Anniversary College on Soil Physics

Lecture Notes

In-Situ Determination of Directional Conductivities of Soil

by
Gautam Barua
Department of Civil Engineering
Indian Institute of Technology, Guwahati
India

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Darcy's law mostly governs the flow of fluid in a porous medium. For an anisotropic medium, Darcy's law at a point *P* in a porous continuum (Bear, 1972) may be expressed as

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{bmatrix} -\dfrac{\partial\phi}{\partial x_1} \\ -\dfrac{\partial\phi}{\partial x_2} \\ -\dfrac{\partial\phi}{\partial x_3} \end{bmatrix} \tag{1}$$

where v_1, v_2 and v_3 are the components of Darcy velocity **v** and ϕ is the average hydraulic head over a representative elemental volume (REV) at the point *P*. The second rank symmetric tensor

$$\mathbf{K} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \tag{2}$$

is called the hydraulic conductivity of the porous medium. Thus,

$$v_i = -K_{ij}\frac{\partial\phi}{\partial x_j} = -K_{ji}\frac{\partial\phi}{\partial x_j} = -K_{ji}\frac{\partial}{\partial x_j}\left(\frac{p}{\rho g} + z\right) \; ; \qquad (i=1,2,3) \tag{3}$$

Hydraulic conductivity **K** is a macroscopic parameter, which depends on the properties of both the fluid as well as the porous matrix. At a point P in a porous medium, it may be expressed as

$$\mathbf{K} = \frac{\mathbf{k}\rho g}{\mu} \tag{4}$$

where ρ and μ are the macroscopic averages of density and viscosity of the fluid over an REV at the point P, g is the acceleration due to gravity, and **k** is the intrinsic permeability of the medium. It should be noted that **k** is independent of the properties of the fluid and has the dimensions of L^2. From equations (3) and (4), we obtain

$$v_i = -\frac{k_{ij}}{\mu}\left(\frac{\partial p}{\partial x_j} + \rho g \frac{\partial z}{\partial x_j}\right) \tag{5}$$

Stratified soils are usually anisotropic in nature. In most soils, the water transmitting capacity in the horizontal direction is observed to be higher than the vertical conductivity. However, in many soils (e.g., loess), vertical joints, root holes and animal burrows make the vertical conductivity higher than the horizontal. Accurate estimations of the horizontal and the vertical conductivities of a soil medium in its natural water-saturated state is of considerable importance in obtaining rational solutions to drainage and other groundwater flow problems.

One of the most commonly used and reliable methods for in-situ determination of saturated hydraulic conductivity of soil below a water table is the auger hole method (Dorsey et al., 1990). The method was described in detail, among others, by Van Bavel and Kirkham (1948), Luthin (1957), Bouwer and Jackson (1974), Oosterbaan and Nijland (1994). Barua and Tiwari (1995) gave an extensive review of the various theories and procedures associated with the method. The method essentially consists of pumping a cylindrical hole dug below a water table and noting the rate of rise of water in the pumped hole. The rate of rise is then translated into saturated hydraulic conductivity of the soil by applying a suitable theory obtained from the classical works of Kirkham and Van Bavel (1948), Kirkham (1958) and Boast and Kirkham (1971). For an auger hole dug into a water table aquifer and underlain by an impervious stratum, the theories proposed by Barua and Tiwari (1995) may also be employed to get the directional conductivities values; however, if the hole pierces a confined aquifer, none of the above theories can be safely applied and in that case the theories proposed by Barua and Hoffmann (2002) may be used. This lecture will be mainly concerned with the development of a suitable auger hole seepage theory for the confined situation.

Figure 1 show pumping from an auger hole of radius *a* dug into a homogeneous and anisotropic confined aquifer of infinite radial extent. An impervious layer at a finite distance from the confining stratum underlies the aquifer. The depth to the impervious layer, partial penetration of the auger hole, level of water in the auger hole, and confining pressure of the aquifer are taken as h_i, H_3, H_1 and t, respectively, all distances being measured from the confining stratum as shown in the figure. The saturated hydraulic conductivities of the soil in the horizontal and vertical directions are taken as K_r and K_z, respectively. Because of axial symmetry, we consider only one half of the flow domain for analysis located towards the right of the vertical axis passing through the origin *O*. For convenience, we take the *z* axis to be positive vertically downward and *r* axis to be positive towards the right. Further, in the analysis to follow, we assume the flow to be steady, the drawdown near the vicinity of the hole during one experimental cycle to be negligible the aquifer material and water to be incompressible and the principal directions of anisotropy of the aquifer to coincide with the horizontal and vertical directions, respectively. Following a similar approach like that of Barua and Tiwari (1995), we solve the boundary value problem by dividing each flow domain into two sub-domains: (1) the region between the bottom of the hole and the impervious layer and (2) the rest of the flow domain. The hydraulic head function for region (1) is designated as $\phi_{1(i)}$ and for region (2) by $\phi_{2(i)}$.

The boundary conditions of the flow problem can now be expressed as

$$\frac{\partial \phi_{1(i)}}{\partial r} = 0; \qquad r = 0; \qquad H_3 \le z \le h_i \qquad \text{(B.C. I)}$$

$$\phi_{1(i)} = -H_1; \qquad z = H_3; \qquad 0 \le r \le a \qquad \text{(B.C. II)}$$

$$\phi_{1(i)} = \phi_{2(i)}; \qquad r = a; \qquad H_3 \le z \le h_i \qquad \text{(B.C. IIIa)}$$

$$K_r \frac{\partial \phi_{1(i)}}{\partial r} = K_r \frac{\partial \phi_{2(i)}}{\partial r}; \qquad r = a; \qquad H_3 \le z \le h_i \qquad \text{(B.C. IIIb)}$$

$$\frac{\partial \phi_{1(i)}}{\partial z} = 0; \qquad z = h_i; \qquad 0 \le r \le a \qquad \text{(B.C. IV)}$$

$$\phi_{2(i)} = -z; \qquad r = a; \qquad 0 \le z \le H_1 \qquad \text{(B.C. Va)}$$

$$\phi_{2(i)} = -H_1; \qquad r = a; \qquad H_1 \le z \le H_3 \qquad \text{(B.C. Vb)}$$

$$\phi_{2(i)} = t; \qquad z = 0; \qquad a < r \le \infty \qquad \text{(B.C. VI)}$$

$$\frac{\partial \phi_{2(i)}}{\partial r} = 0; \qquad r = \infty; \qquad 0 \le z \le h_i \qquad \text{(B.C. VII)}$$

$$\frac{\partial \phi_{2(i)}}{\partial z} = 0; \qquad z = h_i; \qquad a \le r \le \infty \qquad \text{(B.C. VIII)}$$

In order to obtain solution to the problem as shown Fig. 1, the hydraulic head functions $\phi_{1(i)}$ and $\phi_{2(i)}$, must be determined such that the governing equations

$$K_r \frac{\partial^2 \phi_{1(i)}}{\partial r^2} + \frac{K_r}{r} \frac{\partial \phi_{1(i)}}{\partial r} + K_z \frac{\partial^2 \phi_{1(i)}}{\partial z^2} = 0 \qquad (6)$$

in region (1) and

$$K_r \frac{\partial^2 \phi_{2(i)}}{\partial r^2} + \frac{K_r}{r} \frac{\partial \phi_{2(i)}}{\partial r} + K_z \frac{\partial^2 \phi_{2(i)}}{\partial z^2} = 0 \qquad (7)$$

in region (2) must be satisfied together with the boundary conditions as listed above.

The quantity of water, $Q_{(i)}$, entering the auger hole is given by

$$Q_{(i)} = 2\pi a \sum_{n=1}^{N} B_{n(i)} \sqrt{(K_r K_z)}\, k_{1(i)} \left[\frac{-N_{n(i)} a}{K_0^{'}} \right] \tag{8}$$

where (for $N_{n(i)} \neq N_{m(i)}$)

$$B_{n(i)} = \frac{-2}{h_i \left(N_{n(i)}\right)^2} \sin(N_{n(i)} H_1) - \frac{2t}{h_i (N_{n(i)})}$$

$$+ \sum_{m=1}^{M} \frac{A_{m(i)}}{h_i} \Bigg[\frac{1}{(N_{n(i)} - N_{m(i)})} \sin\left[N_{m(i)} H_3 + (N_{n(i)} - N_{m(i)}) h_i\right]$$

$$- \frac{1}{(N_{n(i)} - N_{m(i)})} \sin\left[N_{m(i)} H_3 + (N_{n(i)} - N_{m(i)}) H_3\right]$$

$$+ \frac{1}{(N_{n(i)} + N_{m(i)})} \sin\left[N_{m(i)} H_3 - (N_{n(i)} + N_{m(i)}) h_i\right]$$

$$- \frac{1}{(N_{n(i)} + N_{m(i)})} \sin\left[N_{m(i)} H_3 - (N_{n(i)} + N_{m(i)}) H_3\right] \Bigg]$$

(9)

$$A_{m(i)} = \sum_{n=1}^{N} \frac{P_{nm(i)} B_{n(i)}}{(h_i - H_3)} \Bigg[\frac{1}{(N_{n(i)} - N_{m(i)})} \sin\left[N_{m(i)} H_3 + (N_{n(i)} - N_{m(i)}) h_i\right]$$

$$- \frac{1}{(N_{n(i)} - N_{m(i)})} \sin\left[N_{m(i)} H_3 + (N_{n(i)} - N_{m(i)}) H_3\right]$$

$$+ \frac{1}{(N_{n(i)} + N_{m(i)})} \sin\left[N_{m(i)} H_3 - (N_{n(i)} + N_{m(i)}) h_i\right]$$

$$- \frac{1}{(N_{n(i)} + N_{m(i)})} \sin\left[N_{m(i)} H_3 - (N_{n(i)} + N_{m(i)}) H_3\right] \Bigg] \tag{10}$$

$$P_{nm(i)} = \frac{(-N_{n(i)})}{(N_{m(i)})} \frac{k_{1(i)}\left(\dfrac{-N_{n(i)} a}{K_0^a}\right)}{i_{1(i)}\left(\dfrac{-N_{m(i)} a}{K_0^a}\right)} \tag{11}$$

$$N_{m(i)} = \left[\frac{(1-2m)\pi}{2(h_i - H_3)}\right] \tag{12}$$

$$N_{n(i)} = \left[\frac{(1-2n)\pi}{2h_i}\right] \tag{13}$$

$$i_{0(i)}\left(\frac{-N_{m(i)}r}{K_0^a}\right)=\frac{I_0\left(\frac{-N_{m(i)}r}{K_0^a}\right)}{I_0\left(\frac{-N_{m(i)}a}{K_0^a}\right)} \tag{14}$$

$$i_{1(i)}\left(\frac{-N_{m(i)}r}{K_0^a}\right)=\frac{I_1\left(\frac{-N_{m(i)}r}{K_0^a}\right)}{I_0\left(\frac{-N_{m(i)}a}{K_0^a}\right)} \tag{15}$$

$$k_{0(i)}\left(\frac{-N_{n(i)}r}{K_0^a}\right)=\frac{K_0\left(\frac{-N_{n(i)}r}{K_0^a}\right)}{K_0\left(\frac{-N_{n(i)}a}{K_0^a}\right)} \tag{16}$$

$$k_{1(i)}\left(\frac{-N_{n(i)}r}{K_0^a}\right)=\frac{K_1\left(\frac{-N_{n(i)}r}{K_0^a}\right)}{K_0\left(\frac{-N_{n(i)}a}{K_0^a}\right)} \tag{17}$$

$$K_0^a=\left(K_r/K_z\right)^{1/2}; \tag{18}$$

$I_0(.)$, $I_1(.)$, $K_0(.)$ and $K_1(.)$ are zero order and first order modified Bessel functions of first and second kinds, respectively, *m* and *n* are summation indices 1,2,3, …. $M=N\rightarrow\infty$.

Example

Let us now apply the developed theory to the auger hole experimental data obtained from a real field situation. An auger hole experiment was performed near “De Nieuwelanden” at Wageningen University (The Netherlands) in 2001, where the following data were observed: soil isotropic, i.e. $K_0^a = 1$, confining pressure, t=42cm, h_i=300cm, H_1=30.5cm, H_3=82cm, a=5cm, and rate of rise of water in the auger hole =0.2680 cm/sec; therefore, $Q_{(i)} = \pi(5)^2\, 0.2680 = 21.049$ cm^3/sec.

Substituting the above values in equation (8), we get

$$21.049 = 2\pi \times 5 \times \sum_{n=1}^{N} B_{n(i)} \frac{K}{1} k_{1(i)} \left[\frac{-N_{n(i)} \times 5}{1} \right] \tag{19}$$

where

$$N_{m(i)} = \left[\frac{(1-2m)\pi}{2(300-82)} \right] \quad \text{and} \quad N_{n(i)} = \left[\frac{(1-2n)\pi}{2 \times 300} \right]$$

The constants $B_{n(i)}$ of equation (19) can now be determined by means of equations (9), (10) and (11). By expanding up to 100 terms (M=N=100) of the infinite series, we find K to be 0.96 m/day.

If the entire computation is repeated (again by taking M=N=100) by neglecting the confining pressure, i.e. t=0, the K value now turns out to be 2.90 m/day. It can be observed that this value differs considerably from that of 0.96 m/day obtained by considering the confining pressure of the aquifer. As can be seen, an error of about 200% [(2.90-0.96)x100/0.96] occurs for this flow situation due to neglect of this confining water head.

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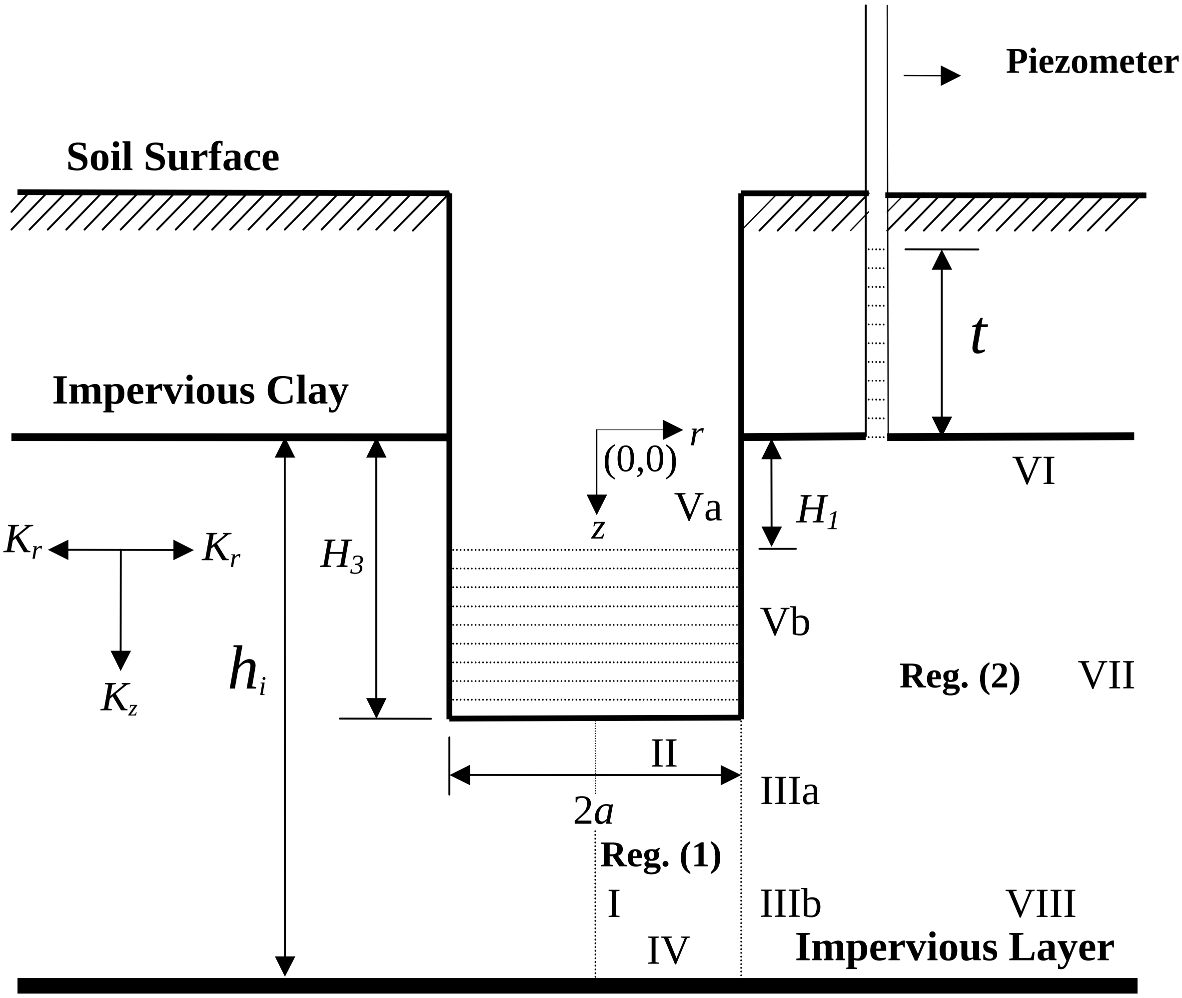

Fig. 1. Geometry of the flow system of a partially penetrating auger hole in a confined aquifer underlain by an impervious layer – water level of the pumped hole lying below the confining clay layer.