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ELECTRON LOCALIZATION IN DISORDERED SYSTEMS
(Part II)

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6. Thouless conductance :-

References :- D.J. Thouless, Phys. Rep. 13C 94 (1974)
J T Edwards and D.J. Thouless J. Phys C 5 807 (1972)
D C Licciardello and D J Thouless :- J Phys C 8 4157 (1975), 11, 925 (1978)
D J Thouless, M-NM Transition ed. Friedman & Tunstall.

Thouless argued that ~~the~~ sensitivity of the eigenstates of a random finite system to changes in the boundary condition is a measure of its conductance e.g. for example, if the eigenstates are localized with localization length α^{-1} much smaller than system linear dimension L, ~~the~~ a change in the boundary conditions will have an exponentially small effect on the energy levels. Specifically, Thouless proposed that

$$\frac{\overline{\delta E}}{\overline{\Delta E}} = \frac{\hbar}{e^2}\, g(L) \qquad (6.1)$$

where $\overline{\delta E}$ is the spread of energy level shifts as the boundary condition is changed say from periodic to antiperiodic, for energies in a narrow neighbourhood. $\overline{\Delta E}$ is the average spacing between levels in the same neighbourhood. The result (6.1) can be made plausible in several ways, and we mention two below.

i) Consider a wave packet diffusing through the system of size L. It takes a time $t_L \simeq L^2/D(L)$ to do this where $D(L)$ is the diffusion constant. This is the time for ~~perturbation~~ effects due to change in boundary conditions to eventuate; by an uncertainty principle argument then

$$\overline{\delta E} = \frac{\hbar}{t_L} = \frac{\hbar\, D(L)}{L^2} \qquad (6.2a)$$

Now

$$\overline{\Delta E} = \frac{1}{N(E)}\,\frac{1}{L^d} \qquad (6.2b)$$

density of states per unit energy per unit volume

so that

$$\frac{\overline{\delta E}}{\overline{\Delta E}} = \frac{\hbar}{e^2}\,\underbrace{e^2 N(E)\, D(L)}_{\sigma(L)}\, L^{d-2} = \frac{\hbar}{e^2}\, g(L)$$

(ii) A more formal analysis considers the effect of a given rate of ~~change of~~ wave function phase change, i.e. a gauge transformation. Consider the transformation

$$p_x \rightarrow (p_x - \hbar K) \qquad (6.3)$$

This changes the phase of the wave function at a rate K so that the relative phase of the wave function on two sides of the box of size L is KL. Introducing (6.3) into the system Hamiltonian, we see that a perturbation

$$\delta H = \frac{\hbar K^2}{2m} - \frac{\hbar K}{m} p_x \qquad (6.4)$$

has been introduced. This shifts a given energy level E_i by the amount

$$\delta E_i = \left[\left\{ \frac{\hbar^2}{m^2} \sum_j \frac{|\langle i | p_x | j \rangle|^2}{(E_i - E_j)} + \frac{\hbar^2}{2m} \right\} \frac{1}{L^2} \right] K^2 L^2 \qquad (6.5)$$

where the quantity in square brackets is the shift due to a (phase change)^2 $\simeq$ $(KL)^2$. It has been discussed extensively by Edwards and Thouless. Under certain assumptions of statistical independence, its spread ($\sqrt{\langle \delta E_i^2 \rangle_{average}}$ can be related to the Kubo' expression for the conductivity

$$\sigma(E_F) = \frac{e^2}{m^2} \sum_{i,j} |\langle i | p_x | j \rangle|^2 \, \delta(E_i - E_j)\, \delta(E_i - E_F) \qquad (6.6)$$

The Thouless conductance formula is very suggestive. For example, by solving for the energy eigenvalues (say numerically) under different boundary conditions and determining δE_i, the conductance can be calculated. This was the route taken by Licciardello and Thouless in their work on σ_{min} or g_{min} in two dimensions. Qualitatively, it argues strongly for a universal measure of disorder. Using this idea, Thouless suggested that the behaviour of wires with $R \gtrsim (\hbar/e^2)$ should be non ohmic at low temperatures. The point is that in the localized regime,

$$\delta E / \overline{\Delta E} \sim e^{-\alpha L} \quad \text{for } L \gg \alpha^{-1}$$

i.e. $$R(L) \sim (\hbar/e^2)\, e^{\alpha L} \qquad (6.7)$$

where the localization length is α^{-1}. Since the natural scale is set by $(\hbar/e^2) \sim 4000\ \Omega$, (6.7) suggests that the resistance of wires with $R > 10^4\ \Omega$ say, will increase exponentially with length rather than according to Ohm's law, i.e. linearly. The wire is thin so that its diameter is much less than α^{-1}. This behaviour is of course not observed, and Thouless points out that the reason is inelastic scattering.

Inelastic collisions change the energy of an electronic state. If the rate at which inelastic collisions occur is Γ_i, there is an intrinsic width Γ_i to the levels, and there are no meaningful scale dependent shifts $\delta E_i < \Gamma_i$. That is, if the inelastic collision time is τ_i, a wave packet diffuses a distance L_{Th} such that

$$L_{Th}^2 / D(L_{Th}) = \frac{\hbar}{\delta E(L_{Th})} = \tau_{inelastic} \qquad (6.8)$$

before such a collision dephases it. Diffusion of the wave packet begins anew. Equation (6.8) sets a length scale L_{Th} for scale dependent (L dependent) resistance such as described in (6.7). Beyond L_{Th}, the resistance behaviour is Ohmic, or more precisely, a wire can be thought of as bits of length L_{Th} joined together, the combined resistance being determined according to Ohm's law.

Nonohmic effects are cut off at a length scale

$$L_{Th} = \sqrt{D(L)\,\tau_{inel}} = \sqrt{\frac{v_F l}{d}\,\tau_{inel}}$$

$$\simeq \sqrt{l\, l_{inel}} \qquad (6.8)$$

If this is small (say compared to the length λ of wire with resistance $\sim 10^4\,\Omega$), nonohmic effects will be small. However, as temperature decreases, l_{inel} increases (say as T^{-p}) so that L_{Th} increases. Thus nonohmic, localization effects ~~are~~ ~~more~~ become more prominent at lower temperatures. For example, for $L_{Th} \ll \lambda$, a guess for the wire conductivity $\sigma(L)$ as a function of length based on the above ideas and Eq. (6.7) is

$$\sigma(L) = \sigma_0 - \frac{a_1 e^2}{\hbar}\,\frac{1}{A}\,L \qquad (6.9)$$

where a_1 is a numerical constant and A the cross-sectional area of the wire. We derive this formula later. (Eq. 8.7c is the basis for it).

Now as L increases, σ decreases, so that the resistivity increases with decreasing temperature. The size of the decrease has a natural scale $(e^2/\hbar)$. Thouless presented estimates in his 1977 paper for the size of this anomalous term, due to breakdown of Ohm's law. In a wire with diameter 500 Å and a localization length ($R(L) \sim 10^4\,\Omega$) 10^{-3} cm, a temperature of about 1°K or lower is

needed for these effect to become significant. The effect was searched for, and was finally found. It seems to be much smaller than estimated [illegible]. The main reason is that inelastic [illegible] much shorter than the estimates [illegible] clean limit. We will come to this question later. (Section 10).

The most important qualitative consequence of the Thouless idea of conductance is that it naturally leads to another idea namely that conductance is a universal function of scale size. As a matter of fact, such a scaling ~~theory~~ was suggested by Thouless (Scottish Universities Summer School 1978). A scaling function whose asymptotic forms are determined by an appropriate perturbation theory, was first described by Abrahams, Anderson, Licciardello and Ramakrishnan who worked out its consequences. We discuss this now.

7 Scaling Theory of Localization :-

Refs :- Abrahams, Anderson, Licciardello and Ramakrishnan Phys. Rev. Lett. 42, 673 (1979).

Anderson Abrahams and Ramakrishnan

Abrahams and Ramakrishnan, J. Noncryst. Solids 35, 15 (1980),

Gorkov, Larkin and Khmelnitskii JETP Letters L 30, 228 (1980)

The relation between conductance and sensitivity to boundary conditions suggests a scaling theory for the former. Imagine fitting hypercubes of size L together to form larger blocks. We compute the Thouless conductance for the larger block. Its sensitivity to boundary condition change would depend on the new scale, and how the perturbation is transmitted across the common boundary. The latter is measured by $g(L)$, so that if the new block has dimension bL,

$$g(bL) = f(b, g(L)) \qquad (7.1)$$

or in differential form (i.e. for $b = 1+\epsilon$)

$$\frac{L}{g}\frac{dg}{dL} = \frac{1}{g}\left.\frac{\partial f}{\partial b}\right|_{b=1} = \beta_d(g) \qquad (7.2)$$

The idea is that ~~because~~ ~~the~~ since quantum interference

effects are measured by conductance which has a universal scale, i.e. $g_c = (e^2/\hbar) \approx (38.7\ K\Omega)^{-1}$ the evolution of such interference effects with scale size should be a universal function depending only on g in units of g_c. The assumption also is that there is only one relevant ~~to~~ microscopic length scale, the mean free path l or the phase incoherence length. We denote this length scale by L_0. Given the conductance g_0 at L_0, the scaling equation determines in a ~~unique way~~ universal way $g(L)$ at any large L. Clearly, other length scales, ~~depending~~ e.g. inelastic diffusion length L_{Th}, Landau orbit length L_H $(= \sqrt{\Phi/H})$, superconducting coherence length ξ_0 can affect g(L) depending on the physical situation.

We now guess $\beta_d(g)$ based on asymptotic forms ~~and~~ etc.

i) Small g :- In the localized regime, we have already seen in Eq (6.7) that $g(L) \approx g_0 e^{-\alpha L}$. This leads to $\beta(g) = \ln(g/g_0)$ independent of dimensionality. For small g, one can use the locator perturbation theory of Anderson (Anderson 1958) to show that

$$\beta(g) = \ln(g/g_1)\,[\,1 + \gamma g\,] \quad \text{for } g \ll g_c \qquad (7.3)$$

where γ is positive.

(ii) Large g :- Here Ohm's law, i.e. macroscopic or classical transport theory is valid, and

$$g = \sigma L^{d-2} \qquad (7.4a)$$

so that

$$\beta_d(g) = (d-2) \qquad (7.4b)$$

~~Thus~~ We show in the next Section, using many body perturbation theory, that there is a correction to (7.4b) of order $(1/g)$ and that it is negative so that

$$\beta_d(g) = (d-2) - (a_d/g) \qquad (7.5)$$

where $a_d = (e^2/\hbar \pi d)$ for a noninteracting electron gas (spin ½) in a random potential.

To construct the scaling curve, knowing Eq. (7.5) and (7.3), we use the assumptions of continuity and monotonicity. $\beta(g)$ is continuous because it describes the effect of blocking a finite number of sites, i.e. a finite size system. Further, as g decreases, one tends towards a more localized behaviour so that $\beta(g)$ must decrease. ~~as g~~ We ~~because~~ mention later below recent numerical ~~work~~ and analytical work which supports this scaling curve. ~~and~~

We plot the scaling curve in Fig. 1. It implies the following.

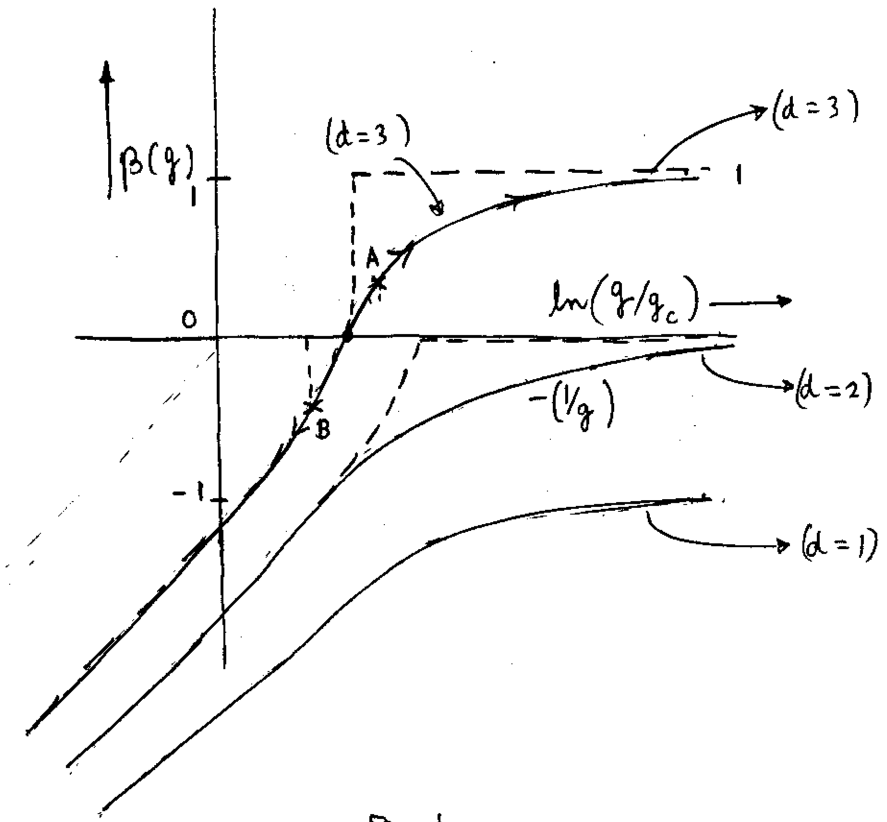


Fig. 1.

i) Three dimensions :- $\beta(g)$ intersects the x-axis at some $g = g_3$. If the system is such that the conductance g_0 at the microscopic scale L_0 is greater than g_3, one has $\beta(g) > 0$, so that g increases with size. ~~For very large length scales, one~~ Starting from the point A (g_0 corresp. $g=g_0$), on scaling to larger ~~distances~~ sizes one finally ends up, for large L, in the Ohm's law limit. On the other hand, starting from B for which the microscopic disorder is ~~much~~ larger and such that g_0 at scale L_0 is less than g_3, ~~on increasing~~ with increasing length scale one finally ~~ends up~~ gets to the localized limit, i.e. $\beta(g) \sim \ln(g/g_1)$. Thus g_3 is the mobility edge, ~~and~~ i.e. ~~the~~ for critical disorder, the conductance at a microscopic scale $L_0 = g_3$. The macroscopic behaviour in the vicinity of critical disorder, i.e. for $g_0 \simeq g_3$, is determined by the scaling curve which describes quantum terference ~~the~~ effects ~~Fig~~ (localization effects).

For $g_0 \simeq g_3$, suppose the $\beta(g)$ curve has a slope $(1/\nu)$, i.e.

$$\beta(g) = \frac{1}{\nu}\left|\frac{g-g_3}{g_3}\right| = \frac{\delta g}{\nu} \qquad (7.6)$$

In perturbation theory (Section 8) $\nu = 1$. Using the form (7.6) and integrating out to near $\beta(g) \simeq 1$, we find that

$$\sigma = A(g_3/L_0)\,(\delta g)^{\nu} \qquad (7.7)$$

A is a constant of order unity. There is no minimum metallic conductivity. The conductivity goes to zero at the mobility edge with a universal exponent ν. There is a natural scale ~~of~~ for σ, i.e. (g_3/L_0). This is to be identified with

σ_{min}. The Mott estimate corresponds to assuming that beyond the microscopic length scale, L_0, one has classical diffusive behaviour so long as $g_0 > g_3$. However, for any $g_0 < g_3$, there are assumed to be strong ~~the~~ ~~quan~~ localization or interference effects because of which ~~changes~~ $g(L) \sim e^{-\alpha L}$. The corresponding $\beta(g)$ curve is shown by ~~a~~ ~~dotte~~ broken lines.

The form Eq. (7.7) was obtained by Wegner in 1976 from general scaling arguments.

~~As~~ The result Eq (7.7) can be written also as

$$\sigma = A\, g_3/\xi \qquad (7.8a)$$

where

$$\xi = L_0\,(\delta g)^{-\nu} . \qquad (7.8b)$$

One can think of ξ as a diverging correlation length. Diffusion is scale dependent or nonclassical over distances $\lesssim \xi$ and is classical ~~big~~ beyond ξ.

Now the quantity g_0 is a smooth function of atomic disorder and one can assume it to be linearly related to other parametrizations of disorder i.e. location of mobility edge E_c with respect to Fermi energy E_F. So one has

$$(g_0 - g_3) \propto (E_F - E_c)$$

so that

$$\sigma \propto |(E_F - E_c)/E_c|^{\nu} \qquad (7.9)$$

in three dimensions.

On the localized side, of the fixed point g_3, one finds using the ~~scales~~ form Eq. (7.6), that

$$g(L) = g_3 \exp\left[- A \, |\delta g|^{\nu} L / L_0 \right] \qquad (7.9a)$$

which means a localization length

$$\xi_{loc.} = (L_0 / A) \; |\delta g|^{-\nu} \qquad (7.9b)$$

We see that like in any continuous transition, the correlation length diverges in the same way on both sides of the transition.

<u>Two dimensions</u> :— The $\beta(g)$ curve is negative, so that no matter what the initial conductance at size L_0, the large length scale behaviour is always that corresponding to localized states, i.e. g decreases continuously with increasing L, and finally one scales into the localized regime. ~~Fo~~ ~~each~~ There is no truly metallic state in two dimensions.

To estimate the characteristic length scale, we use the perturbative form of $\beta(g)$ and integrate it to find

$$g(L) = g_0 - \frac{e^2}{\hbar \pi^2} \ln\left(\frac{L}{\pi L_0}\right) \qquad (7.10)$$

~~class~~ where

$$g_0 = \frac{e^2}{2\pi\hbar} \left(k_F L_0\right). \qquad (7.11)$$

We see from (7.10) that at length scales of order

$$\xi^{2d}_{loc.} \simeq \pi L_0 \exp\left(\frac{\pi}{2} k_F L_0\right) \qquad (7.12)$$

the conductance is strongly reduced by localization effects. Thus we identify ξ_{loc} with the localization lengths in two dimensions. It is exponentially large, i.e. depends exponentially on disorder, whereas in one dimension, the localization length is just the back-scattering mean ~~fr~~ free path, i.e.

$$\xi^{1d}_{loc.} \simeq \pi L_0 \qquad (7.13).$$

This is also the estimate one gets by using $\beta(g)$, Eq. (7.5) in one dimension.

8 Perturbation Theory :-

References :- (see previous section).
Also, J M Luttinger for transport theory using many body methods.
Also, S. Hikami Phys. Rev. B 24, 2671 (1981).

It is well known that the conductivity can be written as a current current correlation function (Kubo formula), ~~is~~. ~~For~~ One has

$$\sigma_{\gamma\gamma'}(0) = \frac{e^2\hbar}{2\pi L^d} \sum_{p,p'} \frac{p_\gamma}{m} G^{II\,+-}_{pp'} \frac{p'_{\gamma'}}{m} \qquad (8.1)$$

for electrons scattered by rigid random impurities (See Luttinger). G^{II} is a two particle Green's function.

a) Transport Equation formula :-

For a zero range potential, the simplest approximation

$$G^{II+-}_{pp'} = G^+_p \, G^-_p \, \delta_{p,p'} \qquad (8.2)$$

gives the transport equation conductivity. Here the single particle propagators G^+_p and G^-_p are given by

$$G^{\pm}_p(\nu) = \frac{1}{\nu - \epsilon_p \pm i\Gamma_p} \qquad (8.3a)$$

where $\Gamma_p = 2\pi |V|^2 \rho(\epsilon_F)\, n_i$ (8.3b)

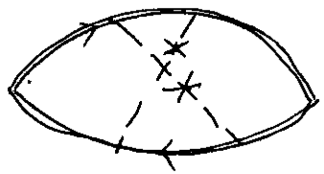
In (8.1) ~~the~~ frequency ν is zero, i.e. ~~and~~ the ~~electrons~~ excitations are at Fermi energy. Γ_p is just the rate at which a state p decays by scattering to other states p'. This scattering is caused by random impurities of concentration n_i. Using this we find

$$\sigma_{\gamma\gamma'} = \delta_{\gamma\gamma'} \frac{ne^2\tau}{m} \quad \text{with} \qquad (8.4)$$

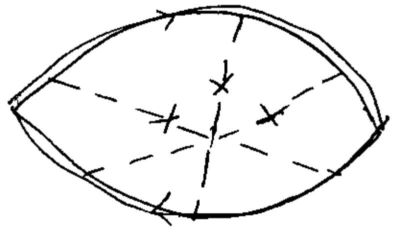
$$\Gamma = k/\tau .$$

(b) Singular backscattering :-

In attempting to go beyond the leading ~~lowest~~ order in concentration and ~~[crossed out]~~ in trying to develop a systematic expansion for σ in powers of n_i, Langer and Neal* found that diagrams of the type Fig. 2 all contribute in 3d, a term of the type $n_i^3 \ln n_i$ to σ.



(Fig. 2a)



(Fig 2b)

* Langer and Neal PRL 16, 984 (1966)

AALR realized that this meant that all terms in this series have to be considered together, and summed. Physically, this means that all such processes leading to singular backscattering have to be considered together. Fig. 3 shows these terms.

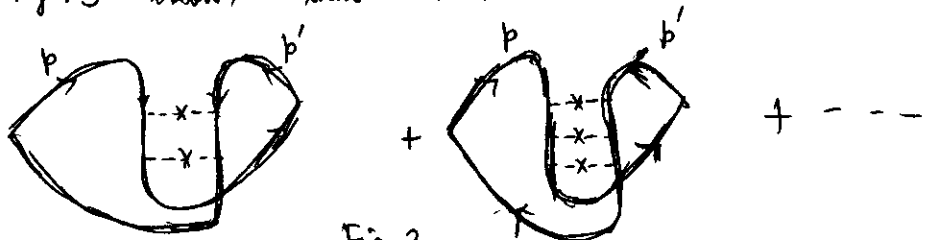


Fig. 3

On ~~sum~~ doing this summation, we find that the scattering amplitude

$$W(p,p') \propto \frac{d\ (n_i v^2)^2}{(p+p')^2 (v_F^2 \tau^2)} \qquad (8.5)$$

i.e. it is singular for $p = -p'$. That is, there is ~~an~~ singular interference between forward motion and backward motion due to scattering from the random impurities.

Vollhardt and Wölfle (Phys. Rev. B 22, 4666 (1980)) have shown how the form (8.5) is related to time reversal invariance and the diffusion propagator.

Including only this process,

which will reduce the conductivity, we find

$$\sigma = \frac{ne^2\tau}{m} - \frac{2e^2}{k\pi}\frac{1}{L^d}\sum_q \left(\frac{1}{q^2}\right) \qquad (8.6)$$

so that ~~with~~ for a finite system where $q_l \simeq \frac{\pi}{L}$ and $q_u \simeq \pi/L_0$, one has

$$\sigma_{3d}(L) = \sigma_0 - \frac{e^2}{k\pi 3}\left(\frac{1}{L_0} - \frac{1}{L}\right) \qquad (8.7a)$$

$$= \sigma_0 - \frac{e^2}{k\pi^2}\ln\left(\frac{L}{L_0}\right) \qquad (8.7b)$$

$$= \sigma_0 - \frac{e^2}{k\pi}\left[L - L_0\right] \qquad (8.7c)$$

(i) ~~3~~ (3d) :-

$$\sigma_{3d}(L) = \frac{e^2}{k\pi^2}\frac{k_F L_0}{3} k_F \left[1 - \frac{3}{\pi}\frac{1}{(k_F L_0)^2}\right] + \frac{e^2}{k\pi L} \qquad (8.8)$$

There seems to be an expansion in $\left(\frac{1}{k_F L_0}\right)^2$.

However, the scale dependent term is significant.

For example, when $k_F L_0 = \sqrt{\frac{3}{\pi}} \simeq 1$,

$\sigma_{3d}(L) = \sigma_{Mott}$ at $L = L_0$, but

$\boxed{5} = 0$ as $L \to \infty$. At this mobility edge, The diffusion constant decreases as L^{-1} with increasing L, a highly nonclassical behaviour.

(c) Negative Magnetoresistance :-

It is clear that the singular backscattering term, depending on the interference between the motion of an electron and its time reversed pair, will be strongly affected by a magnetic field which breaks time reversal invariance. The magnetic field (vector potential) affects the phase of the pair and dephases them in a distance λ_H such that

$$H \lambda_H^2 \sim \frac{c\hbar}{e} \qquad (8.9a)$$

$$\text{or } \lambda_H = \sqrt{c\hbar / eH} \qquad (8.9b)$$

$\boxed{6}$ Typically $\lambda_H \sim 800 \text{ Å}$ for $H \sim 1\,kG$. Thus even moderate fields dephase effectively, ~~there~~ reducing the singular backscattering effect which decreases σ. One thus has a large negative magnetoresistance.

[Altshuler, Aronov, Khmelnitskii & Lee, Phys Rev. B 22, 5142 (1980)

Hikami, Larkin and Nagaoka, Prog Theo. Phys. 1980]

In two dimensions for example,

$$\delta\sigma = -\frac{e^2}{2\pi^2\hbar} \ln\left(\frac{\hbar c}{2eH L_0^2}\right) \qquad (8.10)$$

~~where~~ for $\lambda_H \ll L_{Th}$

so that λ_H is the smallest scaling cutoff length.

The corresponding result in 3d was obtained by Kawabata who found

$$\delta\sigma = 0.605 \frac{e^2}{2\pi^2\hbar} \frac{1}{\lambda_H} \qquad (8.11a)$$

$$\text{i.e. } \delta\sigma \propto \sqrt{H} \qquad (8.11b)$$

This characteristic negative magnetoresistance has been observed in 2 and 3d.

d) Higher Order :-

Higher order terms ~~have~~ in $\beta(g)$ have been looked for by Hikami (Ref. above) and by Khmelnitskii. There are no terms upto $O(1/g^3)$.

9. Beyond Perturbation Theory

a) Numerical Methods.

I will only mention references to recent numerical calculations of $\beta(g)$. These are expected to be more accurate for moderate rather than weak disorder.

(i) D S Fisher and PA Lee Phys Rev. Lett.

(ii) McKinnon and Kramer

(iii) Richard and Sarma

b) Self consistent theory of localization

A theory which generalizes the backscattering term of AALR so that (in 2d) ~~both~~ ~~extended and~~ the localized regime is described as well, is due to

Vollhardt and Wölfle Phys Rev B 22, 4666 (1980).

10. Interaction Effects :-

a) Inelastic cutoff :-

(See refs. in Section 6 and Anderson, Abrahams & Ramakrishnan Phys Rev. Lett. (1979).

The above results are for a disordered system at $T=0$ and in which the electrons do not interact. One effect of electron phonon or electron electron interactions is to induce energy changing collision which dephase the electronic state and thus cut off scaling at a length L_{Th} as discussed in Section 6 [Eq. (6.8)]. Assuming that $L_{inel.} \sim T^{-p}$, one has

$$L_{Th} = C\, T^{-p/2} \qquad (10.1)$$

so that the scale dependent conductance calculated in Sect. 8 for $T=0$ translates into temperature dependent conductance :-

$$\sigma_{3d}(T) = \sigma_0 - \frac{e^2}{\hbar\pi^3}\left(\frac{1}{L_0} - \frac{T^{p/2}}{C}\right) \qquad (10.2a)$$

$$= \sigma_0 + \frac{e^2}{\hbar\pi^2}\frac{p}{2}\ln\left(\frac{T}{T_0}\right) \qquad (10.2b)$$

$$= \sigma_0 - \frac{e^2}{\hbar\pi}\left[C T^{-p/2} - L_0\right] \qquad (10.2c)$$

In 2d for example, a logarithmically temperature dependent σ decreasing as T decreases, with a slope $(e^2 p / 2\pi\hbar^2)$ should be observed. We ~~We shall see later that this is~~

(b) Coulomb interaction effects :-

Altshuler and Aronov† found in 1979 that there are ~~significant~~ characteristic interaction effects on density of states and conductivity in ~~a~~ disordered interacting Fermi system. Related effects, on ~~spi~~ spin susceptibility and quasiparticle lifetime, had been found earlier by Brinkman and Engelsberg (PRL 1969-1970) and by A. Schmid (Z. Phys. 1974) respectively but their significance was not realized. The actual calculation of this effect is a long exercise in many body theory, but its flavour ~~can be~~ ~~obtained~~ comes through in ~~the~~ following calculation of the density of states.

bi) Density of states :-

Consider electrons in a random potential. The exact eigenstates are $|m\rangle$ say with wavefunctions $\psi_m(r)$. In this representation, with a two body interaction potential $v(r-r')$, the Hamiltonian of the system is ††

$$H = \sum_m \epsilon_m a^+_m a_m + \sum_{npqs} a^+_p a^+_n a_q a_s \iint \psi^*_p(r)\psi_q(r)\, v(r-r')\, \psi^*_n(r')\psi_s(r') \qquad (10.3)$$

† Altshuler and Aronov, Solid St. Comm. 30, 115 (1979)
JETP Sov. Phys. 50, 968 (1979)

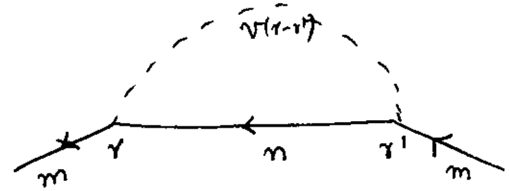
Altshuler, Khmelnitskii, Larkin & Lee, Phys Rev. B 22, 5142 (1980).

†† Abrahams Anderson Lee & Ramakrishnan, Phys Rev B Dec 1981.

Consider the lowest order energy shift of the state m due to interactions.

$$\Sigma_m = - \sum_{n\ \text{occupied}} \int d\vec{r}\, d\vec{r}'\, v(r-r')\, \psi^*_m(r)\, \psi^*_n(r')\, \psi_m(r')\, \psi_n(r) \qquad (10.4)$$

This is diagrammatically shown in Fig. 4.



If we insert a particle at energy E, its energy will be shifted on the average by an amount

$$\Sigma(E) = \frac{1}{N(0)} \Big\langle \sum_m \delta(E-E_m)\, \Sigma_m \Big\rangle_{\text{configuration average}} \qquad (10.5a)$$

$$= - \int_{-\infty}^{0} dE'\, F(E,E'; r,r')\, v(r-r')\, d\vec{r}\, d\vec{r}' \qquad (10.5b)$$

where
$$F(E,E'; r,r') = \sum_m \delta(E-E_m)\,\delta(E'-E_n)\, \psi^*_m(r)\psi^*_n(r')\, \psi_m(r')\,\psi_n(r). \qquad (10.5c)$$

F is simply related to the density density correlation function

$$A(\omega,q)\ \cancel{A(q,\omega)} = \int_{-\infty}^{+\infty} dr\, dt\, e^{i\omega t + iq\cdot r} \langle [\rho(r,t), \rho(0,0)] \rangle . \qquad (10.6a)$$

$$\simeq \frac{\partial n}{\partial \mu}\, \mathrm{Im}\left[\frac{Dq^2}{-i\omega + Dq^2}\right] \qquad (10.6b)$$

The relation is ~~that~~ the following. (25)

$$F(E,E';r,r') = F(E-E';r-r') = F(\omega,\vec{p})$$

$$A(\omega,q) = \int F(\omega,p)\, e^{iq\cdot\rho}\, d\rho$$

One thus has

$$\Sigma_E = \left(\frac{\partial n}{\partial \mu}\right)\int_{-\infty}^{0} dE' \int d\vec{q}\, \frac{Dq^2}{(E-E')^2+(Dq^2)^2}\, v(q). \qquad (10.7)$$

The main physical point is that the interaction is via density fluctuations, and these have a diffusive behaviour. [Eq. (10.6)]. This means large spectral strength at low ω and q, and this leads to a significantly energy dependent shift Σ_E. The change in density of state is

$$\delta N(E) = \left(\partial \Sigma_E / \partial E\right) \qquad (10.8)$$

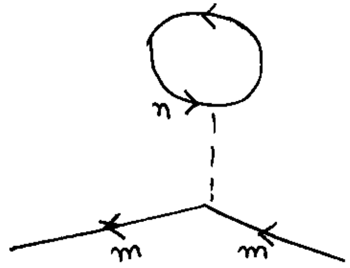
Calculation based on (10.7) leads to a logarithmically divergent term in 2d and a $\sqrt{E}$ correction in 3d. Schematically, in 3d,

$$\frac{\delta N(E)}{N(0)} \sim \frac{v(q=0)}{(Dk_F^2)^{3/2}} \sqrt{|E|} + C \qquad (10.9)$$

where C is a constant such that for $E \sim q_c v_F = (\pi v_F / L_0) \simeq (v_F^2/D)$ the correction ~~vanishes~~ vanishes and the density of states joins on smoothly to the bare value. The sharp cusp described by (10.9) ~~is larger, the smaller~~ increases with decreasing D. Thus with increasing disorder, such diffuson

(26)

effects become larger. In general there are two contributions to the the density of states correction, to lowest order in v. One is the exchange term already discussed above; ~~the~~ other is a Hartree term shown below. (Fig. 5)



This also contributes similarly in energy dependence, but its sign is opposite so that in general (in 3d)

$$\frac{\delta N}{N(0)} \propto (\underbrace{1}_{\text{exchange}} - \underbrace{F}_{\text{Hartree}})\sqrt{E} \qquad (10.10)$$

If v is a screened coulomb interaction and a Thomas-Fermi screening approximation is used, $F<1$ since the Hartree term involves summing over all momentum transfers q (in $v(q)$). However, if the interaction is zero range, it can be only between opposite spins and there is no exchange term. So both the size and sign of the density of states correction depend on the nature of two body interaction.

b ii) Inelastic rates :-

The fact that electrons do not move in a straight line in a disordered system, but diffuse ~~[struck]~~ enhances ~~the~~ coulomb interaction effects because they 'stick around' longer. One dramatic effect is in ~~the~~ quasiparticle lifetime. In a pure Fermi system, the rate Γ at which a fermion with $E=0$ decays into another fermion and a particle hole pair goes as T^2 ~~in all dimens~~ independent of dimension.

It was pointed out by ~~[struck]~~ Schmid ~~the~~ for the three dimensional case that diffusion effects of the sort described above enhance inelastic rates :- in three dimensions, $\Gamma \sim T^{3/2}$. This could be part of the reason why ~~in experiments the~~ localization effects are ~~[struck]~~ experimentally rather small; large Γ means small L_{Th} which sets the scale for localization effects at any nonzero temperature.

The calculation for ~~[struck]~~ Γ proceeds as above, except that ~~a statically screened~~ the static potential is replaced by a dynamically screened potential, e.g. in 3d

$$V_{c3}(q, \omega_n) = \frac{4\pi e^2}{q^2} \, \frac{|\omega_n| + Dq^2}{|\omega_n| + DK_3^2} \qquad (10.11)$$

where K_3^{-1} is the screening length.

The result in (3d) is $\Gamma(T) \sim T^{3/2} T_F^{-1/2}$ (10.12)

The calculation in 2d leads to

$$\Gamma_{2d}(T) \approx \frac{T}{(Dk_F^2)\, N(0)} \ln\left(T_F/T\right) \qquad (10.13)$$

Thus the lifetime is shortened by a large factor $(T_F/T)\ln(T_F/T)$ with respect to the clean limit value $\sim (T^2/T_F^0)$. ~~That~~

b iii) conductivity and magnetoresistance :-

The conductivity corrections ~~were first calculated by AA', and are~~ are

$$\delta\sigma_I = \frac{e^2}{\hbar}\frac{1}{4\pi^2}(2 - 2F)\ln(T\tau) \qquad (2d)\ (10.14a)$$

(Altshuler . . . Lee)

$$= \frac{e^2}{\hbar}\frac{1}{4\pi^2}\frac{3}{\sqrt{2}}\left(\frac{4}{3} - 2F\right)\sqrt{\frac{T}{D}} \qquad (3d)\ (10.14b)$$

(Altshuler and Aronov).

For 2d, we note that the $\ln T$ dependence is similar to that from localization. The size is also comparable. So the observed effects could be due to a combination of localization and interaction ~~effects~~ and it is of special interest to find physical ~~phenomena~~ effects which distinguish between them. Two such are the Hall effect and magnetoresistance. We discuss the latter below.