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MINIWORKSHOP ON
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"Orbital paramagnetism of Andreev electrons"

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These are preliminary lecture notes, intended only for distribution to participants.

Orbital Paramagnetism of Andreev Electrons

Yoseph Imry (WIS)

ith: Christoph Bruder (Karlsruhe)

iscussions with A.C. Mota (ETH)]

INTERESTING SITUATION

BETWEEN "SUPER" & "NORMAL"

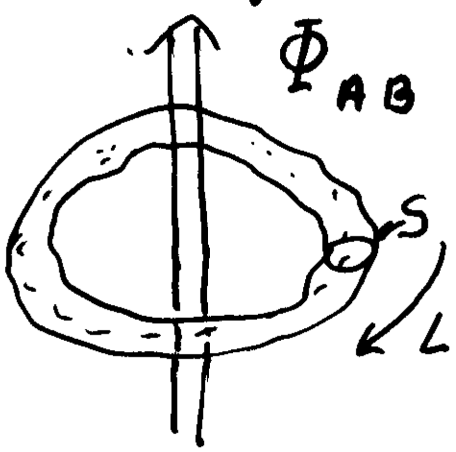
Orbital Magnetism of Conduction Electrons

1. Zero in classical Limit (B-vL)

. Huge in Superconductors (Meissner, Flux Quantization)
"Macroscopic Quantum State"

Nonzero, but small in Macro Metals (Landau)

Nonzero, small, and very interesting in Mesoscopics.

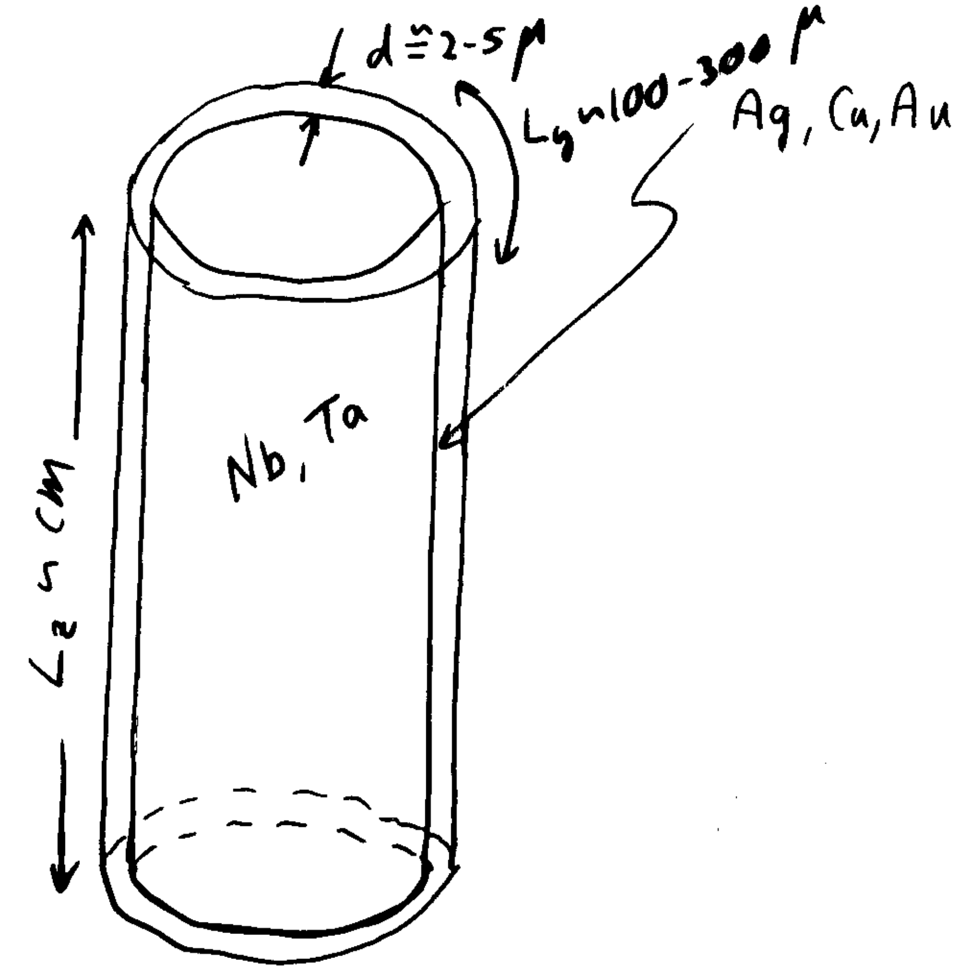


$\Longrightarrow$ Much good Φsics!

Kohn (64):

$\dfrac{\partial^2 F}{\partial \phi_{AB}^2}$ = important measure for conducting states of Matter

$$\frac{\text{super}}{\text{Meso (coherent)}} \sim \begin{cases} Vol/l_{ee} & \text{"clean"} \\ \Delta_s/\Delta & \text{"dirty"} \end{cases}$$



"$l_{el} \cong 2-20\ \mu$"

Mota et al. 1989-97

Reentrant Paramagnetic Response

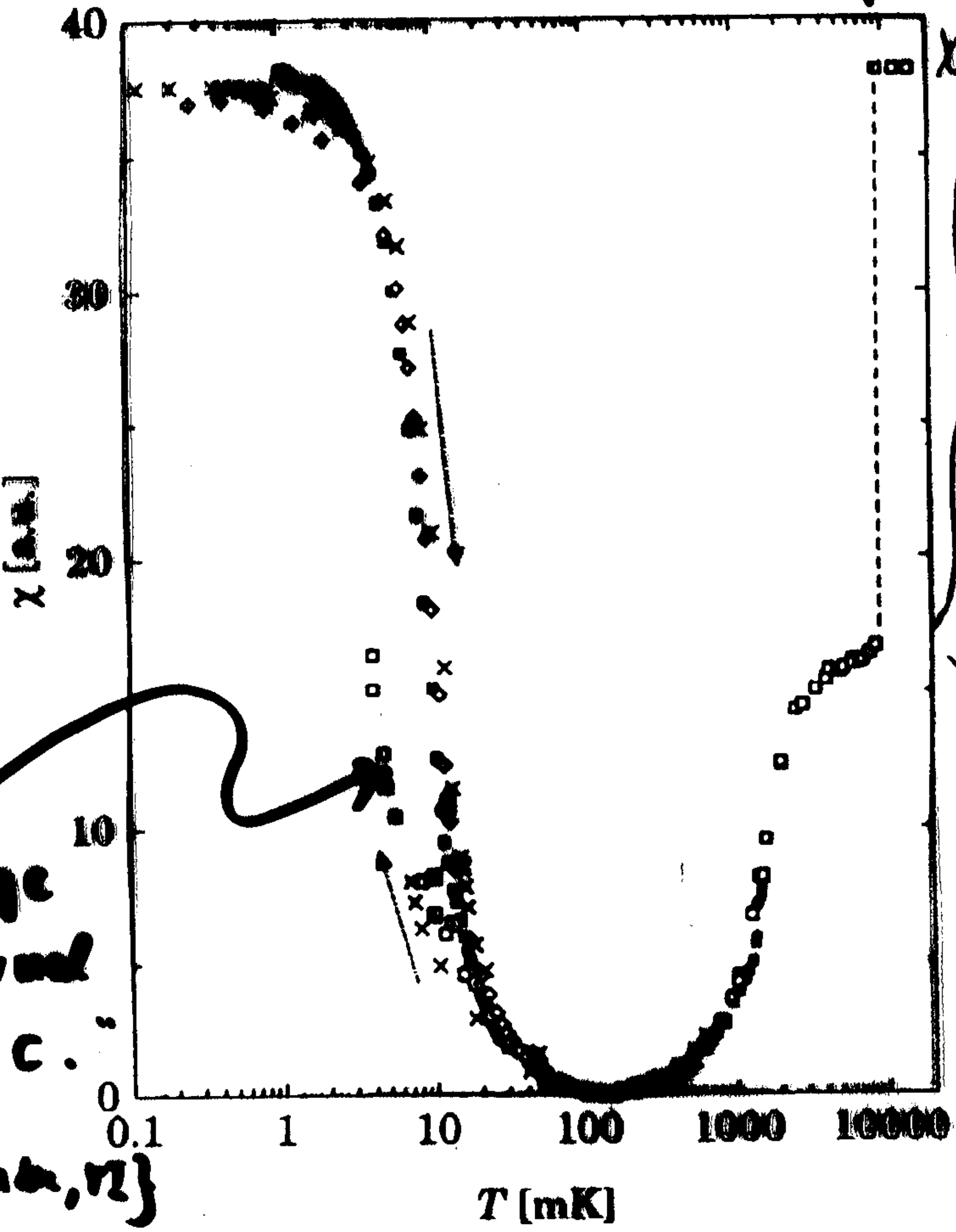
Mesoscopic Effect?

over a fraction of a mm?

- $\exp(-T/T^*) \sim \exp(L/(\hbar v_F/kT))$ clean limit 300 ?!
- Killed by small B ($\lesssim 1$ Oe)

T_c

$\chi = 0$



$\Delta\chi_{Nb}$

$\Delta\chi_{Ag}$

Huge Normal P. C. "

{C. Bruder, 92}

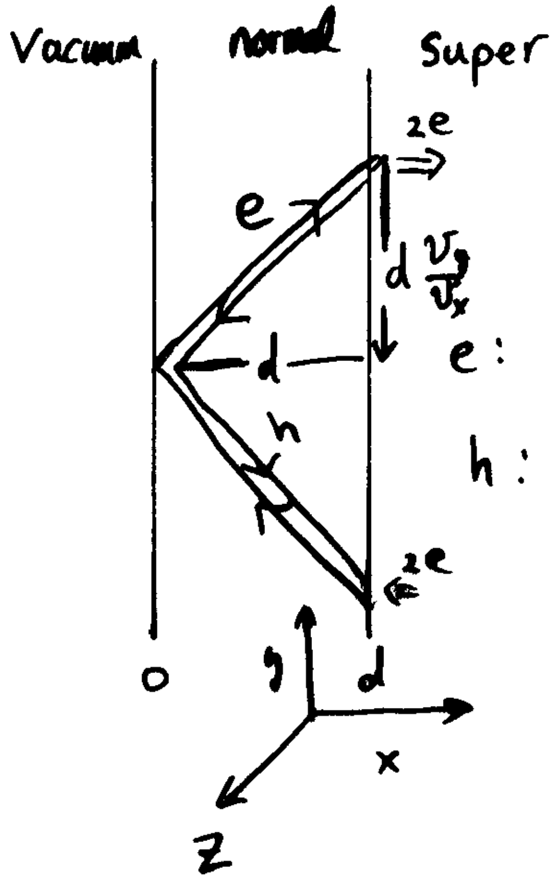
Ag/Nb $L = 72\,\mu m$

$\xi_N = \frac{1.6\,\mu m}{T}\ \frac{K}{k}$

Mota et al. 1990-97

Orbital Paramagnetism – Current Loops

(cf.: Shmuel, Yacoby, YI – P-N junction)



Gor'kov – Andreev Eqs. in N

$$e: \quad (i\vec{v}\cdot\vec{\nabla} + e\vec{v}\cdot\vec{A} + \varepsilon)\, u = 0$$

$$h: \quad (i\vec{v}\cdot\vec{\nabla} - e\vec{v}\cdot\vec{A} - \varepsilon)\, v = 0$$

$\Delta \gg \varepsilon \equiv E - E_F$ (excitation)

$$A = \begin{pmatrix} 0 \\ B(x - x_0) \\ 0 \end{pmatrix}; \quad \vec{B} = \begin{pmatrix} 0 \\ 0 \\ B \end{pmatrix}$$

Linearized around $e^{i\vec{v}\cdot\vec{r}}$, $|v| = v_F$.

Total reflection on vacuum boundry

Possibility of Andreev reflection on S boundry $\begin{pmatrix} r_{ee} & r_{eh} \\ r_{he} & r_{ee} \end{pmatrix}$

Ideal interface $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$; large barrier $\begin{pmatrix} e^{iz} & \frac{1}{2z^2} \\ \frac{1}{2z^2} & e^{-iz} \end{pmatrix}$ $z \gg 1$

Andreev: e retro-reflected to h $\Leftrightarrow$ pair into super
h " to e $\Leftrightarrow$ pair from super

At E_F ($\varepsilon = 0$) h unwinds spatial phase of e, but magnetic phase "$e\int A\cdot dl$" "adds"

$\varphi = \frac{B d^2 v_y}{v_x \Phi_0}$ ($\Phi_0 \equiv hc/e$) A-B phase in triangle.

Thick N: $d \gg \xi = \frac{\hbar v_F}{\Delta} \Leftrightarrow \Delta \gg \frac{\ldots}{d}$

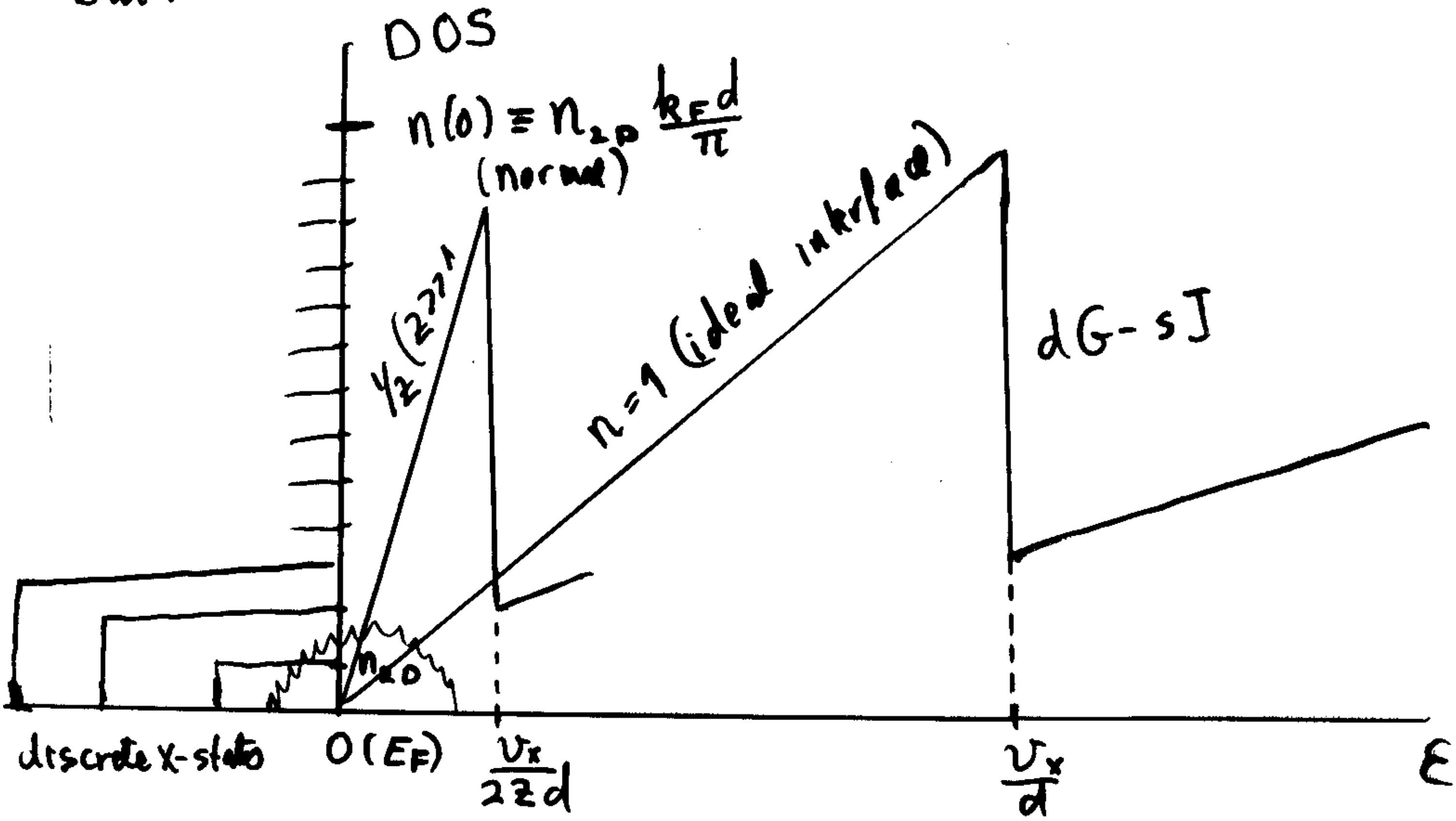
- normal slab ($z=\infty$): $\varepsilon = \frac{v_x}{d} n\pi + k_y v_y$
- Good interface ($z=0$): $\varepsilon = \frac{v_x}{4d}\left(\pi + 2n\pi + 2\varphi\right)$
- High barrier ($z \gg 1$): $\varepsilon = \frac{v_x}{2d}\left(\frac{1}{z} + 2n\pi\right) + \frac{v_x}{2dz^2}\varphi$ $(\varphi > 0$

No φ dependence (normal) full φ dependence ($z=0$)

$O(1/z^2)$ for $z \gg 1$ — lowers ε — increases ε

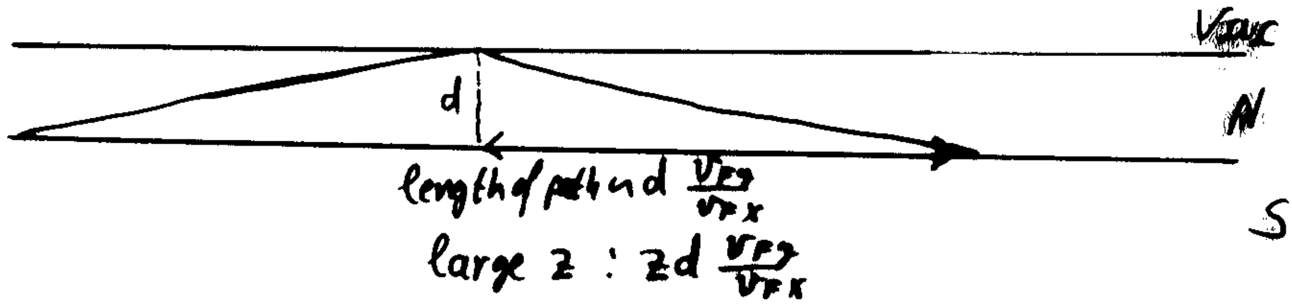
There are paramagnetic current loops !!!

But: More to come !!!



$n(\varepsilon) \hat{\sim} A \frac{\varepsilon}{v_F/d}$ small ε

Due to small density of small v_{Fx} – <u>Glancing States</u>



- Glancing states insensitive to surface roughness ([illegible]

AND:

$\Rightarrow$ Here, these low ε states are immune to impurity scattering

$$\frac{1}{\tau_{el}} \hat{=} \frac{2\pi}{\hbar} |\langle k | V_{imp} | k \rangle|^2 \, n(\varepsilon)$$

$\Downarrow$ Low ε

$$\boxed{l_{el}^{-1}(\varepsilon) \sim l_{el}^{-1} \frac{\varepsilon}{v_F/d(z)}}$$

Can get to ~100 μ at low T!!

Very low ε : Length of orbit ~ $dz \frac{v_F/d(z)}{\varepsilon(\sim kT)} \sim \xi_N$

and: $\boxed{l_{el}(T) \hat{\sim} \xi_N}$ For all $T \ll \frac{\hbar v_F}{zd}$

$\Downarrow$ Low energy ($k_B T$) states are ballistic !!!

(once $l_0 \gtrsim dz$)

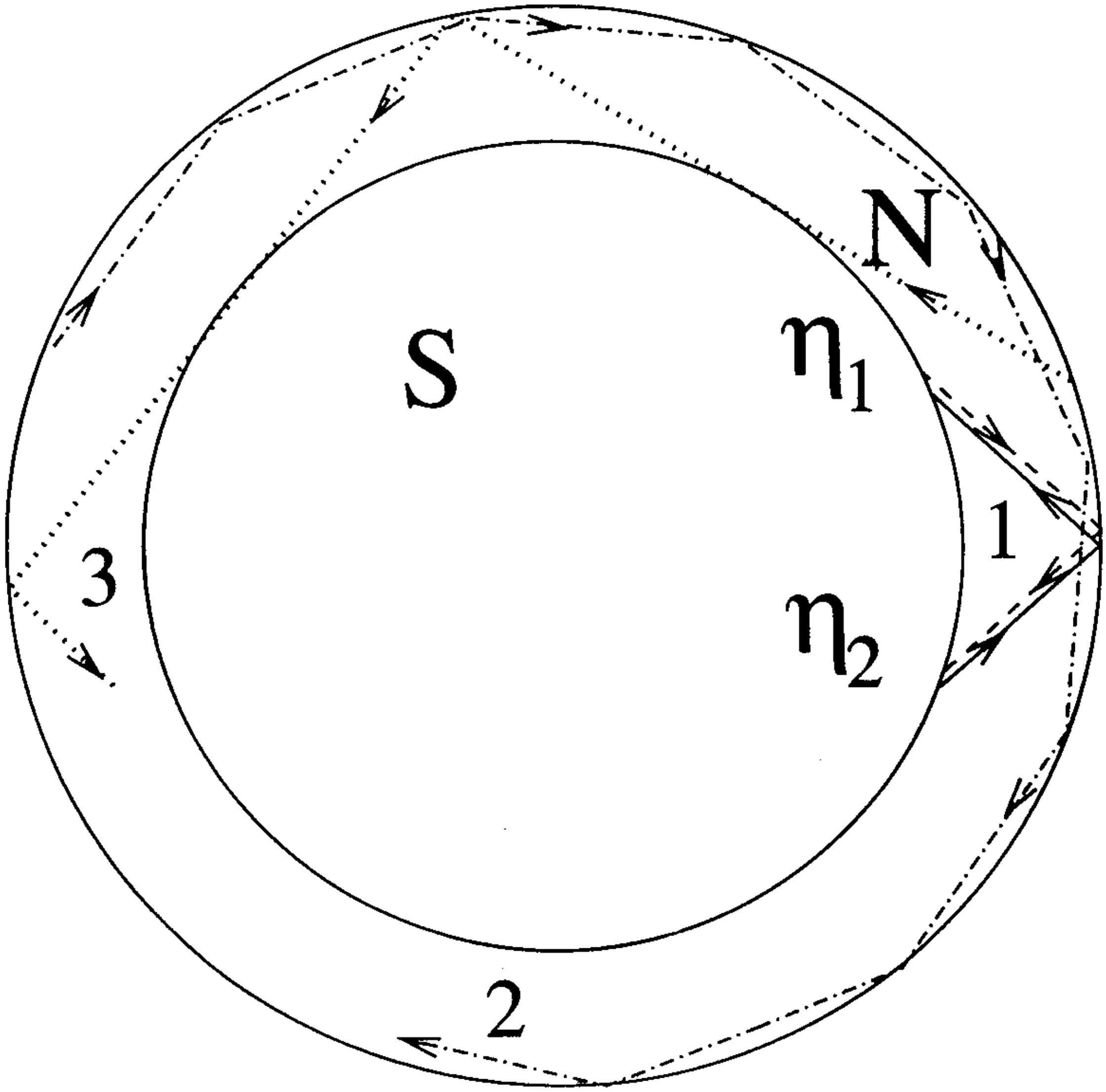
At low enough $\varepsilon \lesssim kT^* \sim \frac{\hbar v_F}{L_y}$,

elastic m.f.p $\ell_{el} \gtrsim L_y$

States w/ $\varepsilon \lesssim kT^*$ are <u>ballistic</u>

They are <u>glancing</u> states.

$\frac{d}{L_y}$ of all states.

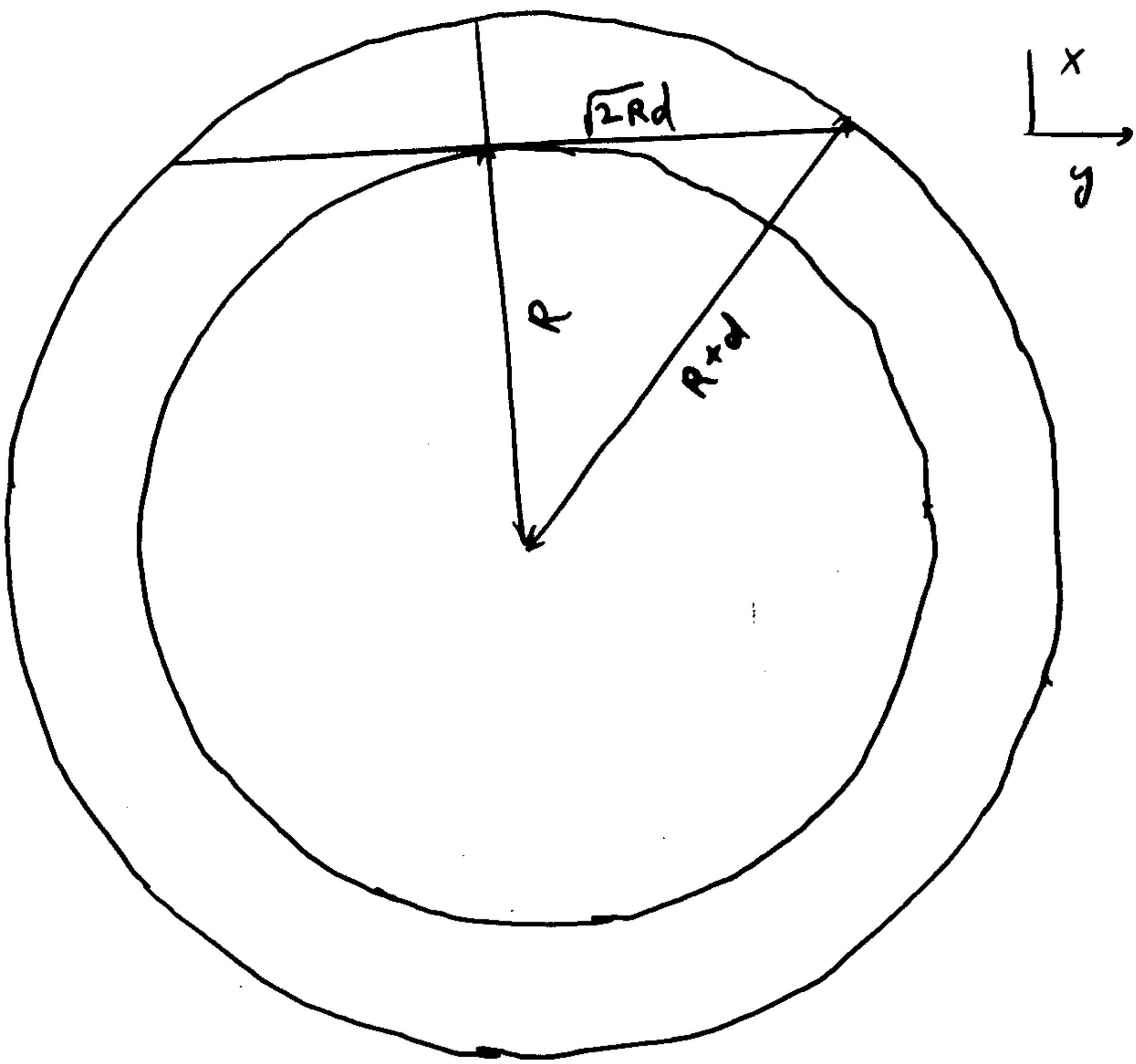


N
S
η_1
η_2
1
2
3

# of Glancing modes.

1. Simple <u>under</u>estimate $\frac{v_x}{v_y} < \frac{d}{L} \Rightarrow N_g = \frac{k_F d^2}{\pi L}$.

2. $\frac{v_x}{v_y} \leq \sqrt{\frac{d}{2R}} \Rightarrow N_g = \frac{k_F d}{\pi} \sqrt{\frac{d}{2R}}$



- consistent with confinement by centrifugal pot. $\frac{m v_y^2}{R} x$.

Orbital (Pauli-type) paramagnetism

Every state with $|\varepsilon| \lesssim k_B T$ which splits to $\varepsilon \pm \mu B$ gives $\chi = \dfrac{\mu^2}{kT}$

$\mu \equiv$ magnetic moment of current loop $\sim e v \times$ length (compact)

usual pauli: $N(0)\, kT$ such states $\underline{\chi_p = N(0)\,\mu_B^2}$

Here: "$N(0)$" $\sim N(0)_{\text{normal}} \times \dfrac{kT}{\hbar v_F/(2d)}$

$\boxed{\mu \sim e v_F d}$ Andreev states

glancing states $\Rightarrow$ significant.

But:

Once $T \lesssim T^* \sim \dfrac{v_F}{L}$ $\quad (\xi_N \sim L)$

$\boxed{\mu \sim e v_F L}$ Glancing States

larger by $\sim 10^2$
Qualitative X'over!

Or: Normal P.C. of

$$\frac{e v_F}{L_y} \cdot k_F^2 L_z d \;\cong\; \text{Meissner!!!}$$

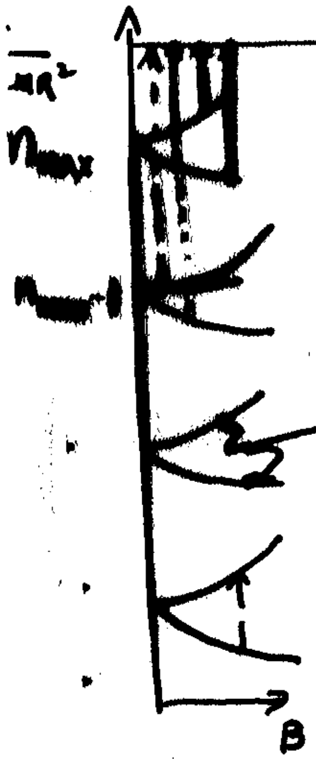
(up to d/ξ)

ballistic — all channels para

Normal P.C. in a Long Cylinder, $T=0$

Gunther + YI 1968-9
+ ...

[Gogadze, Kulik, Cheung Riedel & Gefen ...



φ^2 - diamag.

population diff ($\int K$)
paramagnetic

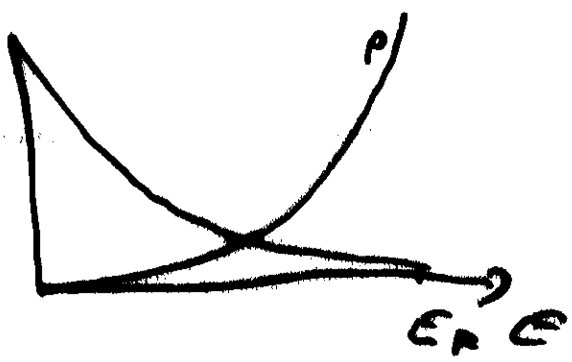
$$E_{n,K} = \frac{\hbar^2}{2mR^2}\left[K^2 + (n-\varphi)^2\right]$$

$$K = R k_z$$

up to $\mu = \xi^2 \frac{\hbar^2}{2mR^2}$

$$\varphi \equiv \frac{\Phi}{\Phi_0}$$

$$\frac{I_{tot}}{\varphi} \underset{\varphi \to 0}{=} 2 \frac{L_z}{L_y} \frac{2e}{h} \frac{\hbar^2}{2mR^2} \sum_{n \le n_{max}} -\sqrt{\xi^2 - n^2} + \frac{n^2}{\sqrt{\xi^2 - n^2}}$$



dia

para

= in classical limit !!!

[mag. moment]

$$\frac{n^2}{\sqrt{\xi^2 - n^2}}$$

diff. in $\int_{\varepsilon_n - \mu B}^{\varepsilon_n + \mu B} dk_z$

Energies of Andreev (non-glancing) states

~INDEPENT of k_z

⇒ ~no paramag. contribution

⇒ Meissner in Proximity-layer.

Glancing states ($N_g = \frac{k_F d^{3/2}}{\pi L_y^{1/2}}$), have

HUGE PARAMAGNETISM.

But: Pure Meissner in "Thermodynamic limit" $L_y \to \infty$; $d = \text{const.}$

Glancing States

- $\sim \sqrt{\frac{d}{R}}$ k_x's of them.

- Have a continuum $\longrightarrow E_F$
  $\Rightarrow$ Orbital Paramagnetic Moment!!!

- Have μ larger by $\frac{L}{d}$ than others.

- Survive elastic scattering.
[For l_{nd}, only for $T \lesssim \frac{\hbar v_F}{L_y}$!!!]

-
-
-

Conservative Estimate of $I_{glancing}$

- Total Diamagnetic "Current" without screening:

$$I_d = \frac{n(e)^2}{m^* c} L_z d^2 B$$

- For $T \lesssim T^* \sim \frac{\hbar v_F}{L}$, $I_{glancing}$ is $\sim \frac{d}{L}$ of total

$$\frac{I_{glancing}}{I_d} \gtrsim \frac{d}{L} \sim 1\%$$

For strong screening ("λ_L" $\ll d$):

$$I_{Meissner} = \frac{c}{4\pi} B L_z$$

$$\frac{I_{Meissner}}{I_d} = \frac{m^* c^2}{4\pi n d^2} = \left(\frac{\lambda_L}{d}\right)^2$$

$$\left[\frac{1}{\lambda_L^2} \equiv \frac{4\pi n}{m^* c^2} \quad \left(\sim \frac{\alpha^2}{\lambda_{TF}^2}\right) \right]$$

$I_{Glancing}$ is depressed by at most $\sqrt{\frac{\lambda_L}{d}}$

$$\frac{I_{Glancing}}{I_{Meissner}} \gtrsim 1\% \left(\frac{d}{\lambda}\right)$$ [redacted]

• Energy scale of "glancing states" is as

• for "clean limit" $T^* \sim \frac{\hbar v_F}{L}$

Higher temps:

Effect reduced by thermal smearing, and by e^{-L/L_φ}.

Magnetic-field scale:

$$B L \{ {d \atop \lambda} \sim \Phi_0$$

~ 1 oe.

Disclaimers (to be treated)

- No Self-consistency in gap
  in B (only 1^{st} correction)
- . . - .

Summary: "Andreev helps Mesoscopics"

- Large Paramagnetic Orbital response in "proximity layer"
- Decoup strongly for $T > T^*$ (but L_φ no problem)
- Understand why normal electrons are ballistic ("trying to superconduct")
- Tremendous B sensitivity ($B \times \min\{L_d, L_\varphi\}$)

But: Qualitative, no claim to "quantitative fits"

Main Pts.

1. Andreev states are discrete ($< E_F$), do not have para response $\Longrightarrow$ Diamagnetism of Prox. Layer

2. Low DOS at low $\mathcal{E}$ increases m.f.p.

3. "Glancing" states have huge "Para" P.C. (protected from scattering by the above.

4. Energy scale is $T^* = \frac{\hbar v_F}{L}$

5. At "high" T, PC killed by averaging, L_φ.

6. Right B scale (~ 1 Oe)

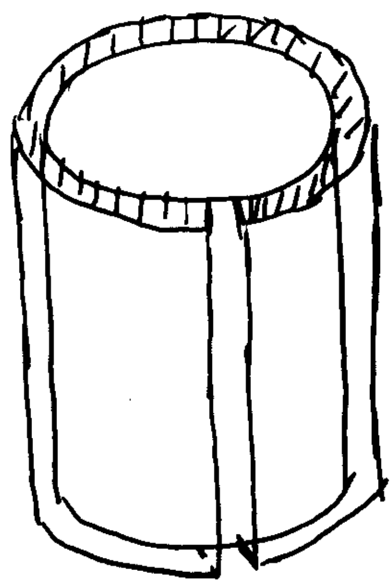
7. Glancing Contribution less sensitive to $\frac{\lambda_L}{d}$.

8. Fully SCF calc. is needed.

Mola Effect is very Natural

for not-too small d/L_y,

Acid Test:



Mesoscopic Effect on scales of 0.3 mm ($\lesssim$ 10 mK)

owing to presence of superconductivity.