



INTERNATIONAL ATOMIC ENERGY AGENCY  
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION



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SPRING COLLEGE ON AMORPHOUS SOLIDS  
AND THE LIQUID STATE  
14 April - 18 June 1982

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ELECTRONIC STATES IN DISORDERED SYSTEMS  
(Part II)

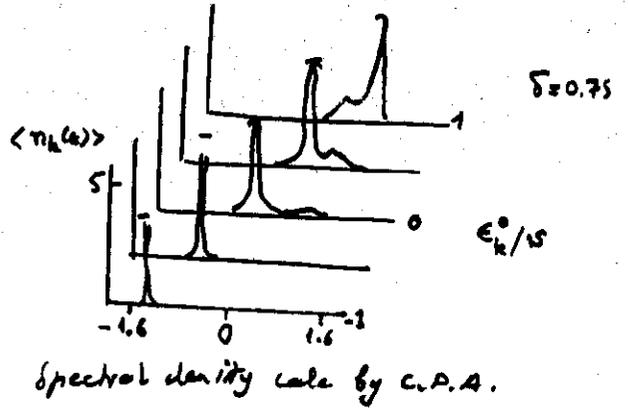
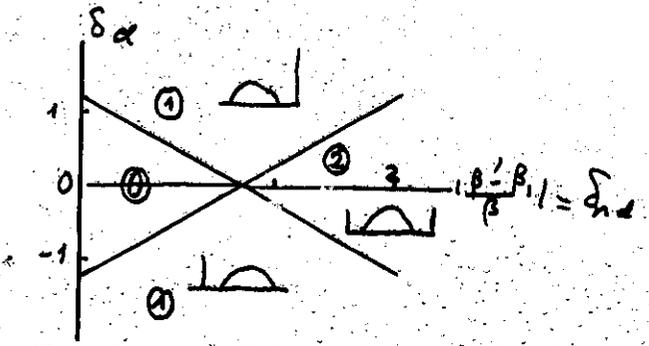
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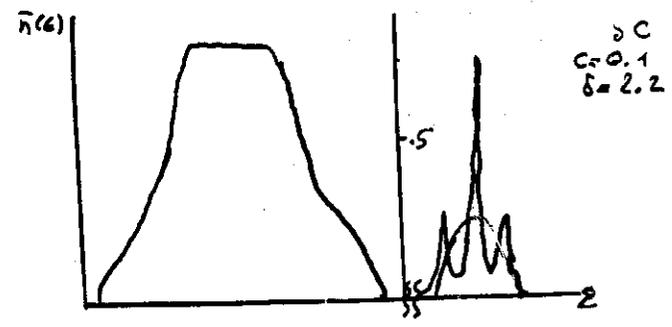
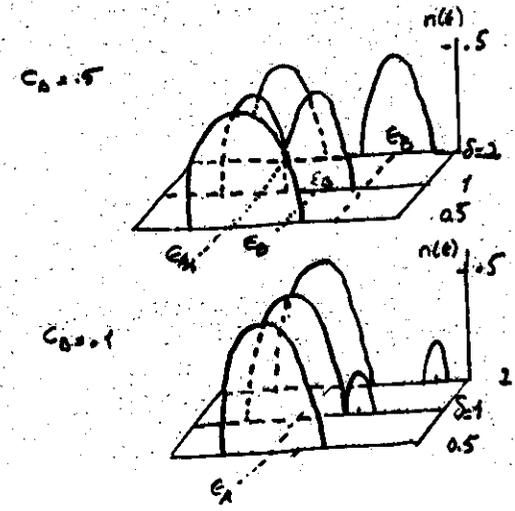
These are preliminary lecture notes, intended only for distribution to participants.  
Missing or extra copies are available from Room 230.



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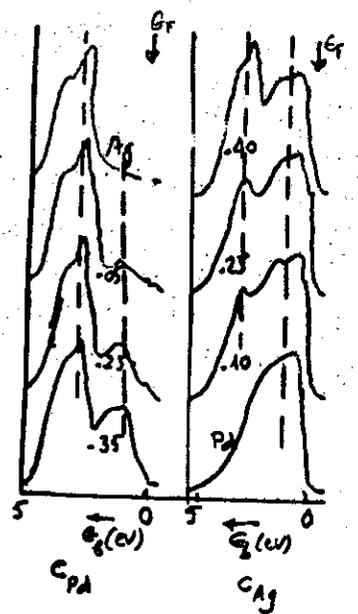


2



ELECTRONIC STRUCTURE OF SOME DISORDERED SYSTEMS.

III. AMORPHOUS  $\frac{1}{2}$  CVD, METALS AND ALLOYS  
 • SUBSTITUTIONAL BINARY ALLOYS

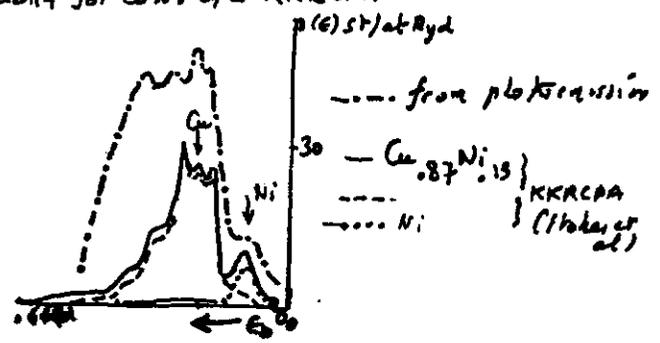


Photoemission spectra of PdAg alloys (see the split band region)

CPA calculations can be done in the same way for opt prop. conductivity... (see ref)

WE PRESENTED TB CPA BUT KKR CPA IS POSSIBLE... (see below: calculation for CuNi in a KKR CPA calc)

occupied states in CuNi alloys (eV)



Introduction

① Topological disorder Amorphous  $\frac{1}{2}$  CVD

- small fluctuations of local envt ⇒ small fluct of  $\rho(E)$
- difficult to study: (recursion moments) ⇒ opt...?
- Stability  $\sim g(E)$ , local envt?
  - orders of magnitude of energy
  - no pair interactions to study the structural part of the energy (compare with Hohen's lectures)

② Crystalline Binary d.m. alloys | Chemical disorder

- can be large ⇒ large variations of  $\rho(E)$  (split band regime)
- C.P.A. alloys to study physical properties (transport, optical, magnetic...)
- Perturbation theory allows to determine structural part of the energy w/out structural SRO (local order) but investigate stable phases.
- ⇒ simple model to study the effects of d.T.

# ① Amorphous 1/2 cond. (tetracond) IV

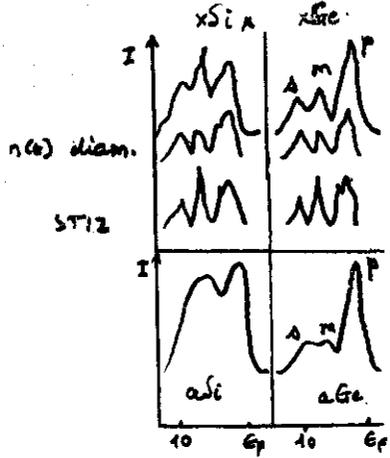
## ②. STRUCTURE

- small fluctuations of the bond lengths (~1%)
- angles (~10%)
- z = 4 ideal = without dangling bonds
- defects?

### CONTINUOUS RANDOM NETWORK (CRN)

contains usually five fold seven fold rings (Polk)  
 However Conwell Temkin's model does not contain odd numbered rings.

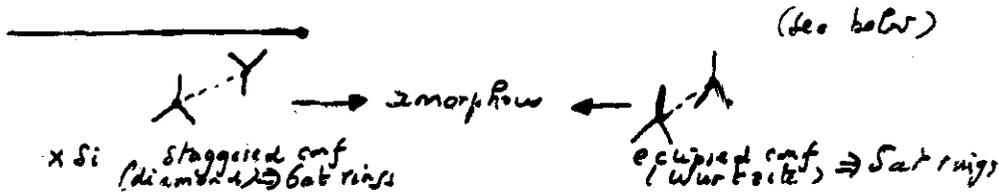
## ③. Photoemission and electronic structure



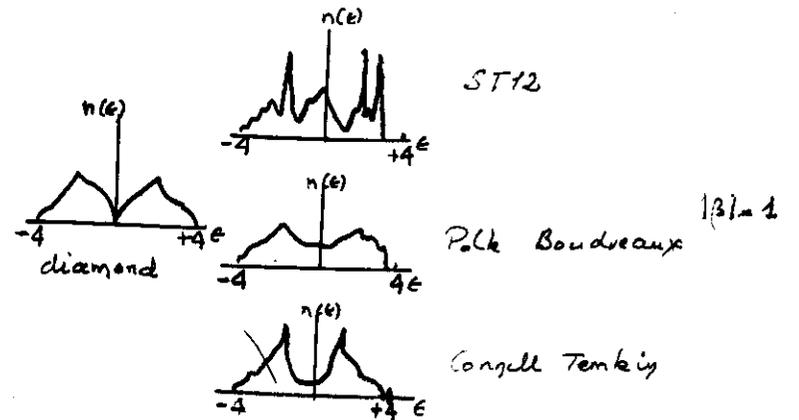
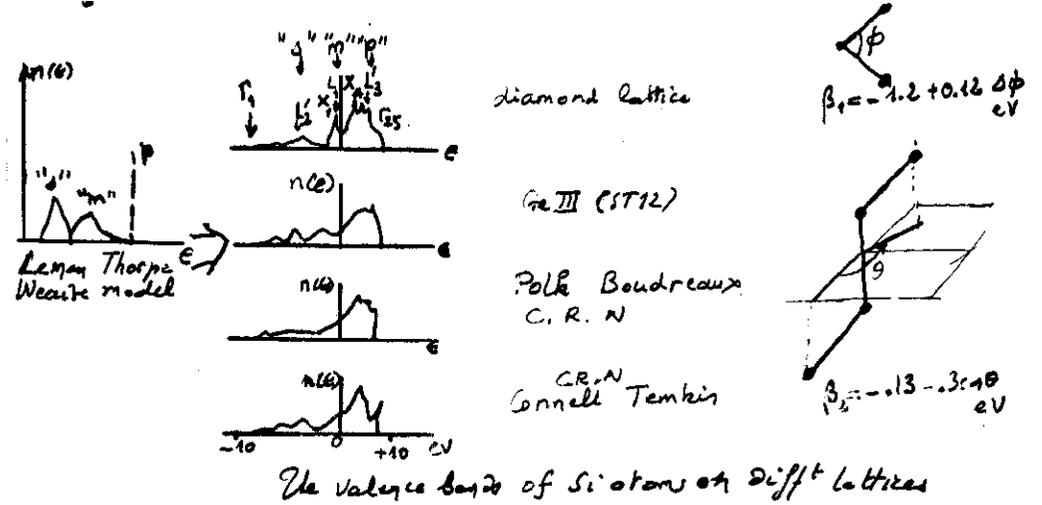
in crystalline Si 3 peaks  
 's' bottom of the valence band  
 'p' top  
 mixed in the middle

in amorphous Si 2 peaks

- note that the 'p' peak is not modified by disorder ( $E_p - |p|$ ) of the simplified LFTW model
- Relation with the one bond TB model  $\Rightarrow$  disorder affects only the central dip and the width of the VB according to the presence of odd numbered rings.



(see below)



TB band issued from 's' from degenerate orbitals (CBO) odd numbered rings prevent the construction of the totally antibonding states (ST12 and Polk B.)

The filling of the pseudo gap can be understood as follows (Gaspard): (Crude model)

$n(\epsilon)$  = average of eigenvalues of rings of diff lengths broadened (from their interaction)

But the  $E_\ell$  of a ring of length  $n$  are

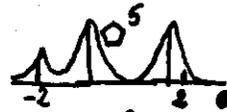
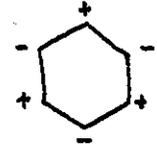
$$E_\ell = -2 \cos \frac{2\pi \ell}{n} \quad (\ell = 0, 1, \dots, n-1)$$

$\epsilon = -2$      $\ell = 0$     (bonding state)

$\epsilon = 2$      $\ell = \frac{n}{2} = \frac{n}{2}$      $E_{\frac{n}{2}} = -2 \cos \pi = 2$

$\ell = p$      $E_p = -2 \cos \frac{2\pi p}{n}$   
 $= -2 \cos(\pi) \left(1 + \frac{1}{2p+1}\right) \dots$

even  
 $n = 2p$   
 odd  
 $n = 2p+1$

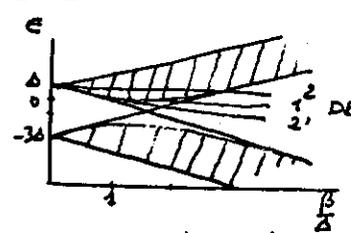


© Hydrogenated amorphous Si

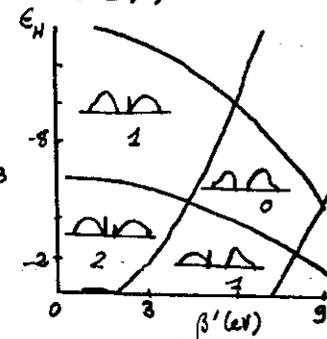
- removal of the states from the gap
- widening of the optical gap
- alteration of both the VB and CB  $n(\epsilon)$

Density of states of hydrogenated a Si (crude model)

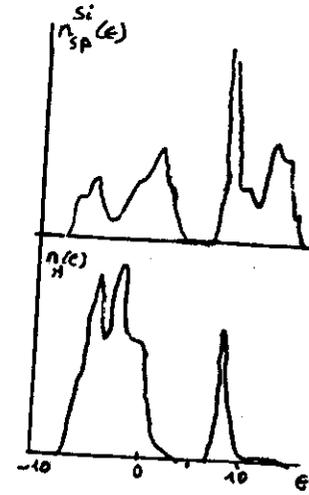
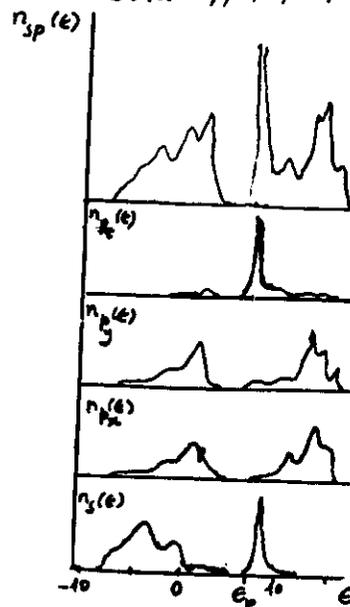
②  $\frac{1}{2} DB + H$



— D.B states in the Bethe Approximation



Nb of Localized states in the gap



$E_g = 0$   
 $E_p = 7.2 \text{ eV}$   
 $SSC = 2.03 \text{ eV}$   
 $SP = 2.55 \text{ eV}$   
 $HP = 4.5 \text{ eV}$   
 $PP = 1.09 \text{ eV}$

$SSC_{HSC} = -2.5 \text{ eV}$   
 $SP_{HSC} = 4.1$   
 $E_H = -1 \text{ eV}$   
 resolution 1 eV

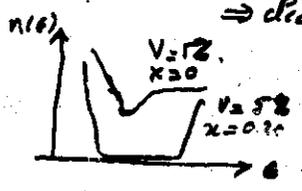
Density of state on a Si atom bearing a D.B pointing in the z direction and with 3 sp<sup>2</sup> back bonds (see the peak in the  $n_p(\epsilon)$  for  $\epsilon = (\epsilon_0 + 3\epsilon_p)/4$ )

By hydrogenation the "p" DB is pushed in the conduction band and the local DOS for H has a 3 peak structure.

$sp^{n-1} \Rightarrow G_{db}(z) = \frac{1}{z - (n-1)\alpha^2}$  (Dette Lattice)  
 $\Downarrow$   
 $G_{db}(z) = \frac{1}{z - (n-1)\alpha - \beta^2 G_{db}}$  ;  $G_H(z) = \frac{1}{z - \epsilon_H - \beta^2 G_{db}}$   
 db peak for  $\beta/\alpha > \sqrt{n-1}$  at  $E_{db} = (\epsilon_0 + (n-1)\epsilon_p)/4$   
 with a weight  $\frac{1}{2 - (n-1)\alpha^2 \beta^2}$

Lemaire Gaspard J. de Phys 64, 765 (1982)

⑥ "Simulation" with  
 -  $x$  Si + random vacancies  
 $\Rightarrow$  D.B filling the gap  
 + H (1, 2, 3, 4) attached to one of the DB (random)  
 $\Rightarrow$  clear the gap, increase the gap  
 SiH<sub>x</sub>

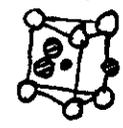


② Amorphous metallic alloys

① Structures

- + Dense Random Packing of hard spheres (DRPH)
  - absence of directional bonds
  - tendency for at to be packed as tightly as possible (without LRO)
  - starting from a triangle max of tetrahedra  $\rightarrow$  icosahedra...
- + Alloys  $\rightarrow$  • DRPH with two hard sphere diameters

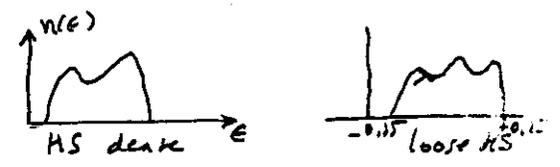
(He)<sub>3</sub> in  
'prismatic'  
units



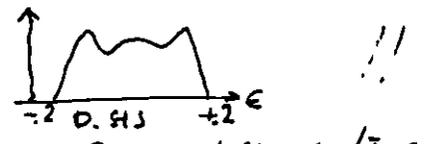
$\Rightarrow$  • Gasbelle's model (for metal metalloids)  
 SRO is the same as the corresponding crystalline compound. (with the same composition) This latter model is supported by a lot of experiments (ME, NMR... see Durand's lectures

④ Electronic structure

From FINNEY'S MODELS 6 (Amorphous metals)



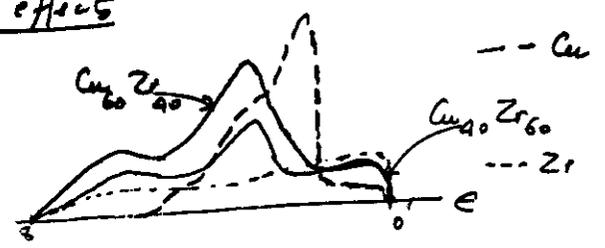
W<sub>F</sub> WITH RELAXATION LOCAL RADIAL DISTRIBUTION GOVERNS THE SHAPE OF n(E) (shape of Voronoi Polyhedra related Morse potential)



③ Electronic structure of amorphous metallic systems

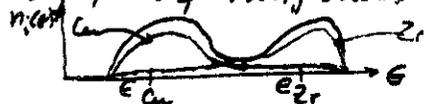
Photoemission  
 MOSTLY alloy effects

METAL-METAL



⇒ Split band regime (cf PdAg etc...)

AA

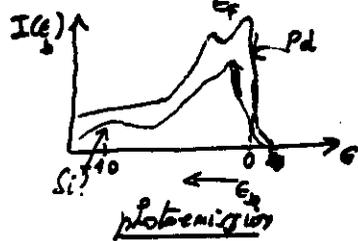
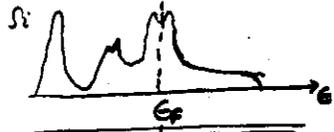
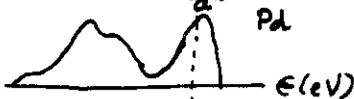
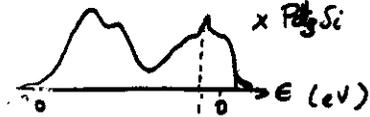
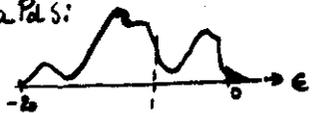


Metals Metalloid system

Ni P, Pd Si ...

"Bandstructure figures"

a. Pd Si

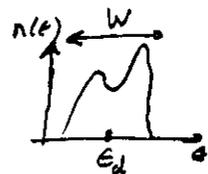


- small differences between both calculations (A)
- note that the 'sp' states on Pd are not correctly treated (only one Pd s, p basis added)

12

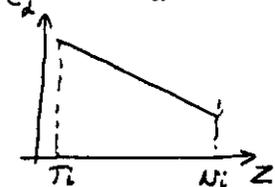
③ ALLOYS OF TRANSITION METALS

NEGLECT of sp STATES ⇒ T.B. 'd' bands  
| Canonical bands / dep on struct and scale  $W E_d$



② The importance of disorder

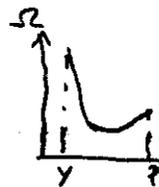
D.D  $E_d \approx -Z eV + E_d^0$



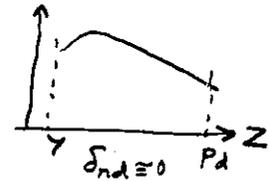
ODD

of latt

for equilib.



Example VRH  $\delta_d = .9$



④ Split band regime

for a number of cases

Pd Ag, Cu Ni, ... Ni Ru ... Pt V etc..

⑤ Phase stability and local order.

①. Orders of magnitude

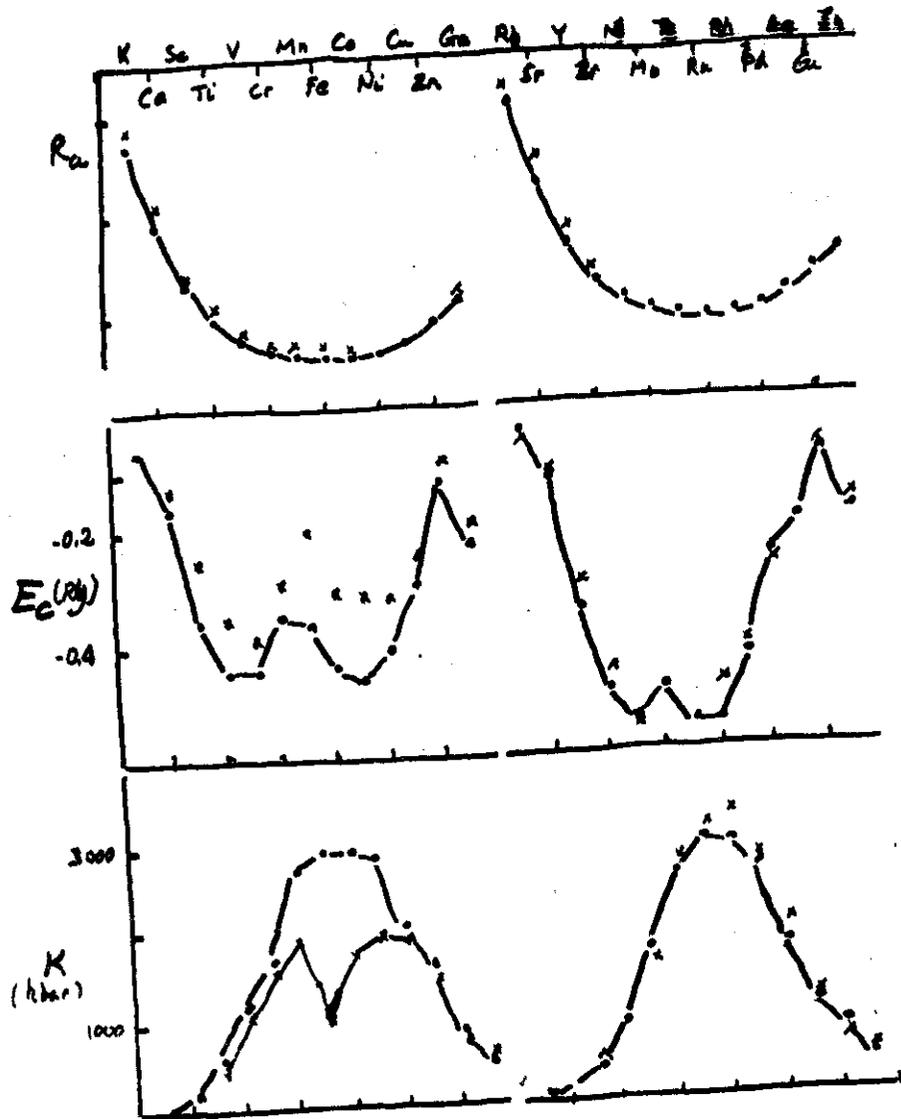
$E_i$	PURE METALS	$\sim eV/at$	CONSERVE ENERGY
	DISORDERED ALLOYS	$\sim \frac{eV}{10}/at$	
	ORDER ON A LATTICE		ORDERING ENERGIES OF DIFFERENT PHASES
	RELATIVE STABILITY	$\sim \frac{eV}{100}/at$	

$$E_c^i = |E(\text{crystal})/N_a - E_{at}| = |E_c^i - E_{at}|$$

$$E_{\text{form}}(A_c B_{1-c}) = E_{\text{dis all}}(A_c B_{1-c}) - c E_A - (1-c) E_B$$

$$E_{\text{ord}}(A_c B_{1-c}) = E_{\text{ord all}}(A_c B_{1-c}) - E_{\text{dis}}(A_c B_{1-c})$$

$$\Delta E_{\text{sp}}(1 \rightarrow 2) = E_{\text{ord}}^{(1)} - E_{\text{ord}}^{(2)} \quad \text{two phases 1, 2}$$



1st serie  
2nd serie  
Cohesive energy, Wigner Seitz radius  $R_a$  and bulk compressibility of transition metals of 1st and 2nd serie (oth. x exp)

2. For the cohesive energy: filling of the 'd' bands explains most of the  $|E_c|$  value (70%)

$E_c = \int n(E)[E - E_{at}] dE$

$n = 10 \int n(E) dE$

$|E_c| \approx \frac{W}{20} (n)(10-n)$

EXPLAINS THE PARABOLIC VARIATION OF  $|E_c|$  (Friedel)

For the formation energy. C.P.A

Qualitatively change of  $E$  comes from change of bond width

$W_{cell}(c) = z\beta^2 + C_A C_B (E_A - E_B)$

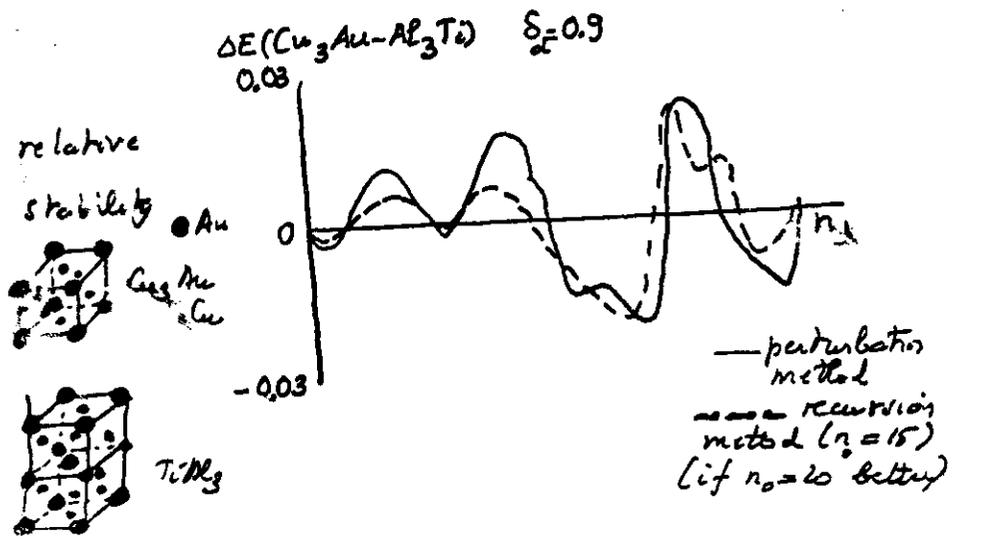
$= W \left[ \frac{1}{4z} + \frac{2C_A C_B}{4} \delta^2 \right]$

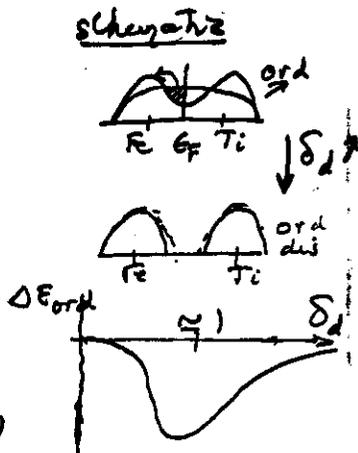
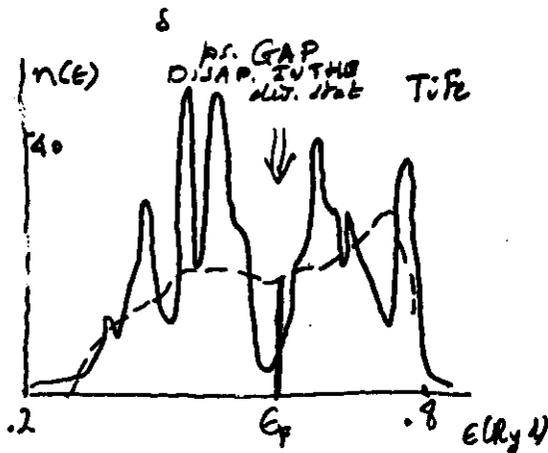
$\Rightarrow$  increasing of bonding energy (at const volume)

For the ordering energy, relative stability!

band structure calculations?

"perturbation" method --- valid for strong disorder





5) Generalized Perturbates Method

G.P.M.  $\leftrightarrow$  Cluster expansion for the total energy from a reference medium: i.e. THE DISORDERED STATE FOR THE CONSIDERED CONCENTRATION C

$$E(\{P_{ij}\}) = \bar{E} + \sum_{ij} P_{ij} P_{ji} v_{ij}(1-k)$$

Binary alloy

$$+ \sum_{ijkl} P_{ij} P_{kl} P_{ji} P_{lk} w_{ijkl}(1-k, 1-l) + \dots$$

structural part

It's convergent keeping only pair interactions - up to 6th nn  
(see previous by Luzhu - Ti Fe)

$\rightarrow$  Deriv. key ( $\delta n_d = 0$ )

G.P.M. (simple derivatives) for a non deg TB band

$$E = -\frac{1}{N} \int f(\epsilon) \text{Tr} \log G(\epsilon+i0) d\epsilon$$

$f(\epsilon)$  F. Dirac function  
 $G(\epsilon+i0) = \frac{1}{\epsilon+i0-H}$  noted G

- Ref medium  $\bar{H}$  (CPA)
- Hamiltonian for a configuration  $\{P_{ij}\}$   
 $H(\{P_{ij}\}) = \bar{H} = \bar{h} + V(\{P_{ij}\})$

Diagonal / off Diag. G

fluctuating part  
 $(V = \sum P_{ij}^i E_j^i \leftarrow DD)$

$$G = G^d + G^{nd} \quad \langle \alpha | G^d | \beta \rangle = \delta_{\alpha\beta} \langle \alpha | G | \alpha \rangle$$

$$\begin{cases} E = \bar{E} + \Delta E \\ \bar{E} = -\frac{1}{N} \int f(\epsilon) \text{Tr} [\log \bar{G}(\epsilon) - \log (1 - \bar{G}_d V)] \\ \Delta E = \frac{1}{N} \int f(\epsilon) \text{Tr} \log (1 - G^{nd} V) \end{cases}$$

$$(t_d = V / (1 - G^d V))$$

- Expansion of  $\Delta E$  (struct part)

$$\Delta E = -\frac{1}{N} \int f(\epsilon) \text{Tr} \left[ \begin{aligned} & G^d V^d \\ & + \frac{1}{2} \text{Tr} G^{nd} V^d G^{nd} V^d \\ & + \frac{1}{3} \text{Tr} G^{nd} V^d G^{nd} V^d G^{nd} V^d \end{aligned} \right]$$

pair triplets

- Pair interactions for DD

$$v_{ij}(1-k) = -\frac{1}{N} \int d\epsilon (\Delta \epsilon)^2 \bar{G}(R)^2 d\epsilon$$

$$G(R) = \langle \alpha | G(\epsilon) | R \rangle$$

- It's a cluster expansion in terms of  $(t_d G^{nd})^n$

Small parameter  $(t_d G^{nd}(R))^2$   
 Better convergence for large  $\delta$   
 $|G(R)| \sim \exp(-R/\delta) \quad \delta = \text{mean free path}$

6 Pair interactions for phase stabl if

$$V_{2-n} = V_1, V_2 \dots V_n$$

nth nn.

• scaling property for all binary alloys

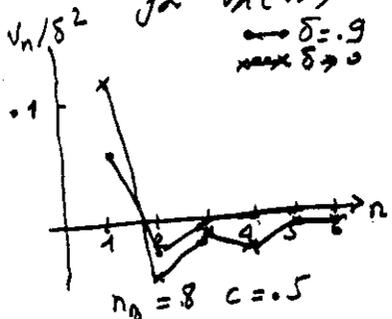
$$V_n = \bar{W} V_n$$

$$V_n = V_n(c, \bar{W}, \delta_d, \delta_{nd})$$

(i) range : short-ranged for large disorder

(ii) band filling  $\bar{N}$

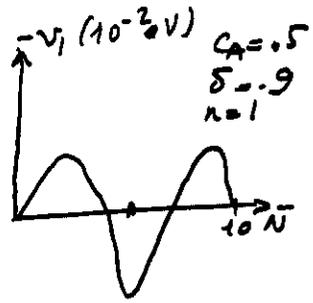
sum rules  $\Rightarrow$  existence of a min. number of 0 for  $V_n(\bar{N})$



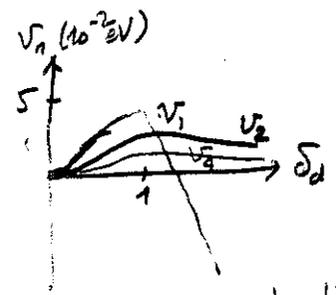
$n_0 = 8$   $c = 0.5$

"range"

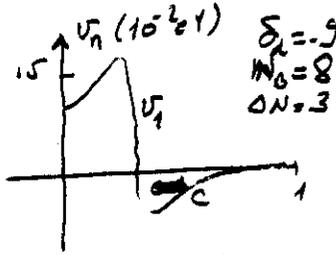
(iii) disorder ( $\delta$ ) (can change of sign)



$c_A = 0.5$   
 $\delta = 0.9$   
 $n = 1$



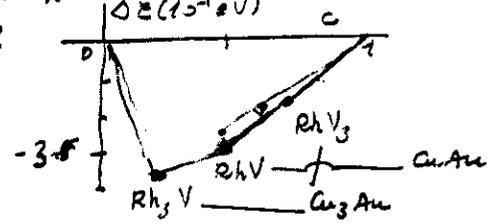
(iv) concentration (can also change of sign)



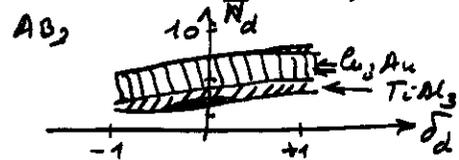
$\delta_d = 0.9$   
 $N_0 = 8$   
 $\Delta N = 3$

18 7 Most stable O.S (Te OX) on a swigg lattice for a quic

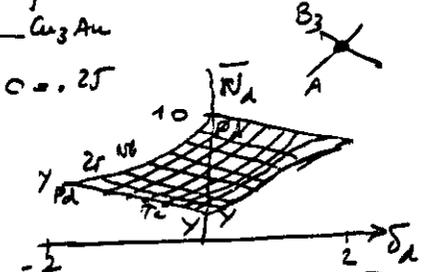
{ MAPPING OF THE ELECTRONIC PB. ON AN EQUIVALENT 3D ISING MODEL WHOSE GROUND STATES IS KNOWN FOR SHORT RANGED INT. }  
 $V(2+r) = V_n$  ( $V_n = 0$   $n > 4$ )  
Ex VRh



8 Structural maps Ex.  $c = 0.25$



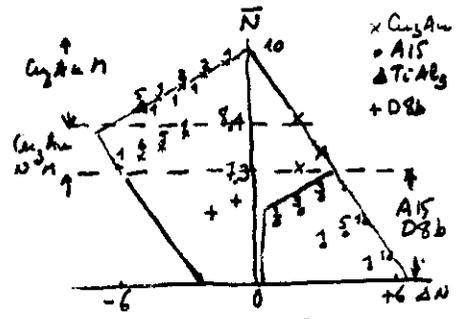
MOST STABLE PHASES FOR GIVEN ( $\delta_d, N_d$ )



A PECULIAR POINT AB2 CORRESPONDS TO A POINT IN THIS DIAGRAM

$\Rightarrow$  Syst predictions for the phase stability and for ERC

- in such crystalline alloys (no. mag)
- magnetic?
- self-organizing? ...



Experimental Structural map

for  $c = 0.25$

