Determination of Hydraulic Properties of Unsaturated Soil via Inverse Modeling

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Abstract.

The method for determining the hydraulic properties of unsaturated soil with inverse modeling is presented. A modified cone penetrometer has been designed to inject water into the soil through a screen, and measure the progress of the wetting front with two tensiometer rings positioned above the screen. Cumulative inflow and pressure head readings are analyzed to obtain estimates of the hydraulic parameters describing K(h) and θ(h). Optimization results for tests at one side are used to demonstrate the possibility to evaluate either the wetting branches of the soil hydraulic properties, or the wetting and drying curves simultaneously, via analysis of different parts of the experiment. The optimization results are compared to the results of standard laboratory and field methods.

1. Introduction

The soil-moisture characteristic, *θ(h)*, and hydraulic conductivity, *K(h)*, curves are two basic hydraulic properties of soils. Current direct laboratory and in-situ methods for their determination are often time consuming and costly. Parameter optimization is an indirect approach that makes it possible to obtain *K(h)* and *θ(h)* simultaneously from transient flow data [*Kool et al.*, 1987]. In this case, a flow event is modeled with an appropriate governing equation and analytical expressions of *K(h)* and *θ(h)*. The unknown parameters of *K(h)* and *θ(h)* are obtained by minimization of an objective function describing the differences between some measured flow variables and those simulated with a numerical flow code. This methodology was originally applied to laboratory one-step column outflow data [*Kool et al.*, 1985; *Parker et al.*, 1985; *van Dam et al.*, 1992] and multi-step column outflow data [*van Dam et al*., 1994; *Eching and Hopmans*, 1993; *Eching et al.*, 1994]. Parameter estimation has also been used with data obtained with the evaporation method (see for example, *Santini et al.*, [1995]; *Ciollaro and Romano* [1995]; *Šimůnek et al.* [1998a]). All of these laboratory methods provide information about the drying branches of the soil-moisture characteristics.

For field determination of the wetting branches of soil hydraulic properties, parameter estimation methods were applied to ponded infiltration flow data [*Russo et al.*, 1991; *Bohne et al.*, 1992], and tension disc infiltrometer flow data [*Šimůnek and van Genuchten*, 1996; 1997; *Šimůnek et al*., 1998b]. Another technique for gaining information about the drying branches of the soil hydraulic properties via multi-step soil water extraction and parameter optimization was developed by Inoue et al. [1998]. The field methods described above are applicable only in the near surface. *Gribb* [1996] proposed a new cone penetrometer tool (e.g., cone permeameter) and use of parameter optimization to estimate soil hydraulic properties at depth. A prototype was further developed by *Leonard* [1997]. A detailed description of the prototype, and its use under saturated and unsaturated conditions were previously presented by *Gribb et al*. [1997]. The cone permeameter is placed in the soil, and a constant head of water is then supplied to the 5-cm long screen. Cumulative inflow volume is determined from scale readings of the mass of water removed from the source. Progress of the wetting front is measured with tensiometer rings 5 and 9 cm above the screen. After the water supply valve is closed, the tensiometers monitor the redistribution of water in the soil profile. *Kodešová et al*. [1998] discussed results of the numerical analysis of data from the wetting parts of cone experiments which were performed for one type of soil but under different initial and boundary conditions. *Šimůnek et al.* [1999] finally examined both the wetting and redistribution parts of cone permeameter experiments to find the wetting and drying branches of the soil hydraulic properties. *Kodešová et al*. [1999] presented results of field testing in two types of sandy soil. Test procedure and results from one side are briefly discussed here.

2. Theory

Flow Equation

The governing flow equation for radially symmetric, isothermal Darcian flow in an isotropic, rigid porous medium, assuming that the air phase plays an insignificant role in the liquid flow process is (*Richards*, 1931):

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\,K\,\frac{\partial h}{\partial r}\right] + \frac{\partial}{\partial z}\left[K\left(\frac{\partial h}{\partial z}+1\right)\right] = \frac{\partial \theta}{\partial t} \qquad \textbf{(1)}$$

where r is the radial coordinate [L], z is the vertical coordinate positive upward [L], t is time, h is the pore water pressure head [L], K is the unsaturated hydraulic conductivity [LT^{-1}], and θ is the volumetric moisture content [L^3L^{-3}].

Soil Hydraulic Properties Functions

The *van Genuchten* (1980) expressions for moisture content and hydraulic conductivity, $\theta(h)$ and $K(\theta)$, are used in this work:

$$\theta_e = \frac{\theta(h) - \theta_r}{\theta_s - \theta_r} = \frac{1}{\left(1+|\alpha h|^n\right)^m}, \quad h<0 \qquad \textbf{(2)}$$

$$\theta_e = 1, \quad h \geq 0$$

$$K(\theta) = K_s\ \theta_e^{0.5}\left[1-\left(1-\theta_e^{1/m}\right)^m\right]^2, \quad h<0 \qquad \textbf{(3)}$$

$$K(\theta) = K_s, \quad h \geq 0$$

where θ_e is the effective moisture content [L^3L^{-3}], K_s is the saturated hydraulic conductivity, θ_r and θ_s are the residual and saturated moisture contents [L^3L^{-3}], respectively, and α [L^{-1}], n and m (= 1 - 1/n) are empirical parameters [-]. The equations contain 5 unknown parameters: K_s, θ_r, θ_s, α, and n.

Objective Function

To derive estimates of the hydraulic parameters using parameter optimization, an objective function, Φ, expressing the differences between flow responses measured with the permeameter and those predicted by a numerical model with hydraulic parameter inputs, is minimized:

$$\Phi(b,q,p) = \sum_{j=1}^{m_q} v_j \sum_{i=1}^{n_{qj}} w_{i,j}\left[q_j^*(x,t_i) - q_j(x,t_i,b)\right]^2 + \sum_{j=1}^{m_p} \bar{v}_j \sum_{i=1}^{n_{pj}} \bar{w}_{i,j}\left[p_j^*(\theta_i) - p_j(\theta_i,b)\right]^2 \qquad \textbf{(4)}$$

where the first term on the right-hand side represents deviations between measured and predicted space-time variables (e.g., observed pressure heads or moisture contents at different locations and/or times, or the cumulative infiltration rate versus time). In this term, m_q is the number of different sets of measurements, and n_{qj} is the number of measurements in a particular measurement set. Specific measurements at time t_i for the jth measurement set at location $x(r, z)$ are represented by $q_j^*(x, t_i)$, $q_j(x, t_i, \boldsymbol{b})$ are the corresponding model predictions for the vector of optimized parameters $\boldsymbol{b}$ (e.g., θ_r, θ_s, α,

n, K_s), and v_j and $w_{i,j}$ are weights associated with a particular measurement set or point, respectively. The weighting factor, v_j, is given by the inverse of the number of measurements multiplied by the variance of those observations, and $w_{i,j}$ is equal to 1 in this work. The second term represents differences between independently measured and predicted soil hydraulic properties (e.g., $\theta(h)$, $K(\theta)$ or $K(h)$ data), while the terms m_p, n_{pj}, $p_j^*(\theta_i)$, $p_j(\theta_i, \boldsymbol{b})$, v_j and $w_{i,j}$ have similar meanings as for the first term, but are now for the soil hydraulic properties.

3. Field Testing of the Cone Permeameter (Poinsett State Park, South Carolina)

Test Procedure

- 4 Guelph permeameter tests for determining field saturated hydraulic conductivities were performed.
- Soil anchors were placed into the Guelph test holes and the insertion frame was secured.
- The soil core sampler was then inserted. Soil samples of known volume were removed from the barrel of the core sampler. Measured volumetric moisture contents were paired with initial cone permeameter tensiometer readings and used in the inversion process as known points of the retention curve, $\theta(h)$.
- The cone permeameter was inserted into the core sampler hole.
- A constant head (in one or two steps) was then applied to the 5-cm long screen to inject water into the soil. 5 Tests were performed:
    - A, B, C with applied pressure heads of 30 and 50 cm
    - D with applied pressure heads of 21 and 108 cm
    - E with applied pressure heads of 21 and 80 cm
- Cumulative inflow volume was measured.
- The advance of the wetting front was detected as pore water pressure increases were measured with tensiometer rings 5 and 10 cm above the screened section.
- The redistribution of water in the soil profile was monitored with tensiometers after the source of water was shut off.
- Undisturbed soil samples were taken near the permeameter for pressure plate, hanging column, and falling head permeability tests to determine the drying soil-moisture characteristic curves and saturated hydraulic conductivities.
- An inverse solution method was used to predict the soil hydraulic properties.
- Independent measurements were carried out:

  Retention curve:
    - Pressure plate test
    - Capillary rise test
    - Hanging column test

  Saturated hydraulic conductivity:
    - Guelph permeameter test (mentioned before)
    - Falling head permeability tests

Inverse Simulations

- Inversse simulation was performed to obtain parameters of *van Genuchten* (1980) expressions for $K(h)$ and $\theta(h)$. The unknown parameters of $K(h)$ and $\theta(h)$ were obtained by minimization of an objective function describing the differences between some measured flow variables and those simulated with a numerical flow code HYDRUS-2D (*Šimůnek et al.*, 1996)

- Inputs were:
  - cumulative inflow
  - pressure heads at two locations
  - point of the retention curve given by initial moisture content and initial tensiometer reading at the corresponding depth
- Performed inverse solutions:
  - one-step test, applied pressure head of 30 (or 21) cm
  - two-step test, applied pressure heads of 30 and 50 (or 21 and 108, 21 and 80) cm
  - three-step test, two applied pressure heads and redistribution
- Obtained results:
  - wetting soil hydraulic properties
  - wetting soil hydraulic properties
  - wetting and drying soil hydraulic properties

4. Results

Measured data and resulting hydraulic parameters are discussed in detail in *Kodešová et al.* [1999]. Therefore only resulting soil hydraulic parameters are presented here in Table 1.

Table 1. Hydraulic parameters obtained from different tests for sandy soil, Bulk density: 1.45 ÷ 1.68 g/cm^3, Porosity: 0.350 ÷ 0.452

Test Method	**Hydraulic Parameters**				
	α^w / α^d [cm^{-1}]	n [-]	θ_r [-]	θ_s [-]	K_s [cm/sec]
Pressure Plate (9 Samples)	0.068	1.54	0.000	0.423	-
Capillary Rise (1 Column)	0.068	3.57	0.000	0.446	-
Hanging Column (1 Sample)	0.034	3.29	0.113	0.420	-
Guelph Permeameter (4 Test Holes)	-	-	-	-	0.0024 ÷ 0.0038
Laboratory Falling Head (9 Samples)	-	-	-	-	0.0013 ÷ 0.0044
Cone Permeameter A, h_o = 30 cm	0.037	3.97	0.088	0.379	0.0022
Cone Permeameter A, h_o = 30, 50 cm	0.035	4.81	0.089	0.377	0.0020
Cone Permeameter B, h_o = 30 cm	0.037	3.95	0.088	0.400	0.0018
Cone Permeameter B, h_o = 30, 50 cm	0.035	4.79	0.089	0.393	0.0016
Cone Permeameter B, h_o = 30, 50, 0 cm	0.035 / 0.026	4.46	0.088	0.390	0.0016
Cone Permeameter C, h_o = 30 cm	0.034	3.65	0.082	0.433	0.0011
Cone Permeameter C, h_o = 30, 50 cm	0.033	4.04	0.083	0.449	0.0011
Cone Permeameter D, h_o = 21cm	0.047	2.53	0.055	0.443	0.0040
Cone Permeameter D, h_o = 21, 108 cm	0.044	3.11	0.069	0.447	0.0036
Cone Permeameter E, h_o = 21cm	0.035	3.19	0.087	0.333	0.0011
Cone Permeameter E, h_o = 21, 80 cm	0.031	4.09	0.089	0.350	0.0010
Cone Permeameter E, h_o = 21, 80, 0 cm	0.031 / 0.026	4.02	0.089	0.349	0.0010

The soil was very homogeneous, without obvious layering or anisotropy, so optimization of parameters K_s, α, n, θ_r and θ_s was sufficient for describing observed flow responses. Initial moisture content paired with the initial tensiometer reading allowed for realistic estimation of θ_r and θ_s. Analysis of one- and two-step tests yielded similar parameters, due to the influence of the first step on the inverse solution. However, addition of the second step stabilized the solution for Test A of Site 2. The wetting hydraulic parameters obtained from analysis of the wetting and redistribution parts of the experiment were consistent with those obtained from analysis of the wetting parts of the two-step experiments. The drying α parameter was lower, as expected. The different α values clearly described the effects of hysteresis. It is obvious that the optimized parameters are in the range of the independently measured data.

5. Conclusions

The inverse modeling technigue has proved many times to be an efficient tool for determination of soil hydraulic properties. For more details about this method see for instance *Hopmans et al.* (2002), *Šimůnek et al.* (2002a, 2002b).

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