with a tight-binding calculation

Result: First $S = 1$ excitation is found with a gap= $.4J_4$ (for $J_2^{eff} = -2J_4$)

Mean-field failure⇒ ∃ large resonances:



Results: A RVB-like spin liquid

- No long range order: none of the spectra (in the classical collinear region) display any 'tower of states'.

- S=0 ground state and gap

- Stabilized states in the $S = N/4$ and $S = N/4 - 1$ subspaces:

  4-spins plaquettes.

Conclusions:

We determined the physical range of parameters relevant for the description of the 2^{nd} layer magnetism of ^{3}He and pointed out the possibility of a **gaped spin liquid phase**.

We recognized the presence of low energy **plaquettes degrees of freedom**. These states might be observed experimentally. NMR ?

A few references

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