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ELECTRON LOCALIZATION IN DISORDERED SYSTEMS

(Part III)

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4. Variable Range Hopping .–

A good reference is Mott and Davis.

5 Interaction Effects

i) Correlation and Electron lattice interaction.

The two (+ve U and -ve U) are discussed in depth by Anderson in the Les Houches lectures for 1978 (Ill- Condensed Matter, eds. Balian & Toulouse)

ii) Long range coulomb interactions

Efros and Shklovskii J Phys C L 1975

Efros , Baranovskii . . . JETP 1981
Sov. Phys.

(b iv) Magnetoresistance :-

We have seen earlier that even a weak magnetic field affects localization adversely. The magnetic field ~~affects~~ dephases the the correlated state ~~of~~ consisting of an electron and its time reversed (backscattered) pair moving in the random potential. More formally, the backscattering term is in the particle particle channel, which is sensitive to the magnetic flux (e.g. as in superconductors). However, the effects on the ~~transport~~ properties discussed above for an interacting system have all to do with the propagation of density fluctuations, i.e. they are in the electron hole channel. Consequently, the above orbital sensitivity to magnetic field is absent. ~~However~~, The only effect left is the splitting of up and down spin bands in a magnetic field (Lee and Ramakrishnan, 1981, Kawabata 1981).

The physical idea is most directly illustrated for the self energy correction. This is due to the correlation between the wavefunction of the added electron with the wavefunction of occupied electrons that are nearby in energy. The exchange term and the equal spin Hartree term are insensitive to the magnetic field, since they involve states of same spin. For the 'opposite spin Hartree term, there is an energy difference $g\mu_B H$ between the electron hole states. For $g\mu_B H >> k_B T$, the singularities due to this term

are cut off by $g\mu_B H$ rather than by $k_B T$. Detailed calculation shows that there is a field dependent term

$$\delta\sigma_I''(H,T) = -\frac{e^2}{\hbar}\frac{F}{4\pi^2}\, g_2(h) \qquad (10.15a)$$

$$= -\frac{e^2}{\hbar}\frac{F}{4\pi^2}\, g_3(h) \qquad (10.15b)$$

where $g_2(h)$ is a known function of $h = (\mu H/kT)$ having the asymptotic values

$$g_2(h) = \ln(h/1.3) \qquad h >> 1 \quad (10.16a)$$

$$= 0.084\, h^2 \qquad h << 1 \quad (10.16b)$$

The function $g_3(h)$ has the asymptotic forms

$$g_3(h) = \sqrt{h} - 1.3 \qquad h >> 1 \quad (10.17a)$$

$$= 0.053\, h^2 \qquad h << 1. \quad (10.17b)$$

We thus see that the magnetoresistance is positive, going as h^2 for small h and as $\ln h$ (2d) and $\sqrt{h}$ (3d) for large h. The field scale is set by $h = (\mu H/kT) = 1$. We discuss later how this can be used to analyze the relative importance of interaction and localization effects in disordered systems.

(b v) Magnetic susceptibility :-

The magnetic field dependence of the Hartree term suggests that the spin susceptibility will be affected characteristically by interaction terms in a disordered electron gas. This term was first discussed by Fukuyama ~~using~~ who considered diagrams for the transverse spin correlation function. However, the same result is obtained more directly and simply by considering the field dependence of the Hartree term for the electron gas free energy, ~~or~~ which for a zero range interaction U is

$$\delta F_0(H) = U \langle \psi^{\dagger}_{\uparrow}(\vec{r})\, \psi_{\uparrow}(\vec{r})\, \psi^{\dagger}_{\downarrow}(\vec{r})\, \psi_{\downarrow}(\vec{r}) \rangle$$

This can be regarded as the equal time equal space limit of the transverse spin spin correlation function, which has a diffusive form in (q, ω) variables. We have a diffusive correction

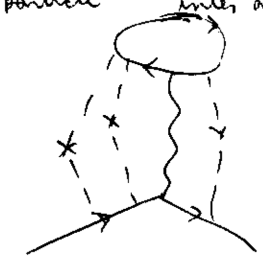
$$\delta F_0(H) = -\frac{U}{\tau^2} \sum_{q,\omega_m} \frac{|\omega_m|\tau}{[|\omega_m| + Dq^2]^2 + 4\mu_B^2 H^2}$$

Since $\delta\chi = -\partial F/\partial H^2$, there is a susceptibility enhancement; e.g. a temperature dependent part going as $\Delta\chi \sim [U\rho^2(\epsilon_F)\epsilon_F / k_F l]\chi \times \ln(T\tau)^{-1}$ in 2d and as $\sqrt{T}$ in 3d. The correction depends on temperature (or on length scale or frequency) and on diffusion constant in exactly the same way as the conductivity correction due to interactions, and becomes ~~zero~~ larger with increasing disorder. Thus one can expect a susceptibility enhancement and slowing down of spin diffusion near the mobility edge.

(b vi) Electron phonon interaction :-

We have not so far discussed the electron phonon interaction in an impure metal. This was done carefully by A. Schmid in a series of papers published in Z. Physik, 1974-76. He showed that because electrons scatter from ~~moving~~ impurities ~~and~~ moving with the lattice and because even diffusing electrons follow the ionic motion adiabatically, the electron phonon coupling is unaffected by disorder. There are no diffuson effects of the kind found for coulomb interactions.

Ramakrishnan (to be published) has pointed out that there is a new disorder induced electron phonon interaction effect. This is the analogue of the Hartree term for two particle interactions (See Fig.).



In the figure, the wavy line is a phonon, and the dotted lines with crosses represent impurity scattering. The diagram represents the effect on an added electron of the lattice distortion produced by electrons in the neighbourhood. This term is absent in a pure system, where because of momentum conservation, the phonon exchanged has $q = 0$. Since this corresponds to uniform translation, there is no such process. However because impurities take up the momentum in a disordered system, the phonons exchanged can have any momentum (but zero frequency, so it's a virtual process) and in fact the interaction is local in space.

It can be shown that this term acts to correlate opposite spin electrons and is best described as a negative U term, i.e. an incipient negative U or lattice distortion term in a disordered metal. It leads to a dip in the density of states, a localizing contribution to the conductivity and a diamagnetic term in the susceptibility. It is tempting to speculate that in systems with large electron phonon coupling, this emerges on the localized side as Anderson's negative U term. If so, the characteristics of many disordered systems (e.g. chalcogenide glasses) close to the boundary of the metal insulator transition, and with large electron lattice coupling can be discussed from the metallic side. There are a number of such systems which show, for example, small activation energies $\approx k_B T$ for conductivity.

II. Beyond perturbation theory with interactions

We have outlined above some characteristic interaction effects in disordered systems. They have all been calculated to leading order in interaction, and to lowest significant order in $(k_F l)^{-1}$ i.e. in disorder or resistivity. Three different types of interaction effects seem to emerge.

(i) Coulomb or charge fluctuation interaction i.e. screened coulomb interaction, with exchange and Hartree terms.

(ii) Short range correlation terms. These are between opposite spins only in the limit of zero range. ~~In principle~~, They are included in the ~~Hartree~~ opposite spin Hartree term, whose size may therefore differ from say the value corresponding to Thomas-Fermi screened coulomb interaction. For the latter, $F < 1$. However, if there are genuine short range correlations U, one can have $F > 1$.

(iii) Electron lattice interaction. This is a short range, disorder induced deformation term, acting effectively like a negative U.

There is as yet no complete theory of what happens when both the disorder and interaction are strong, e.g. as one approaches the metal insulator transition. Localization and interaction effects are strongly coupled to each other, and it is not known whether the consequences are the same for all types of interaction effects etc.

McMillan (Phys. Rev. B 1981) has presented an ~~attractive~~ scaling theory for the metal insulator transition which is a direct combination of the results discussed above, with the additional idea that screening becomes poorer with increasing disorder. He considers the conductance g_L, or diffusion constant D, the effective coulomb interaction u, and the density of states N as functions of scale size L. It is clear that they are coupled to each other. He finds that the ~~dip in the density of st~~ conductivity transition is nearly as in the scaling theory. The dip in the density of states becomes stronger as the mobility edge is approached, and at critical disorder, $\rho(E) \sim |E-E_F|^{1/2}$. This seems to be seen in experiments by Dynes & Garno (Phys. Rev. Lett. 1980) on granular Al, where the density of states is measured by tunnelling.

The screening length diverges near the transition however the static screening is ~~screening regime is very~~ only over a very narrow frequency regime.

The results of McMillan, though appealing are based on lowest order scaling equations for ~~disorder and scaling~~ localization and interaction effects. Further there are no correlation and electron lattice interaction terms. This area is still an early stage of development.

~~12~~. ~~Comparison with Expt~~

12. Experiments on Localization and Interaction Effects —

In the last few years, there have been a number of careful low temperature experiments on the transport properties of disordered metals, ~~begun~~ designed to examine in detail the onset of localization with increasing disorder. We briefly summarize them here.

12a. — Two dimensions :— A large number of experiments on thin films with $R_\square$ ranging from 25 Ω to 10,000 Ω, and on Si inversion layers with $R_\square$ ranging from 300 Ω to 8,000 Ω show the presence of a $\ln T$ term in the conductivity. The term has very nearly the universal size expected.

In a number of cases magneto-resistance ~~not~~ has been measured. The ~~most~~ ~~thanks~~ most definitive work is on Si inversion layers, by Dynes, Bishop and Tsui (Phys. Rev. Lett. 1981)†, Pepper and coworkers (J. Phys. C Lett.) ~~Kawaguchi et al (J P S J 1980)~~. These authors, especially Dynes et al, have measured $\sigma(H,T)$; Hall coefficient $R(H,T)$ for various levels of disorder. They have obtained quantitative fit to a theory in which

† Also R.G. Wheeler et al, Phys Rev '81

$$\delta\sigma(H,T) = \delta\sigma_{loc}(H,T) + \delta\sigma_{int.}(H,T).$$

They have determined the relative size of the terms ξ. The unknown parameters are the inelastic length L_i and its temperature dependence, and the size of the Hartree term F. For a given disorder, $\delta\sigma(H,T)$ over a wide range (including $h \ll 1$ and $h \gg 1$, as well as $l_H \ll L_i$ and $l_H \gg L_i$) is well fitted by theory. The index p is found to be unity; this as well as the size of L_i agree with the theoretical estimate of this quantity which considers diffusion or disorder enhancement of quasiparticle decay rates. The Hartree term F is found to be large and to increase from $F = 1$ at $R_\square \sim 1000\,\Omega$ to $F = 3$ at $R_\square \sim 6000 - 8000\,\Omega$. This is interesting and unexplained. It seems to be clear that :—

i) there is no true metallic state for a disordered two dimensional system

ii) interaction effects are important and

iii) the ~~localization behaviour~~ effect of disorder and of interactions is described in detail by theory.

There is however, no measurement in which, as temperature is lowered, the resistivity first increases logarithmically and finally exponentially according to the variable range hopping characteristic of localized states. That is, there are no direct measurements in which

L_i spans the range $L_i \ll l_{loc}$ to $L_i \gg l_{loc}$ as temperature is lowered. This is very hard to do for a moderately disordered system since l_{loc} is exponentially large so that exponentially low temperatures are required. At these temperatures, electron heating effects require very low measuring voltages,

<u>3d</u> :— Some of the above predictions e.g. $\sigma(0) \ll \sigma_{min}$, $\sigma(H,T) \propto \sqrt{H}$ have been verified [Rosenbaum, Thomas et al. 1980, 1981, 1982 (Phys. Rev. Lett.)]. In case of $\sigma(H,T)$ fits are quantitative to data on <u>P</u>:Si. However, the region very close to critical disorder has not been explored much. There is one measurement of $\sigma(0)$ on (P:Si) and another on $Ge_x Au_{1-x}$ where $\sigma \ll \sigma_{min}$ has been found, in a very narrow disorder or P concentration range for P doped Si. It has been suggested by Mott that macroscopic inhomogeneities could cause average σ to be less than σ_{min} even if there were an intrinsic minimum metallic σ. Since it seems clear that doping cannot be done with the precision needed to study σ near critical disorder, a recent experiment tries stress tuning. Namely, at a particular disorder, the metal

transition in <u>P</u>:Si is spanned by applying stress. The authors quote a critical index $\nu \simeq 1/2$, though most of their data points lie above σ_{Mott}, the lowest σ observed being ($\sigma_{Mott}/5$).

The detailed nature of the metal insulator transition (e.g. as seen in $\sigma(H,T)$, Hall coefficient, density of states, susceptibility, spin diffusion, thermopower) is experimentally an open question.