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OXYGEN TRANSPORT IN WATERLOGGED SOILS
PART II

BY

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APROACHES TO MODELLING SOIL AND CROP RESPONSE TO OXIGEN DEFICIENCY

INTRODUCTION

Many authors have emphasized the complex nature of anaerobiosis in soils produced by waterlogging. Poor aeration may cause the accumulation of various gases and toxic products, but depletion of oxygen below critical levels can be considered as a major effect on plant growth and crop production. From practical, point of view, the questions is “what level of oxygen is critical”. The amount of oxygen in soil is determined by the supply of oxygen from the atmosphere and its consumption in the soil. These aspects are highly dynamic and very difficult to quantify. So, a convenient way to develop quantitative theory on soil aeration has been to split the diffusion process in macro and micro diffusion. In fact, there is abundant literature on mathematical models to describe these two processes. This lecture outlines in a simple way the mathematics of various cases of diffusion which have been widely used in modeling soil aeration. Simplifications of the general equation of diffusion (Fick’s law) giving two possible forms of the problem: planar or one-dimensional diffusion and radial diffusion are given. Furthermore, the solution of diffusion equation is obtained by the analogy to the problem of electrical flow (Ohm’s law). Taking into consideration the soil respiration process, the continuity equation which accounts for the law of conservation of mass is solved. The purpose of this paper has been to review the interrelation soil structure-air movement in waterlogged clay soils, and its consequences on plant growth and crop production. Thus, the mathematics of diffusion is presented, and then its application to specific cases of soil aeration such as diffusion in the soil profile, soil aggregates and roots is given. The following assumptions are taken into consideration.

Gas flow in soils is basically diffusion-dependent. Gas-phase diffusion is the major mechanism for vertical or longitudinal transport (long distance transport); this means, with depth Z in the soil profile (macrodiffusion).

For horizontal transport (short distance transport or microdiffusion) which is assumed to be in X direction; in this case, the geometry of aggregates and the liquid phase are the major components of resistance for diffusion. Soil aggregates and roots are considered to be spherical and cylindrical in shape respectively.

Soil oxygen consumption, Sr, is taken to be independent of the oxygen concentration and considered to proceed at the same rate until oxygen supply drops to critical levels. Thus, aeration problems are assumed to begin when at any time, in the root zone, the oxygen diffusion rate, ODR, becomes less than $30x10^{-8}$ $-g.cm^{-2}.sec^{-1}$, or the value of redox potential Eh is less than +525 mv (Obando, 1990).

Planar Diffusion

The first stages in the soil aeration process in the short term can be described by assuming simple steady-state one-dimensional diffusion system in which there is no net lateral movement and where a linear gradient of oxygen concentration is developed between the atmosphere and an adjacent or more remote sink in the soil profile. As indicated in figure 1 the soil frofile in considered bounded by two parallel planes, e.g. the layers at Z=0 and Z=L. Obviously, these will apply in practice to diffusion into the soil profile where effectively all oxygen enters through the surface layer. An equation describing the steady-state linear diffusion from planar source to planar sink of equal area, through an isotropic medium in which there is no lateral loss or gain of molecules of the gas can be derived from the differential equation for planar diffusion (equation 1, first Fick's law), which for vertical transport (in Z direction) becomes:

$$\frac{\delta c}{\delta t} = D \cdot \frac{\delta^2 c}{\delta Z^2} \qquad (1)$$

The steady state

Let us consider the case of diffusion through a soil profile of depth L and coefficient diffusion D_ε, whose layers at Z=0 and Z=L, are maintained at constant concentrations Co and Cz respectively. After a time, a steady state is reached in which the concentration remains constant at all points of the profile and provided that Dg is constant, equation (1) reduces to: (Crank, 1975)

$$\frac{\delta^2 c}{\delta Z^2} = 0 \qquad (2)$$

on integrating with respect to Z we have

$$\frac{\delta c}{\delta^2} = \text{constant} = K_1 \qquad (3)$$

Where K_1 is a constant > 0 confirming that the concentration must change linearly through the soil profile from Co to Cz. Introducing the conditions at Z=0, Z=L, and integrating a second time we obtain

$$C = K_1 z + K_2 \qquad (4)$$

Where K_2 is a second constant of integration.

Applying the boundary conditions C=Co on Z=0 and C=Cz on Z=L then $Co=K_2$ and $Cz=K_1L + Co$ and hence

$$K_1 = \frac{\delta c}{\delta z} = \frac{(C_Z - Co)}{L} \qquad (5)$$

both equations (3) and (5) show that the concentration changes linearly from Co to Cz through the soil profile. Also, the rate of transfer of diffusing substance is the same across the whole profile and is given by

$$F_\varepsilon = - D_\varepsilon \frac{\delta c}{\delta z} = - D_\varepsilon \frac{(Cz - Co)}{L} \qquad (6)$$

where F_ε , the one-dimensional flux of oxygen is constant through the length of diffusion path from Z=0 to Z=L.

If the depth L and the surface concentration Co and Cz are known, D_ε can be deduced form an observed value of F_ε by using equation (6). Grafically the plot of C against depth Z for this system has the form illustrated in fig (1).

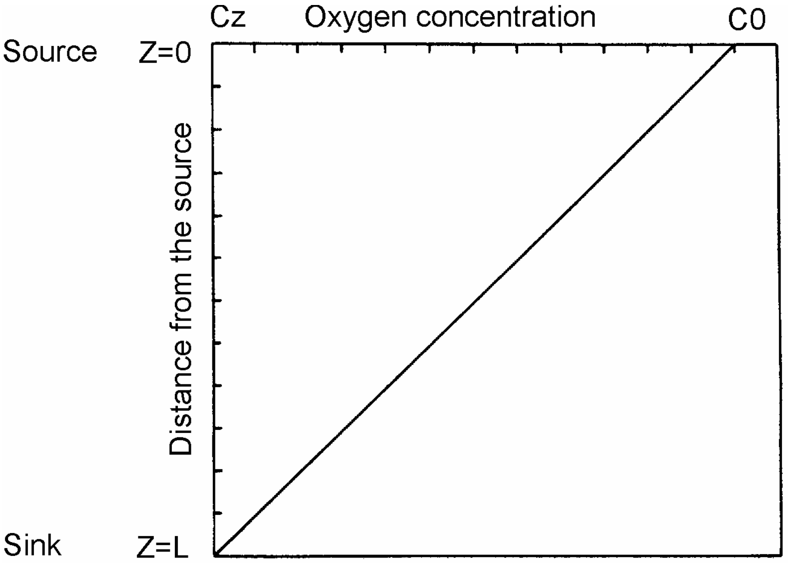

Fig. 1. Linear diffusion gradient in simple planar source-sink system. Oxygen source, Co, on Z = 0 and sink, Cz, on Z = L (Armstrong, 1979).

In equation (6) δc can be likened to a "force" analogous in part to the potential difference in an electric circuit while the term D/ δz is a measure of the conductance (reciprocal to resistance).

In equation (6) there is no restricting area term and the conductance D/L is simple a measure of the linear conductance between planar layers or compartments. Providing that the areas of these surfaces are equal the units of D/L are (m/s). The oxygen flux, which has dimensions of quantity, area and time may also be written as

$$F_\varepsilon = \frac{Q}{A\,.\,t} = D_\varepsilon \frac{(Co - Cz)}{L} \qquad (7)$$

where Q = grams, A in m^2 and t = seconds, hence equation can be rearranged as

$$\frac{Q}{t} = A\,.\,D_\varepsilon \frac{(Co - Cz)}{L} \qquad (8)$$

where the term Q/t is referred to as the diffusion rate ($g.s^{-1}$) for the finite system of planar sectional area A (cm^2). Under steady conditions the flux of gas Q/t, across any surface is contant, though the flux density may change if the area available for flow changes. In fact, equation (7) specifies the area through which diffusion takes place and the conductance K becomes Dg.A/L and has units of m^3/s. Thus, conductance K is better expressed as a function of area by

$$K = D_{\varepsilon} \cdot \frac{A}{L} \qquad (9)$$

then equation 8 becomes

$$\frac{Q}{t} = K\,(Co - C_Z) \qquad (10)$$

from equations 6 and 7 we obtain

$$F_{\varepsilon} = - D_{\varepsilon} \frac{\delta c}{\delta z} = \frac{Q}{A\,.\,t} \qquad (11)$$

by integrating it yields

$$\frac{Q}{t}\int_{0}^{z}\left[\frac{1}{A(z)}\right]\delta z = - D_{\varepsilon}\int_{0}^{z}\delta c = D\varepsilon\,(Cz - Co) \qquad (12)$$

using equations 8 and 10

$$\frac{Q}{t} = \frac{D_{\varepsilon}.\,A}{L\,(co - Cz)} = K\,(Co - Cz) \qquad (13)$$

by combining equation 12 and 13 it yields

$$K.\,(Co - Cz)\int_{0}^{z}\left[\frac{1}{A\,(z)}\right]\delta z = D\varepsilon\,(Cz - Co) \qquad (14)$$

$$K = \frac{D_{\varepsilon}}{\int_{0}^{z} \left[\frac{1}{A\,(z)}\right] \delta\, z} \qquad (15)$$

for simple geometries the integration is straightforward and the conductances determined. When diffusion is purely one-dimensional, the area available for flow remains constant with distance. The conductance is therefore planar diffusion: $K = D_{\varepsilon}/(Z_1 - Zo)$ where the subscript following the distance Z indicates a location of measurement.

Radial diffusion

Radial diffusion is an interesting case for modeling gas transport in soils. It forms the foundation for the development of the so-called micro models. In fact, the theory of the micro site concept for diffusion in soil crumbs, microorganism colonies and roots are based on radial diffusion. In particular, it constitutes the basis for the assessment of data obtained using the platinum electrode technique. The mathematics of the process is outlined below.

Difussion in a cylinder: steady state

Considering a long circular cylinder in which diffusion is everywhere radial, concentration is then a function of radius r and time only. If the medium is formed by a system of coaxial cylinders whose radii are r_a and r_b as indicated in fig. 2, the differential equation describing the steady-state condition is: (Crank, 1975)

$$\frac{1}{r} \cdot \frac{\delta}{\delta r} \left(\frac{r \,.\, D\delta c}{\delta r} \right) = 0 \qquad (16)$$

If the diffusion coefficient D is constant, equation (16) becomes:

$$\frac{\delta c}{\delta c} \left(\frac{r.\ \delta c}{\delta c} \right) = 0, \qquad a < r < b \qquad (17)$$

the general solution of this is

$$C = K_1 + K_2 \ln r, \qquad (18)$$

where K_1 and K_2 are constants to be determined for the boundary conditions: $C = C_1$ on $r=a$, $C=C_0$ on $r=b$ and $a \le r \le b$. Hence

$$C = \frac{C_1 \,.\, \ln\left(\frac{b}{a}\right) + Co \,.\, \ln\left(\frac{r}{a}\right)}{\ln\left(\frac{b}{a}\right)} \qquad (19)$$

the quantity of diffusing substance Q which diffuses through unit area of the cylinder A in time t is equal to the diffusion coefficient times the concentration gradient (Letey and Stolzy, 1964; 1967):

$$\frac{Q}{A.\,t} = \frac{D_{\varepsilon}(Co - C_1)}{\ln\left(\frac{b}{a}\right)} \qquad (20)$$

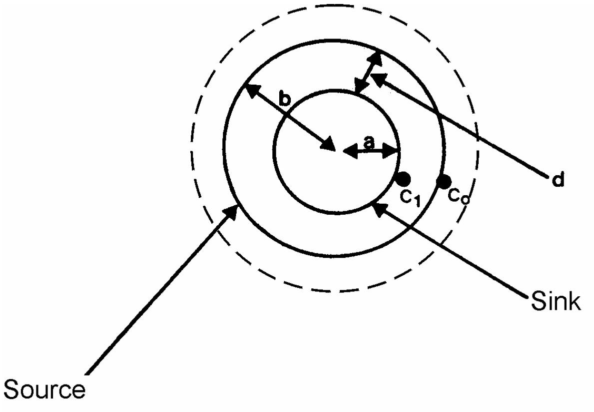

Fig. 2 The model which is applied in order to explain cylindrical diffusion (After Phene, 1986)

The oxygen flux for such a coaxial cylindrical model is given by

$$\frac{Q}{t} = D_{\varepsilon}\ .\ 2\pi\, r\ .\ h \frac{Co - C_1}{r\ .\ \ln\left(\frac{b}{a}\right)} \qquad (21)$$

By analogy with equation (8) it may be noted that the conductance K is given by the term Dg 2π r.h/rln (b/a) where 2π r.h is the surface area of a cylinder or radius r and length h.

If our observation concerns the diffusion incident upon the inner cylinder of r=a, the conductance term becomes Dg. 2π ah/aln(b/a). We may also note that the terms aln (b/a) and blnb/a) are analogous with Lin equations 4.7 and 4.8. It may be thought of as the effective path length for diffusion.

As indicated in fig. 4.3 for equal increments of the path (b-a), the conductance component of DgA/rlnb/a) is distributed in a curvilinear manner and hence at equilibrium the concentration prodile between b and a is also curvilinear (Crank, 1975). This contrast with the linear profile in the corresponding planar system. Application of these equations to modeling soil aeration is presented by Letey and Stolzy (1967), Kowalk, (1985).

By analogy with equation 4.13 for x direction

$$K = \frac{D_{\varepsilon}}{\displaystyle\int_{0}^{x} \left[\frac{1}{A\,(x)}\right] dx} \qquad (22)$$

when diffusion is cylindrical
$A(x) = 2\pi rh = 2\pi xh$ and $A(x)_{\pi} = 2x$ per meter of root, where x is the distance from the centre. Using equation 4.18 gives

$$K = \frac{D_{\varepsilon}}{\displaystyle\int_{0}^{x} \left[\frac{1}{2\pi x}\right]} \qquad (23)$$

Assuming the boundary conditions x = a (root or platinum electrode radius) and x = b (radial distance from the root axis to the outer boundary of the liquid phase of the soil matrix surrounding the root) the solution for equation 19 yields

$$K = \frac{2\pi h D_{\varepsilon}}{\ln\left(\dfrac{a}{b}\right)} \qquad (24)$$

for cylindrical systems, the flux per unit length is constant, so K has units of m^2/sec and flux is per meter of length.

Diffusion in a sphere

Spherical diffusion, as it may be from soil aggregates or fungal masses, has been widely used in soil aeration studies. Its derivation is also presented by Crank (1975). Mathematically it is similar to the linear case which can be derived by simple transformation.

The differential equation for radial diffusion coefficient is expressed by equation (11) for steady-state it becomes

$$\frac{\delta}{\delta r}\left(r^2 \cdot \frac{\delta c}{\delta r}\right) = 0 \qquad (25)$$

of which the general solution is

$$C = K_2 + K_1/r \qquad (26)$$

Where **K₁** and **K₂** are constants to be determined from the boundary conditions. If in the hollow sphere, $a \leq r \leq b$, the surface r=a is kept at a constant concentration C_1, and r=b on C_2 , then

$$C = \frac{aC_1\,(b - r) + bC2\,(r - a)}{r\,(b - a)} \qquad (27)$$

The final solution is:

$$\frac{Q}{t} = D \,.\, A \,\frac{a \,.\, b}{r^2\,(b - a)}\,(C_1 - C_2) \qquad (28)$$

where $a \leq r \leq b$

The quantity of diffusing substance Q which passes through the spherical wall in time t is given by

$$\frac{Q}{t} = 4\pi\, D\,\frac{a \,.\, b}{(b - a)}\,(C_2 - C_1) \qquad (29)$$

where $a \leq r \leq b$.

The conductance in the radial direction measured in cm^3. sec^{-1} will be

$$K = 4\pi\, D\,\frac{a \,.\, b}{(b - a)} \qquad (30)$$

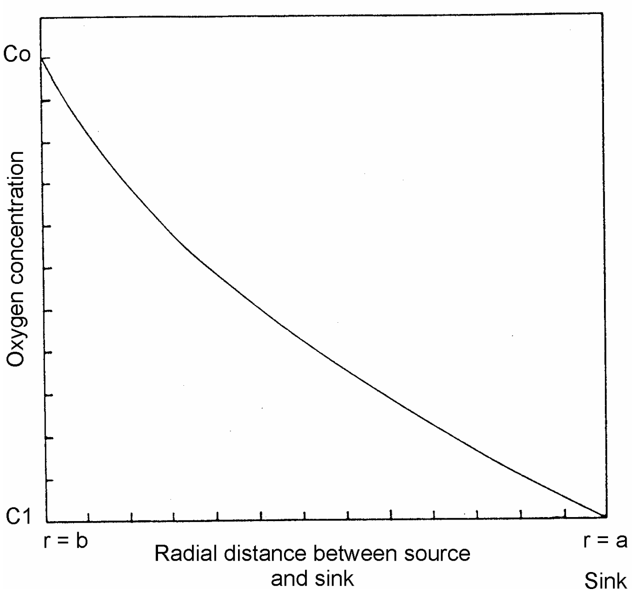

Fig. 4.3 A curvilinear concentration profile characteristic of a diffusive system in which source and sink lie on concentric cylinders (After Crank, 1975).

The equation (30) may be obtained by analogy with equation (15). For spherical diffusion $A(x) = 4x^2$, where x is the radial distance from the centre of the sphere. Integration of equation (15) for the boundary conditions x=a and x=b gives the solution for K as expressed by equation (30).

It is interesting to note that, for a spherical aggregate, the conductance becomes independent of b when b is large compared to the radius of the aggregate. In other words when $b > a$ equation (30) becomes (Campbell, 1985):

$$K = 4\pi D \,.\, a \qquad (31)$$

Continuity equation including sink terms

The flux of oxygen is described by Fick’s law but it does not account for oxygen consumption. According to the law of conservation of mass the change of the amount of oxygen in the volumetric unit of soil and in the unit of time for one-dimensional flow is equal to the change of flux δFx in this elemental volume of soil ∂x mimus the volumetric oxygen consumption of soil Sr, according to the equation:

$$\frac{\delta Q}{\delta t} = - \frac{\delta Fx}{\delta x} - Sr \qquad (32)$$

As the amount of oxygen in the soil Q is taken equal to the amount of oxygen in the gas phase of the soil, after neglecting oxygen in the liquid phase and solid phase ($\delta(C_{\varepsilon} \,.\, \varepsilon)$) and by substitution of this into equation 32 we get:

$$\frac{\delta(C_{\varepsilon} \,.\, \varepsilon)}{\delta t} = - \frac{\delta Fx}{\delta x} - Sr \qquad (33)$$

as $Fx = D_{\varepsilon}\partial c/\partial x$ (first Fick´s law) and substituting the right hand side term of this equation into equation (33) it yields:

$$\frac{\delta(C_{\varepsilon} \,.\, \varepsilon)}{\delta t} = - \frac{D_{\varepsilon}\, \delta c}{\delta x \,.\, \delta x} - Sr \qquad (34)$$

In the three-dimensional space it is written as:

$$\frac{\delta Q}{\delta t} = \nabla\, (D_{\varepsilon} \,.\, \nabla c\,) - Sr \qquad (35)$$

where ∇ is the nable operator indicating three-dimensional flux.

If we consider only longitudinal transport in Z Direction, that means with depth Z, the one-dimensional equation (34) becomes.

$$\frac{\delta}{\delta t}(C_{\varepsilon} . \varepsilon) = \frac{\delta(D_{\varepsilon}\,\delta c)}{\delta z . \delta z} - Sr \qquad (36)$$

and for this equation the initial and bounday conditions are:

for $t = 0$ and $z > 0$, $C = Co$;
for $t > 0$ and $z = 0$, $C = Co$;
for $t > 0$ and $z = L$ $\delta C/\delta Z = 0$

additional assumptions are related to the values of εg, D_{ε} and Sr. It is known, for example, that the oxygen diffusion coefficient Dg is not dependent upon oxygen concentration C. It is assumed that the oxygen respiration Sr is zero if oxygen concentration Cis lower than or equal to a certain critical value. In fact, oxygen uptake Sr is not greatly dependent upon oxygen concentration C is higher than few percent ($Sr > 0$ for $C > 0$).

Additional solution of diffusion equation (36) can give the values of C as a function of z, t, εg, D_{ε} and Sr.

After assuming that ε_g and D_{ε} are constant (case of a homogeneous soil profile) from Fick's second law the continuity equation for z direction can be expressed: (Currie, 1961).

$$\varepsilon . \frac{\delta c}{\delta t} = D_{\varepsilon} \cdot \frac{\delta^2 c}{\delta z^2} - Sr \qquad (37)$$

Equation (37) can also be expressed as follows

$$\frac{\delta c}{\delta t} = \frac{D_{\varepsilon}}{\varepsilon} \cdot \frac{\delta^2 c}{\delta z^2} - \frac{Sr}{\varepsilon} \qquad (38)$$

respiration rate, Sr, may be increased in several ways: by increasing the amount of readily degradable organic matter, by increasing root activity, by watering, in so far this encourages microbial and root activity, and by warming the soil (Currie, 1984).

Ohm's law and diffusion analogy

If we have a multilayered soil composed of n layers or elements of thick nesses z_1, z_2..., z_n and diffusion coefficients $D_{\varepsilon}1$, $D_{\varepsilon}2$,...$D_{\varepsilon}n$, the fall in concentration through the whole soil profile is the sum of the falls through the elements. Since the rate of transfer F is the same across each element, the total drop in concentration can be expressed by

$$\frac{Fz_1}{D_{\varepsilon}1} + \frac{Fz_2}{D_{\varepsilon}2} + ... \frac{Fzn}{D_{\varepsilon}n} = (R_1 + R_2 + \ldots + R_n)\ J, \qquad (39)$$

where $R_1 = Z_1/D_{\varepsilon}1$, etc. may be termed formally the resistance to diffusion of each layer (reciprocal of conductance). Thus, the resistance to diffusion of the whole profile is simply the sum of the resistances of the separate layers; assuming that there are not barriers to diffusion between them.

There are close similarities between equations (7) and (8) and the expression of Ohm's law for the conduction of electricity through a homogeneous conductor and it is often helpful to consider diffusion problems using such electrical analogues; it has also proved helpful to develop functional model of diffusion using electrical system (Campbell, 1985). In its expanded form Ohm's law may be written (Armstrong, 1979):

$$\frac{e}{t} = f\ (Vo - V_1)\ \frac{A}{L} \qquad (40)$$

~~whe~~re e is the quantity of electricity (coulombs) flowing through a conductor in t (seconds), L is the length of the conductor (cm), A is its sectional area (cm^2), V_0 and V_1 represent the electrical potential (volts) at the beginning and end of the conductor, and f is the conductivity constant, the value of which depends on the quality of the conducting material and on temperature. Comparing equations (8) and (36) it will be apparent that Q/t is analogous with e/t, D_{ε} with f, $C_0 - C_z$ with $V_0 - V_1$, and that diffusive resistance $L/D_{\varepsilon}A$ is an analogue of electrical resistance L/f. In the condensed version of Ohm's law e/t is reduced to the term I (amperes), $V_0 - V_1$ reduces to V, and L/fA becomes R, the resistance of the conductor which is measured in ohms (Ω). Ohm's is then written as

$$I = \frac{V}{R} \qquad (41)$$

and is equivalent to a condensed form of equation (8), i.e. $Q/t = \delta c/R$, where δc represents ($C_0 - C_z$) and R represent $L/D_{\varepsilon}A$.

At this stage it may be useful to note that just as in an electrical circuit one may calculate the voltage drop (V') along any section of conductor by applying the relationship V' = IR' where I is the current flowing through the whole conductor and R' is the resistance in the segment; in a diffusion system one may be calculate a localized concentration drop. For homogeneous conductors R' = R/L where R is the length of the segment, L the length of conductor and R its total resistance.

For a number of conductors in series Ohm's law reads:

$$I = \frac{V}{R' + R'' + R''' + \ldots} = \frac{V}{R} \qquad (42)$$

Similarly, diffusive resistances in series become additive and, as with the flow of electricity where only R approaches zero, so too with diffusion: Q/t remains finite at all values of R $< \infty$. Therefore $R \to \infty$ $J \to 0$; for instance when depth z approach the water table position at depth L.

This important principle is illustrated in fig. 4 where the change in diffusion rate consequent upon extending the distance between source and sink across an isotropic medium is plotted against the change in diffusive resistance. While diffusive resistance increases linearly with increasing path length (depth z) the diffusion rate decreases in a curvilinear fashion, certainly, conductors in series are additive in their resistance to flow in both electrical and diffusion systems.

Models on soil aeration following this direction are presented by Campbell (1985) and Kowalik (1985).

Diffusion in soil aggregates

As declared earlier First Fick´s law is fairly realistic for longitudinal transport within the soil profile as a whole. However, if the gas phase is discontinuous somewhere within the soil profile, the assumption of constant Sr/Dg will not hold for a soil profile. Thus, the microconcept is applied for aggregates. Oxygen supply and oxygen flow into the soil aggregates take place in the radial direction from outside to the centre of aggregates, with oxygen consumption in the way.

Mathematical description and solution of oxygen diffusion process in spherical aggregates was given by Currie (1961a) and kowalik (1985). Assuming for simplicity a spherical crumb. For an isotropic concentric shell of radius r, the diffusion equation may written:

$$\frac{\delta \varepsilon . Cr}{\delta t} = \frac{1}{r^2} . \frac{\delta}{\delta r} (D_{\varepsilon} r^2 \frac{\delta c}{\delta r}) - Sr \qquad (43)$$

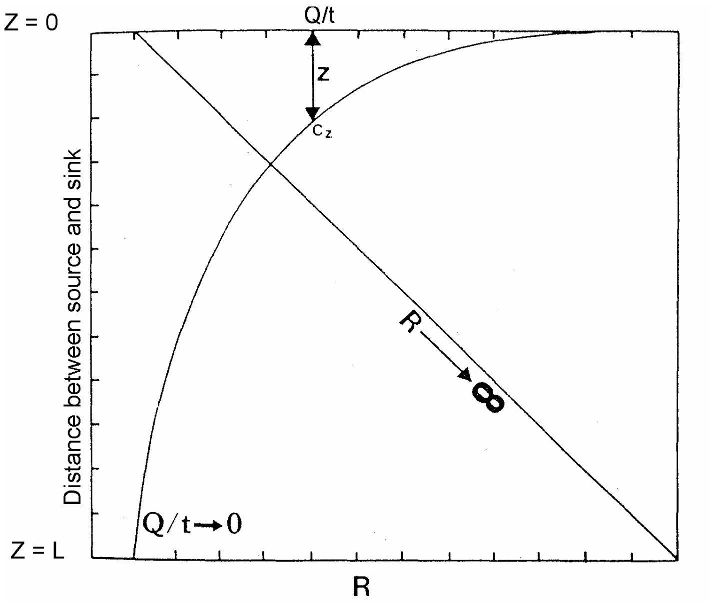

Fig. 4 Showing how a linear increase in diffusive resistance between source and sink in a simple diffusive system is accompanied by a curvilinear decrease in flux (After Armstrong, 1979).

Where the symbols are as before but are referring to the properties exclusive to the crumbs as opposed to those of the soil as a whole. Thus, Cr is the concentration of the gas in the pore space Eg.

In a soil in equilibrium, the term δcrδt may be zero (steady-state) and the equation 3.43 rewritten in the form.

$$\frac{\delta}{\delta r}\left(D_{\varepsilon} \cdot r^2 \cdot \frac{\delta c}{\delta r}\right) = Sr \cdot r^2 \qquad (44)$$

Further development of equation 44 is presented by Currie (1961) and kowalik (1985). By assuming Cr = Cx for r = R at the surface of the aggregate the final solution is:

$$Cr = Cx - \frac{Sr}{6D_{\varepsilon}}(R^2 - r^2) \qquad (45)$$

Equation 45 allows to calculate the oxygen concentration distribution inside the soil aggregate as a function of radius of the aggregate R, diffusion coefficient Dg, biological respiration Sr and oxygen concentration in the gas phase of the soil profile Cx (it is equal to Cz in the equations dealing with oxygen diffusion in the soil profile). Currie (1984) proposed that the difference Cx – Cr could be called an oxygen deficit in the soil aggregate. Taking Cx – Cr = Δ we have:

$$\Delta = Cx - Cr = \frac{Sr}{6D_\varepsilon}(R^2 - r^2) \qquad (46)$$

And for the maximum deficit at the centre of the aggregate, for r = 0:

$$\Delta = \frac{Sr}{6D_\varepsilon}(R^2 - r^2) \qquad (47)$$

Thus, oxygen deficit Δ is directly proportional to the radius of the aggregate in the second power, R^2. This means, lit is quadrupled when the crumb radius is doubled.

For the soil profile, if we want to avoid creating anaerobic conditions inside soil aggregates, then the maximal radius of the soil aggregate is determined by: (Smith, 1977, 1979; Currie, 1984; Kowalik, 1985).

$$R \leq \sqrt{\frac{6D_\varepsilon \,.\, Cx}{Sr}} \qquad (48)$$

Diffusion through the water films

The final segment of the diffusion path to roots takes place through the water films or shell surrounding the roots. The geometry of water- films is depicted in figure 4.5, where the cylindrical coordinates can be applied. For steady-state diffusion in cylindrical coordinates, we have: (Letey and Stolzy, 1967)

$$Fx = D' \frac{Cp}{Rr \,.\, \ln\left(\frac{Rw}{Rr}\right)} \qquad mg.cm^{-2}.s^{-1} \qquad (49)$$

where D' is the coefficient of oxygen diffusion through the water films. The rest of the symbols are indicated in fig. 5. Instead of Fx the symbol ODR (oxygen diffusion rate) has been proposed (Letey and Stolzy, 1964); then

$$ODR = 60x10^{-3}.\ D' \frac{Cp}{Rr \,.\, \ln\left(\frac{Rw}{Rr}\right)} \qquad g.cm^{-2}.min^{-1} \qquad (50)$$

$$ODR = 60x10^{-3}.\ D' \frac{Cp/Rr}{\ln(1 + d/Rr)} \qquad (51)$$

If we consider not only the situation of oxygen respiration at the surface of the root, but also take a cylindrical sample of soil with supply of oxygen from outside and oxygen diffusion and consumption inside the medium the situation from fig 6 is described by the cylindrical form of the diffusion equation:

$$\frac{\delta}{\delta t}(Eg.Cr) = \frac{1}{r}\frac{\delta}{\delta r}\left(D_{\varepsilon} \, . \, r \, . \, \frac{\delta c}{\delta r}\right) = Sr \qquad (52)$$

and for steady state and appropriate boundary conditions (Lemon, 1962; Kowalik, 1985) derived the following equation:

$$Cr = Cp - \frac{Sr}{4\,Dg}\left(R^2 r - r^2\right) \qquad (53)$$

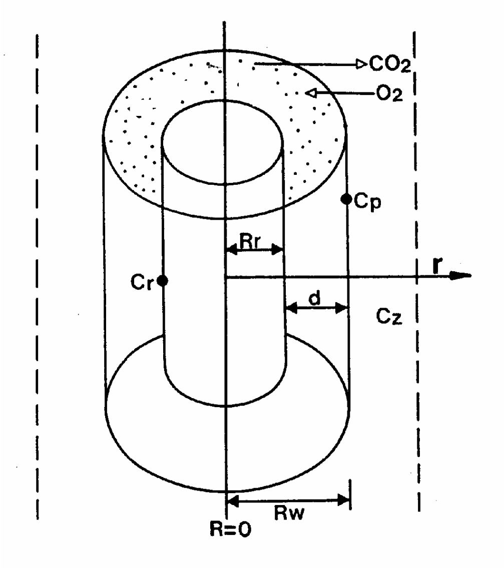

Fig. 5 Plant root surrounded by water-film (After Hillel, 1980; Kowalik, 1985).

Rr: radius of plant root or platinum electrode;
Rw: radius of outside surface of water. film on the plant root or on the platinum electrode;
Cp: oxygen concentration in the soil liquid phase on the border with the gas phase.

From equation (46) it is possible to calculate for which conditions the middle part of the root can be anaerobic, having Sr, Dg, Cp and Rr.

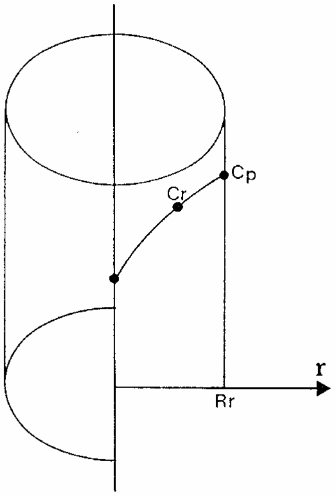

Fig. 6 cylindrical part of the soil body with oxygen consumption inside and oxygen supply from outside (simplified, after Kowalik, 1985)