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I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR.998d - 3

Research Workshop on Condensed Matter Physics
30 June - 22 August 1997

MINIWORKSHOP ON
QUANTUM WELLS, DOTS, WIRES
AND SELF-ORGANIZING NANOSTRUCTURES
11 - 22 AUGUST 1997

"Coherence and Phase Sensitive Measurements of a Quantum Dot"

PART I

A. YACOBY
Bell Laboratories
Lucent Technologies
600 Mountain Avenue
NJ 07974 Murray Hill
U.S.A.

These are preliminary lecture notes, intended only for distribution to participants.

Coherence and Phase Sensitive Measurements of a Quantum Dot

A. Yacoby, M. Heiblum and R. Schuster

V. Umansky, D. Mahalu, H. Shtrikman

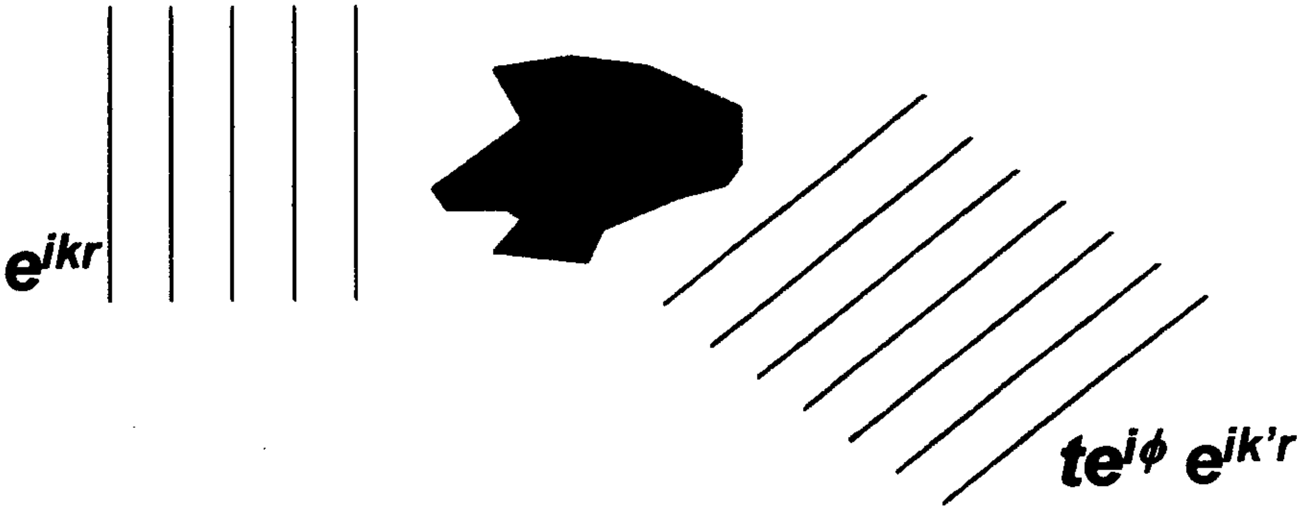
Braun Center for Submicron Research,
Dep. Condensed Matter Physics, Weizmann Institute

Embedding a *Quantum Dot* in an A - B ring as a tool for studying its *phase properties*

➡ Method to Measure Phase of Transmission Coefficients

- Background on : Resonant Tunneling
  Coulomb blockade
- Coherent transport through a *Quantum Dot* - a *Resonant Tunneling* structure
- Dephasing within the dot
- Unexpected phase behavior of the A - B oscillations was found

Scattering Experiments



- Landauer (1957) has shown

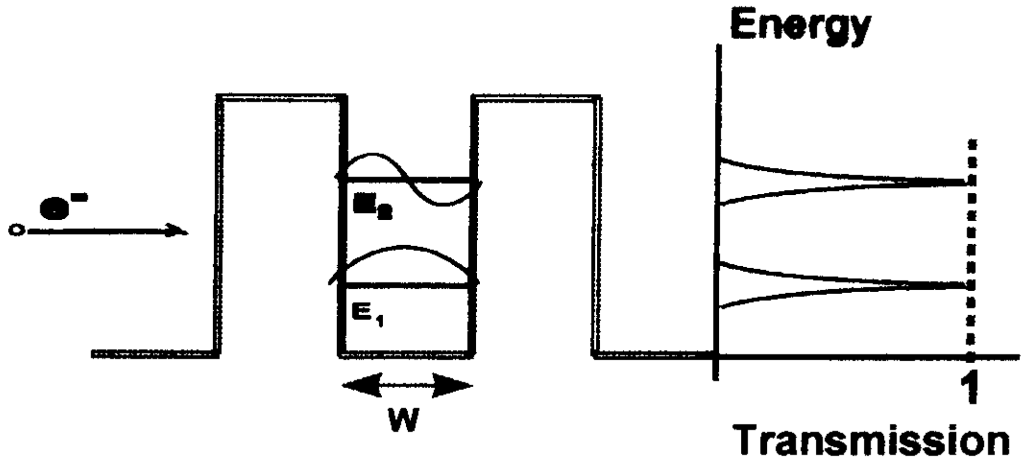
$$I_{kk'} \;\propto\; \left| t\, e^{i\phi} \right|^2 \;=\; \left| t \right|^2$$

- Loss of phase of transmission coefficient (relative phase)

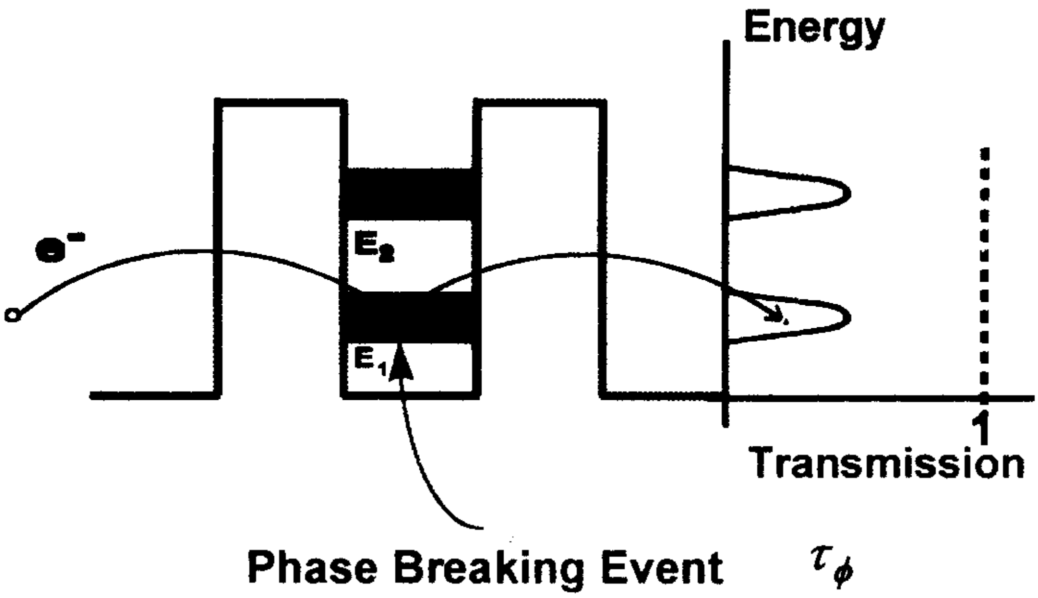
Interference Experiments Provide a Tool to Measure:

- **Actual phase of transmission**
- **Dephasing processes**
- **Phase coherence length**

Coherent vs Sequential Tunnelling



- **Coherency is Maintained During Transport**

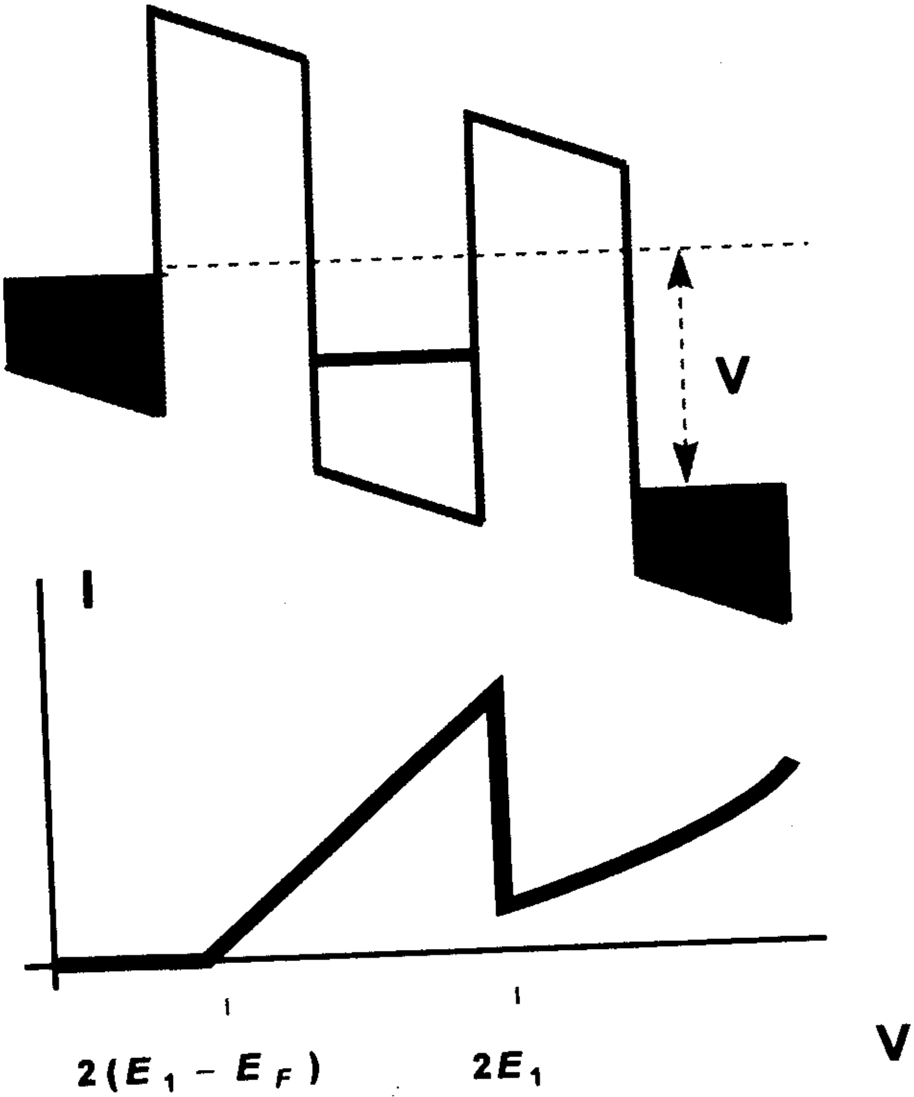


- **No Deterministic Phase Between Input and Output**

$$\tau_\phi \quad < \quad \tau_D \quad \approx \quad \frac{2W}{v} \quad \frac{2Ne^2}{h} \quad R \quad < \quad \tau_\phi$$

Sequential **Coherent**

Experimentally We Measure ...



Can one determine from the I - V characteristics

the nature of transport ?

Landauer's (1957) Formalism leads to :

$$I = v \frac{2e^2}{h} \int |t(E)|^2 \, dE$$

conductivity of one channel

➡ **Only the Area under the Absolute Value of the transmission coefficient is important !**

Weil & Vinter (1987) showed that even though

t (*E*) is different in COHERENT and

SEQUENTIAL mechanisms - - -

The Predicted Conductance is the Same !

There is a Long Lasting Controversy

on the Nature of Transport in

***Resonant Tunnelling (RT)* Devices ...**

Is it Coherent or Sequential ?

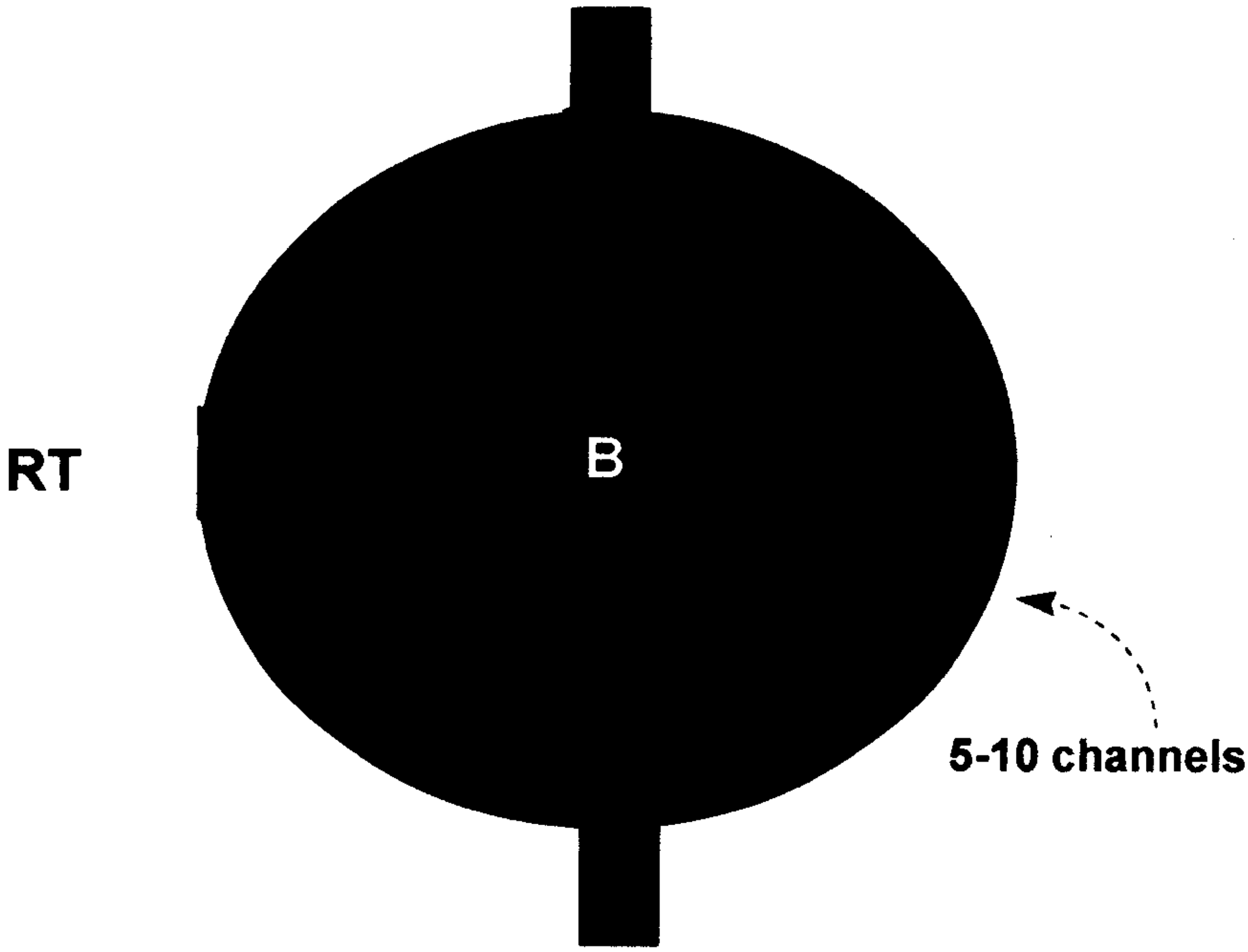
Introducing a RT Structure in an

Interference Experiment and

Measuring Phase Coherency

is a Definite Way to Determine it !

How Can One Determine Coherency of RT ?



Persistence of the Oscillations with : Φ

Expected Periodicity is $\Delta B \cdot A = \Phi_0$

Order of Magnitude :

$$A = 2\ \mu m^2; \quad \Delta B \cong 20\ G$$

Phase coherence in the ring sets the physical dimension of the experiment

A Small RT Structure

Contains a Small Puddle of Electrons

It is Called a Quantum Dot (QD)

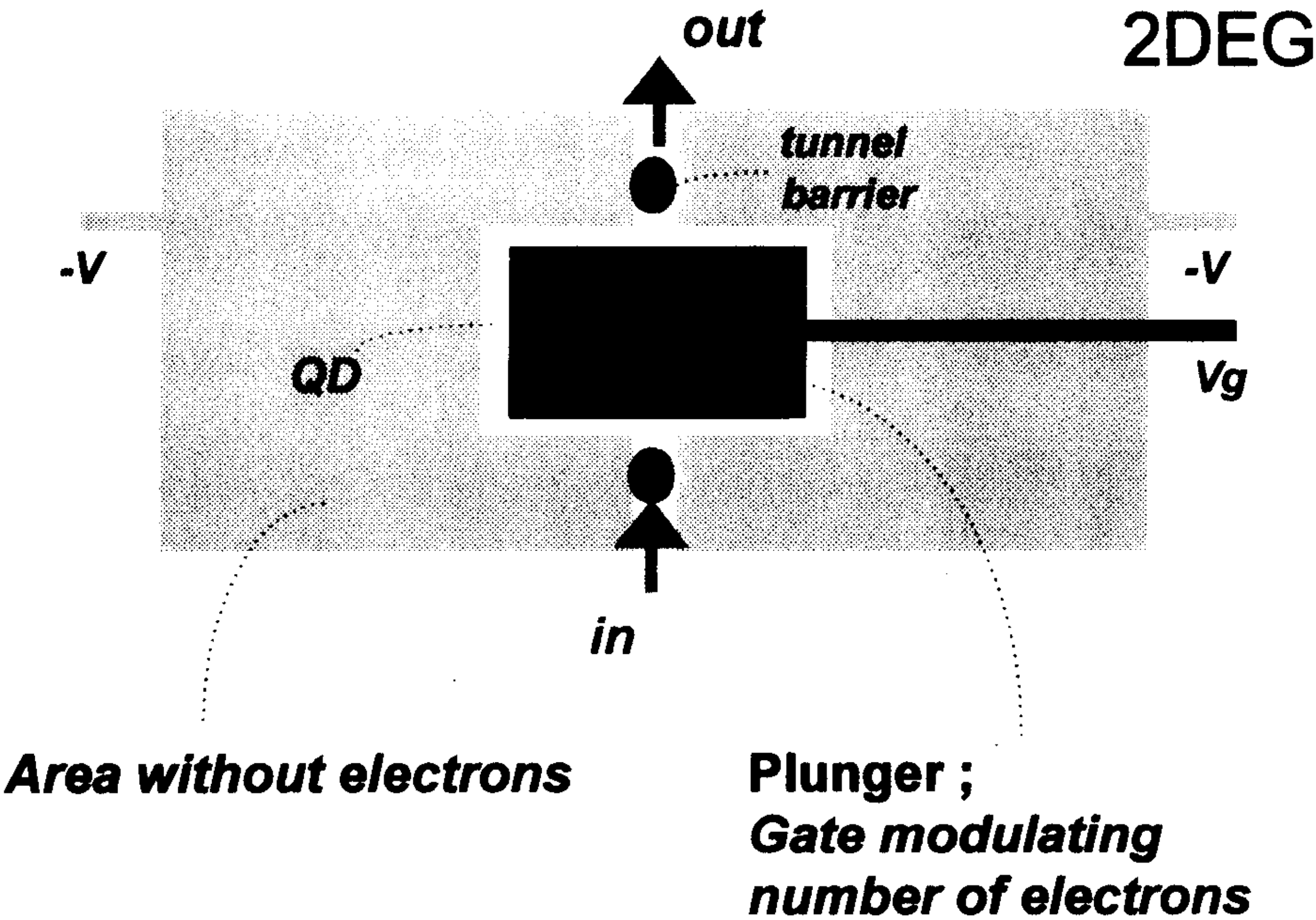
The Small Puddle has a Small Capacitance C

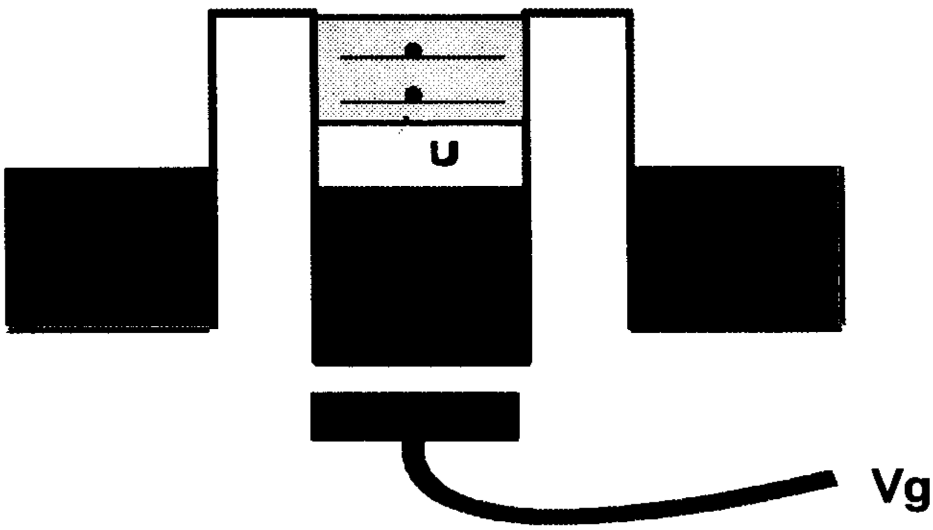
Adding an Electron to the Puddle

Acquires Energy : $$U = \frac{e^2}{C}$$

This is the Coulomb Blockade Regime

The Realization of a QD in 2DEG

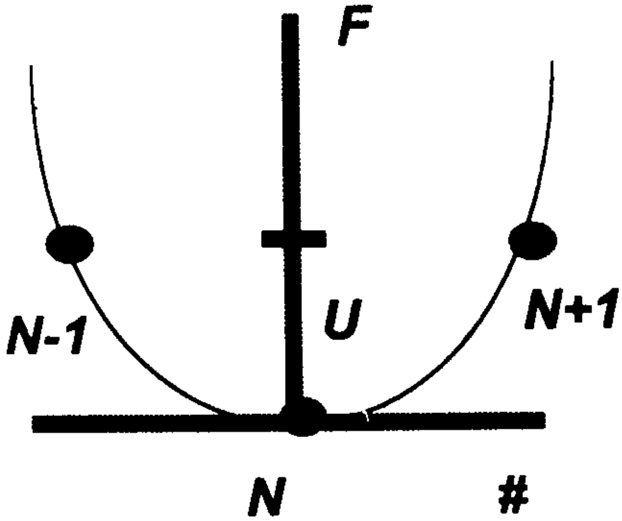




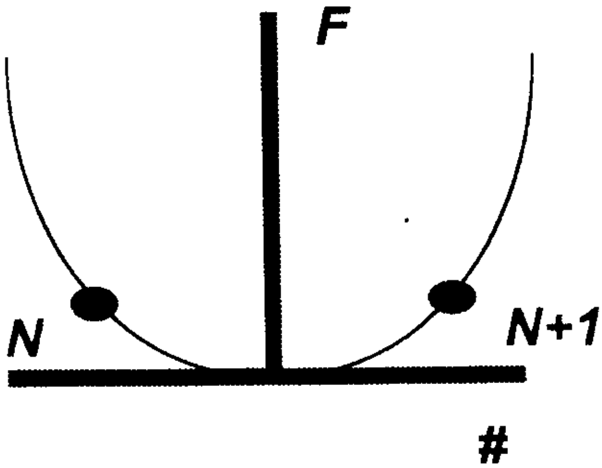
Current Flow Through the QD

-- as a function of gate voltage --

$$F = \frac{(Ne)^2}{2c} + V_P Ne$$

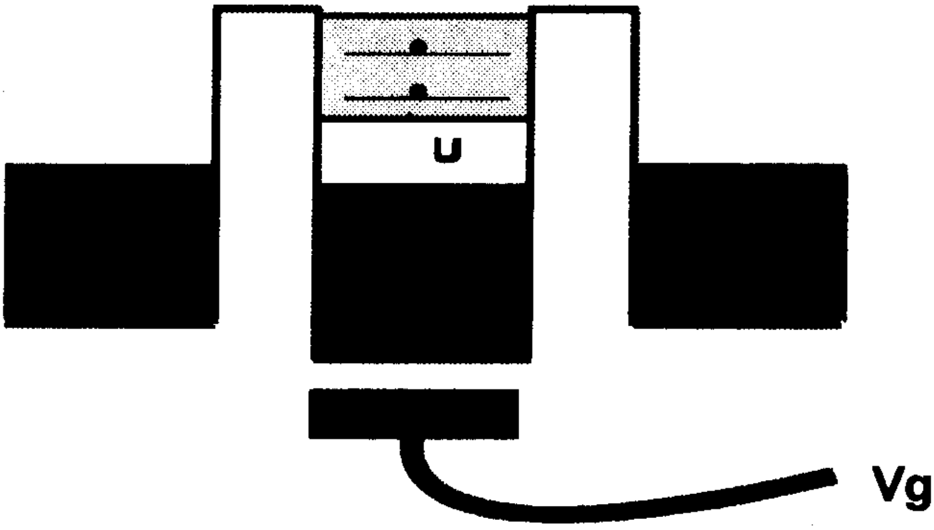


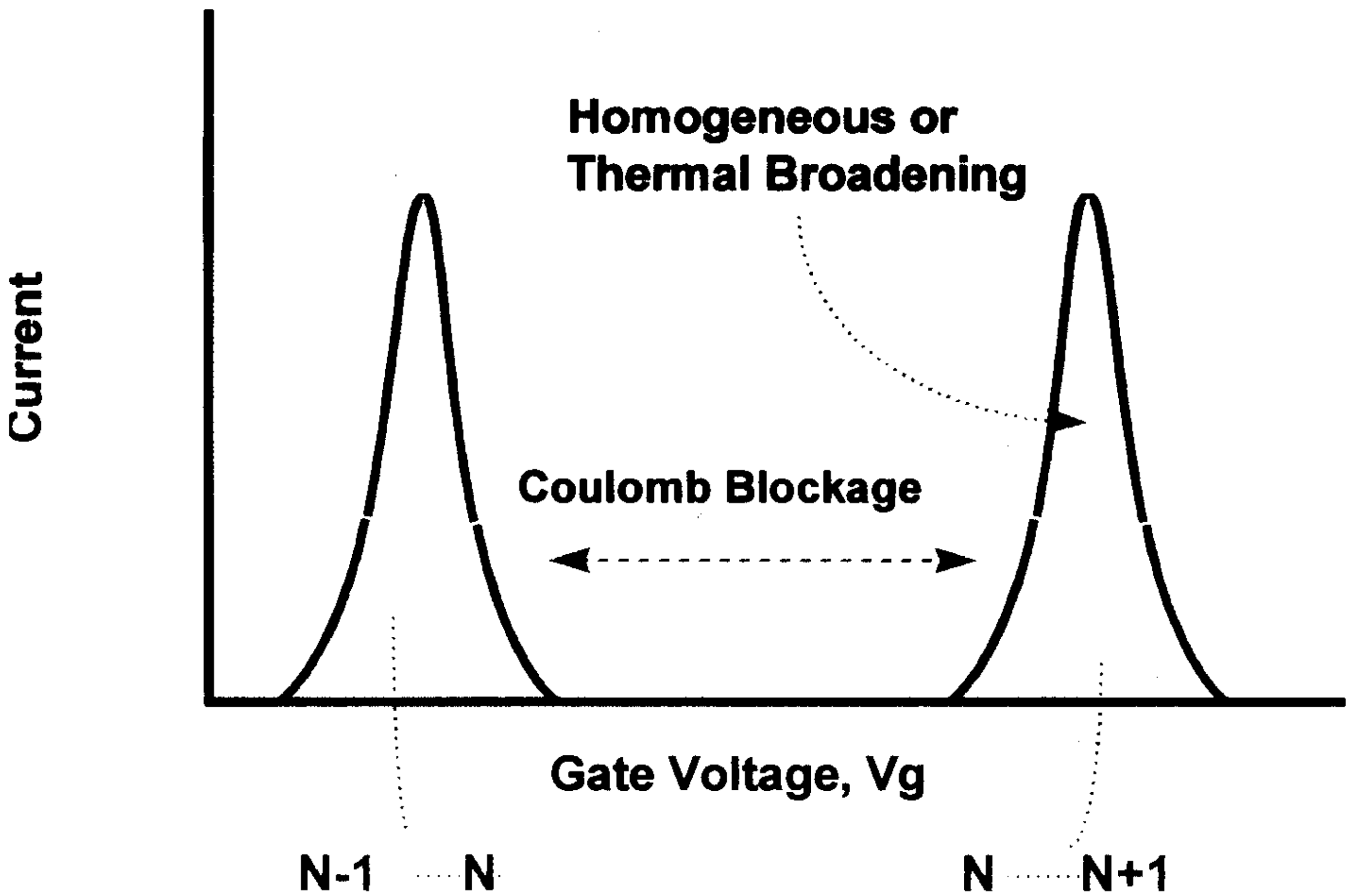
$$V_P = N \frac{e}{c}$$



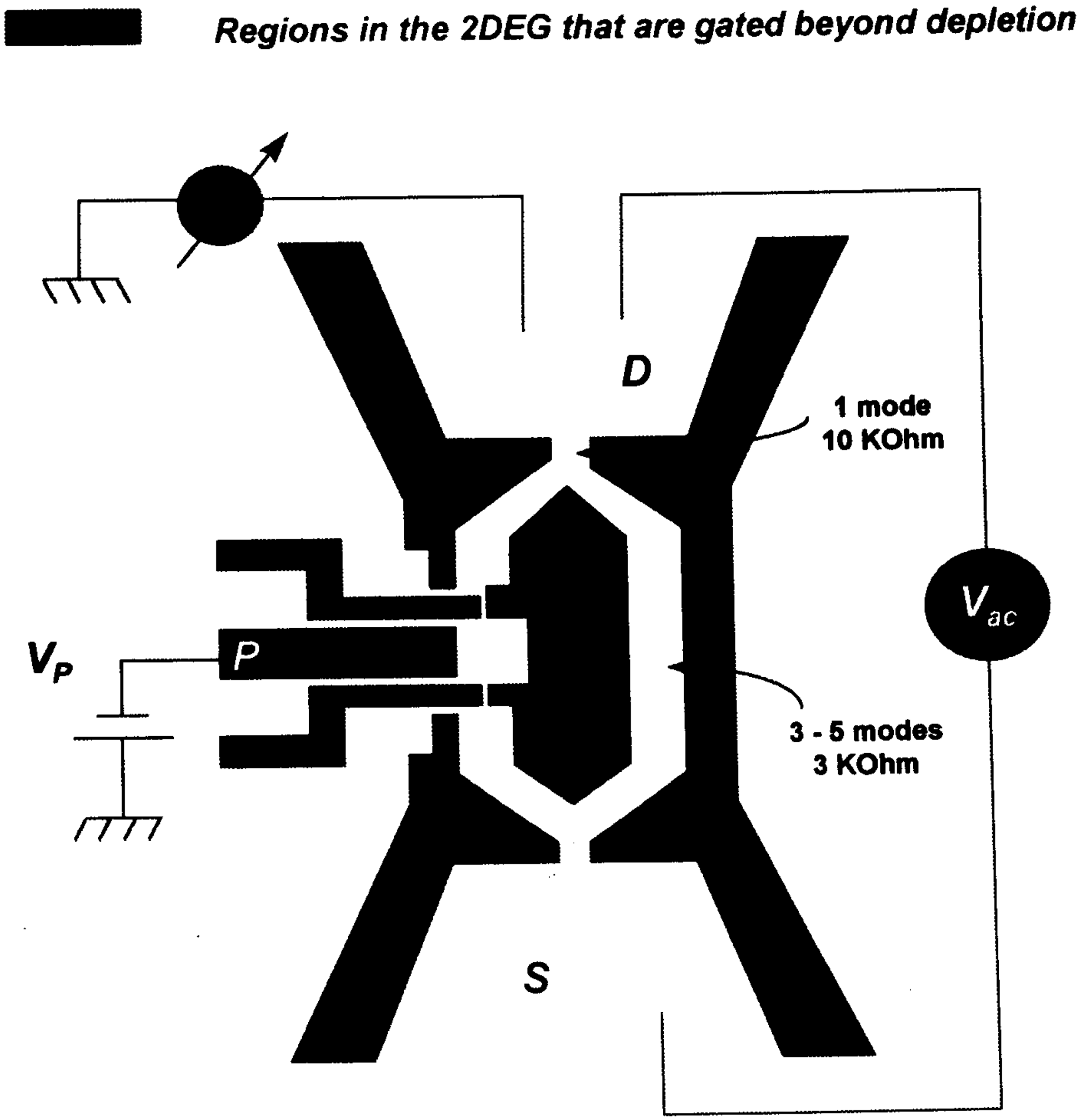
$$V_P = \left(N + \frac{1}{2} \right) \frac{e}{c}$$

The Quantum Dot :
Coulomb Blocked - Resonant Tunnelling

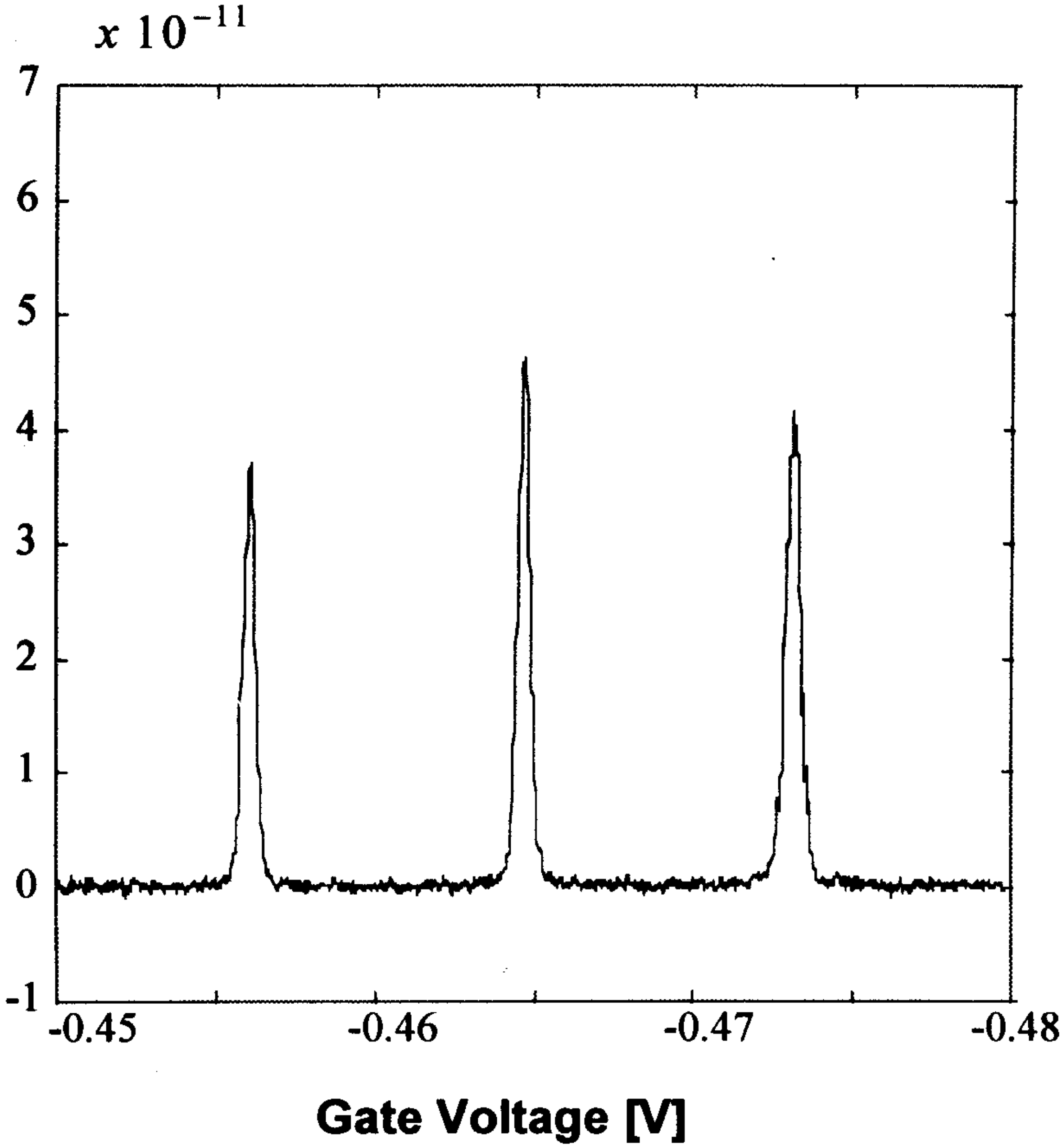




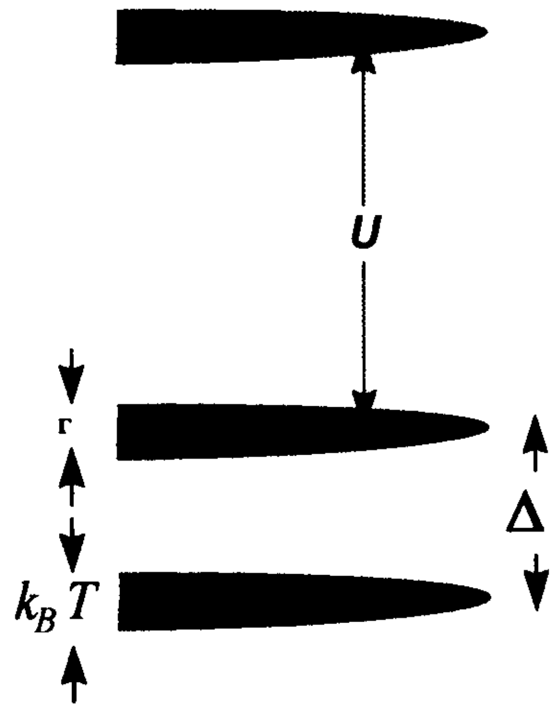
SCHEMATIC DRAWING OF EXPERIMENT



Typical Coulomb Blockade Peaks



Some Important Values:

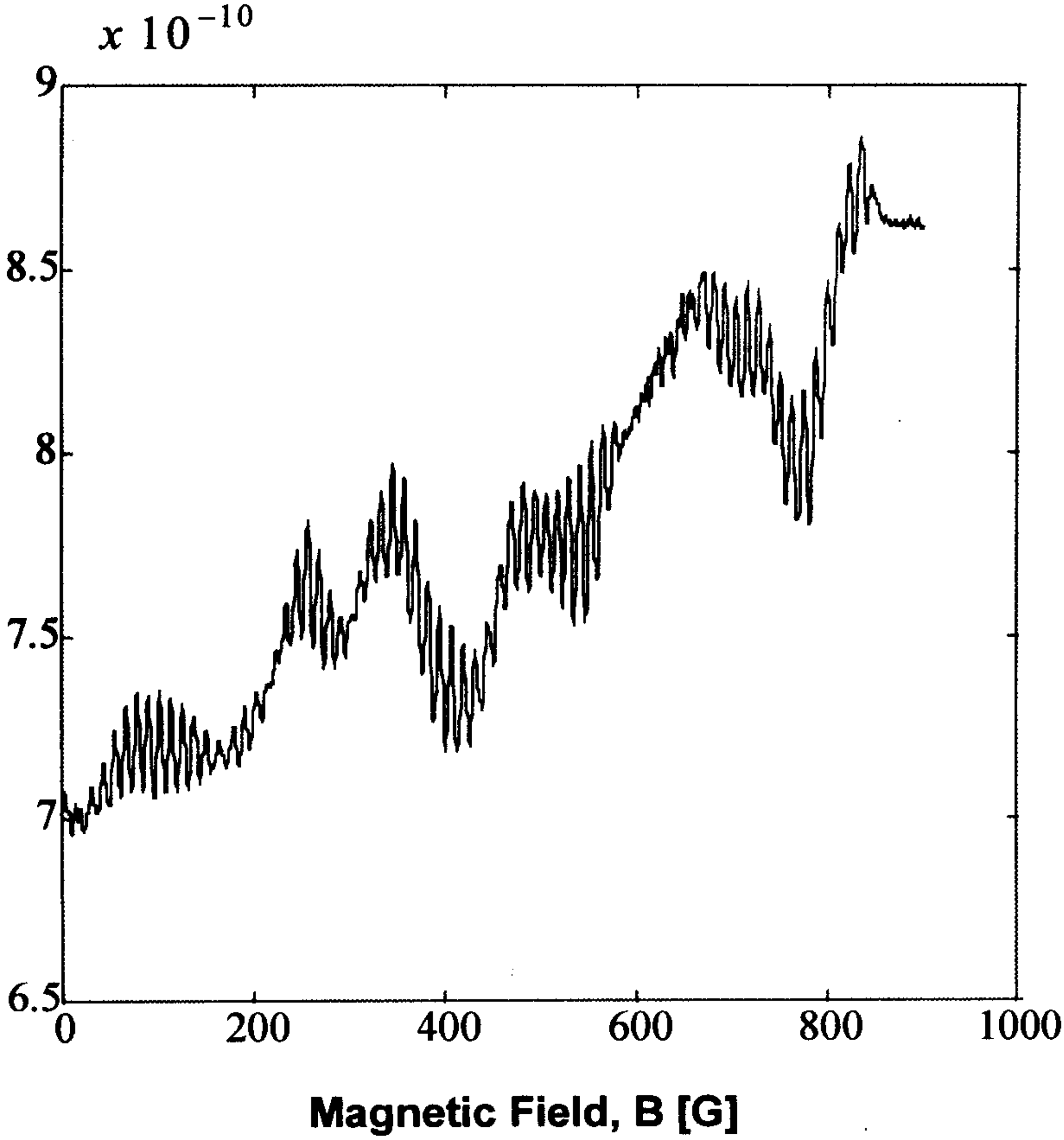


$\Gamma_e \quad << \quad k_B T \quad << \quad \Delta \quad << \quad U$

Elastic Level Width — Level Spacing

- **QD size 300 *nm* x 300 *nm***
- **Number of electrons 100 - 300**
- **Capacitance 160 aF**
- **Charging energy 0.5 meV**
- **Level spacing 50 μ eV**
- **Temperature (100 mK) 10 μ eV**
- **Single particle level width 0.1-1 μ eV**

Typical A - B Oscillations

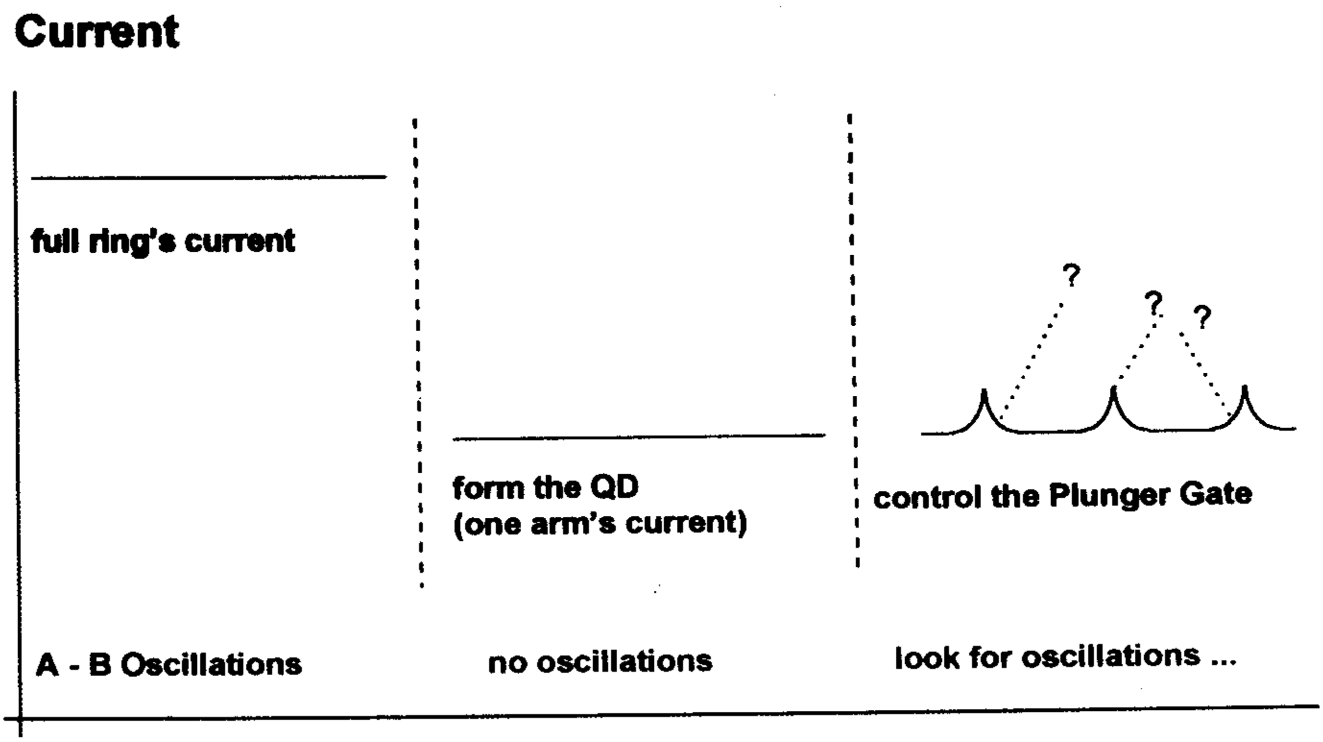


Typical contrast of 10% due to :

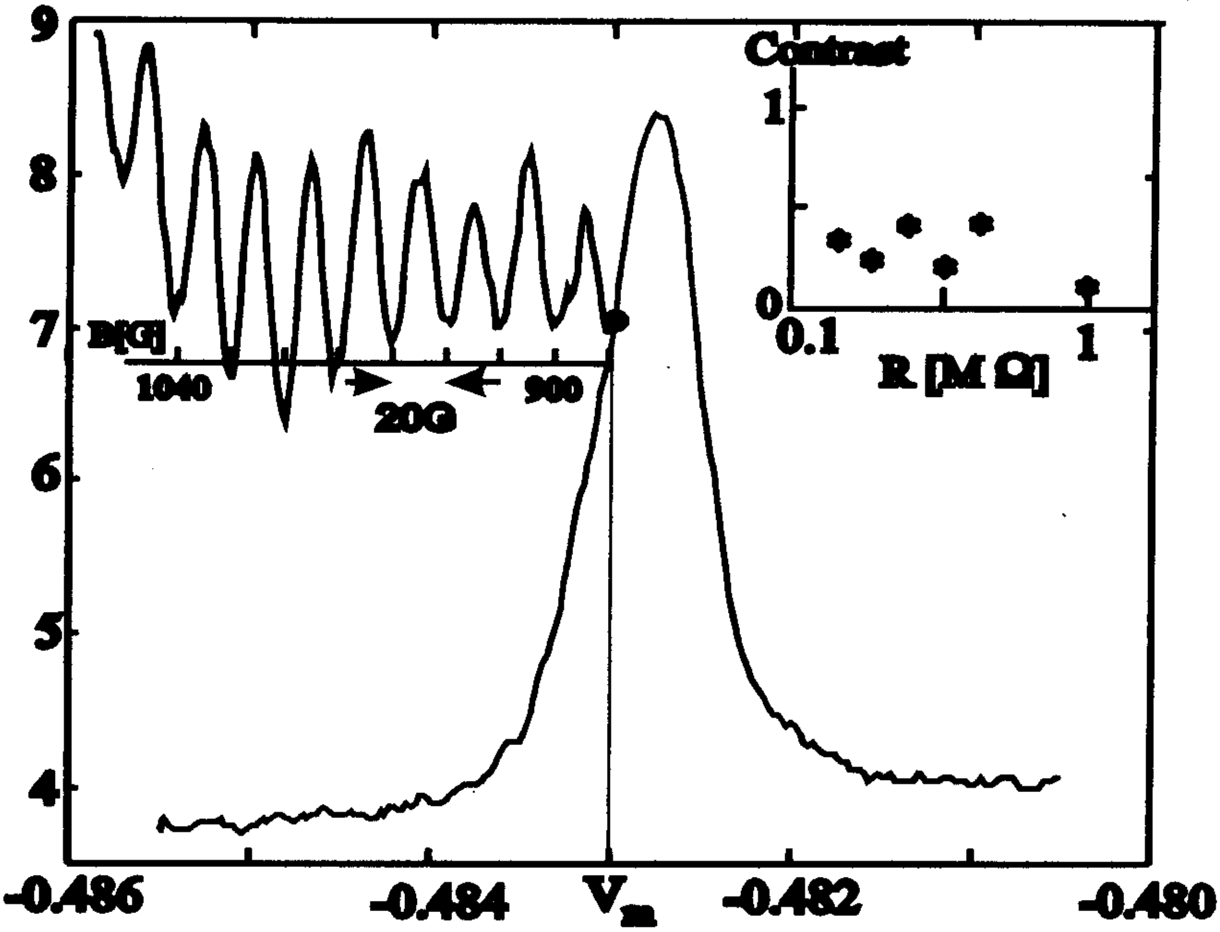
- L_T
- Multiple channels (3 - 5)

Is Transport Through the QD Coherent ?

The Experimental Process ...



Oscillations in a Modified A - B Ring



Plunger Gate Voltage, Vp [V]

∴ The QD Supports **Coherent Transport**

When R = 0.1 - 1 M Ohm ,
Dwell Time Changes 1 - 10 nS

What Contrast Do We Expect ?

- **Using a simple interference picture**

$$I_{Ring} = \frac{e^2 V}{h} \left| t_{Right} + t_D \right|^2$$

- **Case of a coherent - symmetric dot :**

Breit - Wigner

$$t_D = \frac{\Gamma_{elast}}{(E - E_i) + i\,\Gamma_{elast}}$$

$$I_{Ring} = \frac{e^2 V}{h} \left[\left| t_{right} \right|^2 + \alpha \Gamma_{elas} (2 t_{right} \sin \frac{\Phi}{\Phi_0} + 1) \right]$$

The interference contrast is independent of

$$\Gamma_{elast} \approx \frac{h}{\tau_{dwell}}$$

$$Contrast = 4 t_{right}$$

Independent of $t_D \Leftrightarrow R_D$

Coherency Allows the Measurement of ***Phase***, or :

the Complex Transmission Coefficient

$$\boxed{t = |t|\, e^{i \varphi_t}}$$

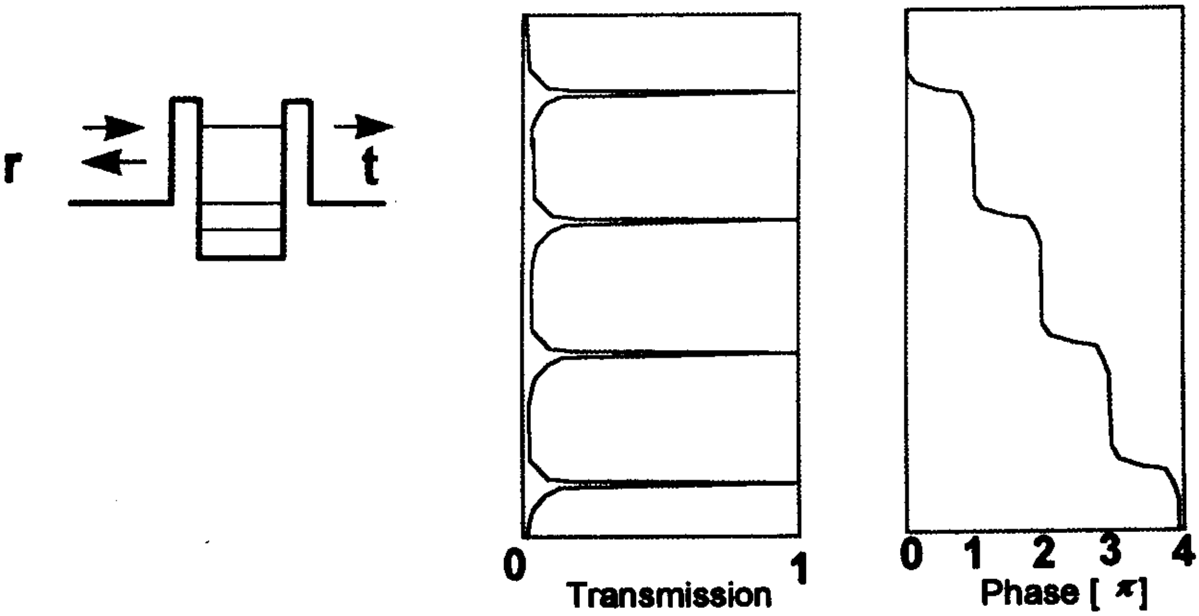
Usually we measure: $G \approx |t|^2$

- **Breit - Wigner Predicts:**

$$t\,(coher) = \frac{\Gamma_{elast}}{E + i\,\Gamma_{elast}}$$

An accumulated phase change of π as a resonance is being crossed !

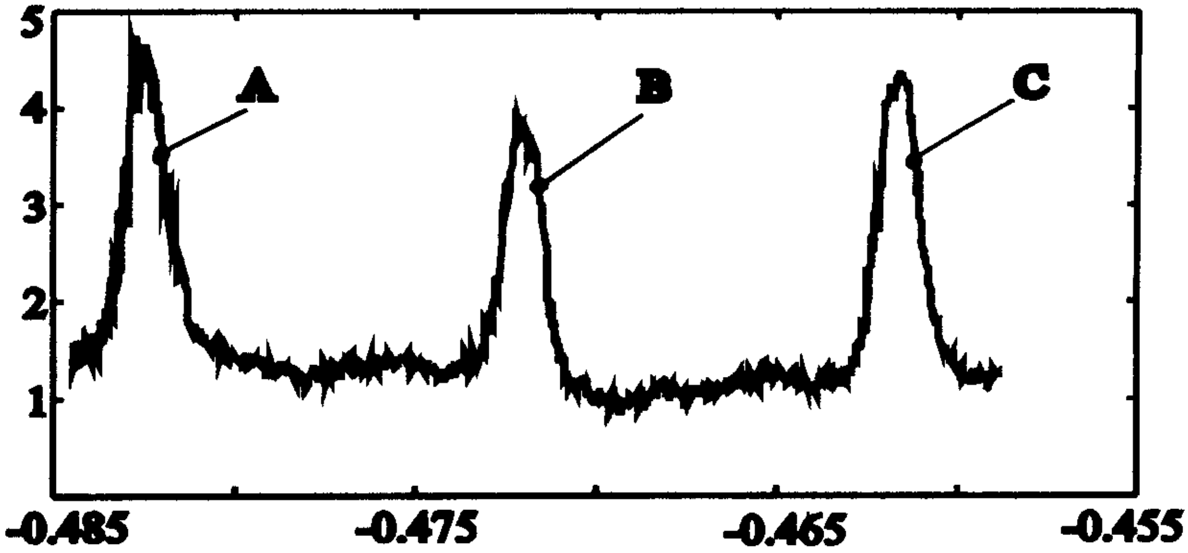
A 1D Model Can Provide a Feel ...

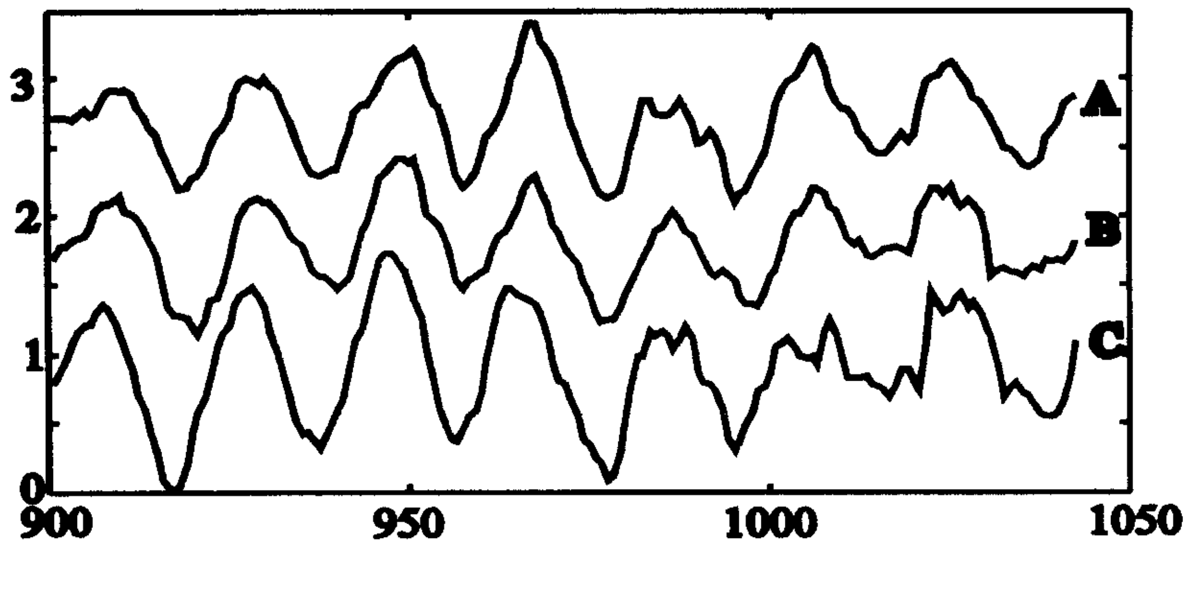


Temperature Smears Changes on the Scale of *kT*

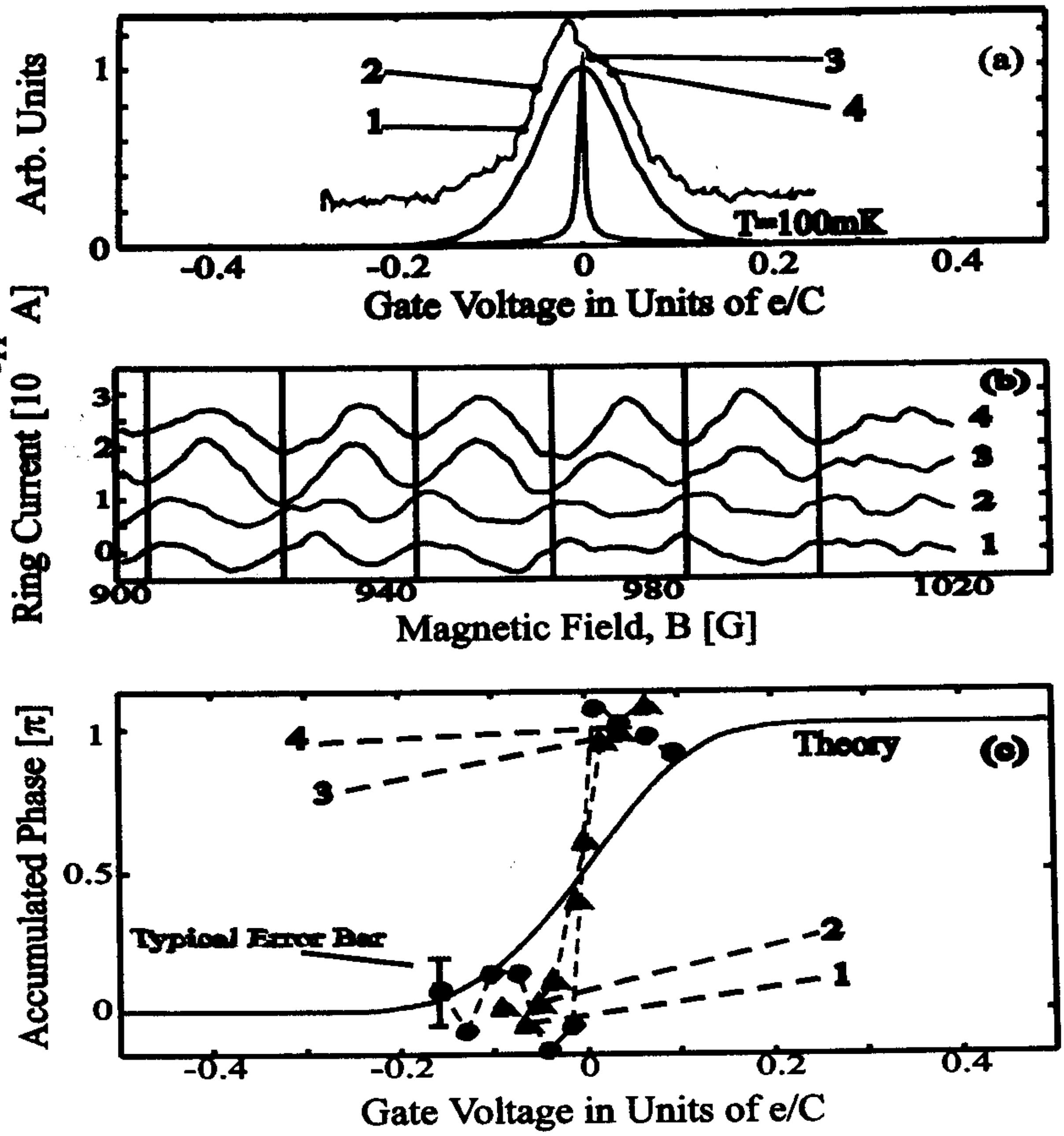
- **Each Resonance Leads to Phase change of π**
- **Consecutive Resonances are Out of Phase**
- **If Spin Degeneracy is Lifted, Every Two Resonances are Out of Phase**

Phase Behavior in Different Resonances





Phase Evolution Through a Resonance Peak



Noise Measurements

- The transition region is also characterized by a large measured noise signal :

1) Amplitude noise

Should be observed with the

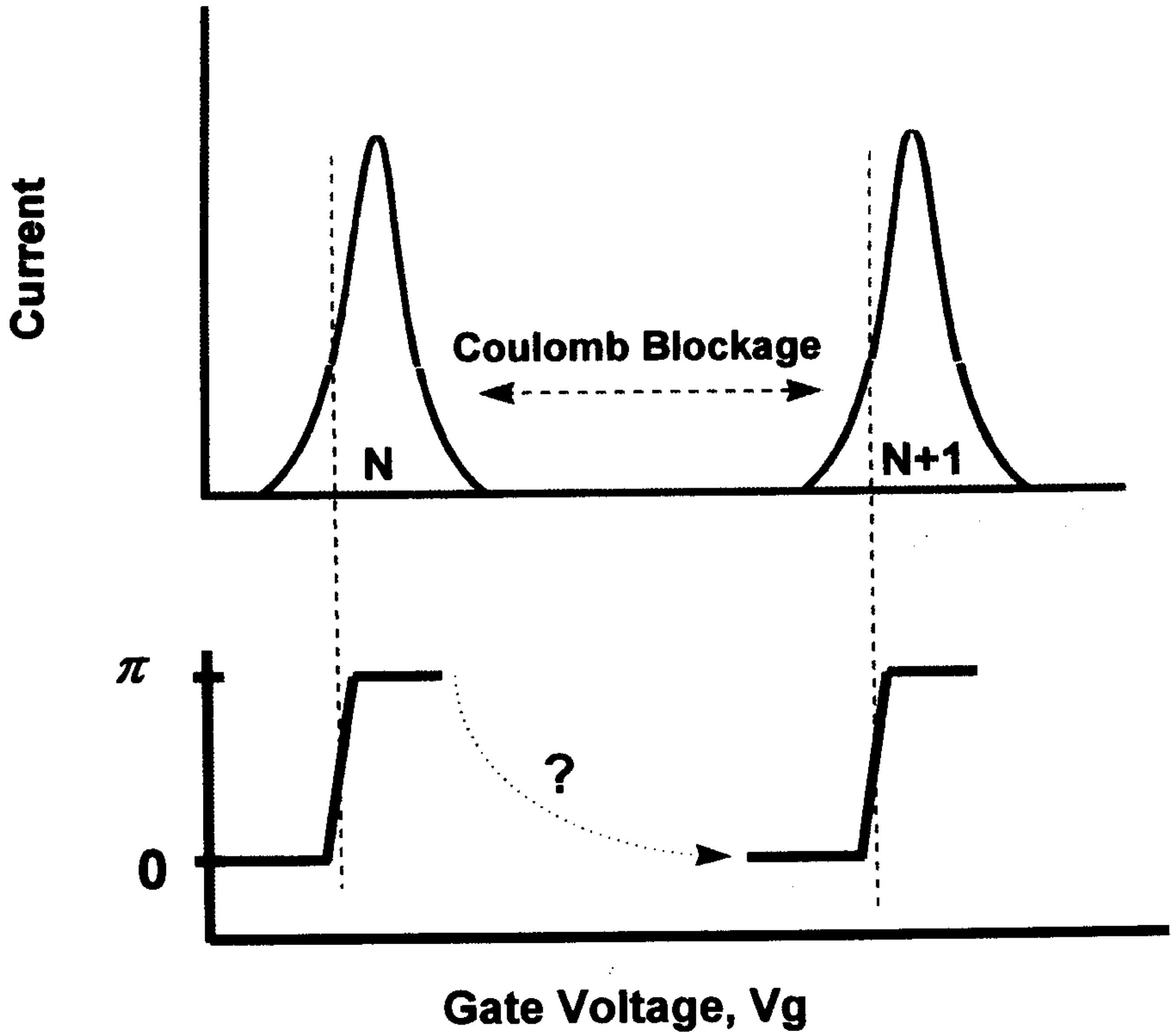
➡ bare dot.

2) Phase noise

Supports the abruptness of

➡ the measured phase change.

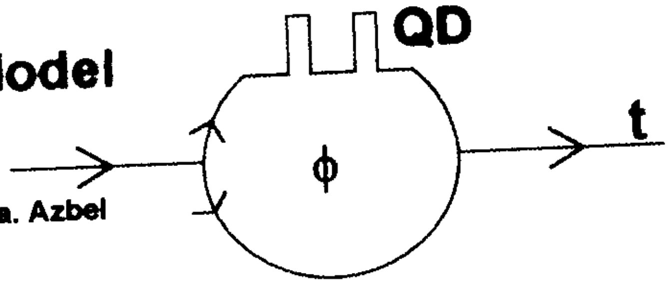
Summary of Phase Behavior



★ **Phase change does not necessarily occur at peak**

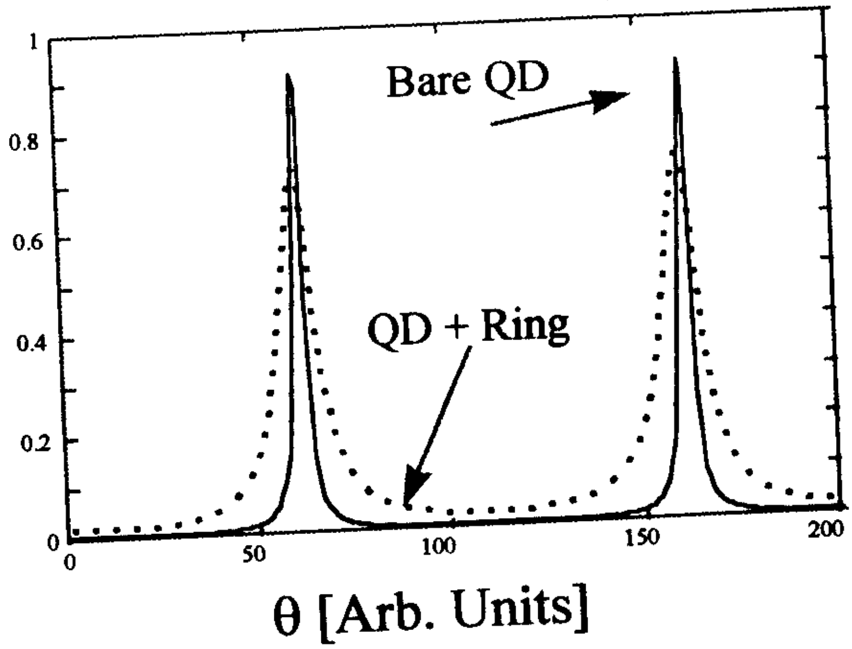
Dot and Ring : One System

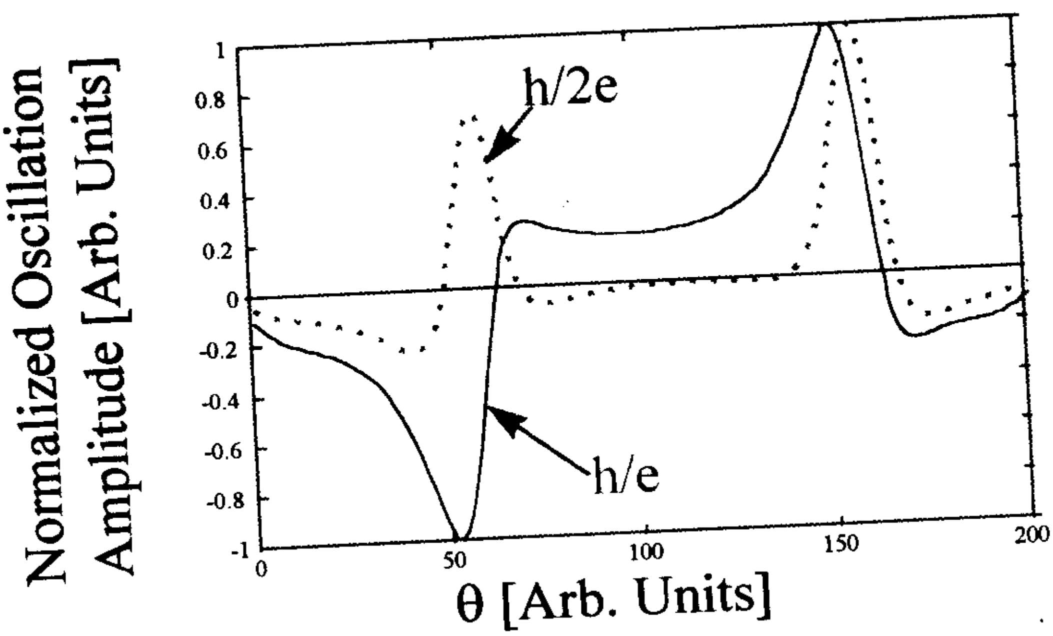
T = 0   1D Model



Y. Gefen, Y. Imry, M. Ya. Azbel
1984

G. Hackenbroich and
H. A. Weidenmuller, PRL 76, (1996)





Underlying Physics

- **Onsager relations :**

$t_{DS}(B) = t_{SD}(-B)$

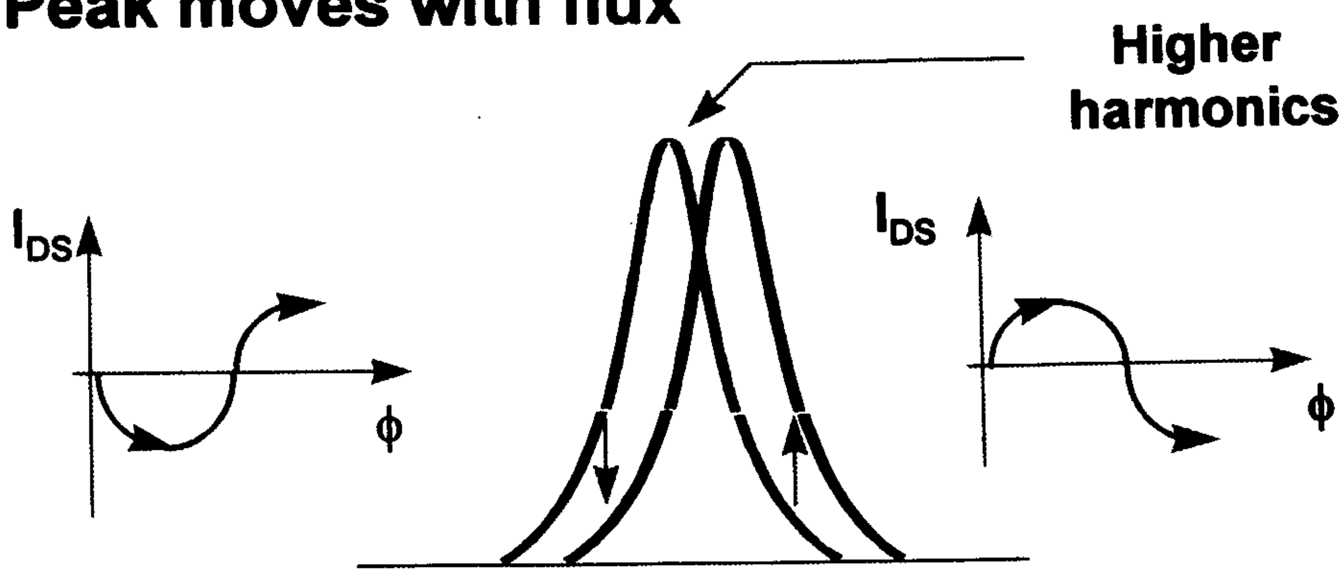
$t_{DS} = t_{SD}$ **Two terminal measurement**

➡ **At B = 0 have a maximum or minimum.**

- **Isolated Ring : System = Ring + Dot**

The system has discrete energy levels that oscillate periodically with flux.

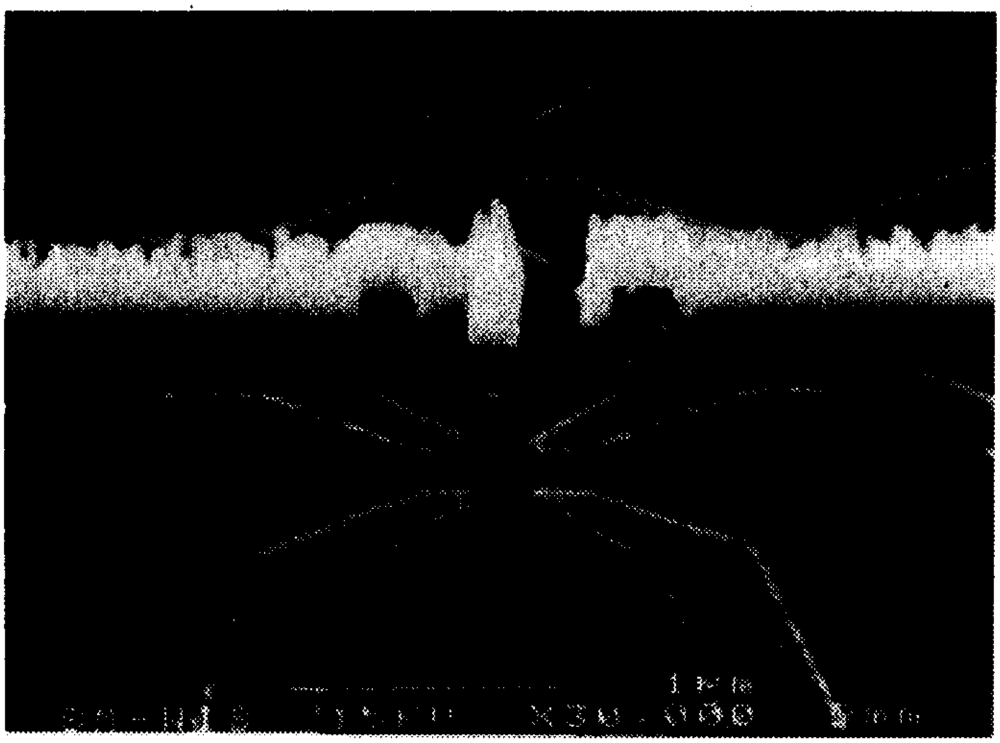
Peak moves with flux



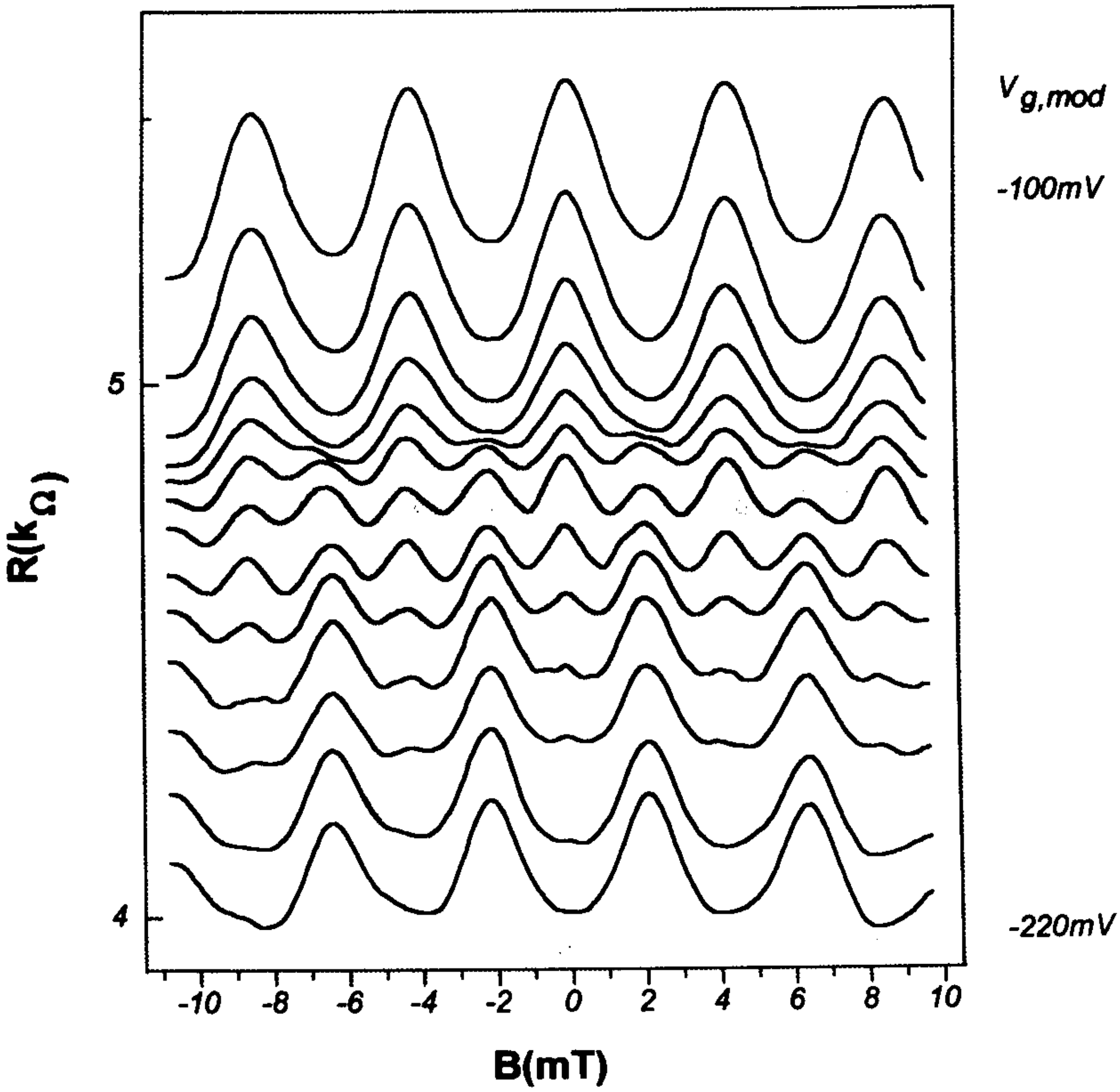
➡ **Related to the Persistent Currents**

Testing the *Two Terminal* Idea

Replace the QD in the AB ring's arm with a phase shifter (an anti dot) .

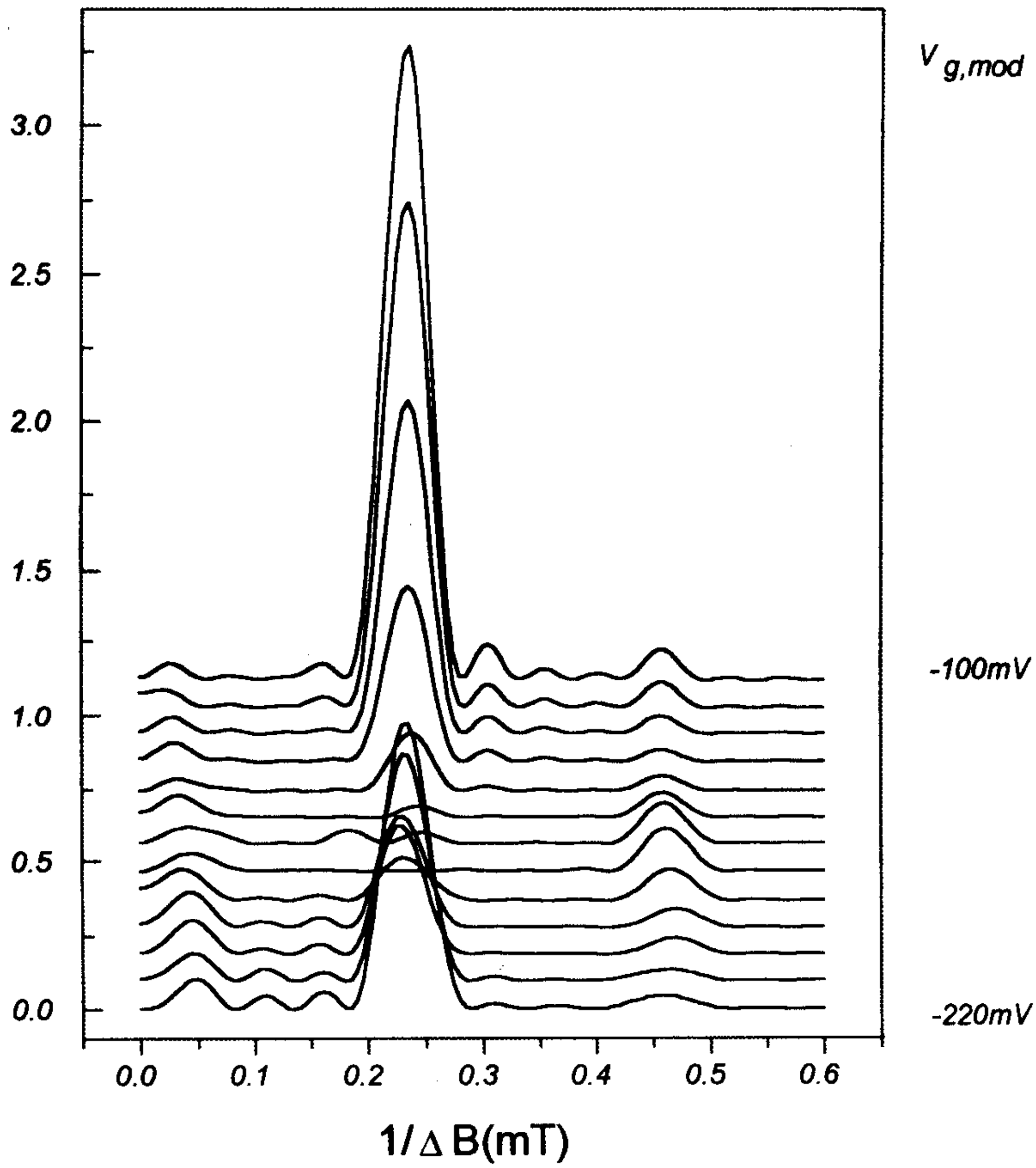


Two Terminal Resistance of an A - B Ring + Antidot in Arm



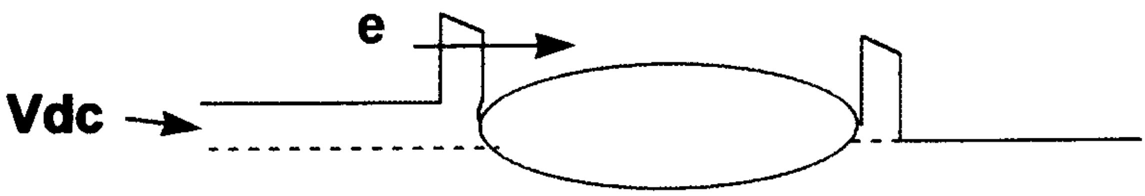
$$G(B) = G(-B)$$

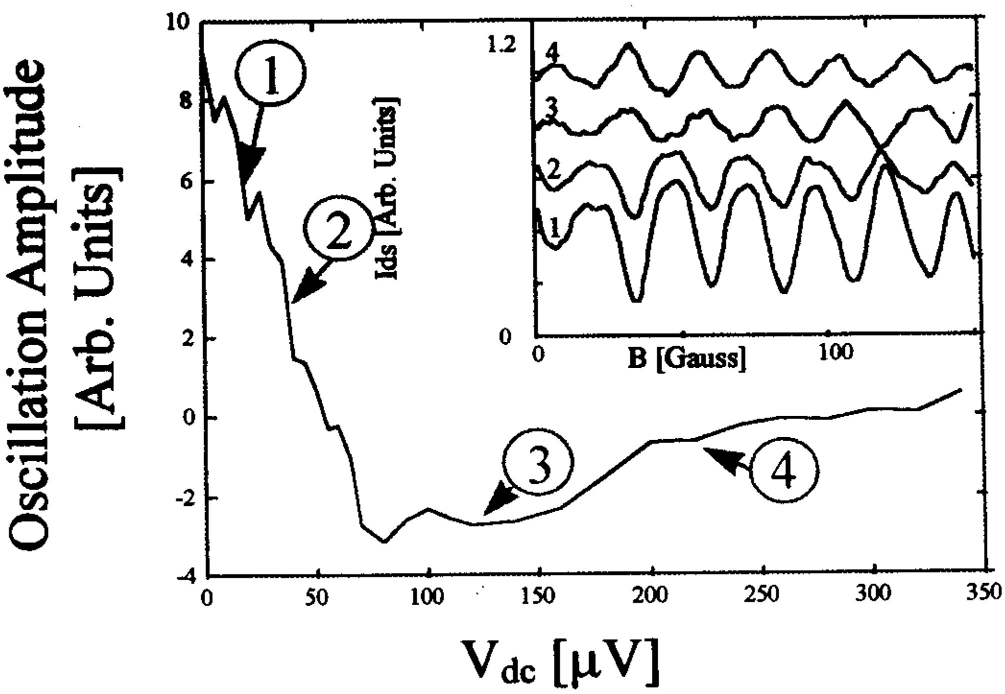
Fourier Plots
of two Terminal Resistance



h/e and h/2e oscillations

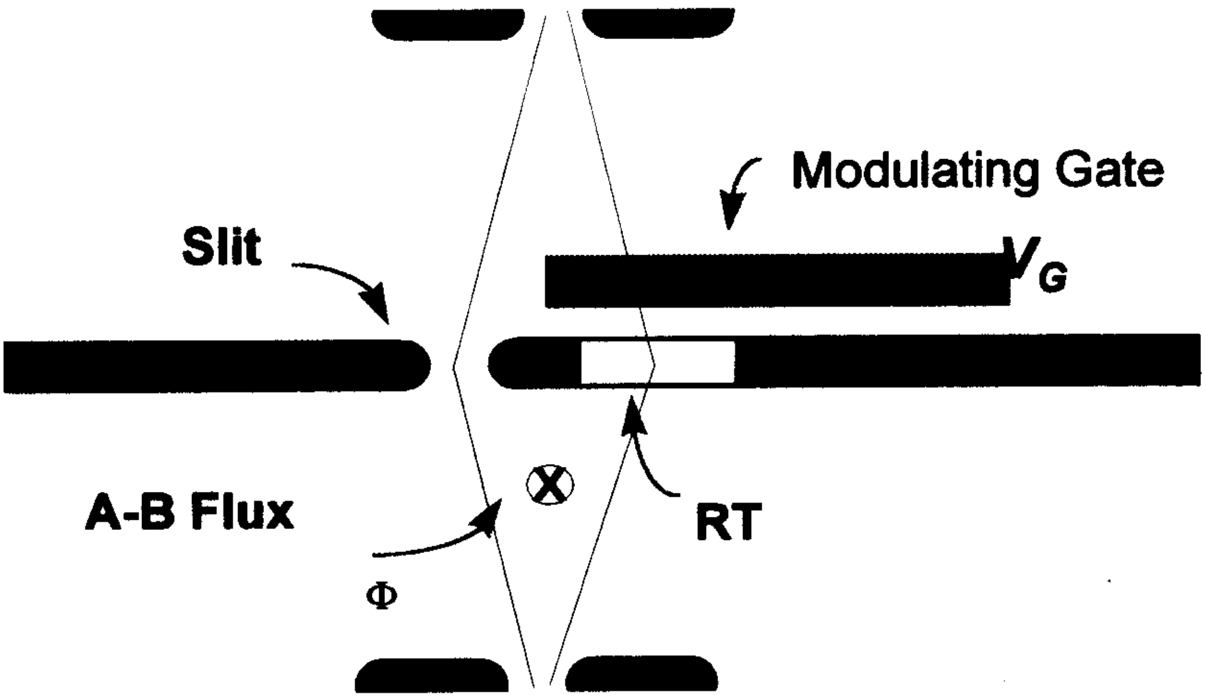
A - B Experiment vs Injection Energy





Adding a QD Should Reveal a Continuos Phase Evolution due to the Strong Coulomb Repulsion.C. Bruder et al PRL 76, (1996)

Open Geometry can Reveal the Details of the Phase Evolution



Study the Shift in Phase Due to the Modulating Gate and the Flux

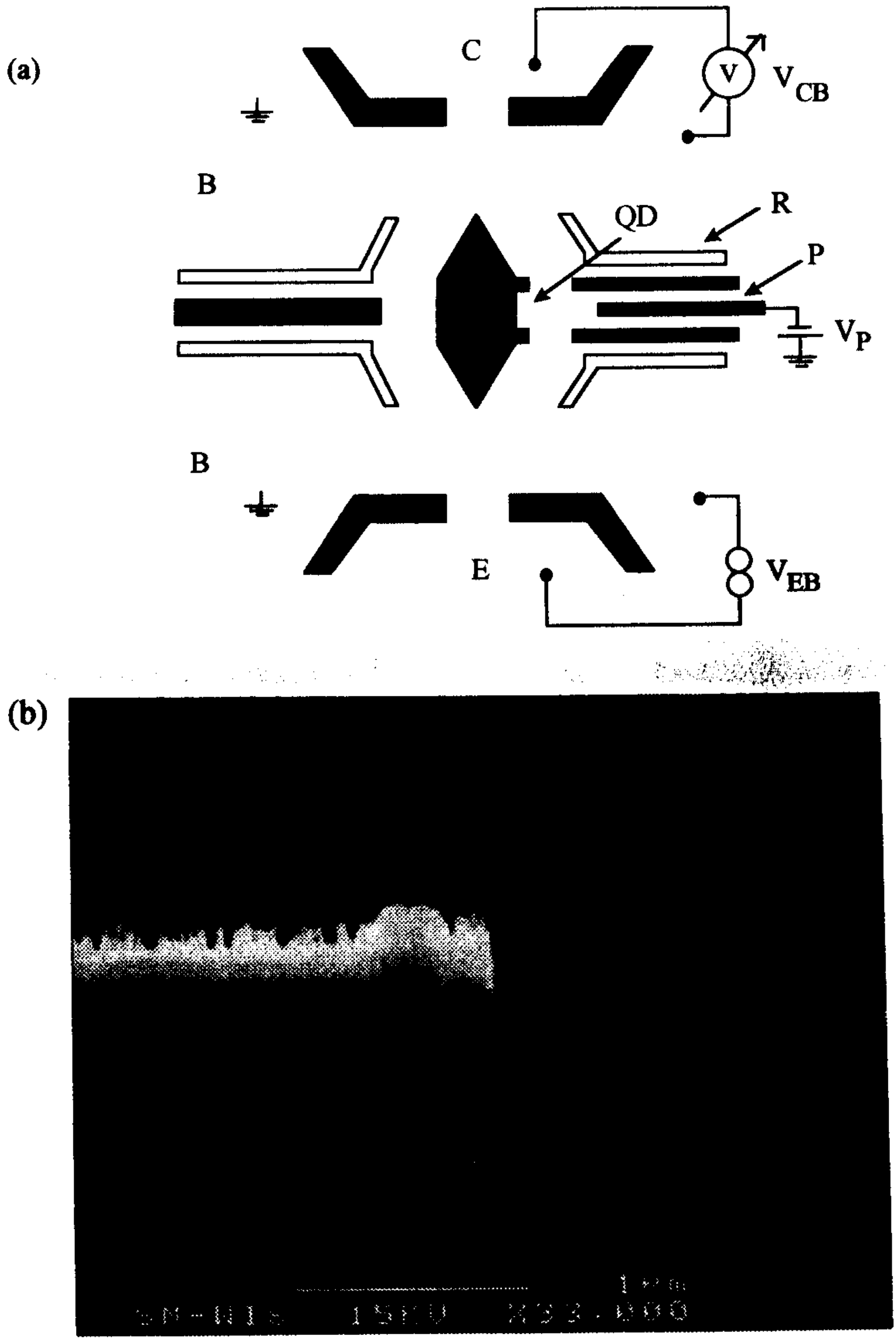
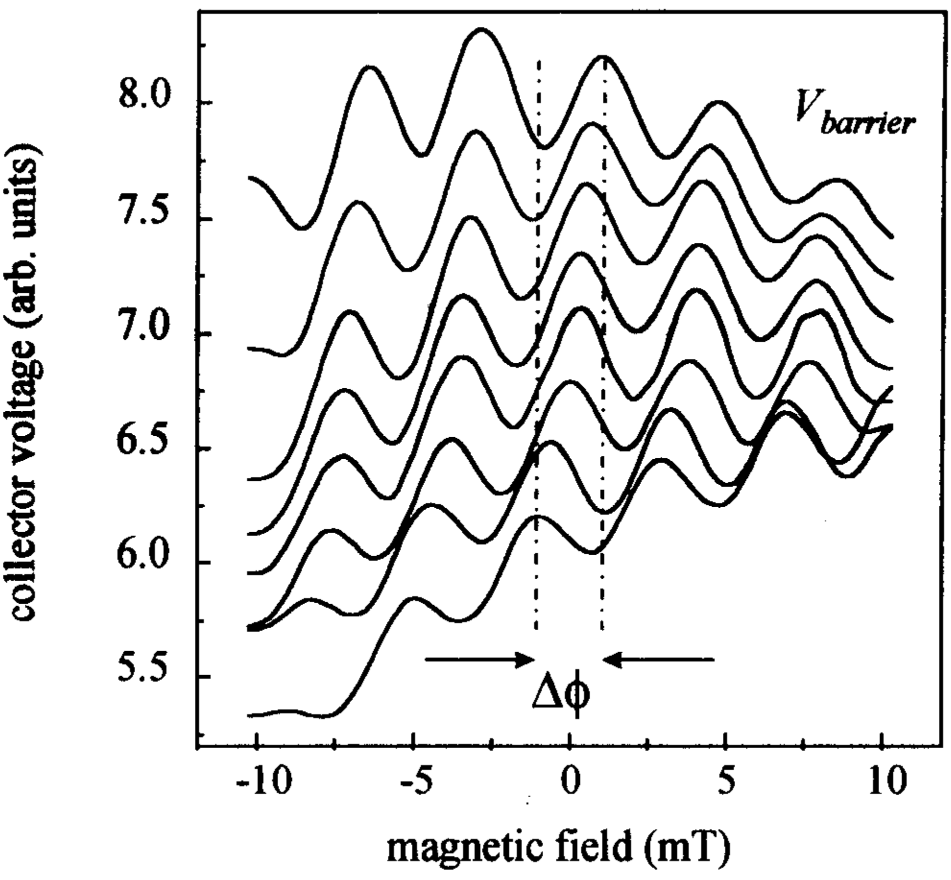


Fig. 1

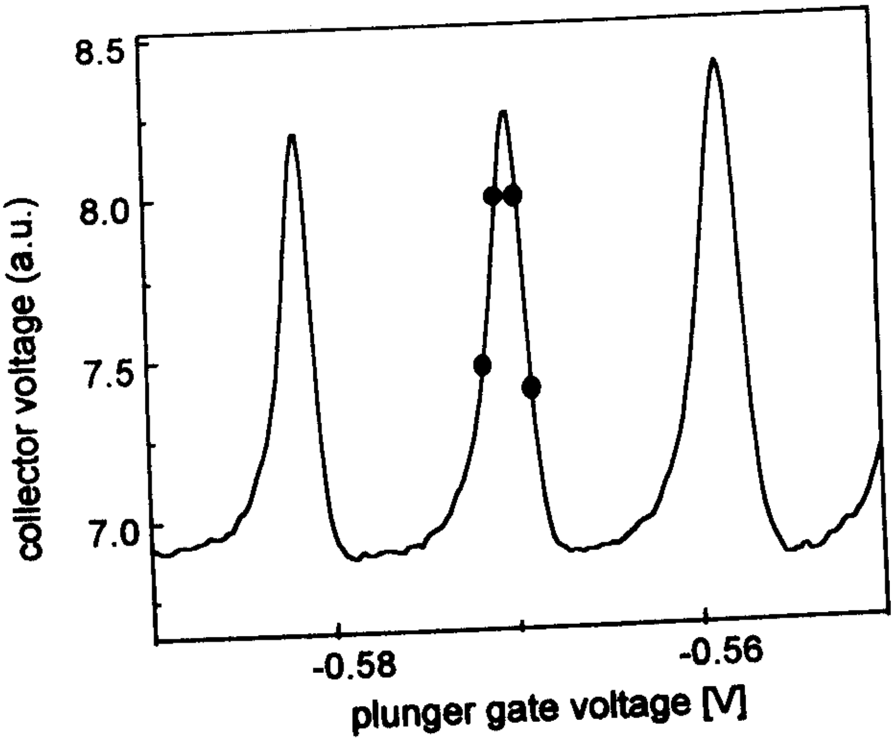
AB Oscillations in :
a Modified Double Slit Configuration

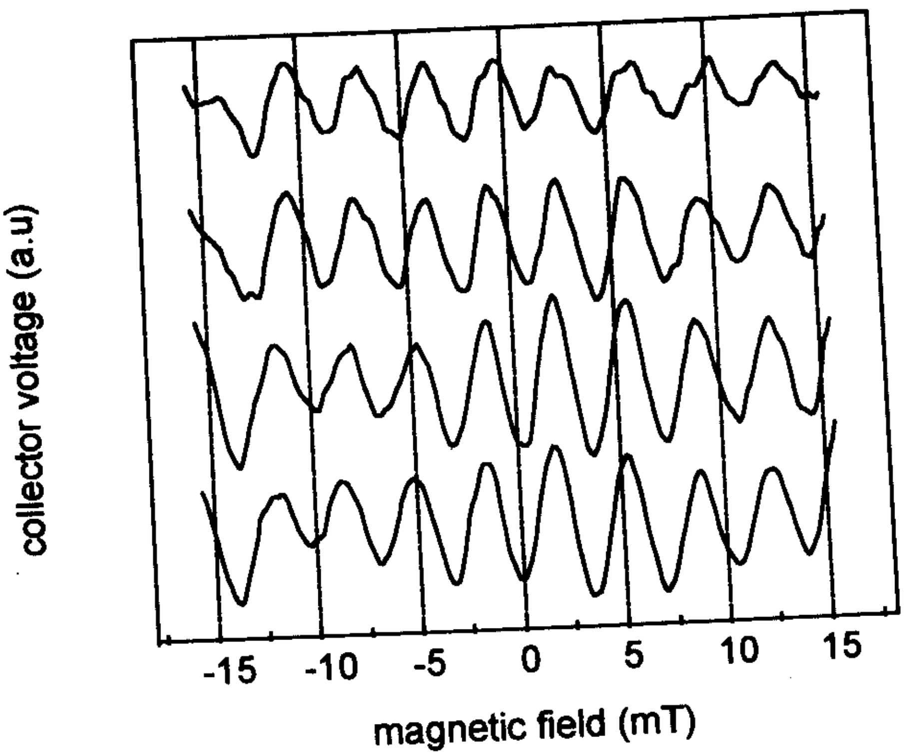
Phase is being changed along one arm



No phase rigidity !

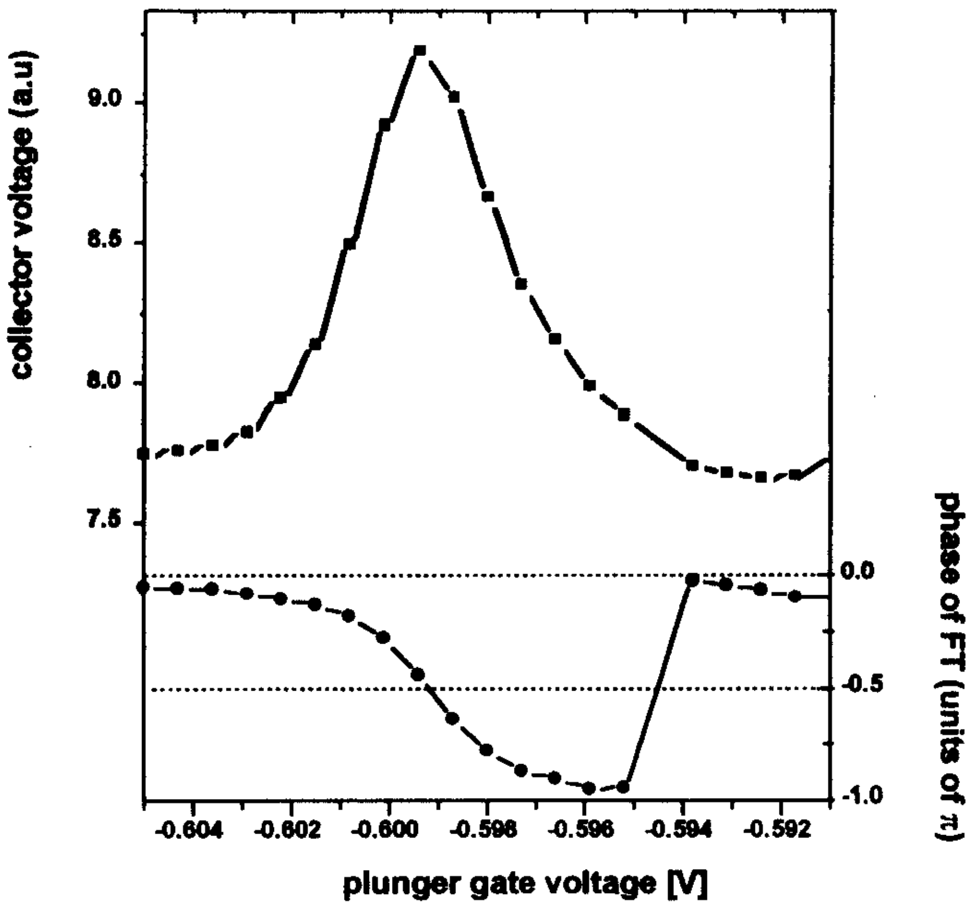
AB Oscillations Along a Resonance Peak





Phase Evolution Along a Resonance Peak

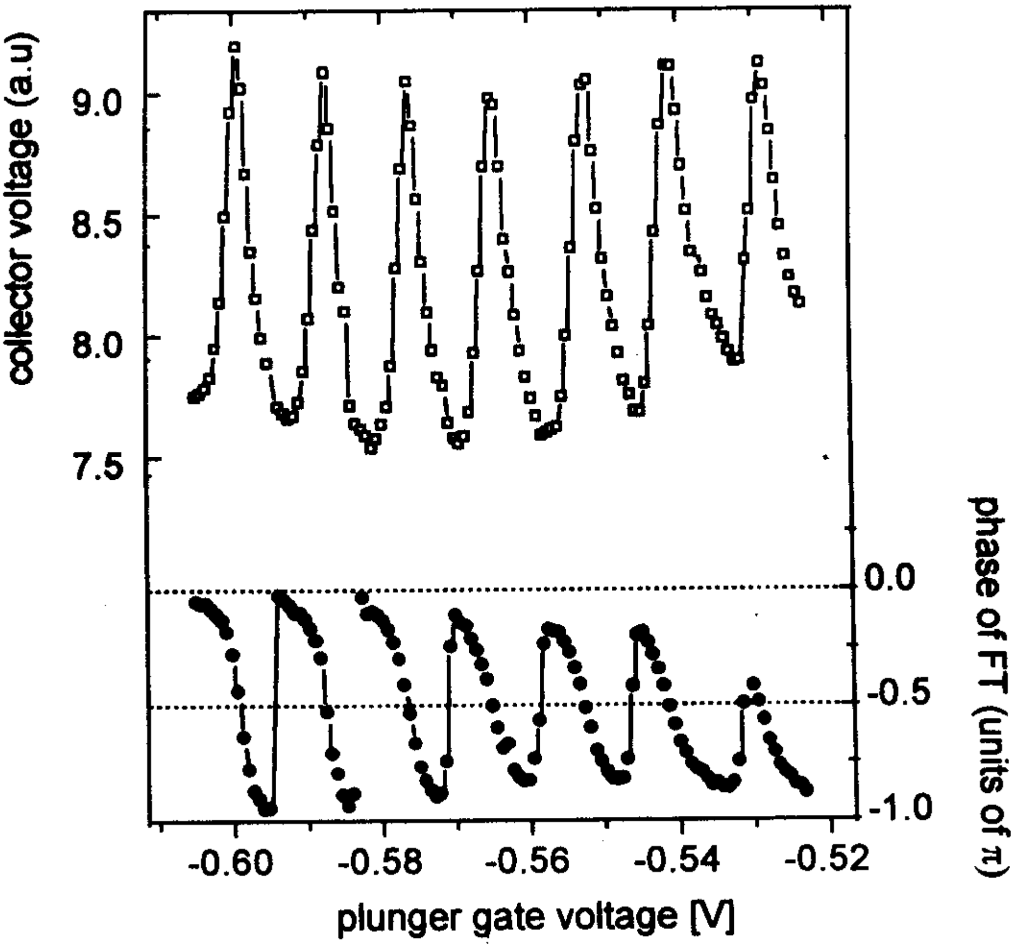
The phase is calculated via a *Fourier transform*



An approximate change of π across a peak !

Phase Evolution Along Several Resonances

Electrons are added to the QD with plunger gate voltage



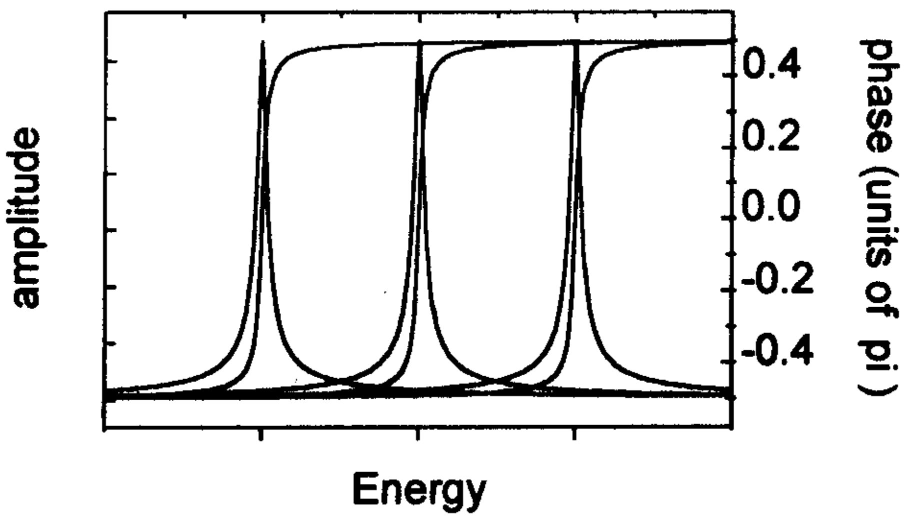
➡ As the plunger gate voltage is more positive :

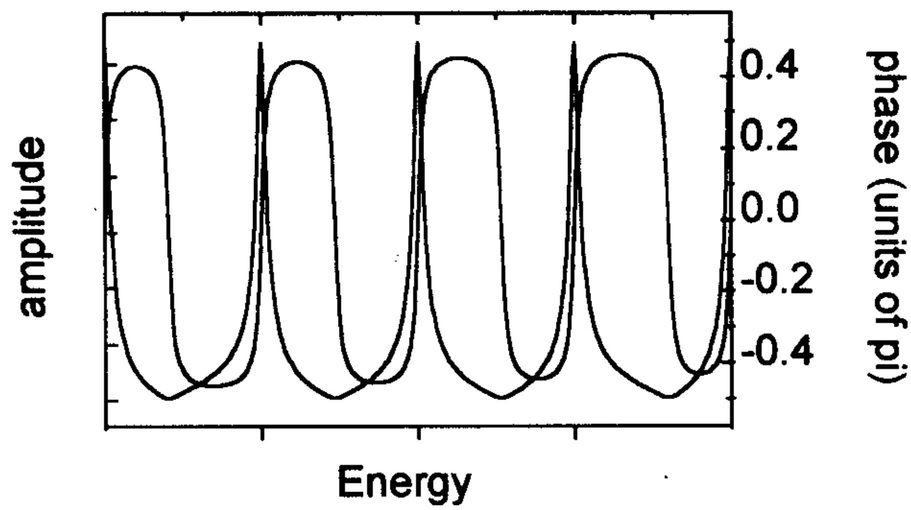
-- the QD couples better to the reservoirs ;

-- The overlapping of resonances reduces the extent of phase change ;

Overlap of Breit - Wigner Resonances

If we assume that each resonance leads to the same phase behavior .....





.... Overlapping resonances lead to smaller phase oscillations .

Summary

- An interference experiment was devised to study the phase properties of a quantum dot

- Observe coherent transport through the dot

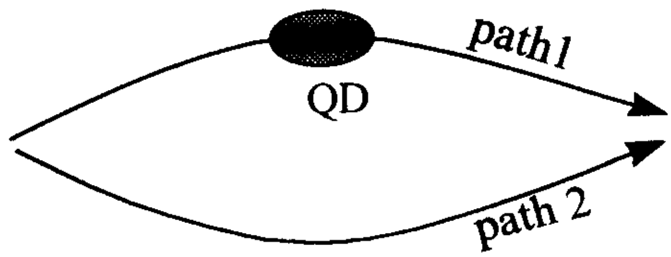
- The phase of the A - B oscillations revealed :

    - An abrupt π change
    - No phase change between resonances

- ID model predicts
    - An abrupt π change
    - Higher harmonics
    - Phase change for successive resonances

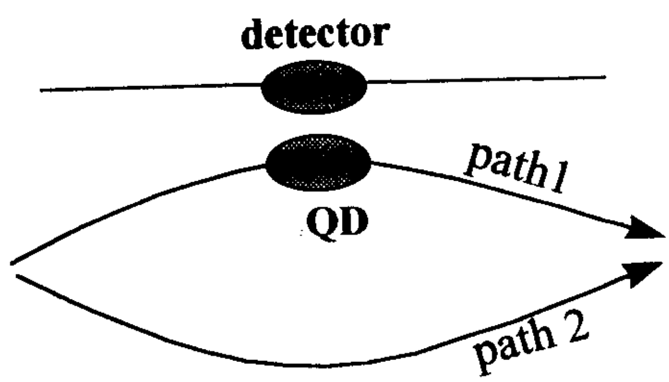
Why do Such Dephasing Experiment ?

Dephasing of QD



With Increasing resistance of QD and dwell time 10 nS ➧ QD remained coherent.

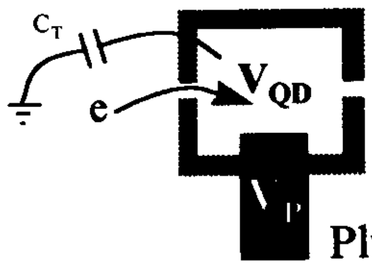
Coherency can be affected by *side detector* , determining *which path* information ➧ Decohering the QD



Determining which path kills interference !

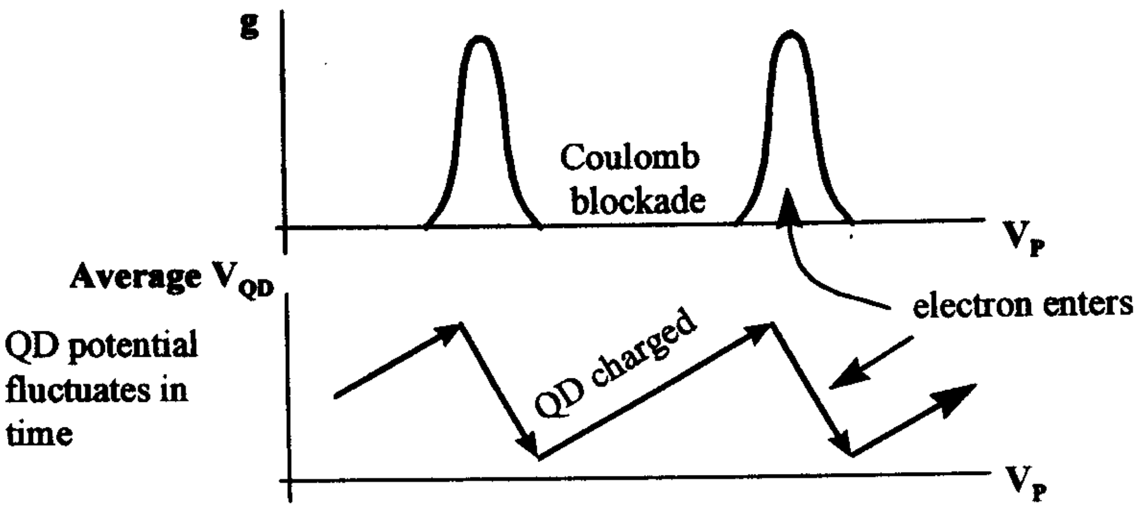
Can an Electron in the QD be Detected ?

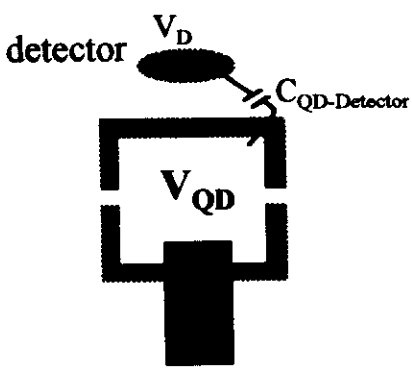
- A distant detector can sense an induced potential ;
- Charging a QD with a single electron changes its potential ;



Total Capacitance: C_T

Charging energy : e^2/C_T



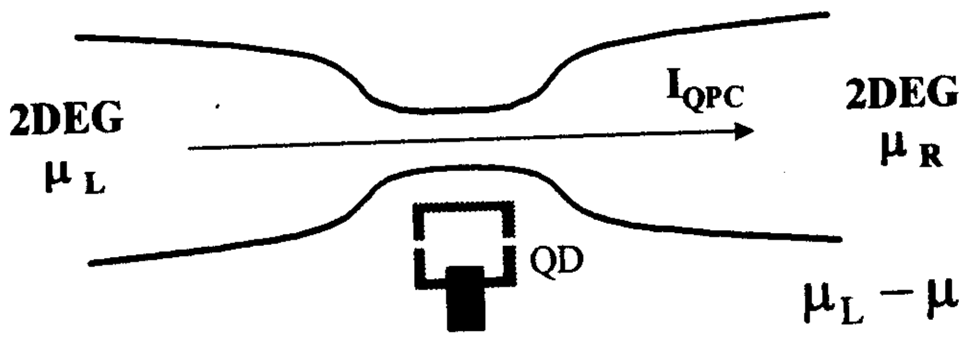


$$V_D = V_{QD} \frac{C_{QD-Detector}}{C_T}$$

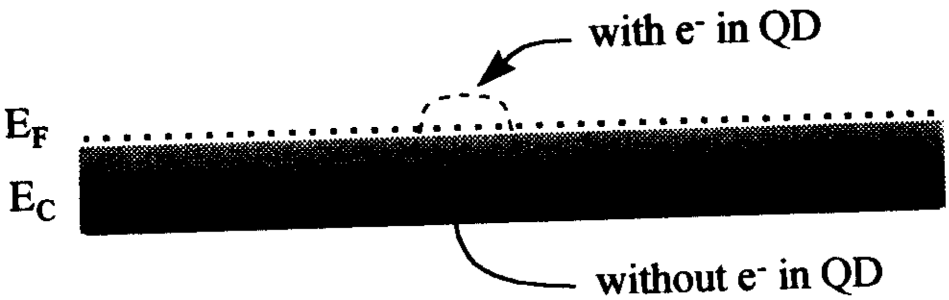
Field et al., PRL **70**, 1311 (1993).
Molenkamp et al., PRL **75**, 4282 (1995).

The Detector

A Quantum Point Contact (QPC) is used as a Detector



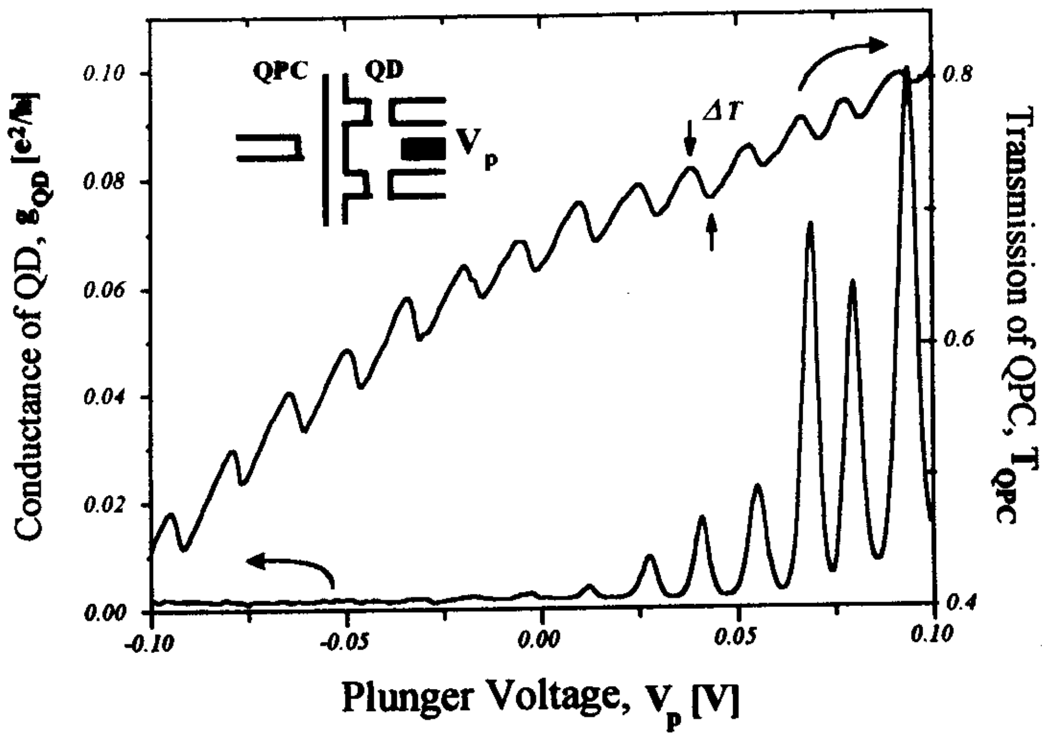
$$\mu_L - \mu_R = eV_{QPC}$$



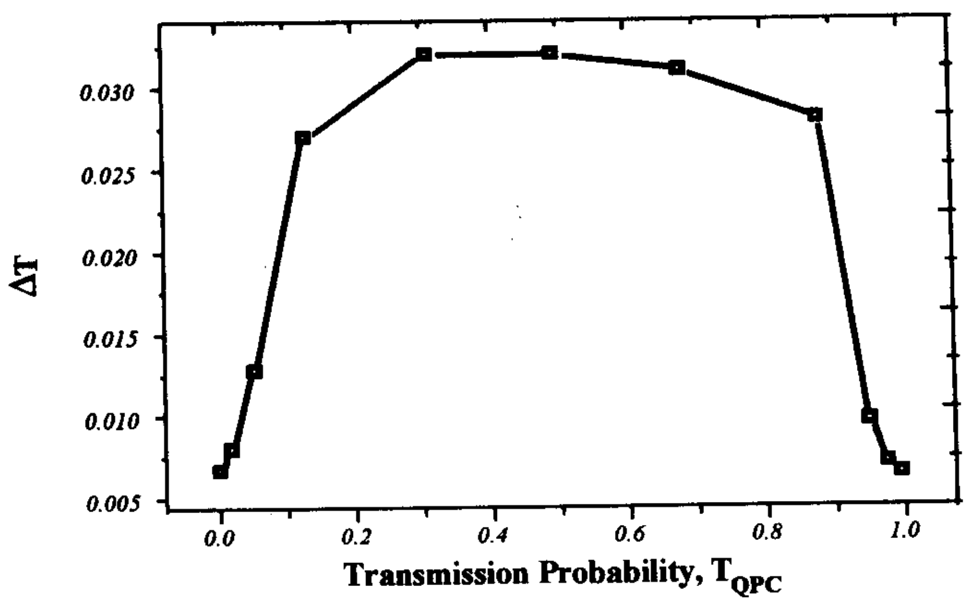
- An electron in the QD changes the potential in the QPC ;
-
- The potential change affects the transmission probability ;
-
- The conductance changes ;
-
- For a fixed current the voltage across the QPC changes,

or, for a fixed voltage the current though the QPC changes.

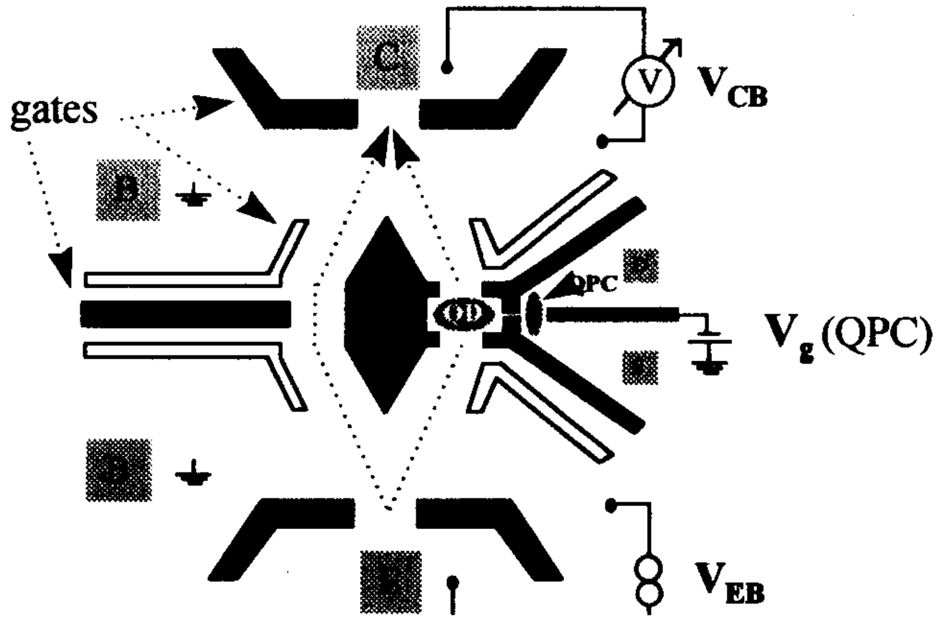
Effect of Electron in QD on QPC Detector

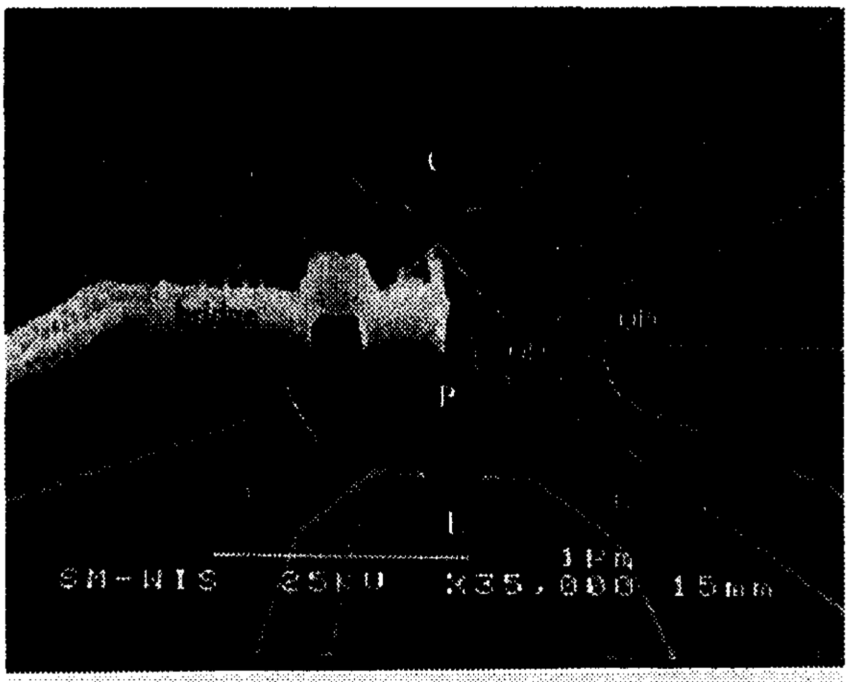


★ The QPC is sensitive to an electron in the QD even when Coulomb Peaks (of the QD) and weak !



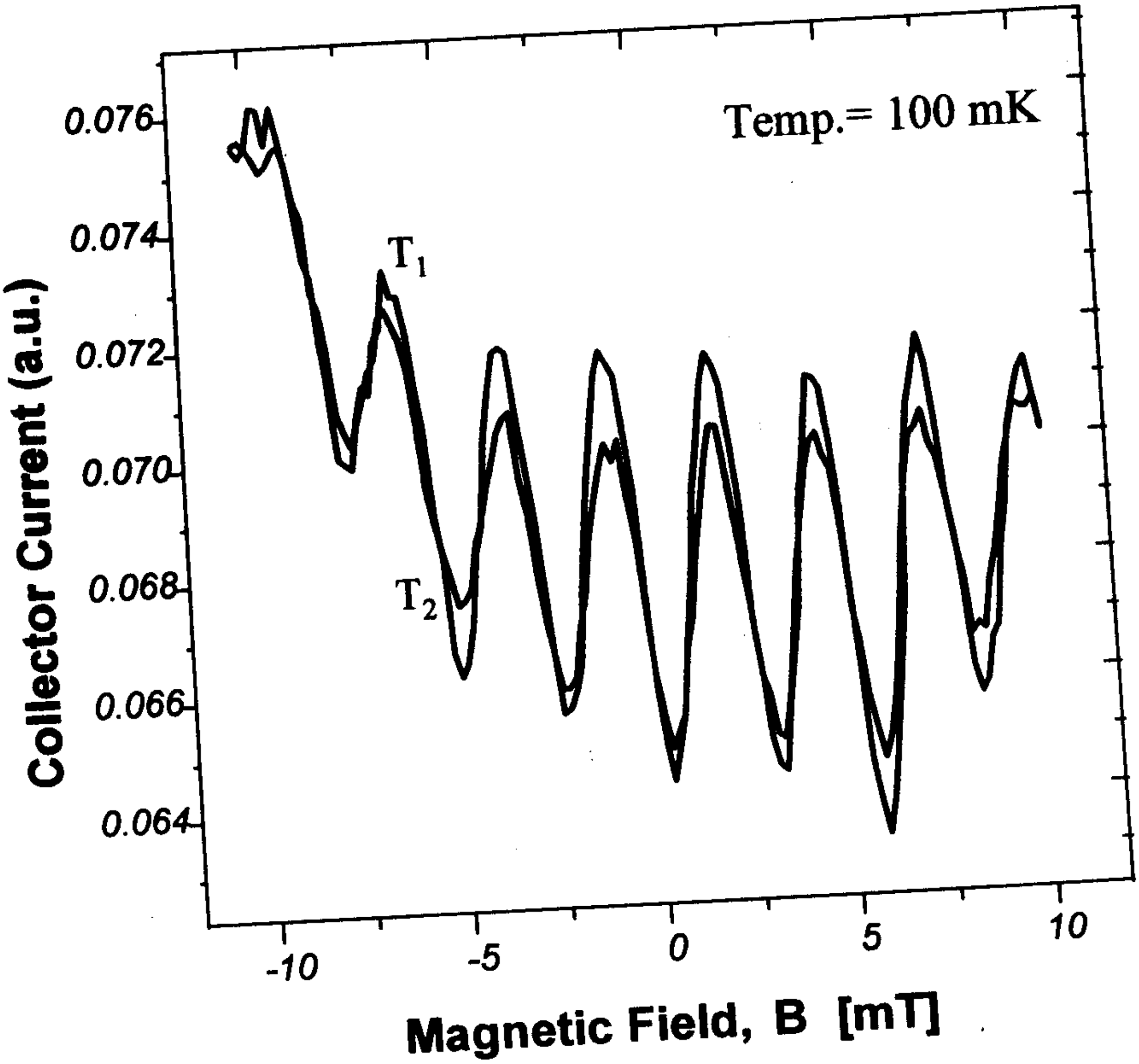
A Realization of the Experiment



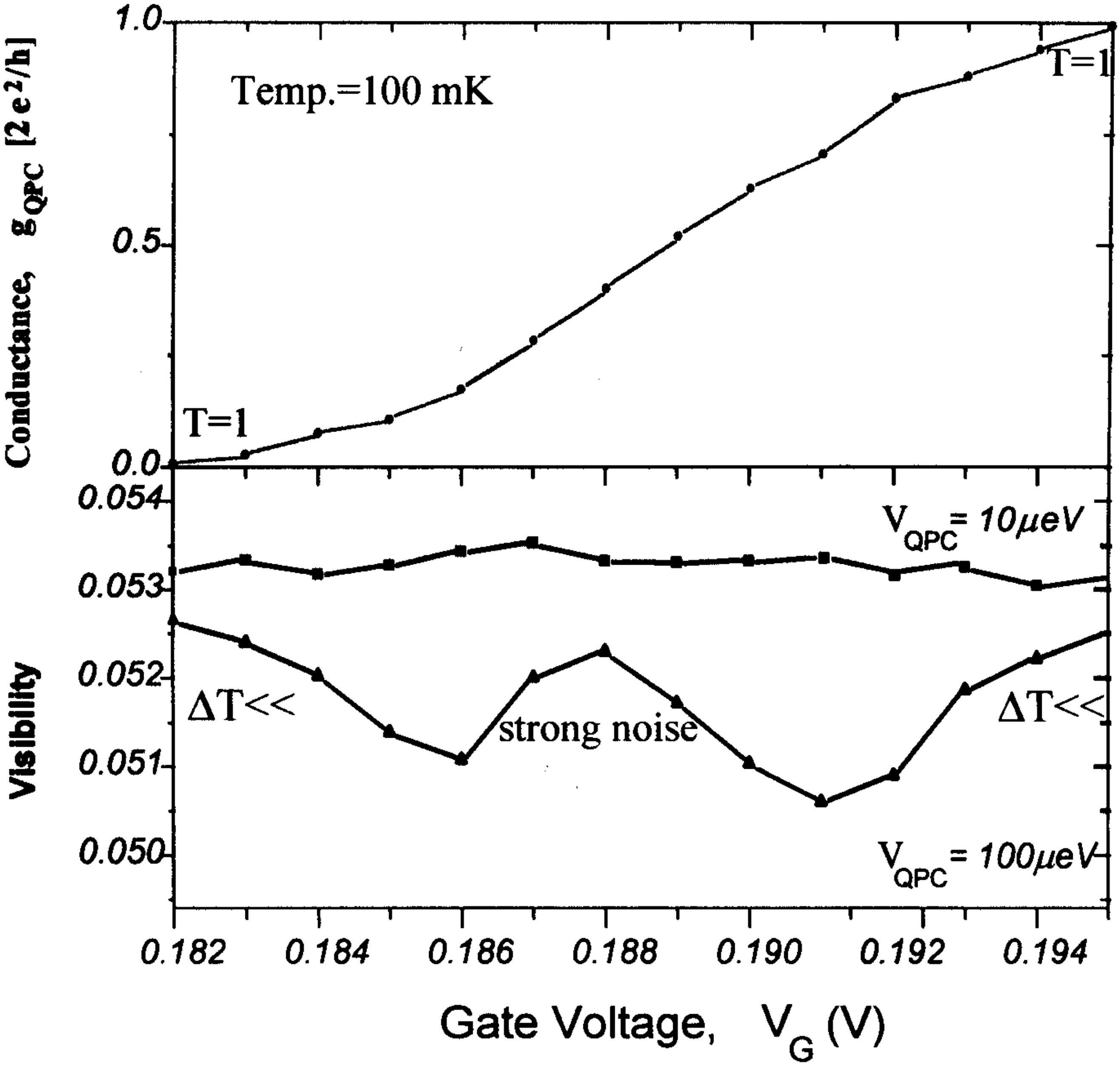


A Four Terminal configuration to prevent multiple paths !

As **T** of **QPC** changes, so does the **visibility** ...



Dephasing the QD with the QPC Detector



Simple Dephasing Arguments

✔ For an applied voltage V_{QPC} across the QPC :

$$I_{QPC} = V_{QPC}\frac{2e^2}{h}T$$

✔ Due to an electron in the QD :

$$\Delta I_{QPC} = V_{QPC}\frac{2e^2}{h}\Delta T$$

✔ During *dwell* time, τ_d, of an electron in the QD, **extra no. of electrons cross the QPC :**

$$\Delta n_{QPC} = \frac{\Delta Q}{e} = \frac{1}{e}\tau_d \cdot V_{QPC}\frac{2e^2}{h}\Delta T$$

✔ This number has to be larger than the Quantum Shot Noise **number** fluctuations.

The spectral density of current fluctuations:

$$S(0) = 2\frac{(\Delta Q)^2}{\tau} = \underbrace{\underbrace{2e}\cdot\underbrace{V_{QPC}\frac{2e^2}{h}T}}_{\text{classical shot noise}}\cdot\underbrace{\underbrace{(1-T)}}_{\text{suppression}}$$

$$\underbrace{2e \bullet I_{QPC}}_{\text{classical shot noise}} \bullet \underbrace{(1\text{-}T)}_{\text{suppression}}$$

No fluctuations at	T=1
Max. fluctuations at	T=0.5

Simple Dephasing Arguments :

Continue...

✔ The intrinsic number charge fluctuations is :

$$\Delta N = \frac{\Delta Q}{e} = \sqrt{\tau_d \frac{S(0)}{2e^2}}$$

$$\Delta N = \sqrt{\frac{1}{e}\tau_d V_{QPC} \frac{2e^2}{h} T(1-T)}$$

✔ For $\Delta n = \Delta N$ the detector dephases

$$\underbrace{\frac{1}{\tau_d}}_{\text{Measurement rate}} \approx \underbrace{\frac{2eV_{QPC}}{h} \frac{(\Delta T)^2}{T(1-T)} \equiv \frac{\Gamma_{\text{dephasing}}}{\hbar}}_{\text{Dephasing rate}}$$

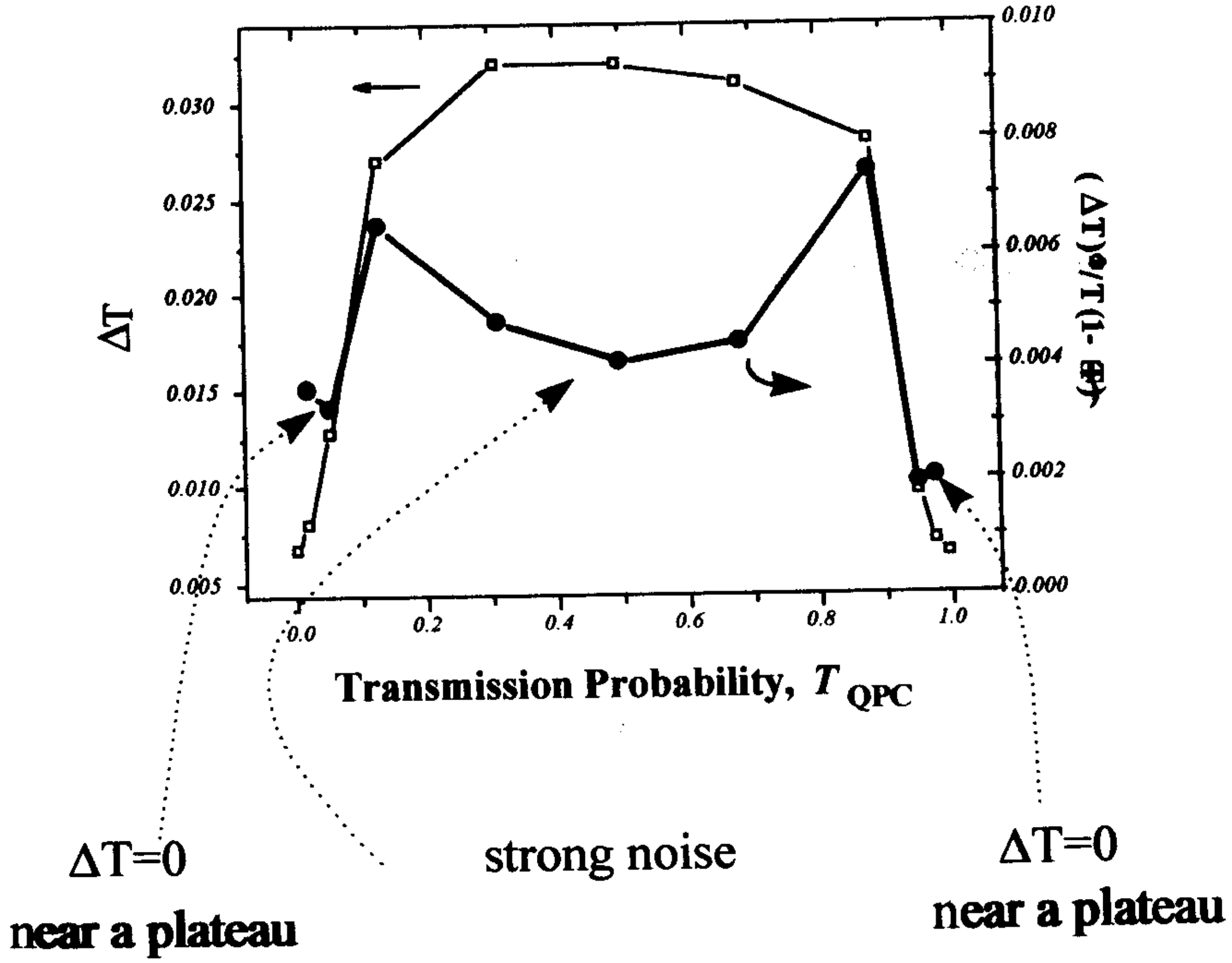
✔ The visibility of the Interference Oscillations, for a practical detector :

$$\nu = e^{-\frac{\tau_d}{\tau_{\text{dephasing}}}} \cong 1 - \frac{\Gamma_{\text{dephasing}}}{\Gamma_{QD}}$$

What is Expected ?

More exact calculations lead to :

$$\nu = 1 - \frac{1}{8\pi}\frac{eV_{QPC}}{\Gamma_{QD}}\frac{(\Delta T)^2}{T(1-T)}$$



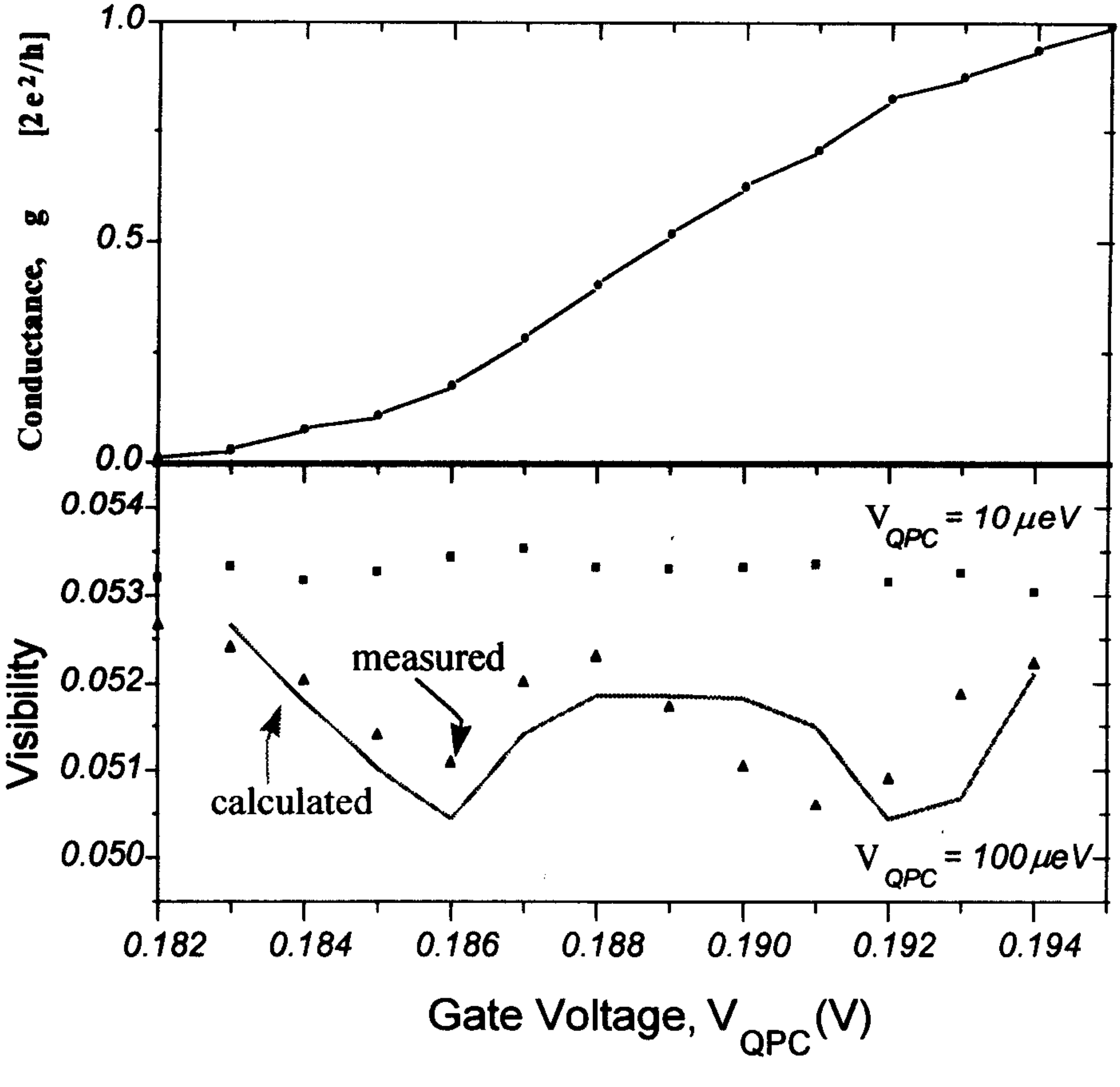
Recent calculations are :

-- I. L. Aleiner, N. S. Wingreen, and Y. Meir, cond-mat / 9702001

-- Y. Levinson, cond-mat / 9702164

-- S. Gurvich, quant-ph/960729 and private communications.

Comparison with Theory



Γ_{QD}=0.5 μeV ($\tau_d \cong$1nS) was used !

What Have We done thus Far ?

✔ A quantum point contact used as a *which path* detector ;

✔ The *which path* detector led to dephasing ;

✔ Reasonable agreement with theory ;

✔ A method to dephase electrons in a QD .

(Bare QD was found before to resist dephasing …)