



UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL ATOMIC ENERGY AGENCY
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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SMR.998d - 4

Research Workshop on Condensed Matter Physics
30 June - 22 August 1997
**MINIWORKSHOP ON
QUANTUM WELLS, DOTS, WIRES
AND SELF-ORGANIZING NANOSTRUCTURES
11 - 22 AUGUST 1997**

"Non-Universal Conductance Quantization in a Quantum Wire"

PART II

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U.S.A.**

These are preliminary lecture notes, intended only for distribution to participants.

MAIN BUILDING STRADA COSTIERA, 11 TEL. 2240111 TELEFAX 224163 TELEX 460392 ADRIATICO GUEST HOUSE VIA GRIGNANO, 9 TEL. 224241 TELEFAX 224531 TELEX 460449
MICROPROCESSOR LAB. VIA BEIRUT, 31 TEL. 2249911 TELEFAX 224600 TELEX 460392 GALILEO GUEST HOUSE VIA BEIRUT, 7 TEL. 2240311 TELEFAX 2240310 TELEX 460392
ENRICO FERMI BUILDING VIA BEIRUT, 6 (TELEPHONE, FAX AND TELEX THROUGH MAIN BUILDING)

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Non-Universal Conductance Quantization in a Quantum Wire

A. Yacoby, H. L. Stormer, K. W. Baldwin,
L. N. Pfeiffer, and K. W. West,
Bell Labs, Lucent Technologies.
N. Wingreen, NEC Research Labs.

Conductance quantization in point contacts
complicates identification of true 1D behavior.



To distinguish true 1D behavior from that
of a Point Contact:

- Long mean free path.

- Large subband separation.

(conventionally obtain 1 meV)

Cleaved Edge Overgrowth

- Mean free path of $10\ \mu m$

- Subband separation in excess of 20 meV

- Wavefunctions of the 1D modes

Observe strong deviations from conventional
conductance quantization.

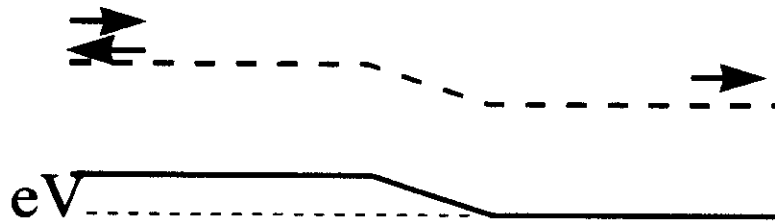
- Effects of e - e interaction. Luttinger liquid

Outline

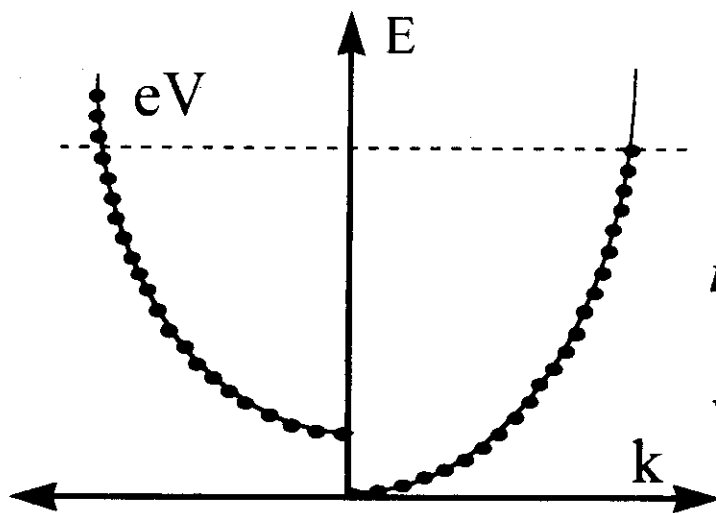
- Non interacting vs Interacting electrons
- Cleaved Edge Overgrowth
 wire characteristics
- Transport properties
 T - dependence
 I - V
- Several theoretical scenarios.

Conductance Quantization

non interacting electrons



Landauer formula,
(1957)

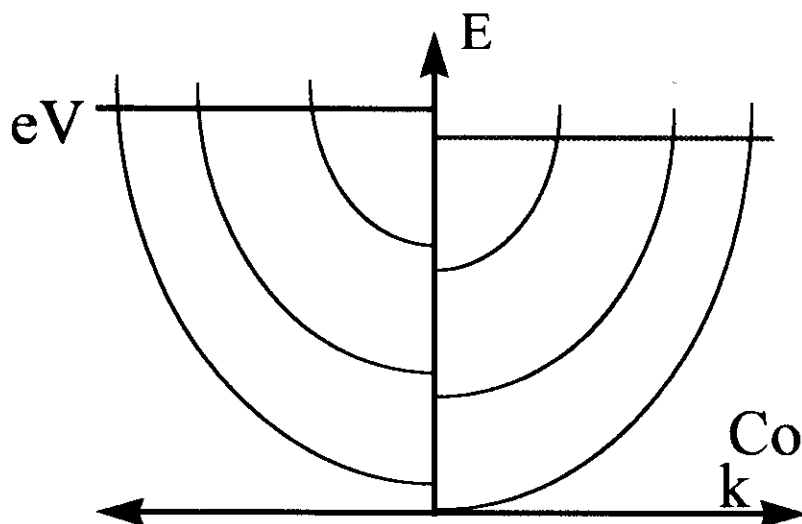


$$I = e n v$$

$$n = \frac{\Delta n}{\Delta E} eV \propto \frac{1}{\sqrt{E}} eV$$

$$v \propto \sqrt{E}$$

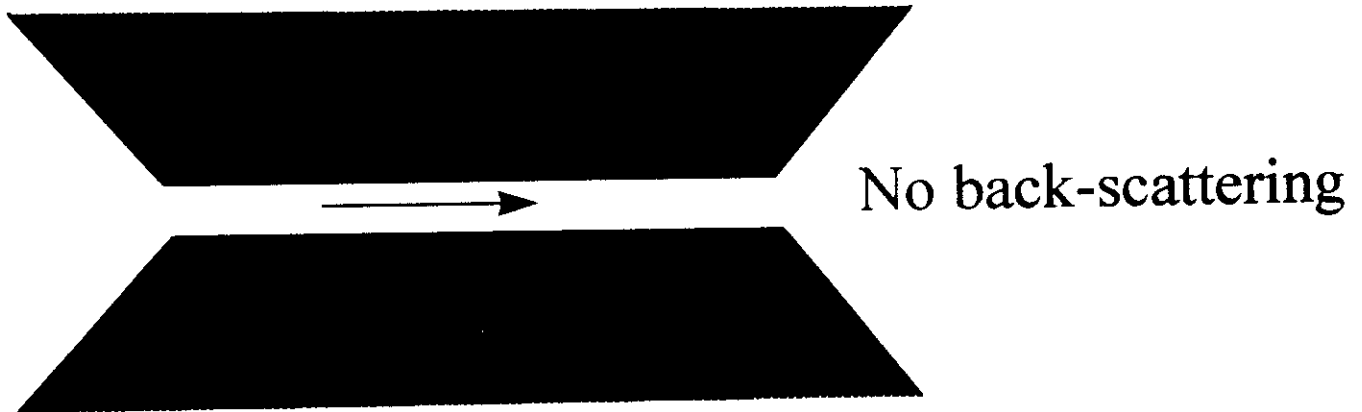
$$I = e \frac{\Delta n}{\Delta E} eV \cdot v = \frac{e^2}{h} V$$



$B=0$ ↙ spin

$$G = N \frac{2e^2}{h}$$

Length Dependence

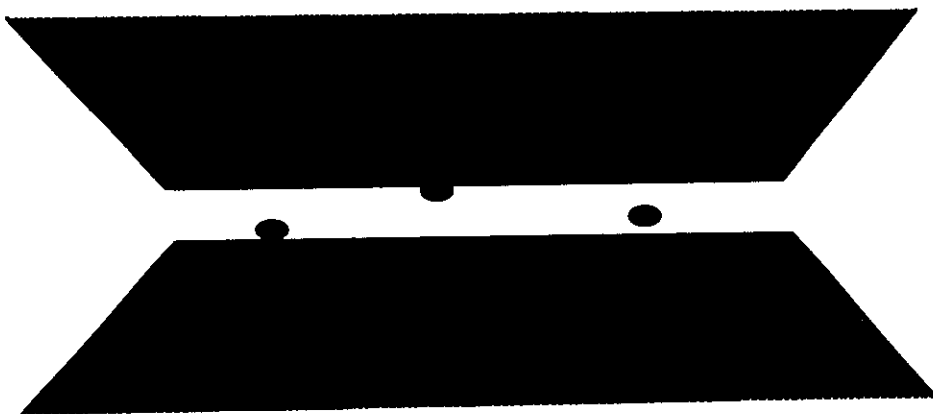


$$G = N \frac{2e^2}{h}$$

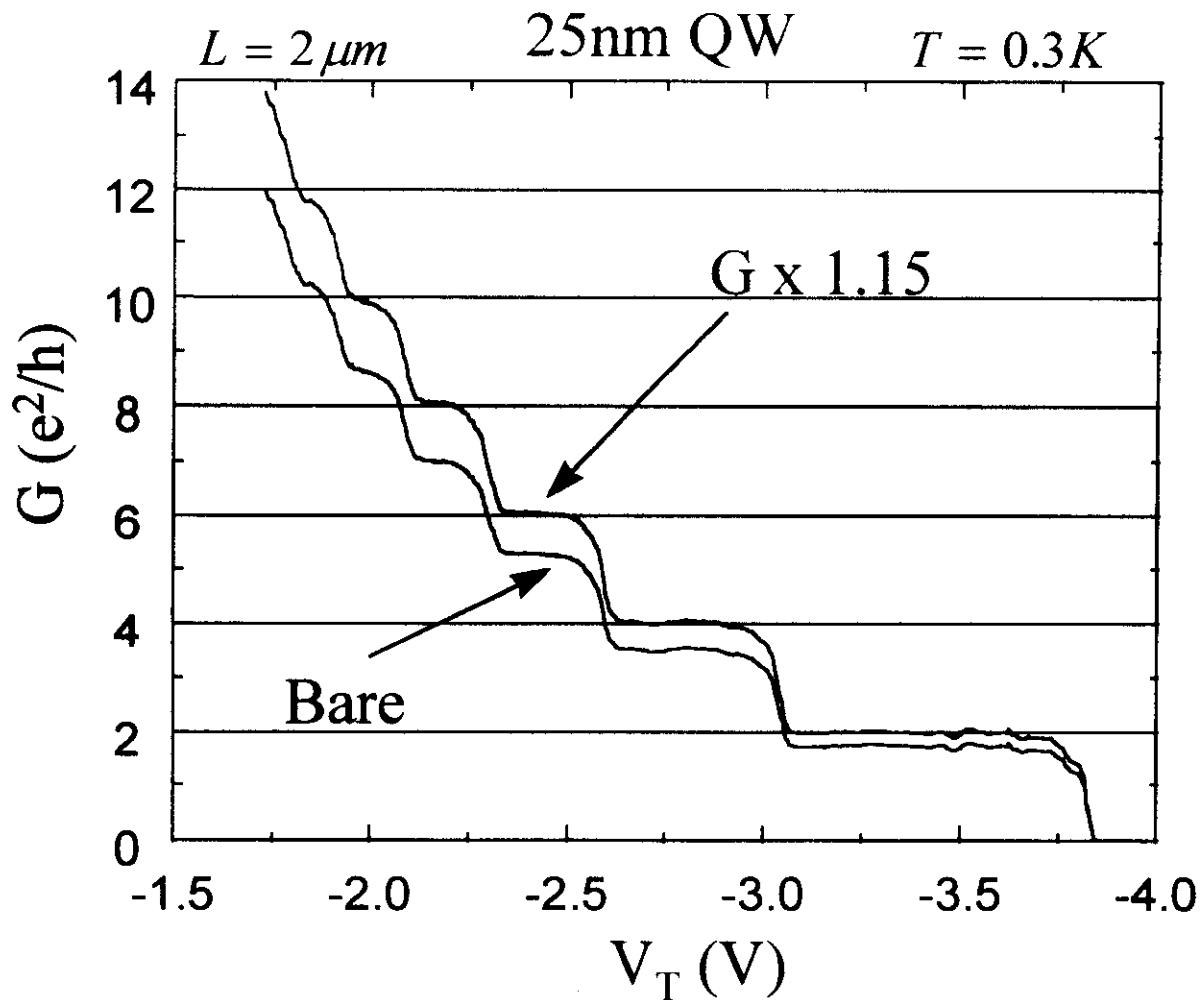
Independent of Length

Contact resistance (Imry, 1986)

- Observe conductance quantization in point contacts (only a few wavelengths long)
- Extremely robust. Accidental point contacts dominates transport.



Non Universal Conductance Quantization

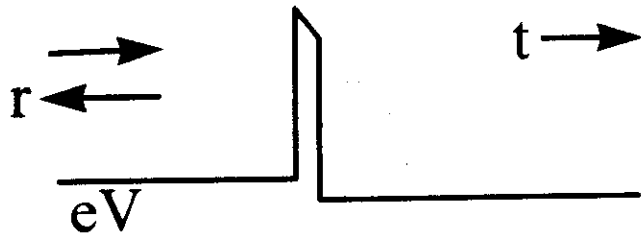


Conductance quantized in units of $0.85\ 2e^2/h$

Conductance values reproduce on all wires fabricated from the same material

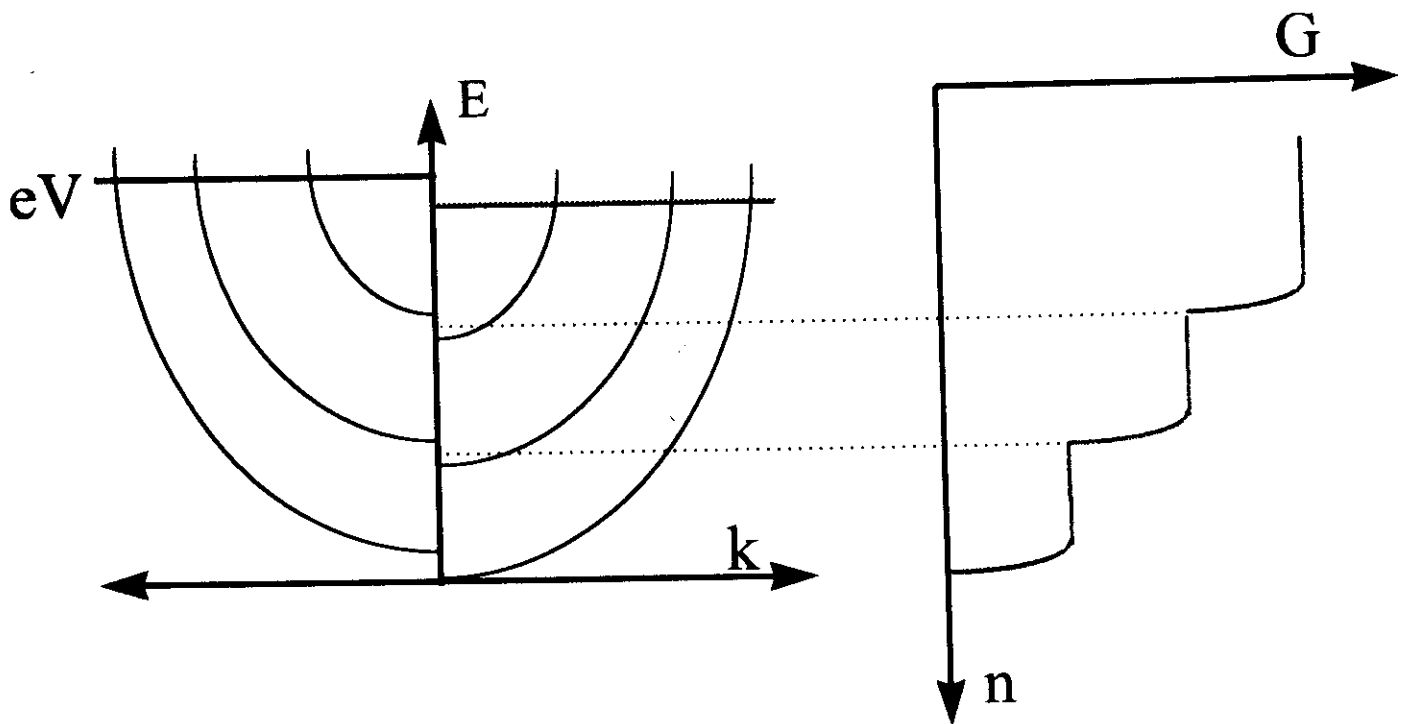
Elastic Scattering

Landauer formula



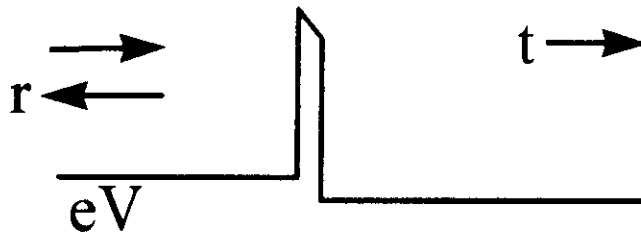
$$G = |t|^2 \frac{2e^2}{h}$$

Plateau $\Rightarrow$ Transmission probability, $|t(E)|^2$, is independent of density.



Temperature Dependence

Landauer formula



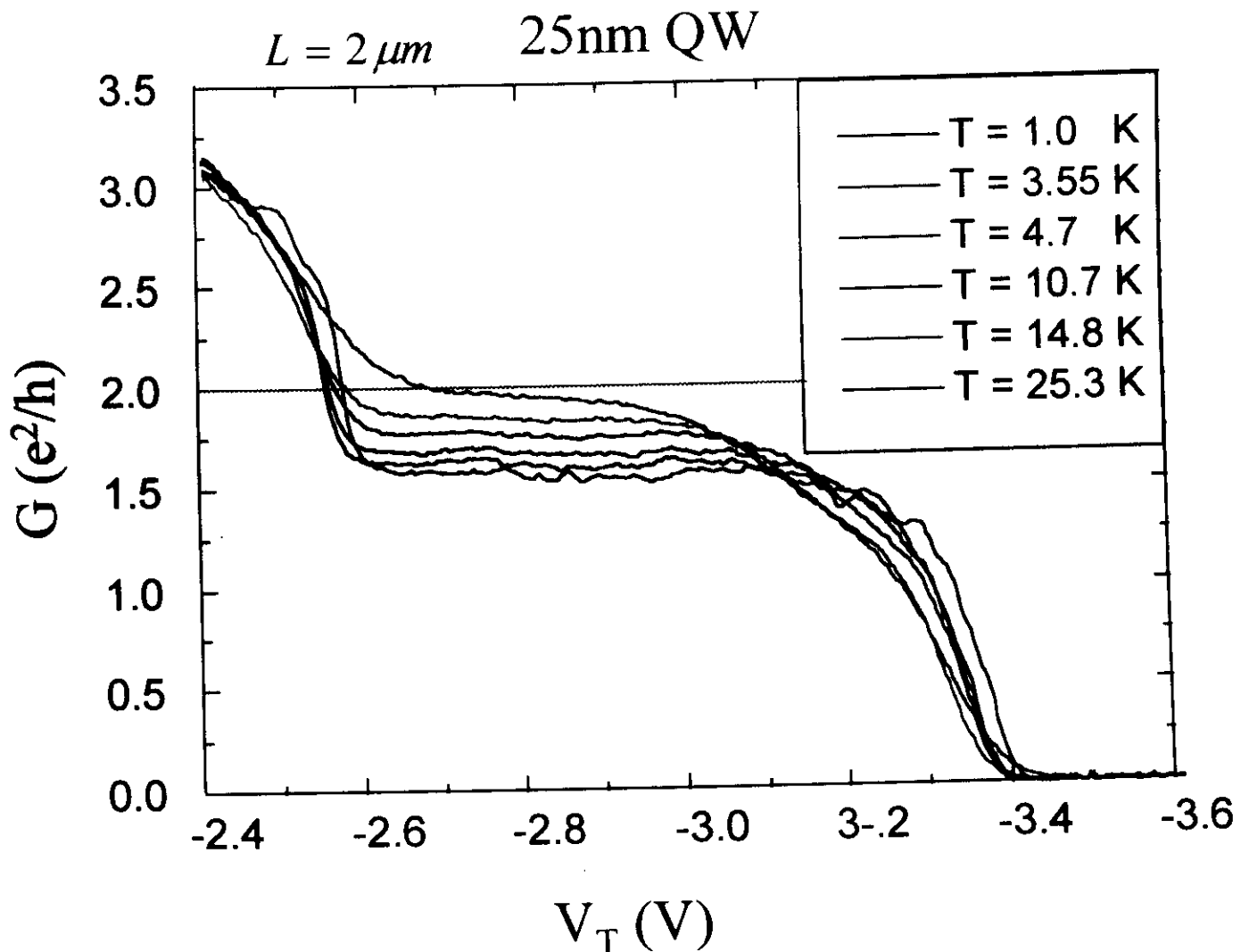
$$G = |t|^2 \frac{2e^2}{h}$$

Finite temperature

$$I = \int e \frac{\Delta n}{\Delta E} \cdot v [f(E + eV) - f(E)] \cdot |t|^2 = |t|^2 \frac{2e^2}{h} V$$

Conductance independent of temperature

Temperature Dependence Last Plateau



Plateau rises rigidly



No observed dependence on wire density

A similar behavior but weaker was observed by Tarucha et al. (SSC. 94, 413 (95))
See also K. J. Thomas et al. (PRL 77,135 (96))

Interacting Electrons

Tomonaga - Luttinger Liquid

Interactions renormalize the velocity and the density of states.

$$G = g \frac{e^2}{h} \quad ; \quad g \approx \left(1 + \frac{U}{2E_F} \right)^{-\frac{1}{2}} < 1$$

W. Apel and T. M. Rice

C. L. Kane and M. P. A. Fisher

repulsive interactions

For $E_F = 20 \text{ meV}$

$$U = \frac{e^2 n}{4\pi\epsilon} = 15 \text{ meV} \quad \longrightarrow \quad g = 0.87$$

Predicted for an infinite wire.

Recently

A. Kawabata

Y. Oreg and A. M. Finkel'stein

predicted $g=1$ also for an infinite wire.

Renormalization of the electric field

Fermi Liquid Contacts

finite length wire

D. L. Maslov and M. Stone
I. Safi and H. J. Schulz
V. V. Ponomarenko

K. A. Matveev and L. I. Glazman

$$g = \frac{v_F}{v_p}$$

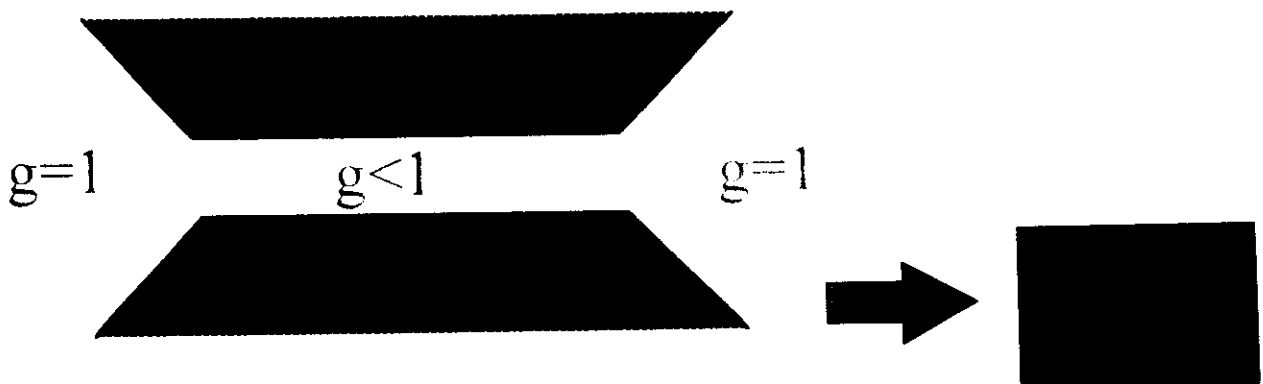
← Fermi velocity
← Plasmon velocity

Finite frequency $\lambda_p = \frac{v_p}{\omega}$

K. A. Matveev and L. I. Glazman

$$G = g \frac{e^2}{h} = \begin{cases} g = \frac{v_F}{v_p} < 1 & \text{for } \lambda_p < L \\ g = 1 & \text{for } \lambda_p > L \end{cases}$$

DC measurement



- Clean finite wire cannot explain the reduced conductivity.

Main Results

Observations

- Non universal conductance quantization.
- Strong temperature dependence.

Theoretical ingredients

- Non interacting electrons with or without disorder.
- Interacting electrons in a clean finite wire.
-
-

Do not have a complete understanding of the experimental results.

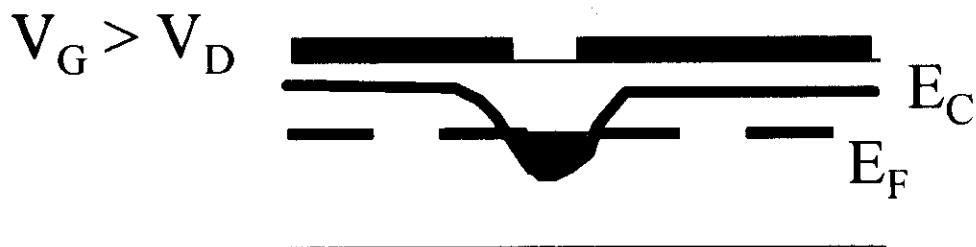
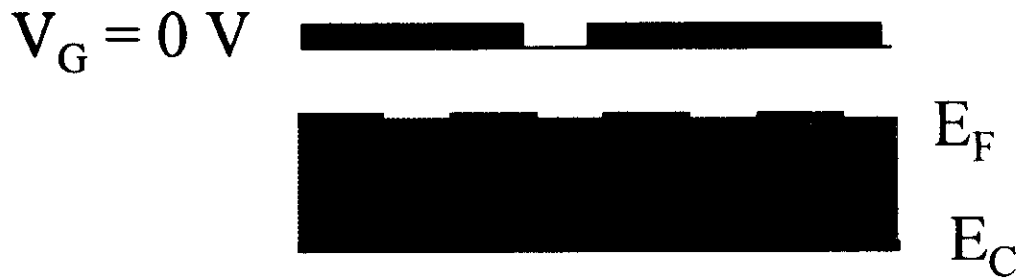
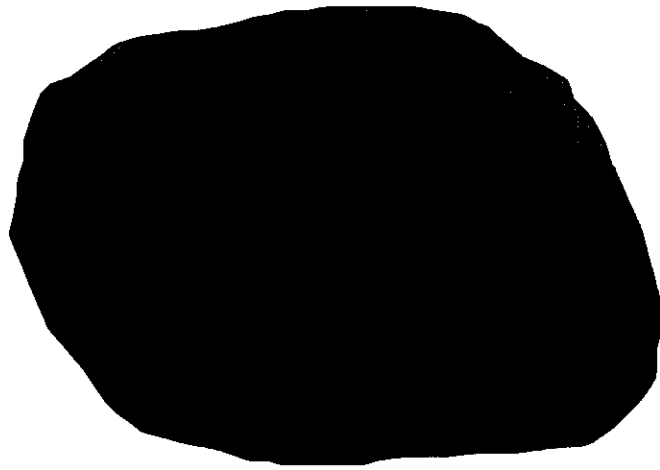
Why GaAs ?

- Organic conductors exhibit 1D behavior
TTF-TCNQ
 - Naturally formed.
 - Many chains in parallel.
 - Disorder.

Advantages of GaAs

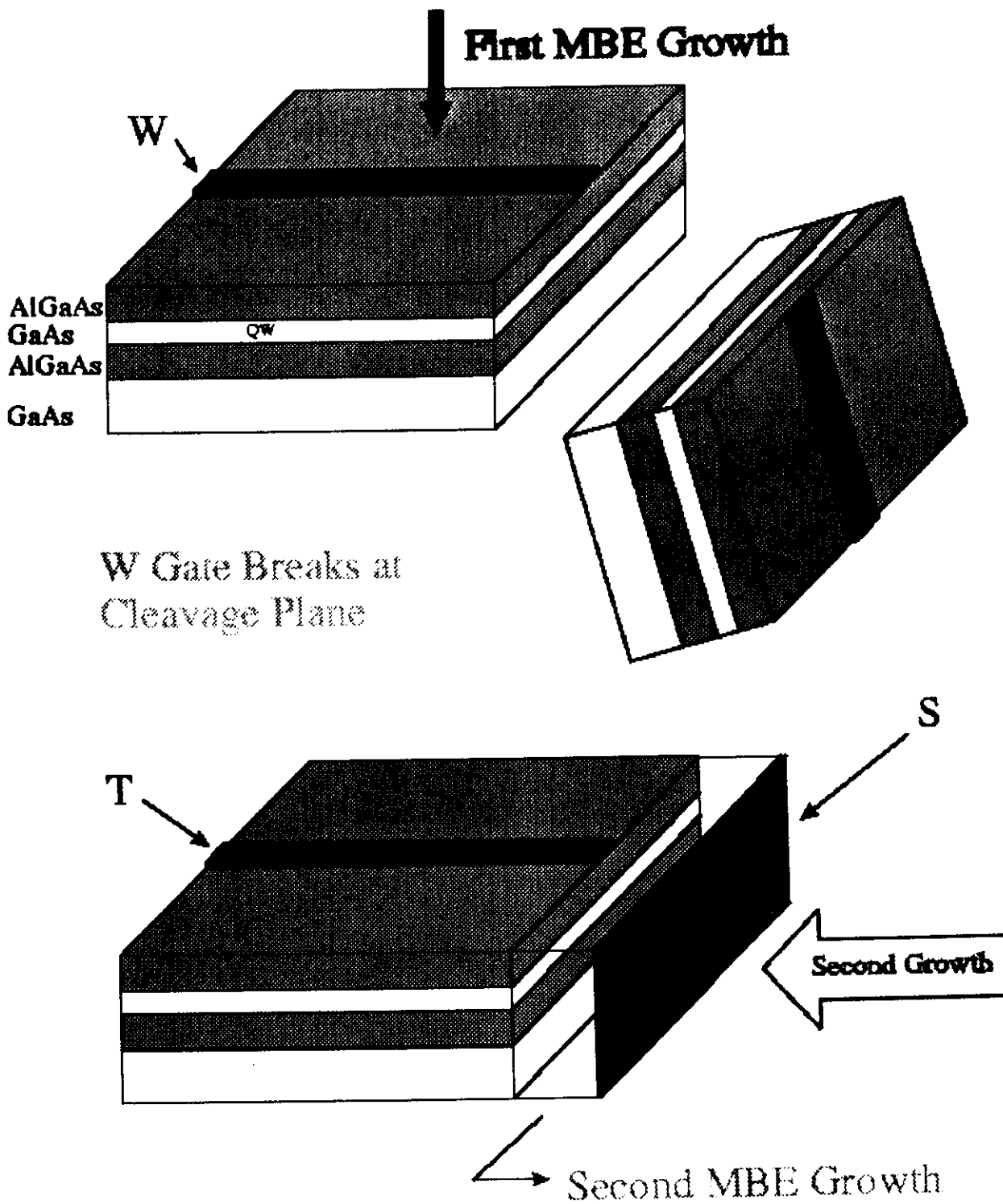
- Can work on a single wire.
- Control over parameters:
 - Disorder
 - Density Low density - Strong e - e
 - Length
- Many theoretical predictions remain untested due to the lack of a suitable 1D system.
 - Chiral Luttinger liquid in the FQHE.
 - 1D formed from a 2DEG.

Lateral Gate Depletion

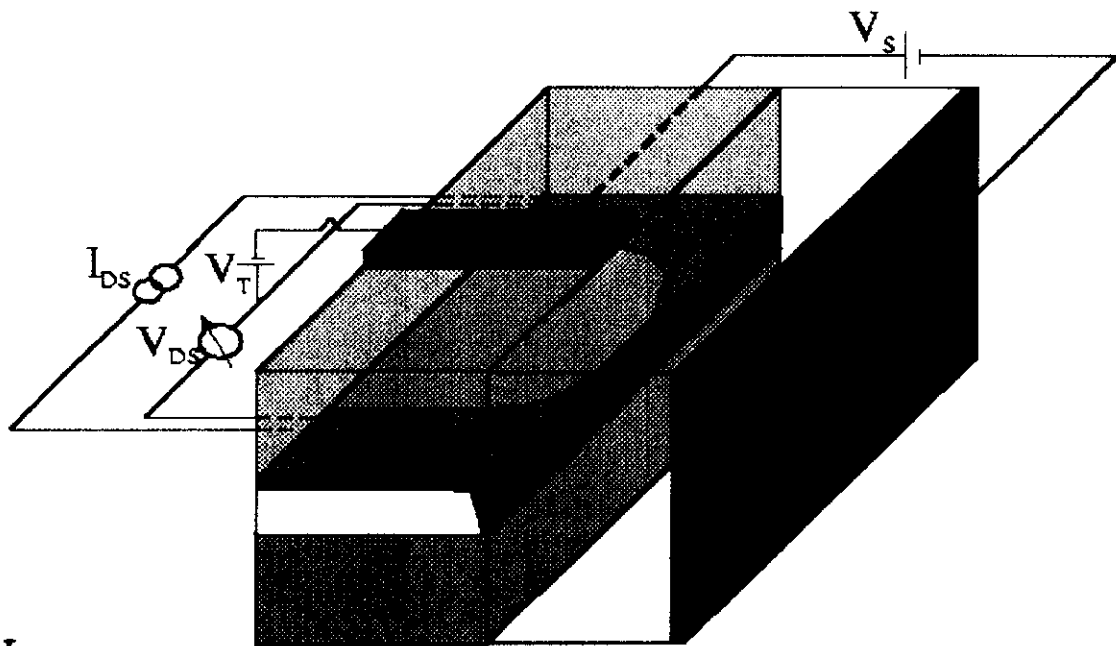


- Subband separation of 1 meV
- Difficult to avoid accidental Point Contacts

Cleaved Edge Overgrowth

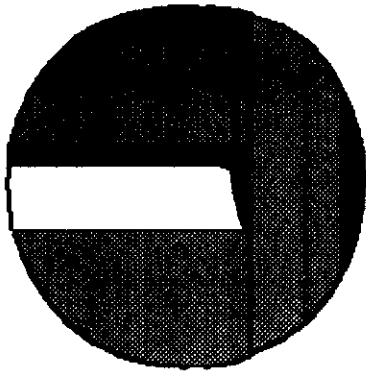


Blow Up of Wire Region

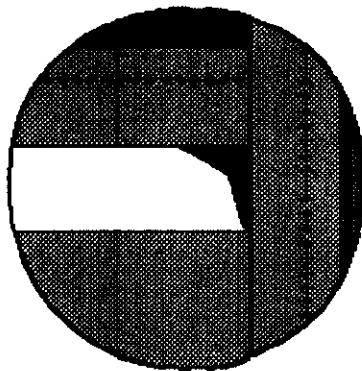


Length of wire determined
by width of gate

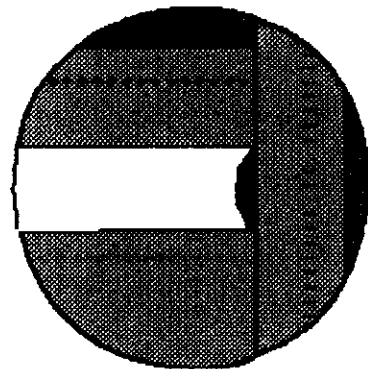
$$V_T < V_D$$



$$V_T = V_D$$

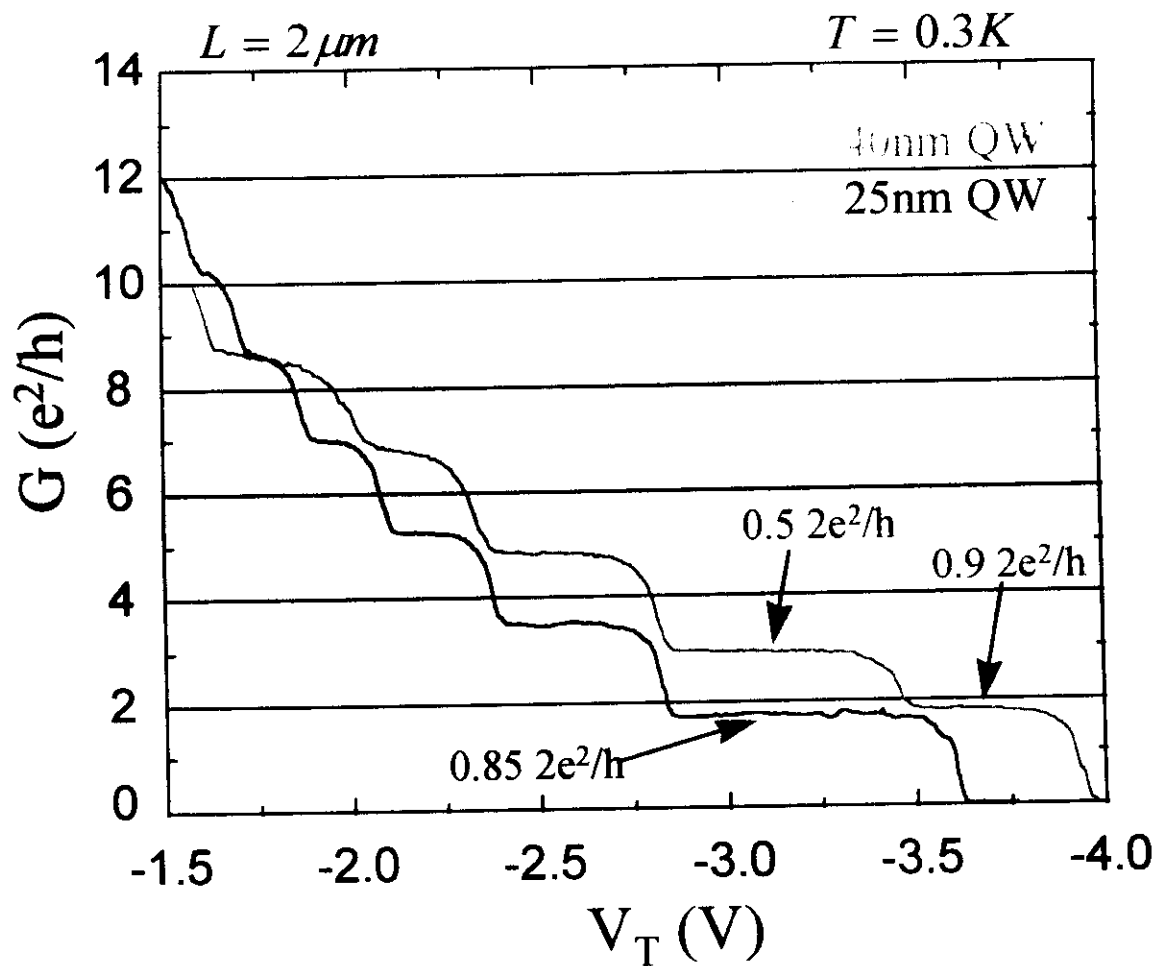


$$V_T > V_D$$



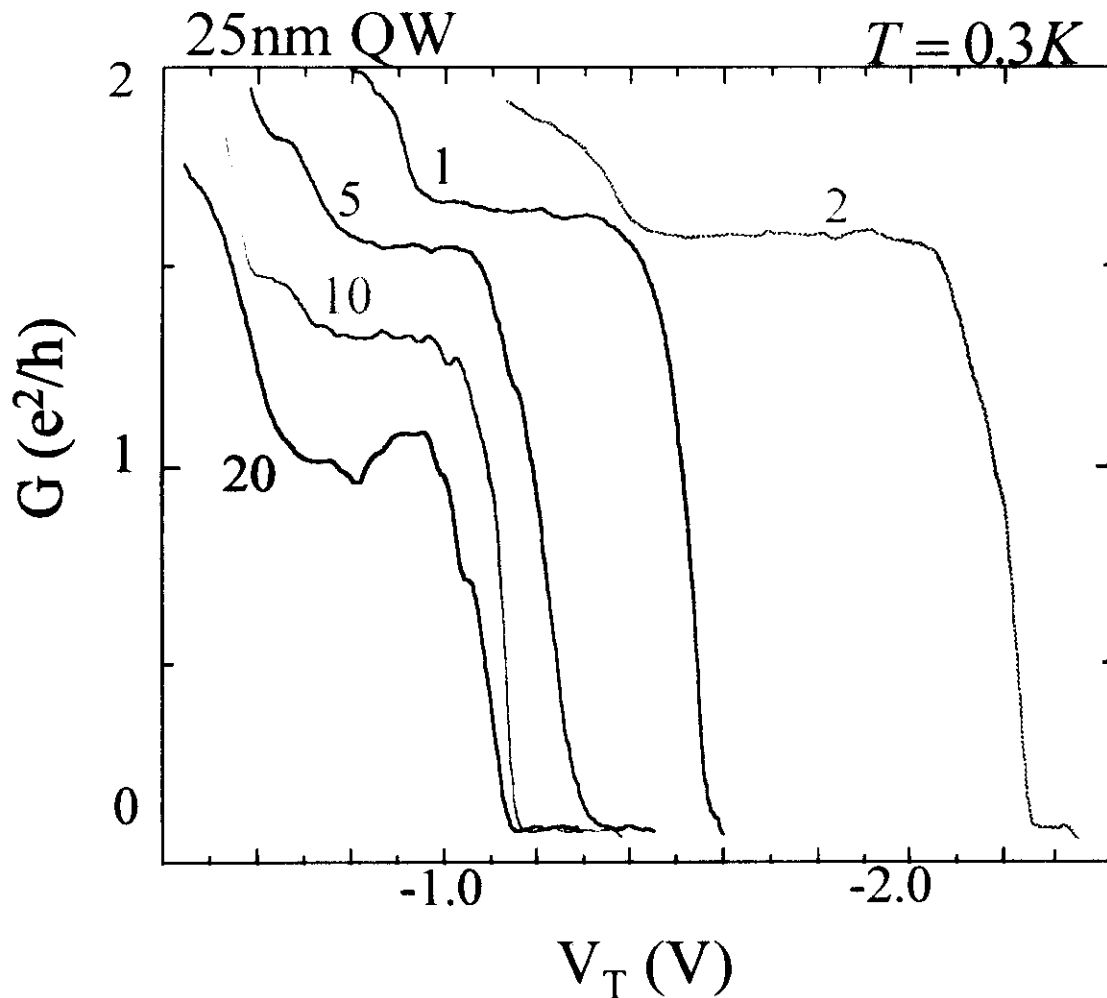
Electrons confined by three atomically
smooth planes and by a strong electric
field in the fourth direction

Conductance Quantization



- Steps are quantized at non universal values.

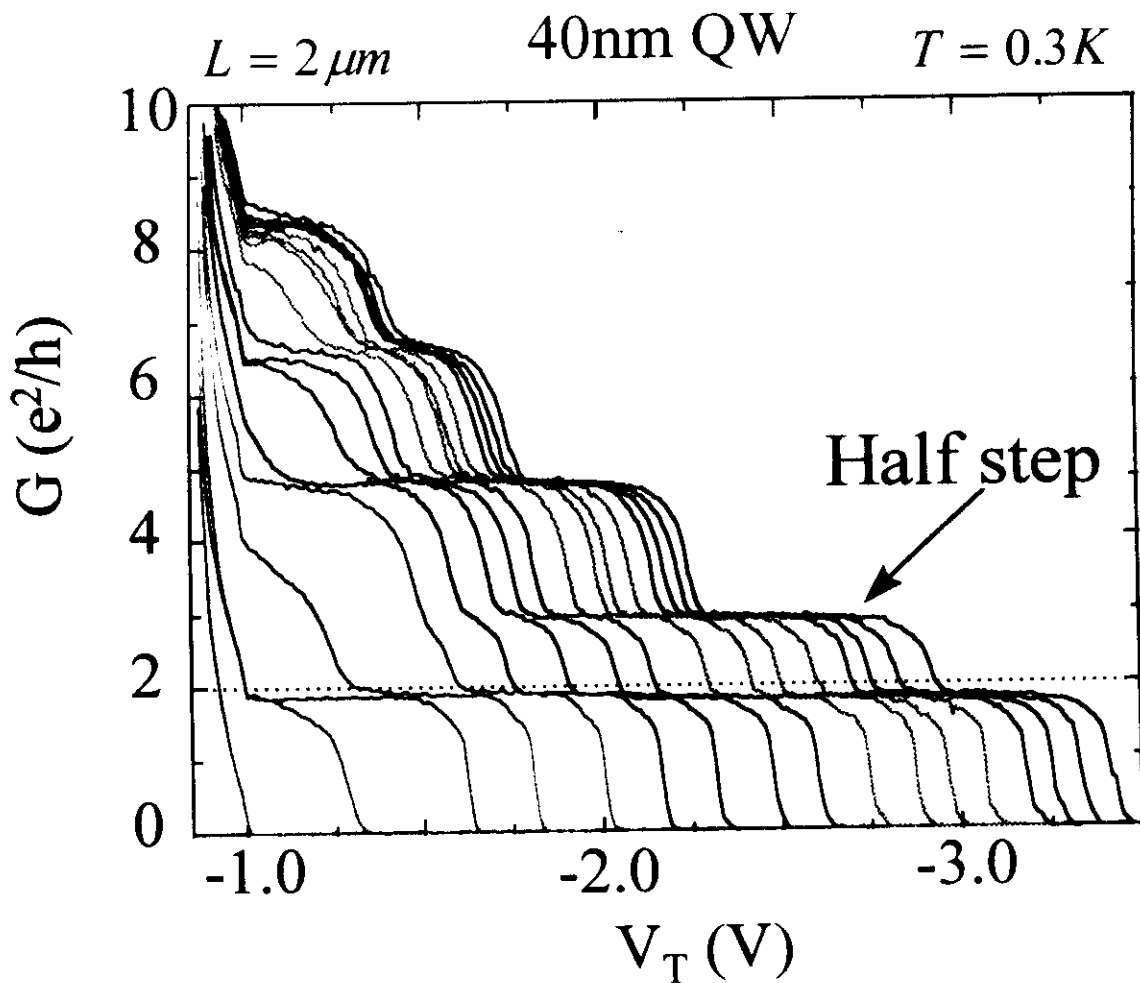
Wire Mean-Free-Path



- Plateaus remain flat for wires up to 10 μm long resulting in a Mean Free Path of 10 μm

Effect of Side Gate

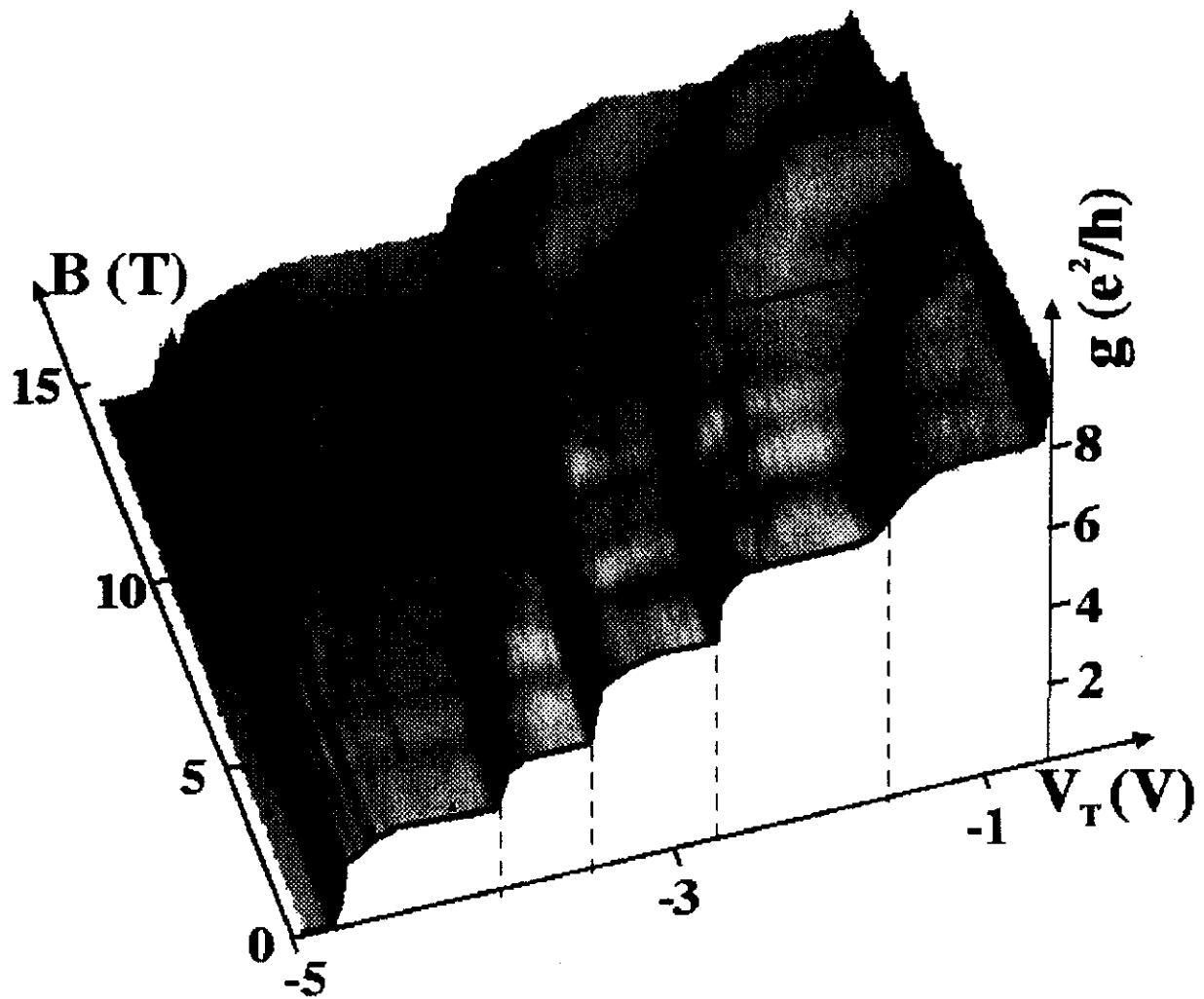
Changing density



- The second mode becomes degenerate with the third mode.
- Conductance values are weakly dependent upon density.

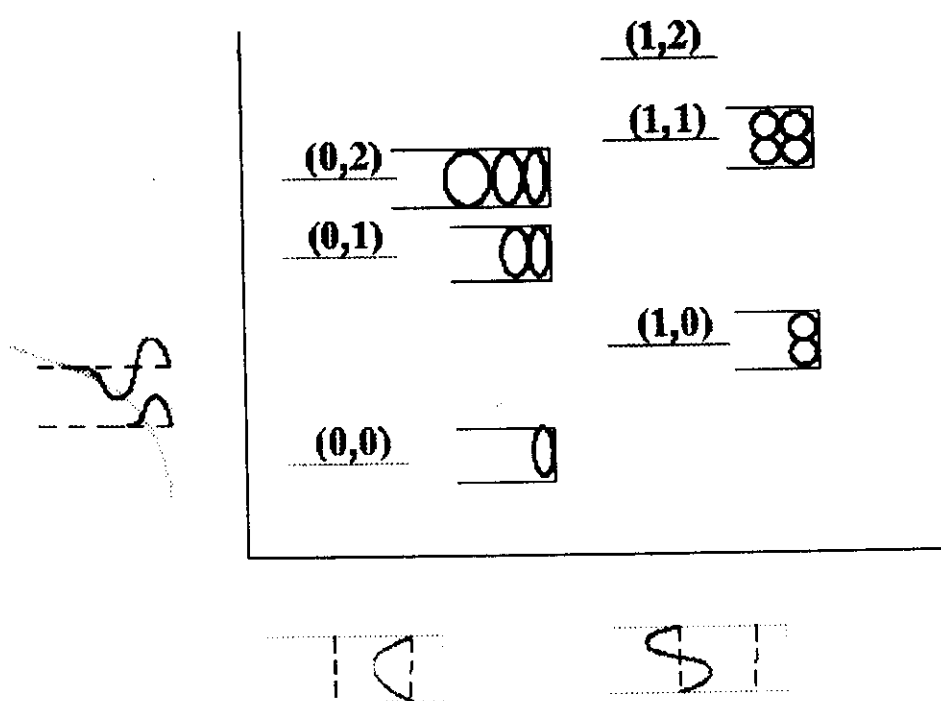
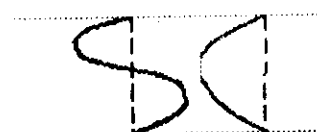
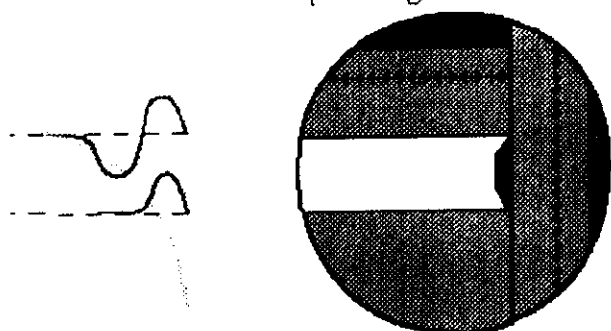
Magnetic Field Dependence

Field directed perpendicular to wire

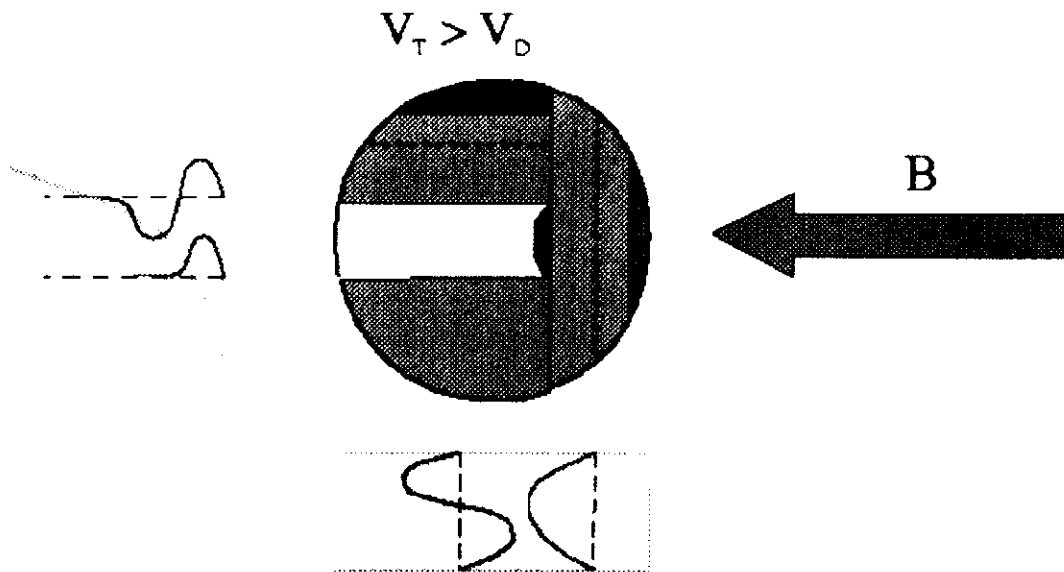


Energies and Wavefunctions

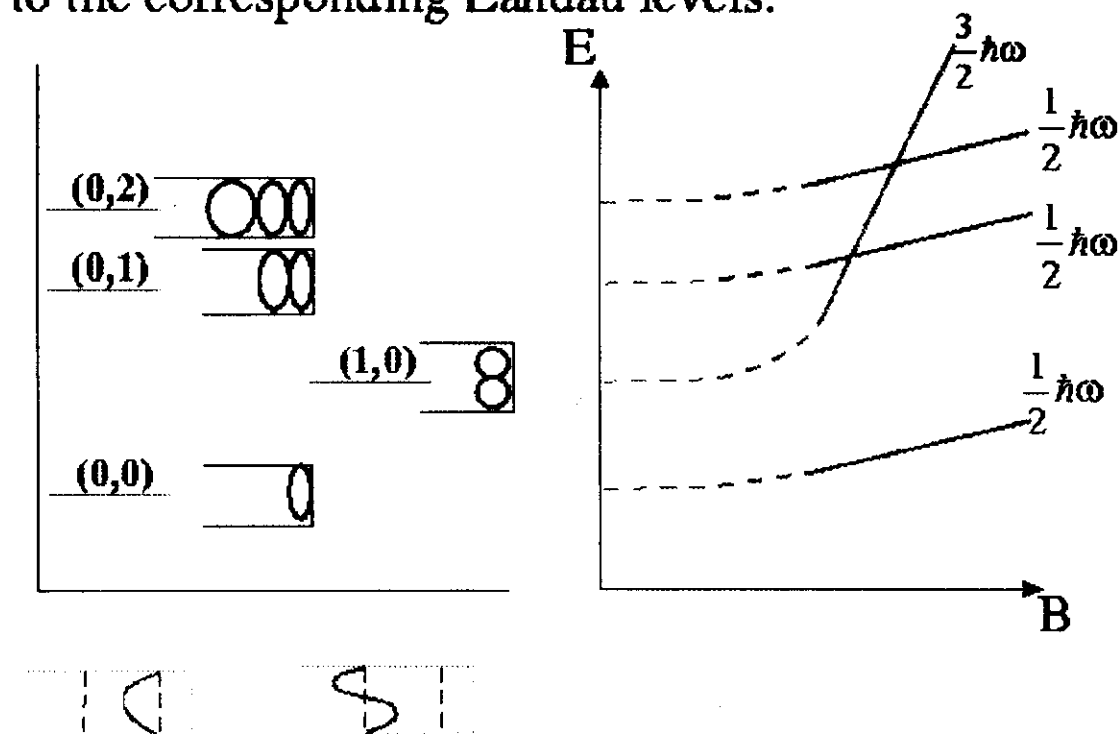
$$V_T > V_D$$



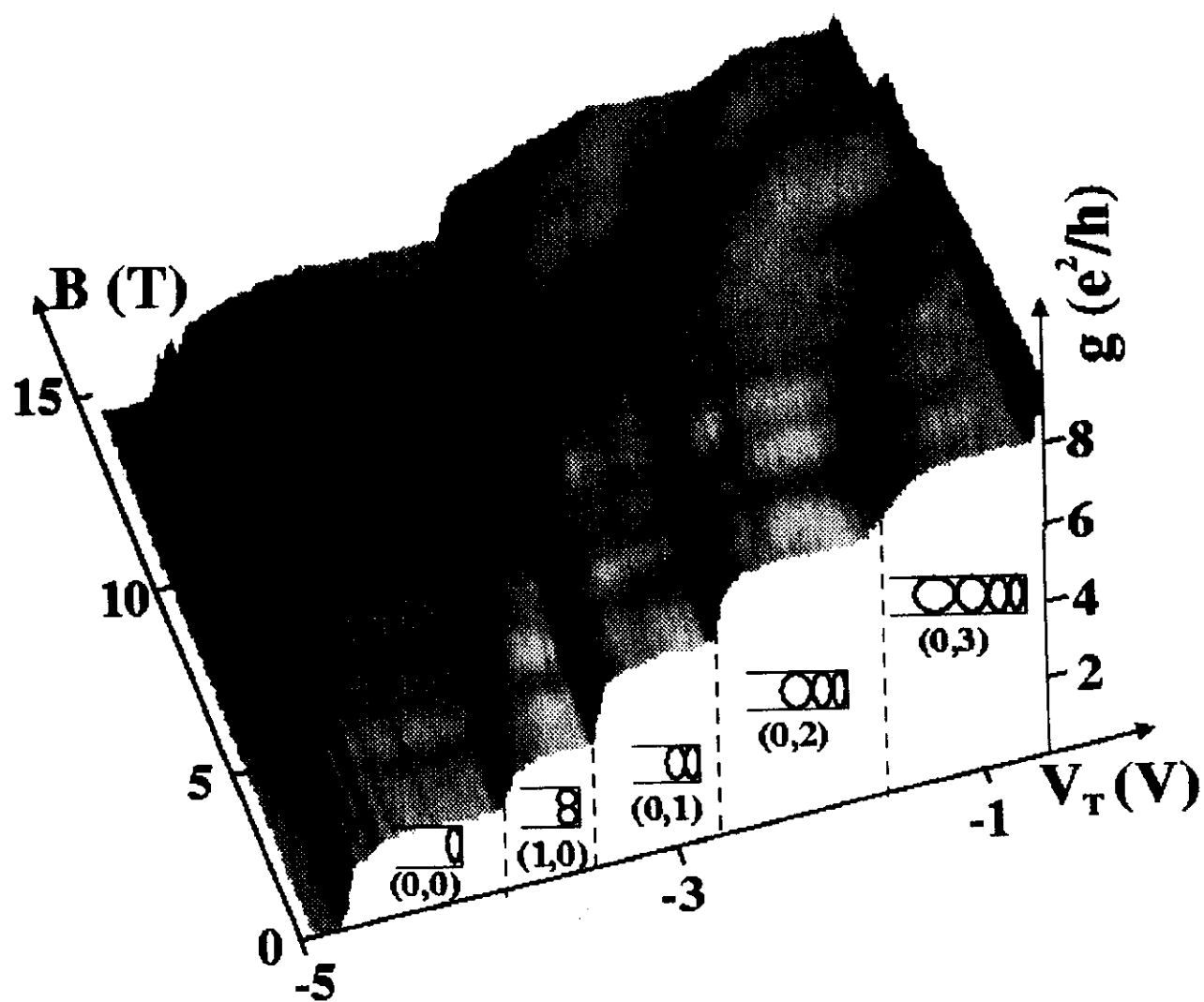
Model for B - Dependence



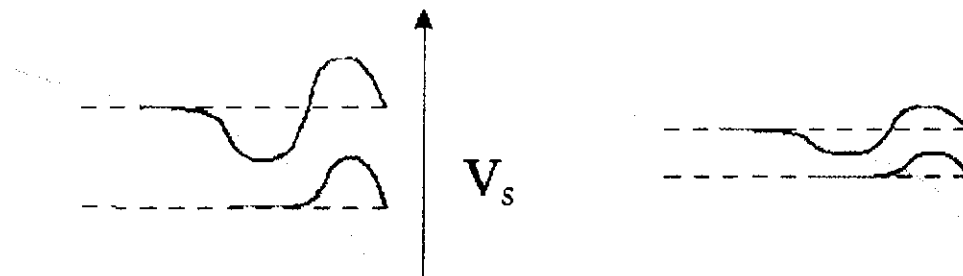
At very high magnetic fields the QW states evolve to the corresponding Landau levels.



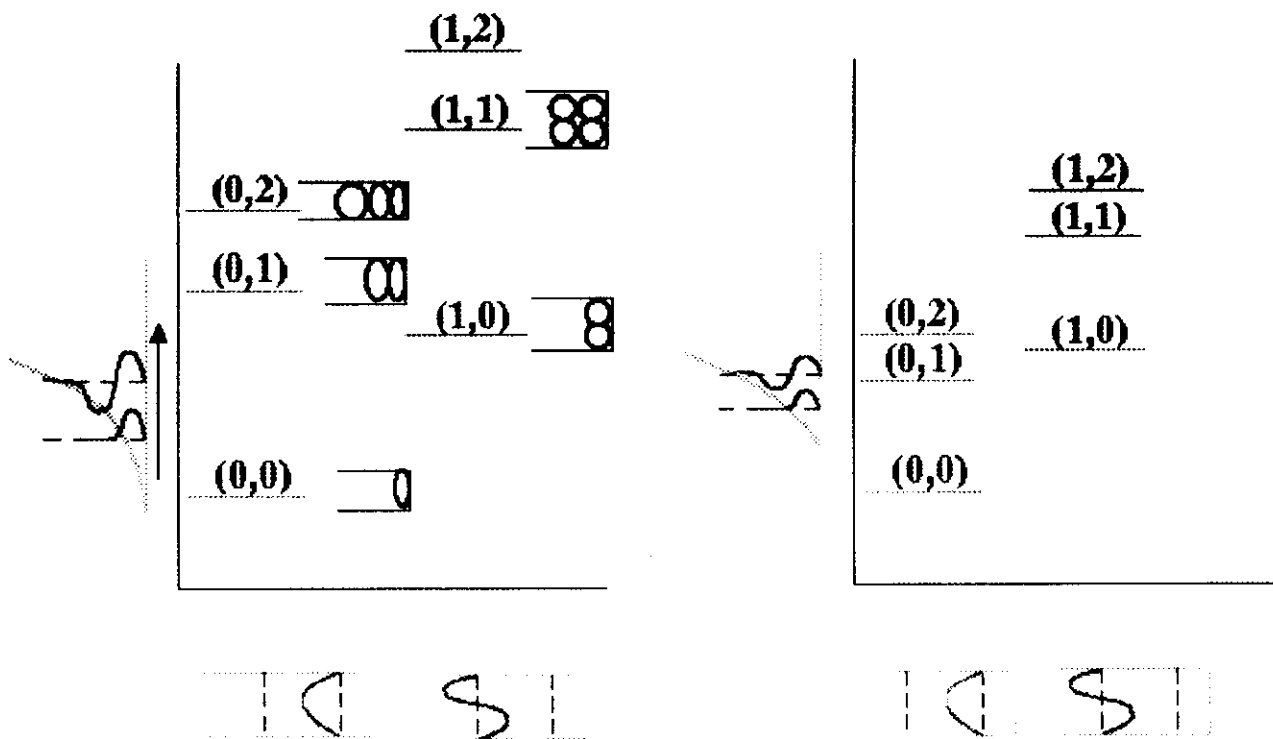
Magnetic Field Dependence



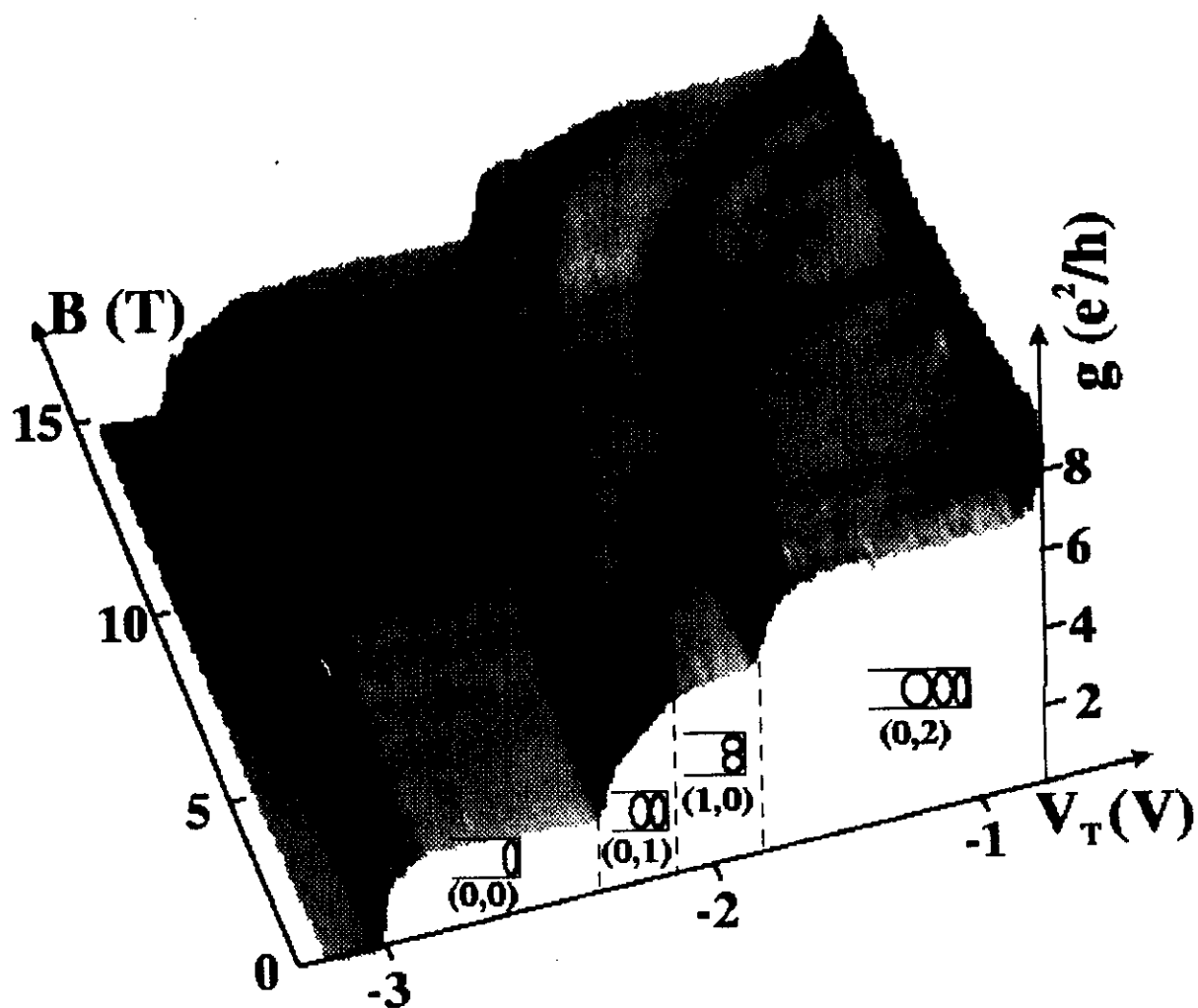
Changing the Side Gate Voltage



Reduces the level separation in the triangular well.
The square well is unaffected.



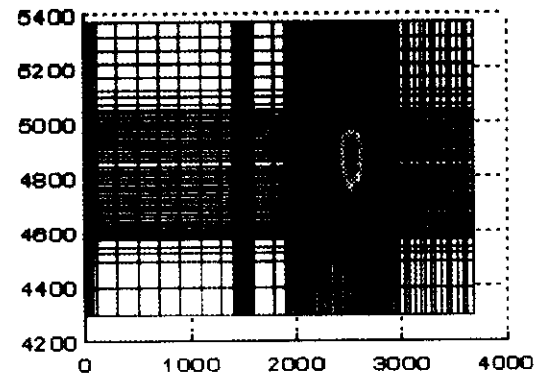
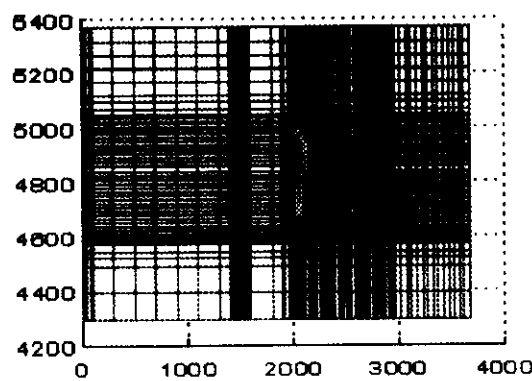
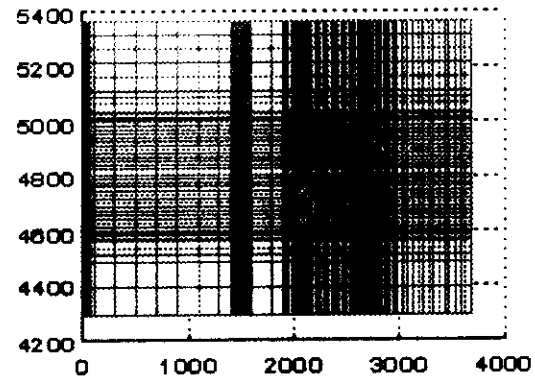
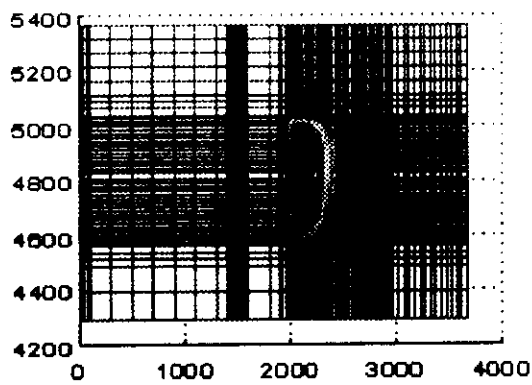
Changing the Side Gate Voltage



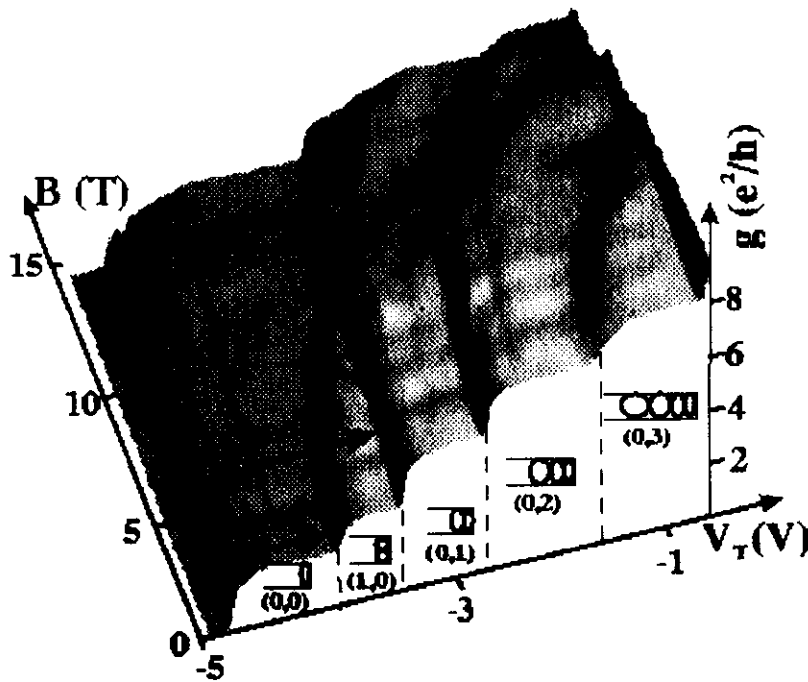
Numerical Simulations

Cedric Gustin
J. P. Leburton

$V_t = -1.0V$
 $V_s = 0.0V$
 $L = 40\text{ nm}$



Extracting Subband Separation



The QW states are insensitive to the applied gate voltages.

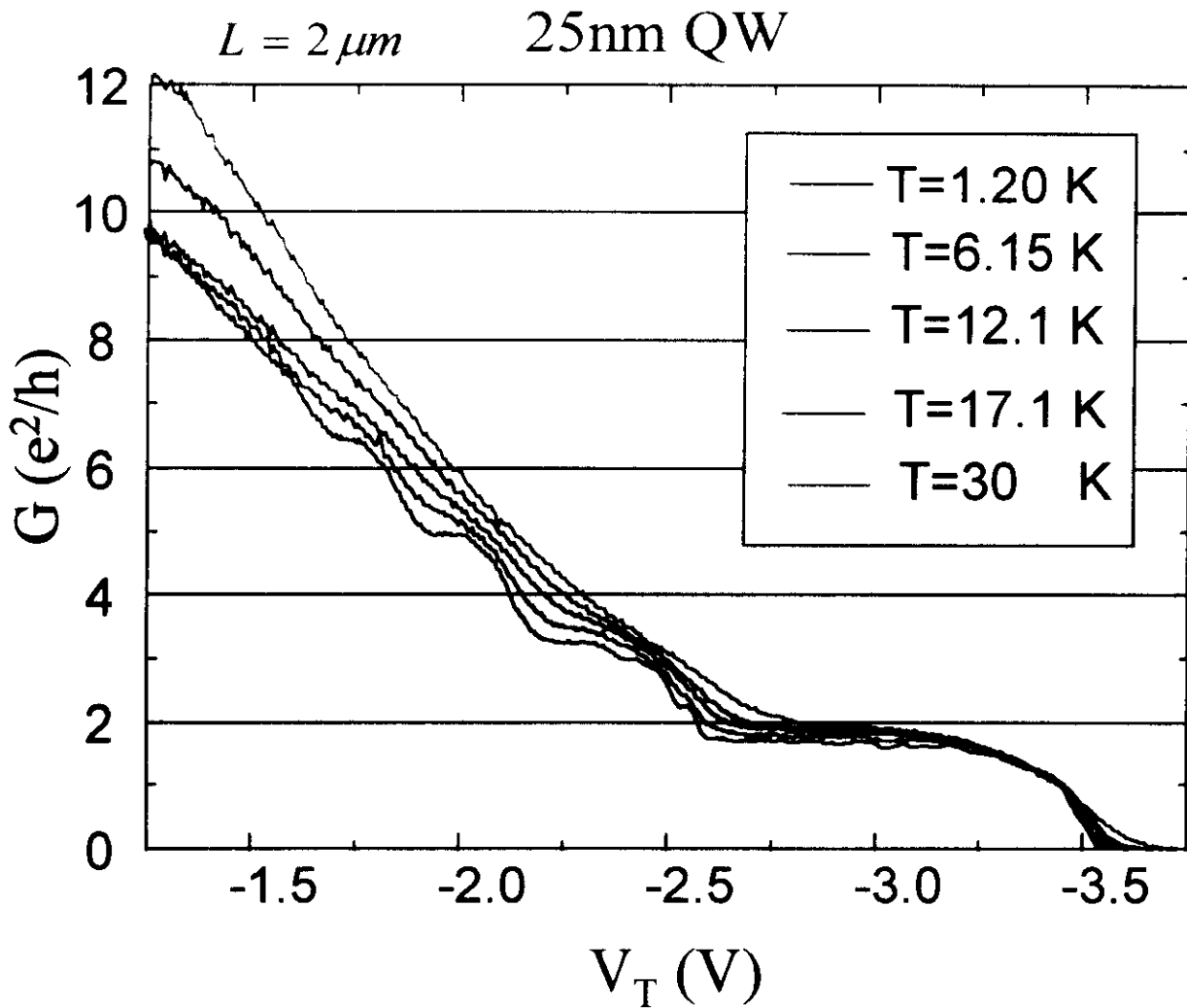
For a 25 nm QW the energy difference between the lowest and first excited states is 22 meV

The energy difference between lowest and first states of the triangular well is therefore 30 meV

The triangular well has a confining dimension of approximately 20 nm.

Electrons confined to an area less than 25nm X 25nm
An order of magnitude larger than conventional methods

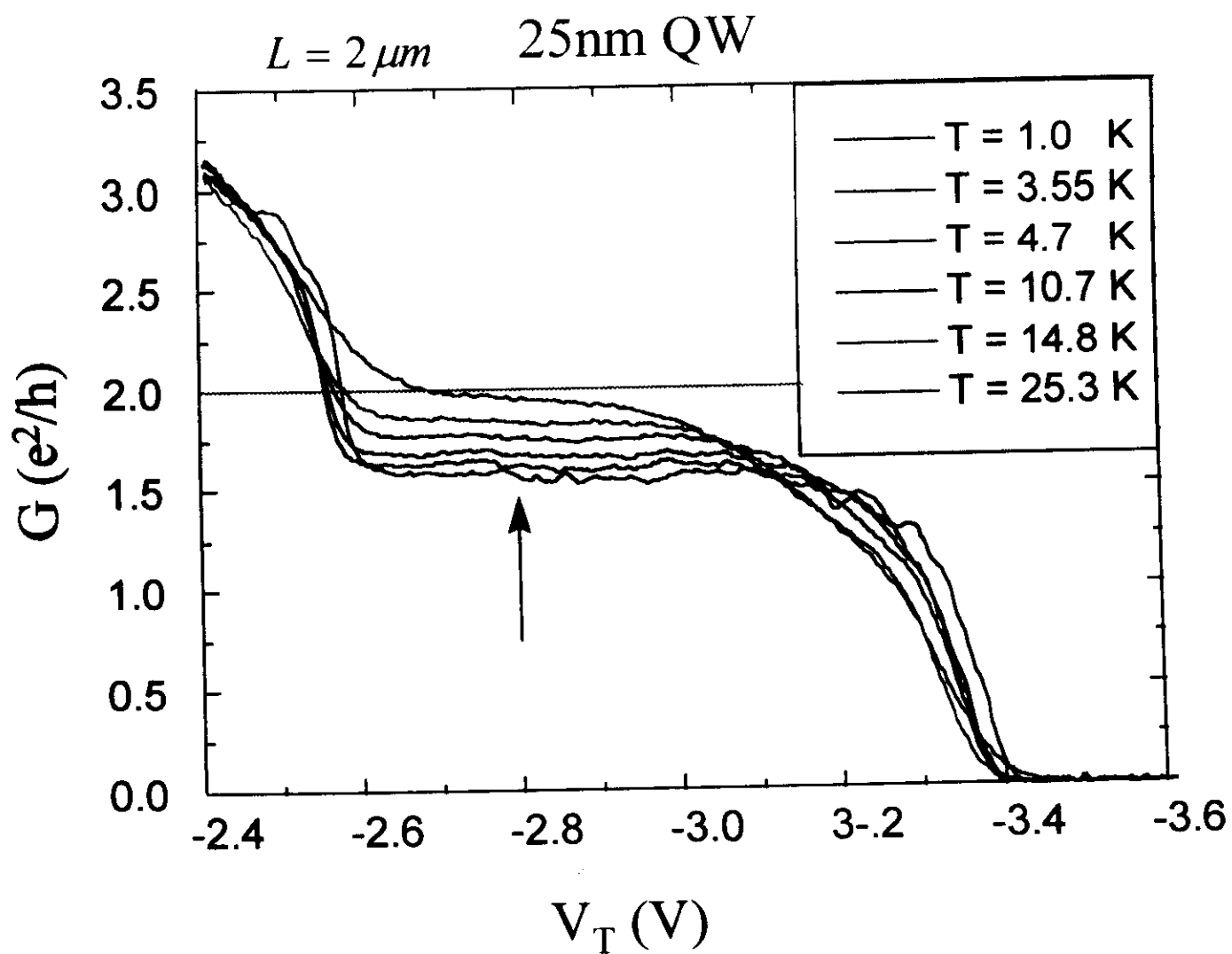
Temperature Dependence



Higher plateau's smear with increasing T

- $G_N = N \cdot G_1(T)$

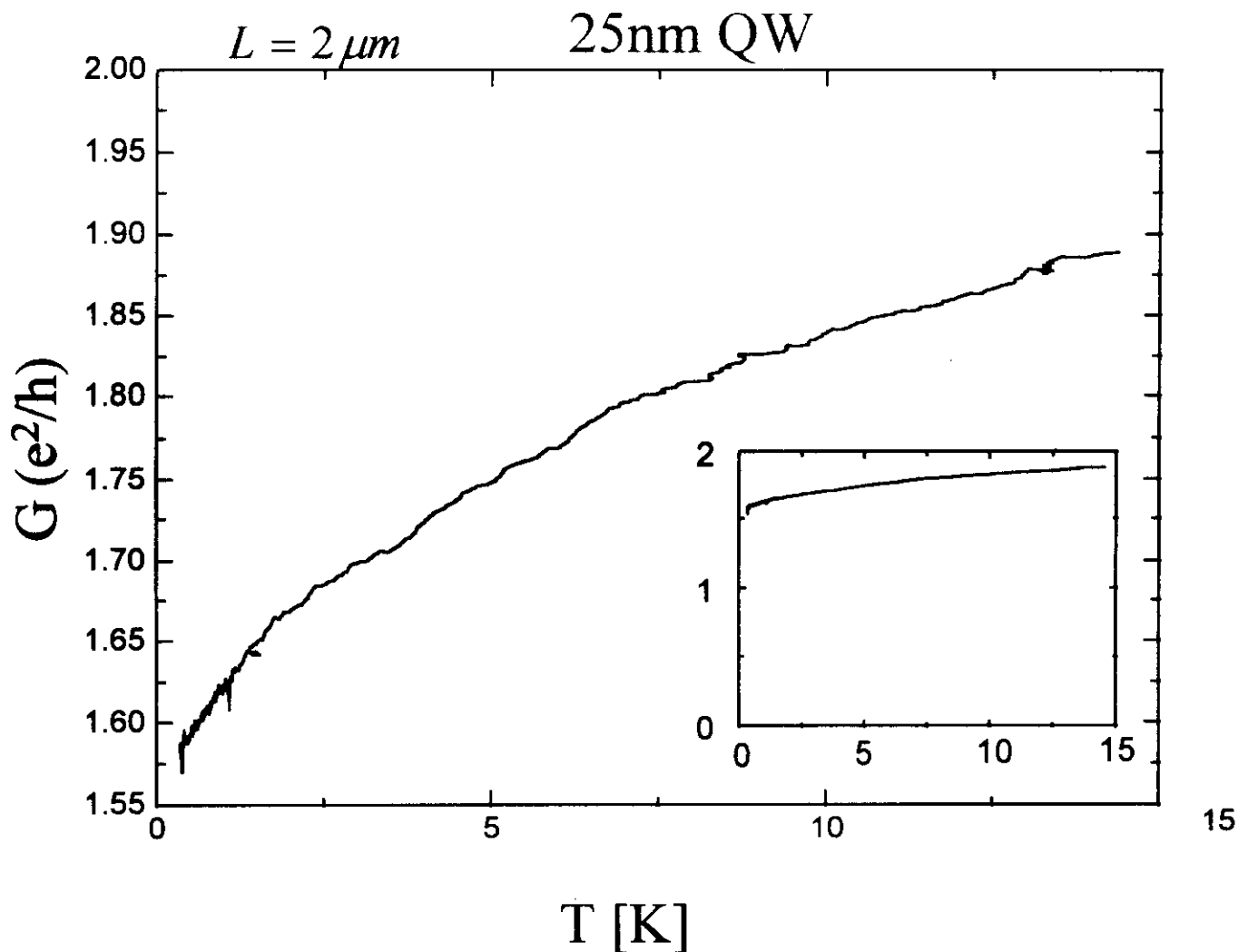
Last Plateau



Plateau rises rigidly

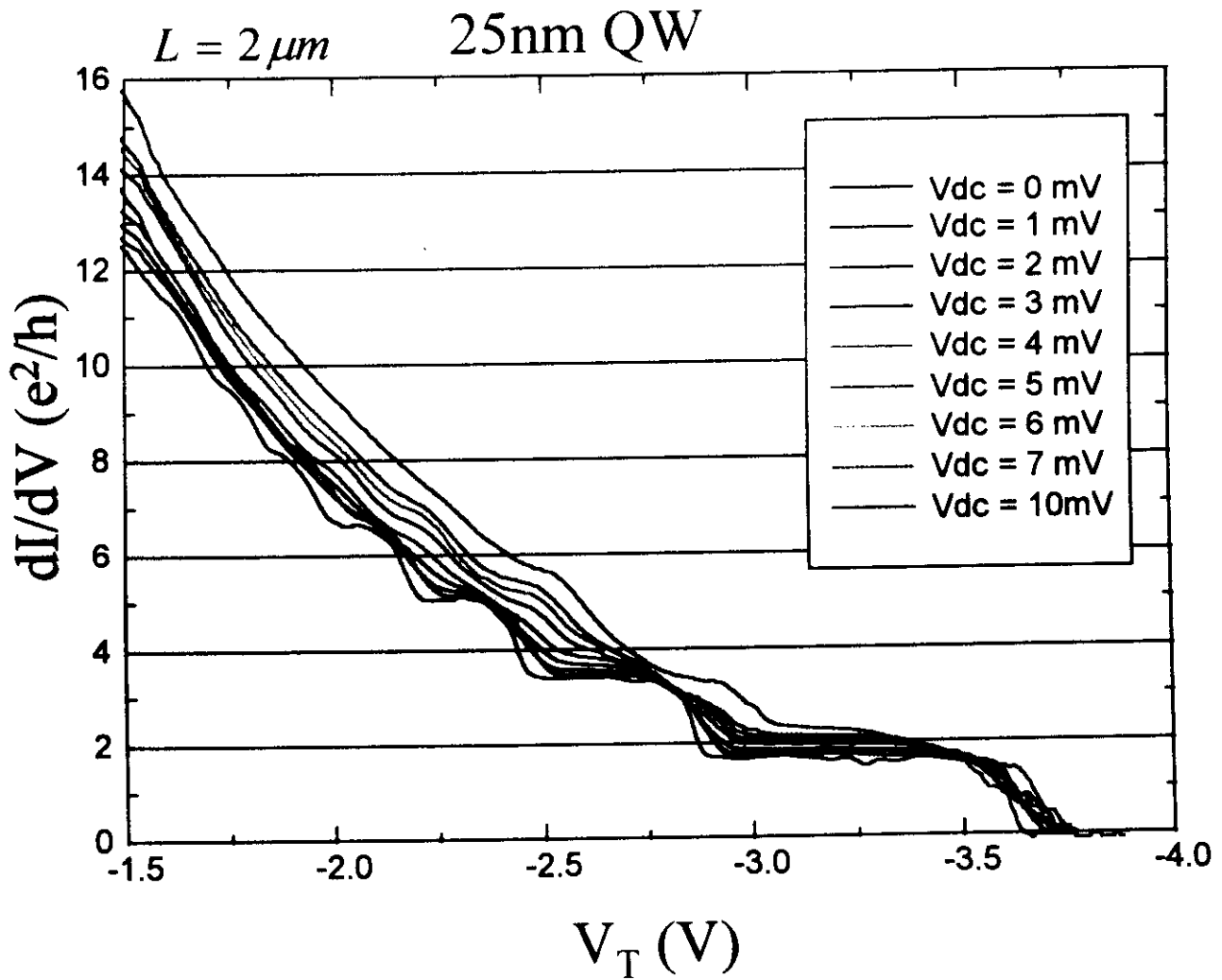
➡ No observed dependence on wire density

Temperature Dependence of Last Plateau



- Conductance approaches the universal value at high temperatures. Luttinger liquid.
- Conductance is finite at zero temperature.

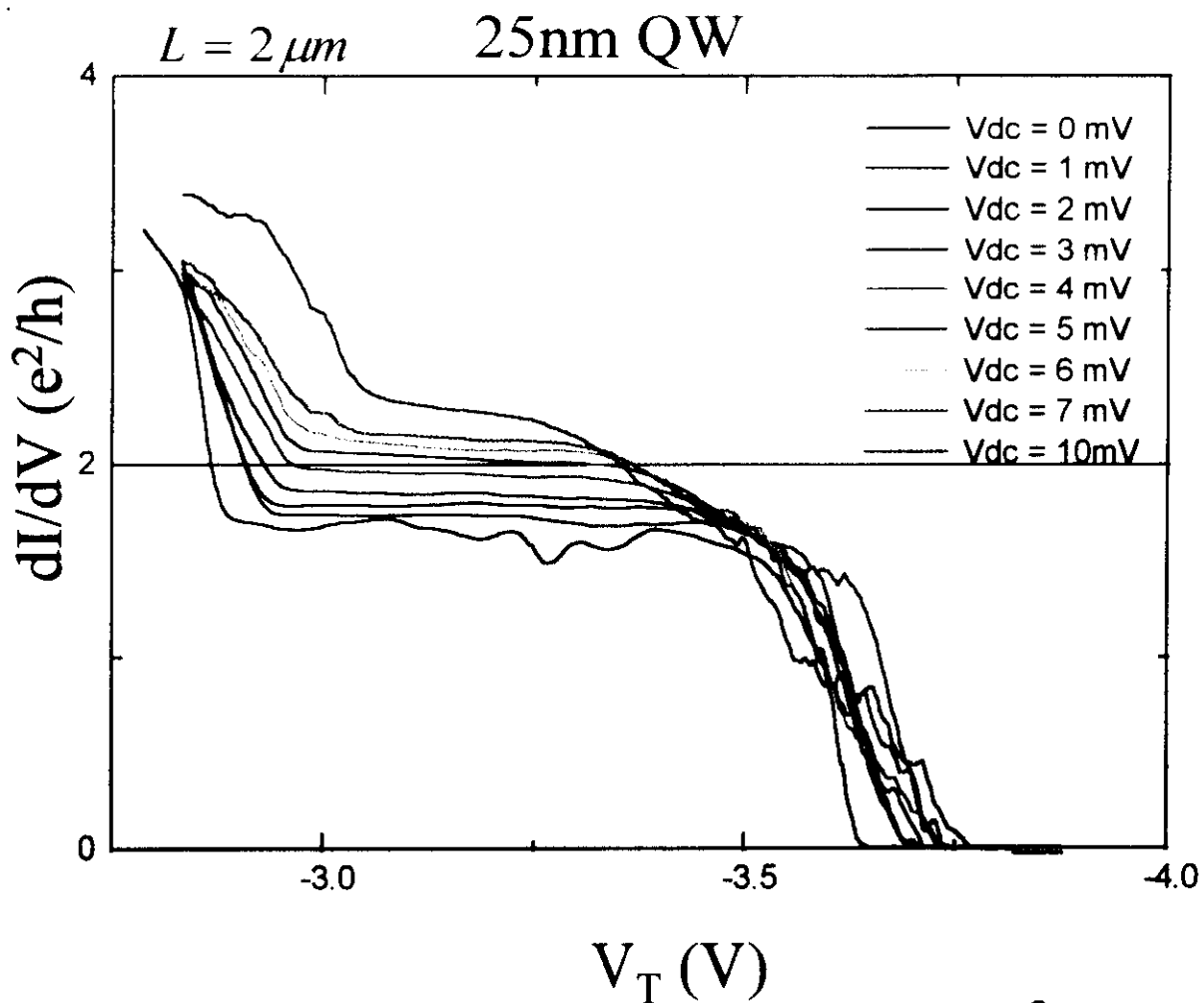
Non Linear I-V



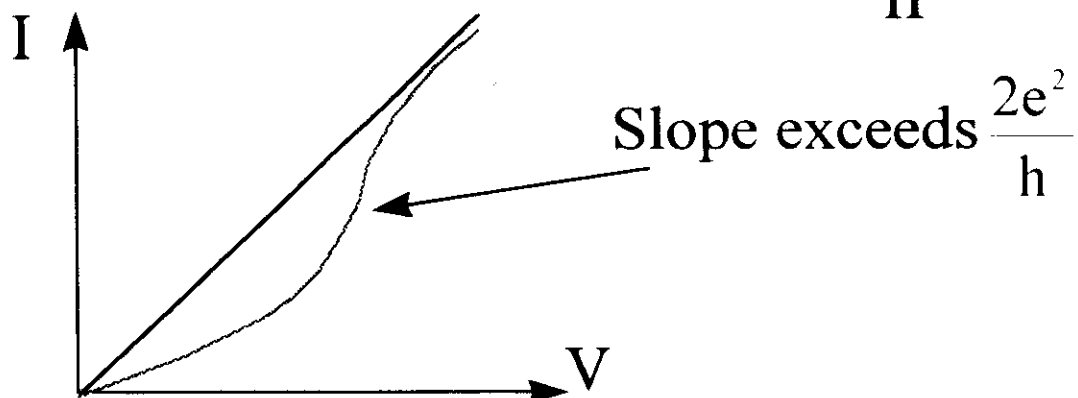
Low V plateau's rise rigidly.
High V observe half plateau's.

No observed dependence on wire density

Last Plateau



Differential conductance exceeds $\frac{2e^2}{h}$



- Flat plateau $\Rightarrow$ No density dependence

First scenario

Non Interacting Electrons

× No disorder  Universal conductance quantization.

× Disorder in the wire:



No plateaus.

No temperature dependence.

× Disorder in the leads:



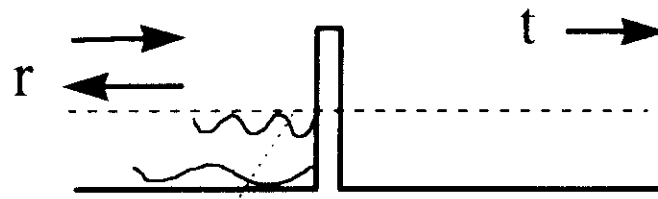
Series resistance.

No temperature dependence.

Second scenario

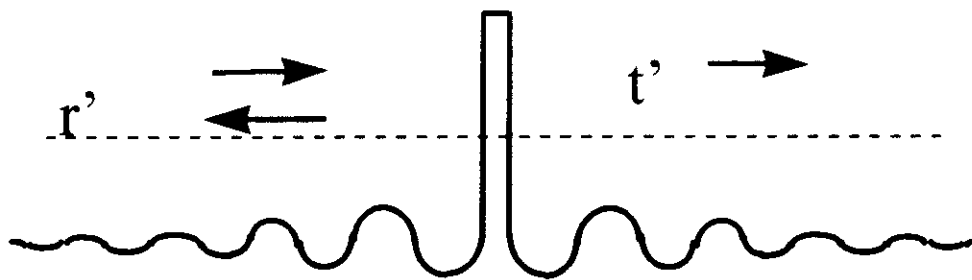
Interactions plus Disorder

C. L. Kane and M. P. A. Fisher



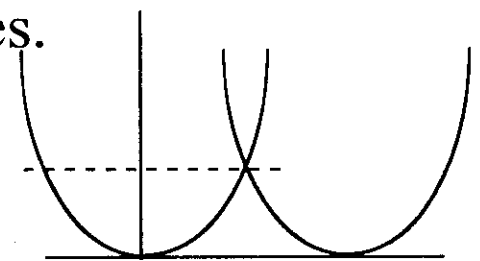
Standing waves

D. Yue, L.I. Glazman,
K. A. Matveev



Friedel Oscillations $\approx \frac{1}{r} \sin(2k_F r)$

- $2k_F$ periodic potential leads to Pierles instability which reduces the density of states at E_F
- Interactions lead to charge density order which is pinned by the impurities.



$$G = \text{const} \cdot \left(\frac{T}{E_F} \right)^{2 \left(\frac{1}{g} - 1 \right)}$$



$$G=0 \quad \text{at} \quad T=0$$

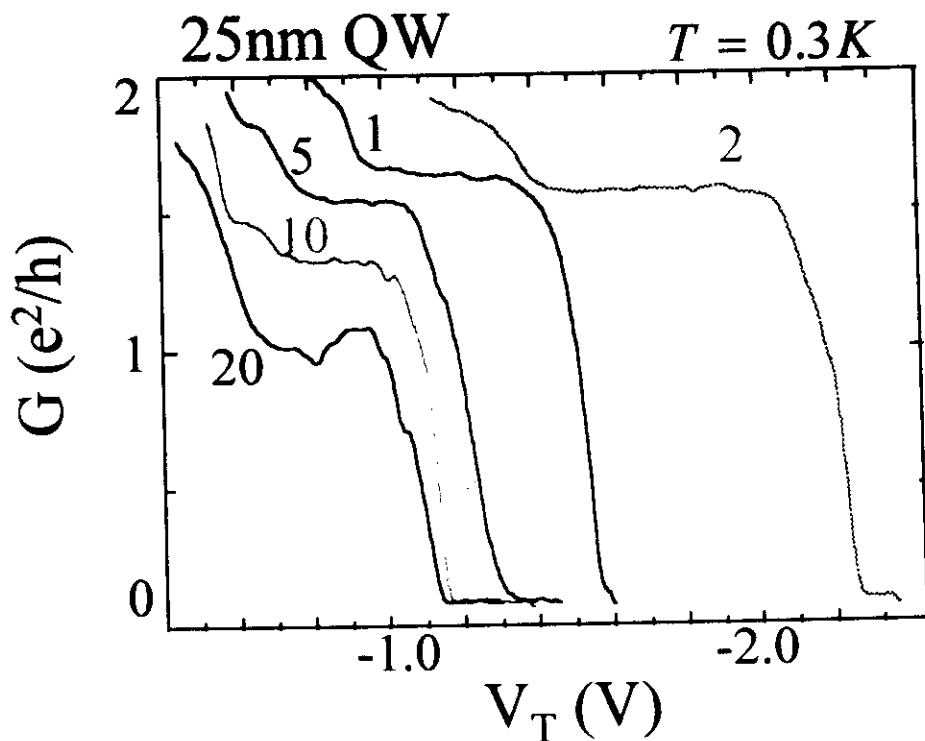
Finite Length

Charge density correlations on length scales :

$$L = \frac{\hbar v_F}{kT}$$

➔ $G_{\min} \approx \text{const} \cdot \left(\frac{\hbar v_F}{E_F L} \right)^{2\left(\frac{1}{g}-1\right)}$

- Expect finite conductance at $T=0$.
- Expect the conductance to depend on L .



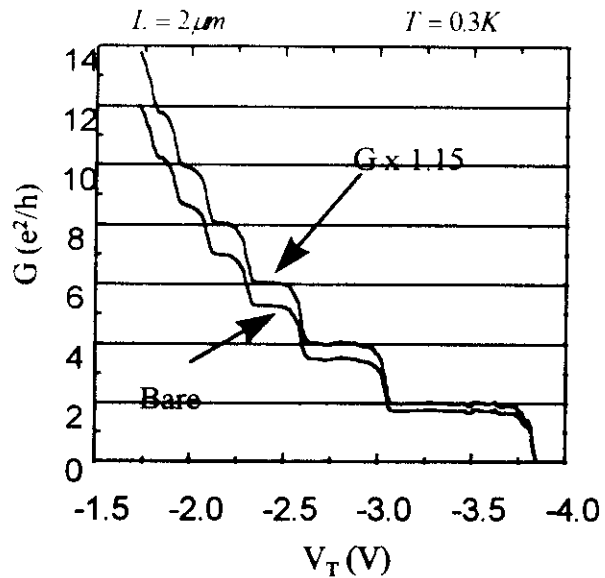
- Flat plateau ➔ No density dependence

e - e Interactions in the wire ?

Flat plateau's suggest no disorder

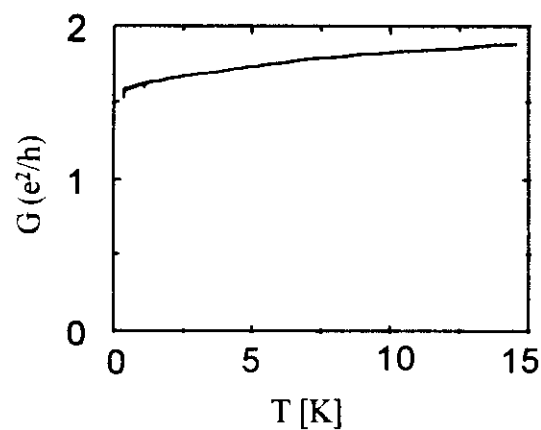
✗ expect $G = N \frac{2e^2}{h}$

observe $G = g \frac{2e^2}{h}$



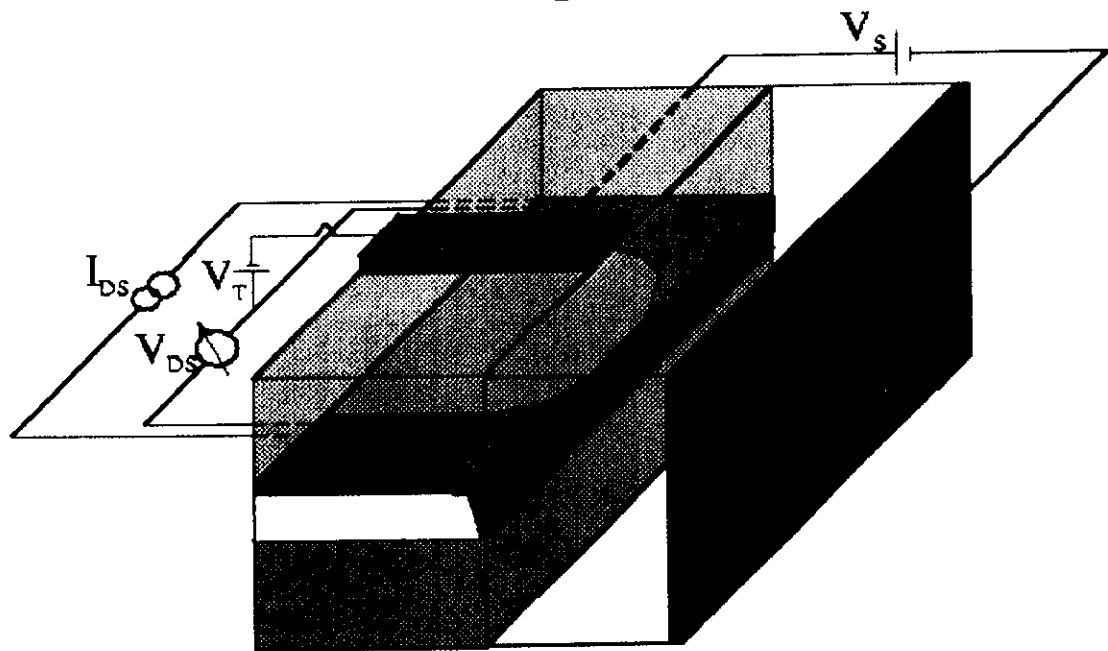
Weak disorder would give

$$G = \text{const} \cdot \left(\frac{T}{E_F} \right)^{2\left(\frac{1}{g}-1\right)}$$

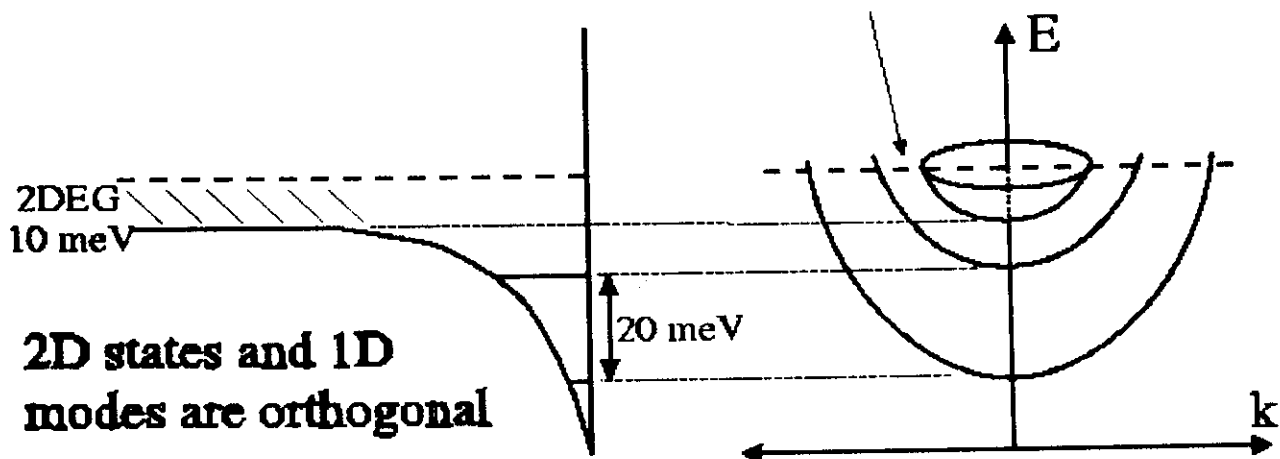


- ✓ • Finite conductance at $T=0$ due to finite L .
- ✓ • Conductance depends on wire length.
- ✗ • Expect dependence on E_F .

The Contact Region



Large momentum difference at the Fermi energy.

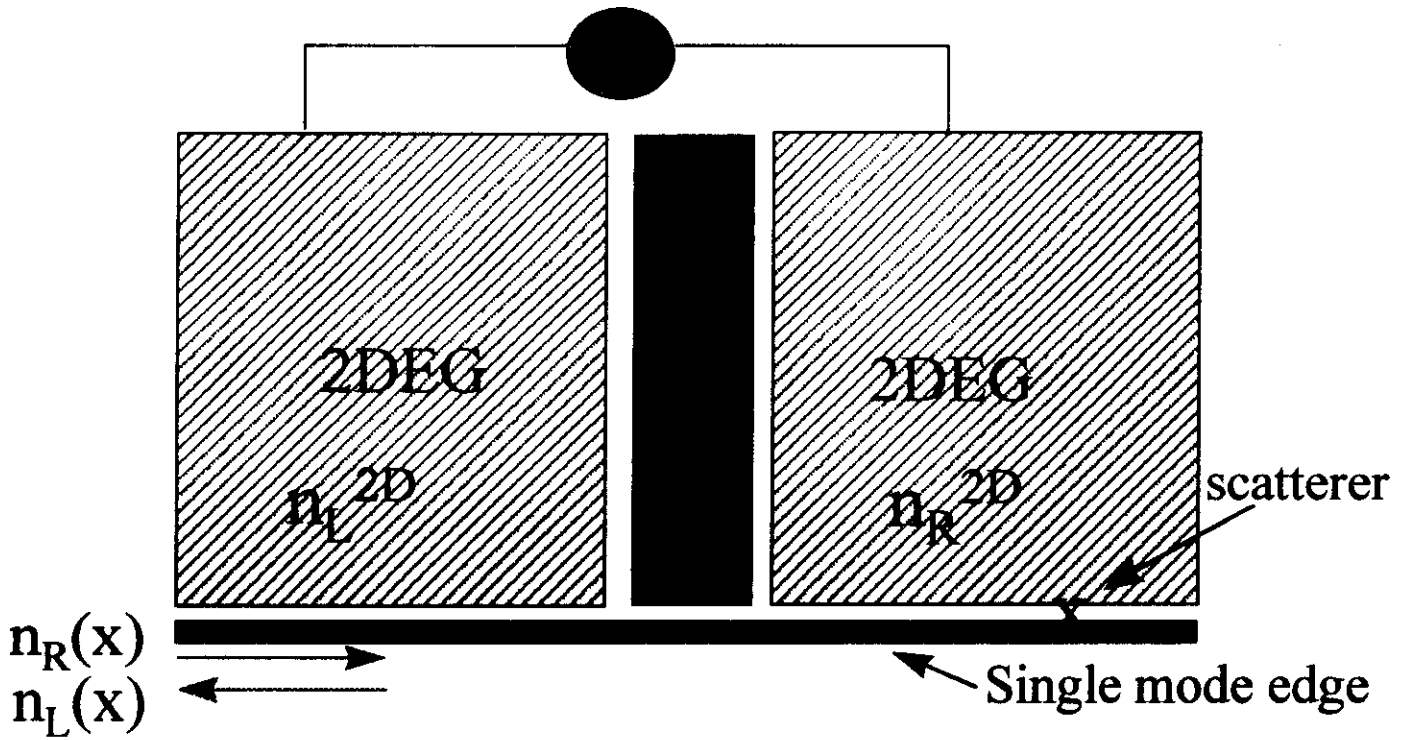


Electrons confined to 1D modes even in the presence of the 2DEG

Third scenario

Model for the Edge

Boltzmann Approach



Phenomenological scattering rates

Γ_{BS} - scattering from R to L and vice versa

Γ_{2D} - scattering from 2D to R or L.

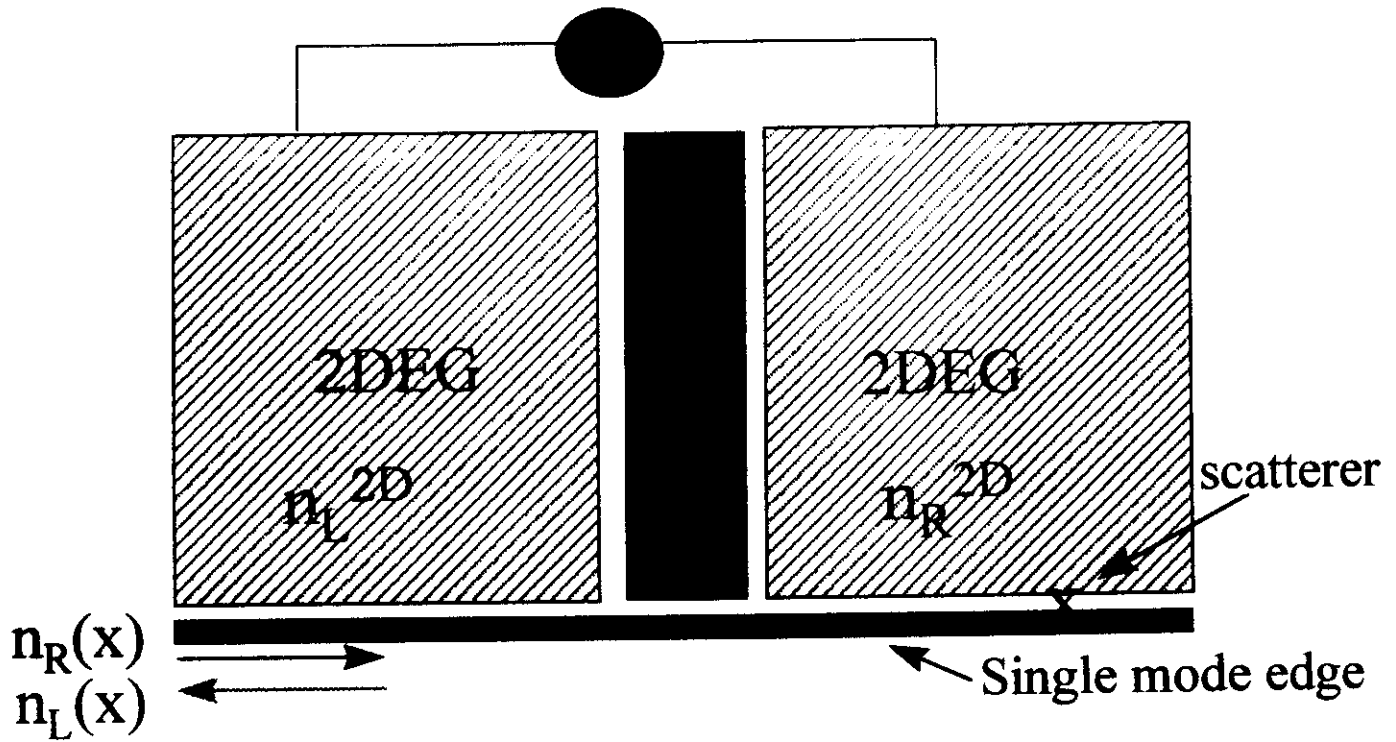
$$v_R \frac{\partial n_R}{\partial x} = n_R^{2D} \Gamma_{2D} + n_L \Gamma_{BS} - n_R (\Gamma_{2D} + \Gamma_{BS})$$

Where v_R is the velocity of the R movers.

The current is given by : $I = e \cdot v \cdot [n_R(L) - n_L(-L)]$

$$I = e \cdot v \cdot (n_R^{2D} - n_L^{2D}) \frac{1}{\sqrt{1 + \frac{2\Gamma_{BS}}{\Gamma_{2D}}}} = G_Q \frac{1}{\sqrt{1 + \frac{2\Gamma_{BS}}{\Gamma_{2D}}}}$$

Interactions plus Disorder in the Leads



$$G = g \frac{1}{\sqrt{1 + \frac{2\Gamma_{BS}}{\Gamma_{2D}}}} \quad g = 1$$

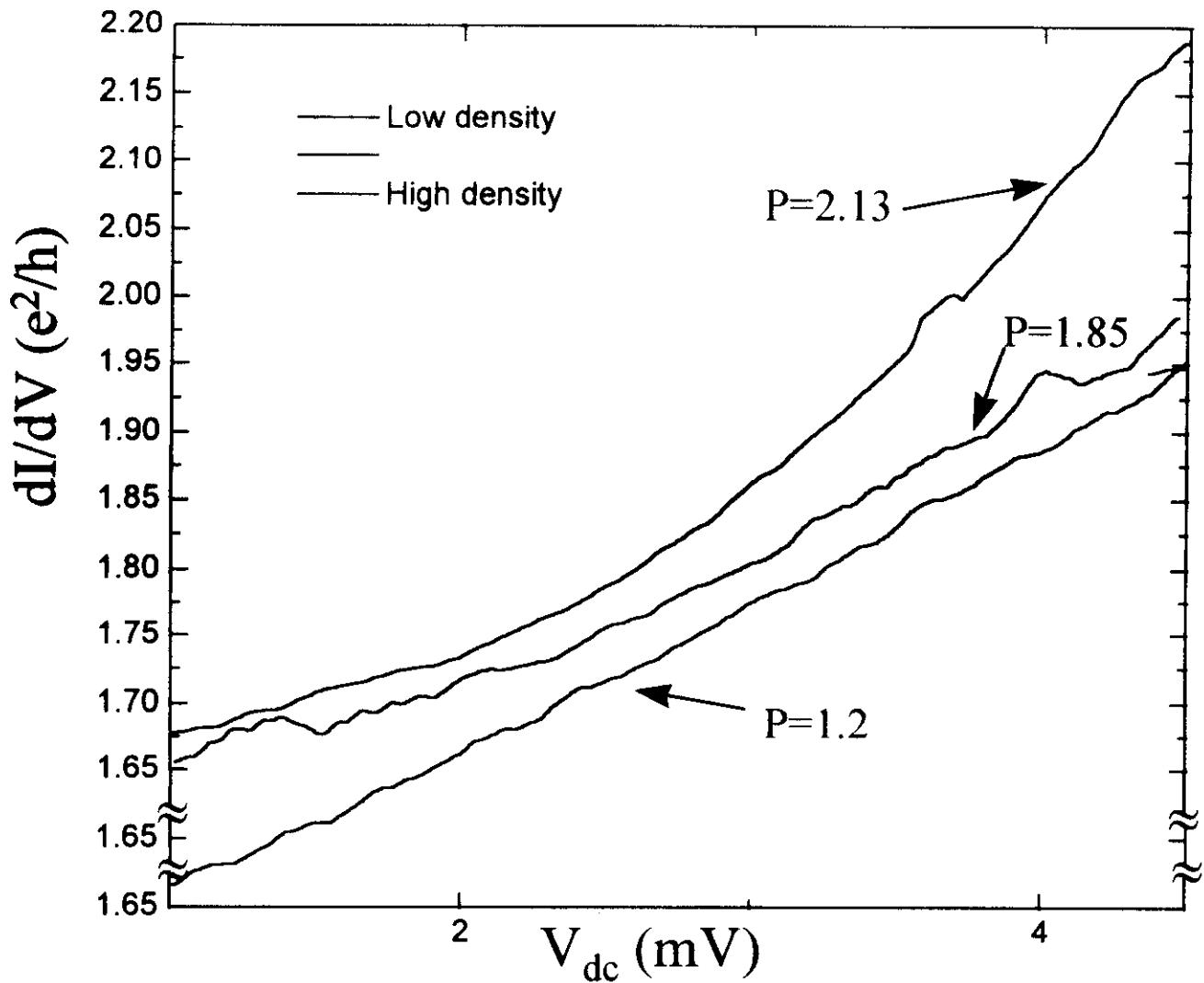
Both Γ_{2D} and Γ_{BS} are affected by the interactions

Γ_{BS} - Increases due to enhanced scattering on impurities.

Γ_{2D} - Decreases due to the suppressed tunneling density of states.

- ✓ • Explains lack of dependence on wire density.
- ✗ • Expect no length dependence.
- ✗ • Expect Zero conductance at $T=0$. (coherence)
- ? • Expect dependence on Lead density.

Lead Density Dependence



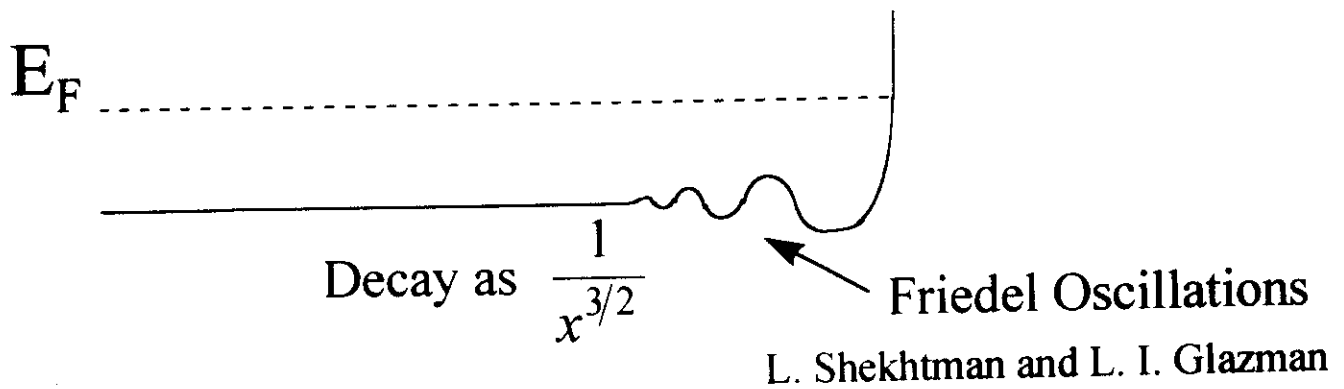
$$\frac{dI}{dV} = c + a \cdot V^P$$

Observe similar trend to the Luttinger liquid predictions

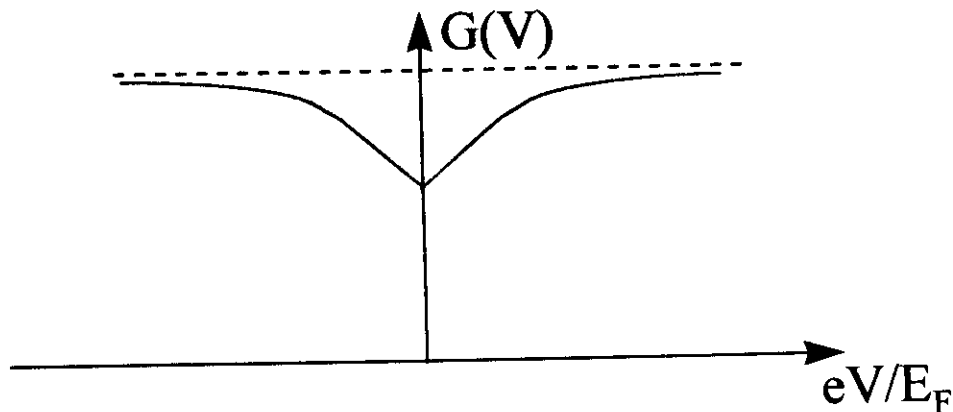
Fourth scenario

Friedel Oscillations in the Edge of the 2DEG

A. Yu Alekseev and V. V. Cheianov



Obtain Zero bias anomaly for tunneling into the edge.



$$G(V) = G_0 \left(1 - \alpha + \gamma \frac{|eV|}{E_F} \right) \quad \text{where} \quad \alpha \approx e^2 / \epsilon h v_F$$

$\gamma \approx 1$

× Expect Linear dependence on V

× Both T and V scales are E_F

Summary

- Electrons confined to an area of less than 25 nm X 25 nm.
- True 1D behavior up to 10 μm long wires.
Aspect ratio of 1:1:400.
- Subband separation in excess of 20 meV.
- Extract the wavefunctions of the 1D modes.
- Conductance is quantized in multiples of
$$g \frac{2e^2}{h} \quad ; \quad g \approx 0.85$$
- Approach the universal value at high kT and dc biases.
- No dependence on wire density.
- Lead density dependent non linear I-V.
- Simple theoretical scenarios are incapable of explaining all the available data.

