Instabilities and Robust Control in Fishery Management

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Preliminary version

Abstract

Demand and supply analysis in fisheries often indicates the presence of instabilities and multiple equilibria, both in open access conditions and in the socially optimal solution. The associated management problems are further intensified by uncertainty on the evolution of the resource stock or on demand conditions. In this paper the fishery management problem is handled using robust optimal control, where the objective is to choose a harvesting rule that will work, in the sense of preventing instabilities and overfishing, under a range of admissible specifications for the stock recruitment equation. The paper derives robust harvesting rules, leading to a unique equilibrium, which could be used to design policy instruments such as transferable quota or landing fees.

1 Introduction

Demand and supply analysis in fisheries has been associated with instabilities and multiple equilibria, both in the context of an open access fishery and a socially optimal managed fishery.[1] The source of instability is the emergence of a backward bending supply curve which is the consequence of biological overfishing that occurs when effort expands beyond the level corresponding

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[1]See for example Clark (1990, Ch. 5).

to the maximum sustainable yield. The combination of a standard downward sloping demand curve with the backward bending supply curve can produce an odd number of interchanging locally stable and locally unstable market equilibria in open access fisheries. There exist locally stable equilibria corresponding to high price and low harvesting, which can be seen as an indication of overfishing. It is interesting to note that a similar picture emerges in a socially optimal controlled fishery. The discounted supply curve is also backward bending for positive discount rates. As a result, there are demand conditions under which multiple equilibria and instabilities are present even in optimally controlled fisheries.

The problems caused by the emergence of instabilities and overfishing in fisheries are further intensified by uncertainty, which is an important aspect of resource economics.[2] Uncertainty in this context can be associated with the evolution of the resource stock or with demand conditions. Thus both supply and demand shocks could disturb a locally stable fishery and lead to instabilities and overfishing. As Clark (1990) points out, many stock-recruitment relationships are poorly understood and difficult to estimate given the existing data, which in most cases are of low quality. This brings into the picture the issue of scientific uncertainty and its effects on fishery management.

In our analysis, scientific uncertainty [3] relates to the stock recruitment equation and the possibility that although the estimated model, often referred to as the approximating or benchmark model, is consistent with the data, there is a set of alternative models describing the evolution of the resource stock, which are also consistent with the data and could be regarded as possibly true. It is important to stress that if the benchmark model is misspecified, and resource stock evolution corresponds to a worse than expected scenario, then the optimal control solution for the benchmark model could result in a fishery with instabilities and overfishing. This observation provides a support for adopting a "precautionary principle" in fishery management in

[2]For some background analysis see for example Clark (1990, Ch. 11), Conrad and Clark (1988, Ch. 5), Conrad (2000, Ch. 7).

[3]Our use of the term uncertainty relates mainly to a situation where the state space of outcomes is known but the decision maker is unable to assign probabilities. The possibility of multiple prior distributions has largely been ignored in recent economic literature, although it is often a more appropriate setting. An example is when a panel of experts is consulted, since a group of people with divergent beliefs will normally not be able to reach a consensus on probability distributions Woodward and Bishop (1997). Their paper analyses circumstances under which rational choices are based in the most extreme possible outcomes, rather than on midpoint values. It also discusses the intermediate case, where some information on the set of probability distributions is known. Introducing the axiom of uncertainty aversion, as in Gilboa and Schmeidler (1989), a maximin model is obtained for in this case.

the event of scientific uncertainty. When the extensive collapse of fisheries over the last century is considered, precaution in designing management rules for regulating fisheries seems to be desirable.

Managing a fishery in this context suggests formulating the management problem as a *robust control* problem along the lines developed by Hansen and Sargent (2001), Hansen and Sargent (2002), where the objective is to choose a harvesting rule that will work, in the sense of preventing instabilities and overfishing, under a range of different model specifications of the stock-recruitment equation. In this case robust control is directly related to precaution and as Hansen and Sargent (2001) explicitly state "a preference for robustness induces context-specific precaution".

The purpose of this paper is to address the issue of scientific uncertainty and the potentially induced instabilities and overexploitation in fisheries by introducing robust control methodologies in fishery management. Our main finding is that by an appropriate choice of the robustness parameter, which is a parameter indicating preference for robustness, a regulator that manages a fishery for the social optimum could eliminate multiple equilibria instabilities and potential overfishing. The robust harvesting rules that lead to a unique equilibrium can be used to design decentralized regulation with policy instruments such as transferable quota or landing fees.

2 Bionomic Instabilities in Fishery Management[4]

We consider a standard harvest fishery model with a logistic growth function where biomass evolves deterministically according to

$$\dot{x}(t) = F(x(t)) - h(t) \tag{1}$$

where $x(t)$ is fish biomass, $h(t)$ denotes the harvest rate and $F(x(t)) = rx(1 - x/k)$ is the logistic growth function for stock recruitment, with biomass stock at the maximum sustainable yield (msy) defined as $x_{msy} = \arg\max F(x)$ and $x_k : F(x_k) = 0, x_k > 0$ denoting the carrying capacity biomass. Let unit harvest cost, $c(x(t))$ be a nonincreasing function of the fish stock x. Then for any price p, the profit flow is determined as[5]

$$\pi = (p - c(x))\,h \tag{2}$$

[4] This section follows Clark (1990, section 5.2), and will serve as background for the development of robust control methodology in the following section.

[5] t is dropped to simplify notation.

The open access supply in equilibrium is determined by the conditions

$$h = F(x) \tag{3}$$

$$p = c(x) \tag{4}$$

Solving (4) for x to obtain $x = x(p)$ and substituting into (3) we obtain equilibrium supply as $h = F(x(p))$. If demand is given by $h = D(p)$, $D' < 0$ the market equilibrium under open access is determined as:

$$\left(p^0, h^0\right) : D\left(p^0\right) = F\left(x\left(p^0\right)\right) \ , \ p^0 = P\left(h^0\right)$$

As shown by Clark (1990) the supply curve is backward bending for typical cost functions, so that when combined with a downward sloping demand multiple equilibria are induced, as shown in figure 1 which reproduces figure 5.11 from Clark (1990). There is bionomic instability at M_2 and overfishing at M_1. Multiple equilibria could be the result of the prevailing demand conditions, curve DD, or a demand shock that shifts demand from $D'D'$ to DD.

To analyze socially optimal fishery management we introduce a social planner or a regulator maximizing net surplus defined as $U(h) - c(x)h$, where $U(h)$ is the area under the demand curve $p = P(h)$ up to h, or $U(h) = \int_0^h P(u)\,du$ with $U'(h) = P(h)$. The welfare maximization problem is defined as:

$$\max_{\{h(t)\}} \int_0^\infty e^{-\rho t}\left[U(h(t)) - c(x(t))\,h(t)\right] dt \tag{5}$$

$$\text{s.t. } \dot{x}(t) = F(x(t)) - h(t) \ , x(0) = x_0 > 0 \tag{6}$$

The current value Hamiltonian for the problem is:

$$\mathcal{H} = U(h) - c(x)h + \lambda\left[F(x) - h\right] \tag{7}$$

with optimality conditions

$$U'(h) = \lambda + c(x) \ , \ U'(h) = P(h) \tag{8}$$

$$\dot{\lambda} = \left[\rho - F'(x)\right]\lambda + c'(x)\,h \tag{9}$$

along with (6) and the transversality condition at infinity. Differentiating (8) with respect to time and substituting into (9) we obtain the dynamic system characterizing the optimal paths of harvest and fish stock. The behaviour of harvest is given by

$$\dot{h} = \frac{1}{U''(h)}\left[(\rho - F'(x))\,(U'(h) - c(x)) + c'(x)\,F(x) \ , \ U'(h) = P(h)\right] \tag{10}$$

whereas stock evolves according to (6). The deterministic steady state equilibrium is defined as $\dot{h} = \dot{x} = 0$. At the steady state, market equilibrium is characterized by

$$P(h) = p = c(x) - \frac{c'(x)\,F(x)}{\rho - F'(x)} = H_\rho(x)\ ,\ h = F(x) \tag{11}$$

Solving the stock equilibrium equation of (11) to obtain $x = G(h)$, market equilibrium when the fishery is optimally managed is defined as

$$(p^*, h^*) : P(h^*) = H_\rho(G(h^*))\ ,\ p^* = P(h^*) \tag{12}$$

The discounted supply curve determined by (11) is backward bending as in the case of open access fishery and induces multiple equilibria and instabilities for demand curves like DD in figure 1 or similar demand shocks. Multiple equilibria of the fishery are presented in the phase diagram of figure 2. For the $\dot{h}_1 = 0$ isocline there is a unique steady state which is saddle point stable. However, a demand shock could shift this isocline to $\dot{h}_2 = 0$ and induce multiple equilibria with the middle one being unstable. Furthermore, if the benchmark model for stock evolution is misspecified, it is possible for a worse than estimated model for the stock recruitment relationship $F(x)$ to be realized. Then, both the $\dot{x} = 0$ isocline and the $\dot{h} = 0$ isocline shift. If these shifts yield a system such as $\dot{x}_2 = 0$, $\dot{h}_3 = 0$, multiple equilibria are also induced.

The possibility of multiple equilibria at the social optimum presents problems for regulation. For example, the regulatory instruments could have been designed to steer the system towards M_1 but due to the realization of a worse scenario for the stock recruitment relationship, the systems converges towards M_3' which is an overfishing steady state. To prevent such cases a different type of regulation is required. The idea behind the robust control methodology as it is applied in this paper to fishery management, is to help designing rules which under the worse possible scenario will prevent instabilities and biological overfishing. As it turns out these rules will be also useful in the presence of demand shocks.

3 Robust Control and Fishery Management

To develop the robust control methodology we introduce uncertainty in the stock recruitment equation. Let $(\Omega, \mathcal{F}, \mathcal{G})$ be a complete probability space, and let $x(\omega, t)$, $h(\omega, t)$ be the stochastic processes for the fish biomass, and harvesting and let $B_t = B(\omega, t)$ be a Wiener process, $\mathcal{E}(dB_t) = 0$, var$(dB_t) = dt$.

The stochastic social optimization problem for the fishery can be defined as the choice of a nonanticipating harvesting process $h(\omega, t)$ that maximizes the expected value of net surplus, subject to the constraints imposed by species growth rate[6]:

$$\max_{\{h(t)\}} \mathcal{E}_0 \int_0^\infty e^{-\rho t} \left[U\left(h(\omega, t)\right) - c\left(x(\omega, t)\right) h(\omega, t) \right] dt \tag{13}$$

$$\text{s.t. } dx(t) = \left[F\left(x(\omega, t)\right) - h(\omega, t) \right] dt + \sigma dB(\omega, t) \tag{14}$$

$$\sigma > 0, \quad x(0) = x_0 > 0 \text{ nonrandom} \tag{15}$$

$$x_t \geq 0, h_t \geq 0 \tag{16}$$

where x_t=$x(\omega, t)$,is the state variable and h_t=$h(\omega, t)$ is the control variable of the stochastic control problem.

In equation (14) the term $F(x_t) - h_t$ represents the expected change in the fish biomass at any given point in time, while the term σdB_t is the random amount of biomass change, with zero mean and variance σ^2. In this setup, which is a typical stochastic control problem, the manager is assumed to know the behaviour of stochastic shocks well enough to fully trust the characterization of the probability distribution implied by 14. This basic assumption leads to a decision on optimal harvest paths. However, it is quite possible (indeed likely, given natural system characteristics and information gaps) that the distribution is only an estimate, so that there is a degree of uncertainty attached not just to the specific realization of the random shock but also to the distribution itself. In other words, the planner might want to consider his own doubts about the model he is using to represent randomness.

Following Hansen, Sargent, Turmuhambetova and Williams (2002), we regard (14) as a benchmark model. If we assume that the social planner knows the benchmark model then there are no concerns about robustness to model misspecification. Otherwise, these concerns for robustness to model misspecification are reflected by a family of stochastic perturbations to the Brownian motion $\{B_t : t \geq 0\}$. The perturbation distorts the probabilities $\mathcal{G}$ implied by (14) and replaces $\mathcal{G}$ by another probability measure $\mathcal{Q}$. The main idea is that stochastic processes under $\mathcal{Q}$ will be difficult to distinguish from $\mathcal{G}$ using a finite amount of data. The perturbed model is constructed by replacing B_t in 14 with

$$B_t = z_t + \int_0^t R_s ds, \text{ or } dB_t = dz_t + R_t dt \tag{17}$$

[6]The basic assumption is that species biomass fluctuates continuously and that these stochastic influences are adequately represented by Wiener processes.

where $\{z_t : t \geq 0\}$ is a Brownian motion and $\{R_t : t \geq 0\}$ is a measurable drift distortion. Changes in the distribution of B_t will be parametrized as drift distortions to a fixed Brownian motion $\{z_t : t \geq 0\}$. The distortions will be zero under the measure G, in which case B_t and z_t coincide.

Now the social planner's concerns about misspecification of the model describing the evolution of fish biomass can be expressed using (17) to write the distorted model

$$dx_t = [F(x_t) - h_t + \sigma R_t]dt + \sigma dz_t \tag{18}$$

Thus, in the fishery management problem under model misspecification, equation (14) is replaced by (18). Now, following Hansen et al. (2002), the corresponding multiplier robust control model for the fishery can be written as:

$$\begin{aligned} &\max_h \min_R \mathcal{E} \int_0^\infty e^{-\rho t} \left[U(h) - c(x)h + \theta \frac{R^2}{2} \right] dt \\ &\text{s.t. } (18),(15) \text{ and } (16) \end{aligned} \tag{19}$$

In problem (19) the social planner is the maximizing agent that chooses harvesting h_t to maximize surplus, while "Nature" is the minimizing agent that chooses the "worst case distortion" to the stock recruitment relationship. The robustness parameter θ can be interpreted as the Lagrangian multiplier associated with an entropy constraint, which determines the maximum specification error in the stock recruitment relationship that the social planner is willing to accept.[7] The robustness parameter will be zero if the constraint is inactive or infinity if the constraint is violated. A value $\theta = +\infty$ signifies no preference for robustness, while lower values for θ indicate such a preference.

Using the Fleming and Souganidis (1989) result on the existence of a recursive solution to the multiplier problem, Hansen et al. (2002) show that problem (19) can be transformed into a stochastic infinite horizon two-player game where the Bellman-Isaacs conditions imply that the value function $J(x, \theta)$ satisfies

$$\begin{aligned} \rho J(x,\theta) &= \max_h \min_R \left\{ \begin{array}{c} \left[U(h) - c(x)h + \theta \frac{R^2}{2} \right] + \\ J_x [F(x) - h + \sigma R] + \frac{1}{2}\sigma^2 J_{xx} \end{array} \right\} \\ &= \min_R \max_h \left\{ \begin{array}{c} \left[U(h) - c(x)h + \theta \frac{R^2}{2} \right] + \\ J_x [F(x) - h + \sigma R] + \frac{1}{2}\sigma^2 J_{xx} \end{array} \right\} \end{aligned} \tag{20}$$

A solution for game (20) for any given value of the robustness parameter θ will determine the socially optimal robust harvesting policy.

[7] Relative entropy must be limited otherwise the distributions $\mathcal{G}$ and $\mathcal{Q}$ would be distinguishable. More rigorously, $\int_0^\infty e^{-\delta u} \mathcal{E}_Q \left(\frac{|R_u|^2}{2} \right) du \leq \eta$ (see Hansen et al. (2002)).

3.1 Robust harvesting rules

The optimality conditions associated with the optimization in the right hand side of (20) imply

$$U'(h) - c(x) = J_x \tag{21}$$

$$R = -\frac{\sigma}{\theta} J_x \tag{22}$$

Equation (21) is the usual result that at the optimal harvest the net marginal benefit of an additional unit of catch must be equal to the resource cost, whereas equation (22) is the worst possible distortion that is admissible, which is negative as expected and depends on θ. When θ is large, R is small and the benchmark model is a good approximation. More specifically, when $\theta \to \infty$ there is no distortion at all and the model yields the same solution as the typical optimal control model.

Differentiating the value function with respect to x and using (21) and (22) we obtain[8]

$$\rho J_x = [F(x) - h + \sigma R] J_{xx} - c'(x) h + F'(x) J_x + \frac{1}{2}\sigma^2 J_{xxx} \tag{23}$$

since $J(x)$ is a function of the stochastic variable x we have by Ito's lemma for $J_x(x)$

$$dJ_x(x) = J_{xx} dx + \frac{1}{2} J_{xxx} (dx)^2$$

Using equation (18), taking expected values, and dividing by dt we obtain

$$(1/dt)\, \mathcal{E} dJ_x(x) = J_{xx} [F(x) - h + \sigma R] + \frac{1}{2}\sigma^2 J_{xxx}$$

Substituting in (23) and rearranging with (21), the expected evolution of the resource cost is

$$(1/dt)\, \mathcal{E} dJ_x = [\rho - F'(x)] (U'(h) - c(x)) + c'(x) h$$

To express the solution in terms of the expected evolution of harvest and biomass, apply the differential operator $(1/dt)\, \mathcal{E} d(\cdot)$ to (21)

$$(1/dt)\, \mathcal{E} d(U'(h) - c(x)) = (1/dt)\, \mathcal{E} dJ_x \tag{25}$$

[8] For a basic explanation of the methods used in this section see for example Dixit and Pindyck (1994, Ch.4).

We need to expand the left hand side of (25), by applying Ito's lemma to $c(x)$ and $U'(h)$, which yields the following second order expansions:

$$\mathcal{E}dc(x) = \left[c'(x)\left[F(x) - h + \sigma R\right] + \frac{1}{2}\sigma^2 c''(x)\right] dt \text{ in expected value terms} \tag{26}$$

$$dU'(h) = U''(h)\, dh + \frac{1}{2}U'''(h)(dh)^2 \tag{27}$$

Since along the optimal path $h = h(x)$, where x is a stochastic variable, using Ito's lemma once again yields

$$dh = \left[h_x\left[F(x) - h + \sigma R\right] + \frac{1}{2}\sigma^2 h_{xx}\right] dt + \sigma h_x dz$$

When taking the expected value, terms of order higher than t go to zero, so that $\mathcal{E}(dh)^2 = \sigma^2 h_x^2 dt$, and (27) becomes

$$\mathcal{E}dU'(h) = U''(h)\,\mathcal{E}dh + \frac{1}{2}U'''(h)\,\sigma^2 h_x^2 dt \tag{28}$$

Using equaitions (26) and (28) into (25), as well as (**??**) we obtain

$$(1/dt)\,\mathcal{E}dh = \frac{1}{U''(h)}\left\{\begin{array}{c} \left[\rho - F'(x)\right](U'(h) - c(x)) + c'(x)\left[F(x) + \sigma R\right] \\ +\frac{1}{2}\sigma^2 c''(x) - \frac{1}{2}U'''(h)\,\sigma^2 h_x^2 \end{array}\right\}$$

substituting the worst case distortion R from first order condition (22), we have the differential equation governing the change of the expected value of robust harvesting along the optimal path.

$$(1/dt)\,\mathcal{E}dh = \frac{1}{U''(h)}\left[\begin{array}{c} \left[\rho - F'(x) - \frac{\sigma^2}{\theta}c'(x)\right](U'(h) - c(x)) + c'(x)\,F(x) \\ +\frac{1}{2}\sigma^2\left(c''(x) - U'''(h)\,h_x^2\right) \end{array}\right] \tag{29}$$

Likewise, the evolution of the expected value of biomass after substituting R from equation (22) becomes

$$(1/dt)\,\mathcal{E}dx = F(x) - h - \frac{\sigma^2}{\theta}\left(U'(h) - c(x)\right) \tag{30}$$

Equations (29) and (30) describe the evolution of the expected values of harvesting and biomass under socially optimal management with robust control.

4 Robust Equilibrium and Stabilization

In equilibrium $(1/dt)\,\mathcal{E}dh = (1/dt)\,\mathcal{E}dx = 0$. Using $U'(h) = P(h)$ the socially optimal expected steady state biomass under robust control is determined by:

$$\rho = F'(x) + \frac{\sigma^2}{\theta}c'(x) - \frac{c'(x)\,F(x) + \frac{1}{2}\sigma^2\left(c''(x) - U'''(h)\,h_x^2\right)}{P(h) - c(x)} \tag{31}$$

Under certainty $\sigma = 0$, in which case (31) is reduced to the well known rule for optimal fishery management, equation (11). Similarly, the management rule under "typical" uncertainty in stock recruitment, without a preference for robustness, is obtained by setting $\sigma \neq 0$ and $\theta \to \infty$.

Solving (31) for $P(h)$ the robust equilibrium market clearing condition becomes:

$$p = P(h) =$$

$$c(x) - \left[\frac{c'(x)\,F(x) + \frac{1}{2}\sigma^2\left(c''(x) - U'''(h)\,h_x^2\right)}{\rho - F'(x) - \frac{\sigma^2}{\theta}c'(x)}\right] = H_\theta(x) \tag{32}$$

$$h = G(x,\theta) \text{ obtained by solving for } h \tag{33}$$

$$h + \frac{\sigma^2}{\theta}U'(h) = F(x) + \frac{\sigma^2}{\theta}c(x)$$

Inverting (33) to obtain $x = G^{-1}(h,\theta)$ and substituting into (32) we obtain the robust supply curve $p = H_\theta\left(G^{-1}(h,\theta)\right) = S_\theta(h,\theta)$. Then market equilibrium is obtained as:

$$(p_\theta^*, h_\theta^*) : P(h_\theta^*) = S_\theta(h_\theta^*,\theta) \text{ and } p_\theta^* = P(h_\theta^*) \tag{34}$$

Setting $\theta \to \infty$ we obtain the corresponding equilibrium condition under typical uncertainty. It is interesting to note that the simpler type of randomness (assuming a known distribution) affects only the supply curve (32), but not the stock equilibrium condition (33). However, once we allow for model uncertainty the stock equilibrium condition is affected by the robustness parameter, so that both harvest and stock expected paths are affected. The chosen equilibrium will depend on σ (which is assumed to be exogenous) as well as θ.

The discussion in section 2 suggests that the dynamic system (29) and (30) could be associated with multiple equilibria and bionomic instabilities in expected values. The idea behind using robust control in this context is to design a management rule that could prevent bionomic instability and overfishing at the social optimum and then use the result to design decentralized instruments. We use the choice of the robust parameter θ as a basis for eliminating multiple equilibria at the social optimum . The idea is that by selecting an appropriate θ the socially optimal solution would lead to a unique stable (in the saddle point sense) equilibrium. The robust parameter could also be chosen on the basis of detection error probabilities, as in Hansen and Sargent (2002), but the stabilization argument seems to provide another plausible way of choosing this free parameter.[9]

We proceed in the choice of θ as follows:

$(1/dt)\,\mathcal{E}dx = 0$ defines, using (30), the curve $h = G(x, \theta)$ with slope

$$\frac{dh}{dx} = \frac{F'(x) + \frac{\sigma^2}{\theta} c'(x)}{1 + \frac{\sigma^2}{\theta} U''(h)} \tag{35}$$

where $F'(x) \underset{<}{\overset{>}{=}} 0$ as $x \underset{<}{\overset{>}{=}} x_{msy}$, $c'(x) < 0$, $U''(x) = P'(x) < 0$.

Assume that for a given (estimated) σ^2 there exists an interval $\left(\underline{\theta}, \overline{\theta}\right)$ such that for any $\theta \in \left(\underline{\theta}, \overline{\theta}\right)$, $\frac{dh}{dx} < 0$ for all $x \leq x_k$. If for the same θ the function $h = K(x, \theta)$ defined by the $(1/dt)\,\mathcal{E}dh = 0$ isocline is monotonic in θ then under appropriate boundary conditions for $G(0, \theta)$ $K(0, \theta)$ there exists a unique steady state in the expected values of harvesting and fish biomass for the fishery as shown in Figure 3. This result can be contrasted with the deterministic solution. As shown in figure 2 with an inverted U-shaped $\dot{x} = 0$ isocline and a monotonic $\dot{h} = 0$ isocline we could have multiple equilibria as a result of a demand shock or the emergence of a worse scenario for the stock-recruitment relationship. Multiple equilibria could also emerge in the stochastic model without preference for robustness or $\theta \to \infty$, since in this case the $(1/dt)\,\mathcal{E}dx = 0$ will also have an inverted U-shape. The $(1/dt)\,\mathcal{E}dh = 0$ isocline will be different from the deterministic one by the factor $\frac{1}{2}\sigma^2 \left(c''(x) - U'''(h)\, h_x^2\right)$. Thus the $(1/dt)\,\mathcal{E}dh = 0$ shifts relative to the deterministic case and could produce one or multiple equilibria.

On the other hand, robust control introduces the factor $\frac{\sigma^2}{\theta}$ both in harvest and biomass dynamics. If a θ^* exists such that $G(x, \theta^*)$, and $K(x, \theta^*)$ have a unique solution then robust control leads to a unique equilibrium. If furthermore $G(x, \theta^*)$,and $K(x, \theta^*)$ are monotonic in x then the uniqueness is preserved under demand shocks. If a unique robust equilibrium is defined then h^R can be used as the quantity limit for designing tradable quota systems, while J_x^R which is the costate variable associated with the

[9] Actually, θ is not a free parameter, as it is the multiplier associated with the constraint that limits the size of the allowable distortion. Depending on the chosen restriction, we will have a specific value for θ. However, we can think of choosing θ directly, thus implicitly setting η in the entropy constraint.

corresponding Hamiltonian representation can be used for designing landing fees. Under these instruments the regulated fishery will reproduce the robust equilibrium avoiding potential instabilities or overfishing.

This result can be related to the safe quota concept introduced by (Homa and Wilen 1997) where the safe quota was determined as $h^F = \max\{0, c + dx\}$. In our case the robust (or safe) quota is determined by a policy function $h^R = \phi(x)$ which is the function describing the stable manifold MM in figure 3.

5 Concluding Remarks

Bionomic instability is an inherent characteristic of fishery models induced by a backward bending supply curve. This instability emerges both in open access and in optimally controlled fisheries. Given the uncertainties associated with fisheries, these instabilities could be intensified by demand shocks or uncertainties associated with the stock-recruitment relationship.

In the present paper we consider the case of scientific uncertainty in the stock recruitment relationship and we introduce robust control methods in fishery management. We show that robust control could act as a tool to prevent instabilities, by an appropriate choice of the robustness parameter. This is obtained by designing a rule so that the optimally managed fishery is stable under a worst possible scenario for the stock-recruitment relationship. Furthermore, the same rule could stabilize the fishery under demand shocks. The robust management rule can be used to design decentralized policy instruments that work better than typical prescriptions at maintaining stable harvests and avoiding biomass collapse.

The basic model developed here can be extended along different lines, such as depensation or non-linear cost effects, or by considering the fishery as a dynamic game between the planner/regulator and the fishermen, and seeking robust solution with possible heterogenous preferences for robustness.

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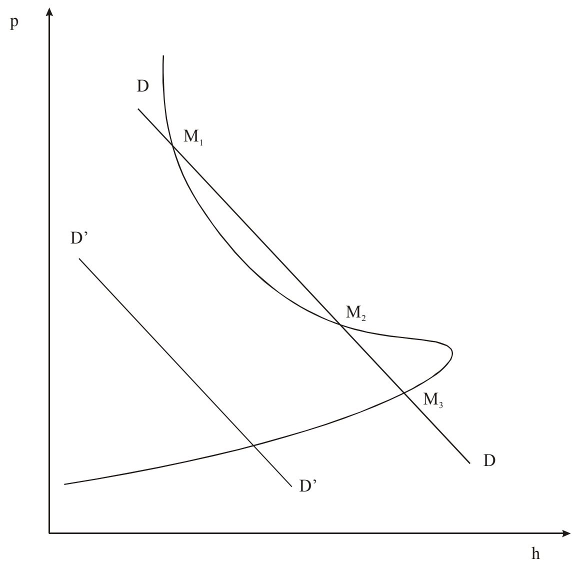

Figure 1.

Figure 1:

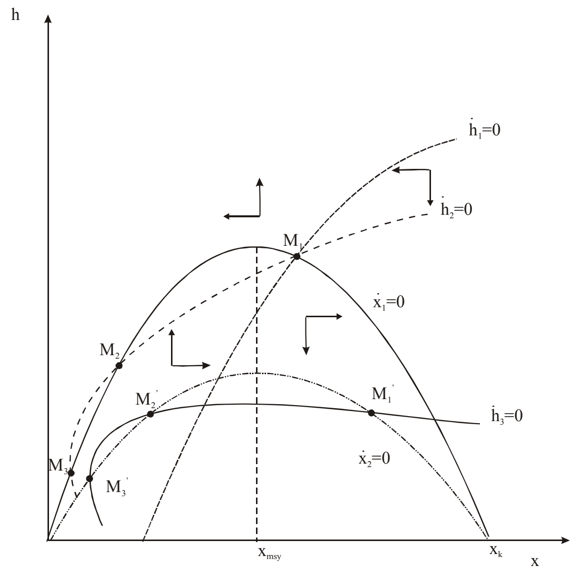

Figure 2.

Figure 2:

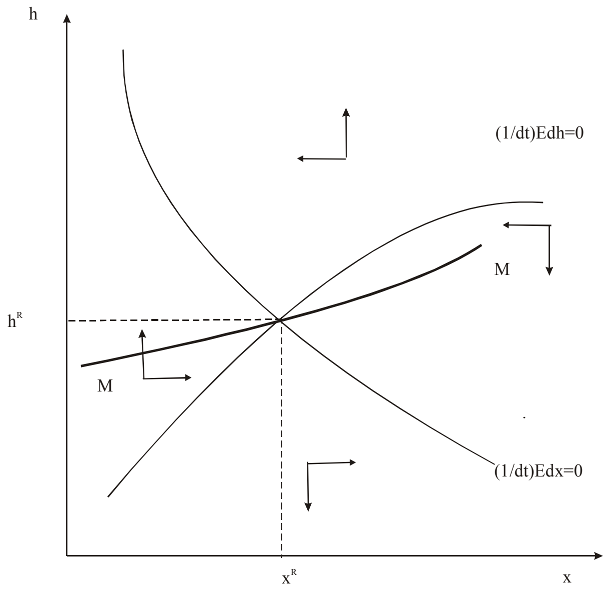

Figure 3.

Figure 3: