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WORKSHOP ON NUCLEAR DATA FOR SCIENCE AND TECHNOLOGY: MATERIALS ANALYSIS

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Physics of the interaction of charged particles with Nuclei

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PHYSICS OF THE INTERACTION OF CHARGED PARTICLES WITH NUCLEI

An atomic nucleus is a strongly bound system of nucleons located in a small domain with a typical size of

$$R \approx (1.1 \div 1.5) \cdot A^{1/3}\,\text{fm} \quad (1\,\text{fm} = 10^{-13}\,\text{cm}) \qquad (1)$$

Nucleons are hold together inside nuclei due to nuclear forces. These forces are strong attractive forces acting only at short distances. They possess property of saturation, due to which nuclear forces are attributed exchange character (exchange forces). Nuclear forces depend on spin, not on electric charge, and are not central forces. The nature of the nuclear forces has not yet fully been clarified.

Nuclear forces are said to be strong forces, in the sense that they are at least 100 times greater than very strong Coulomb forces taken at short nuclear distances of about 1 fm. The short range of nuclear forces leads to a strict demarcation of the regions where only long-range Coulomb forces, or only nuclear forces show up as the latter suppress the Coulomb forces at short distances

The dependence of the force on the space coordinates is described by means of the potential. The presence of one of the interacting bodies is expressed through the potential as a function of the distance from the body center and the force at the point *r*, directed from the first body to the second, is found as a potential derivative with respect to the space coordinates at this point.

Assuming nucleus is a uniformly charged sphere the electrostatic potential energy for the projectile-nucleus system can be written as

$$V_C(r) = \begin{cases} \dfrac{Zze^2}{r} & \text{for } r \geq R \\ \dfrac{Zze^2}{2R}\left(3 - \dfrac{r^2}{R^2}\right) & \text{for } r \leq R \end{cases} \qquad (2)$$

where *Z* and *z* are charge numbers of the nucleus and the projectile respectively.

Nuclear forces are also introduced through the potential energy of the nucleon interaction. The positive potential creates repulsive forces, and the negative potential attractive forces. Therefore, the potential energy is positive if it corresponds to repulsive forces, and it is negative for attractive forces. As a result, the potential energy of the point proton interaction with the nucleus may be presented as is shown in Fig.1

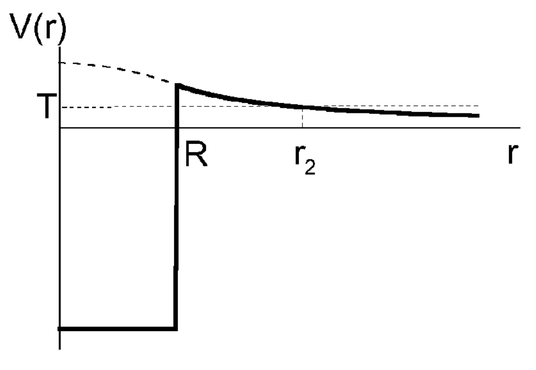

Fig.1

The Coulomb repulsion changes abruptly to attraction at the distance of the radius of action of nuclear forces, i.e. at the boundary of the nucleus *R*. The transition from repulsion to attractions proceeds, though rapidly but continuously, in the region of the space coordinate R. So, to a certain degree of accuracy the nuclear potential is pictured in the form of a square potential well which is about 40÷50 MeV deep.

For a charged projectile to reach the range of action of nuclear forces, it should possess some kinetic energy T sufficient to overcome the Coulomb potential barrier of height

$$B_C = \frac{Zze^2}{R} \qquad (3)$$

which is of the order of 1 MeV even in the interaction of singly charged particles with the lightest nuclei.

According to quantum mechanics the transparency of the Coulomb barrier is given by the formulae

$$D \approx e^{-\frac{2}{\hbar}\int_{r_1}^{r_2}\sqrt{2\mu(V_C - T)}\,dr} \qquad (4)$$

where $\mu = \frac{Mm}{M+m}$ is reduced mass, $r_1 = R$ and r_2 is derived from the relation $T = \frac{Zze^2}{r_2}$.

Thus though electric charge of atomic nuclei hinders the initiation of nuclear reactions with low energy charged particles the reactions are still feasible at energies below the potential barrier. These are so-called under-barrier reactions. The penetrability of Coulomb barrier increases very rapidly as T approaches B_C (eq.3).Therefore, if T does not greatly differ from B_C, the under-barrier reactions take place with remarkable probability.

If the interaction is solely due to electric forces the differential cross section of elastic scattering is derived from energy and angular momentum conservation laws. As far as the law of the interaction (i.e. dependence of the force on the distance) is known it is possible to find a dependence of the scattering angle θ on the impact parameter b which is expressed in a non-relativistic case by the relation

$$\tan\theta = \frac{2Zze^2}{mv^2 b}. \qquad (5)$$

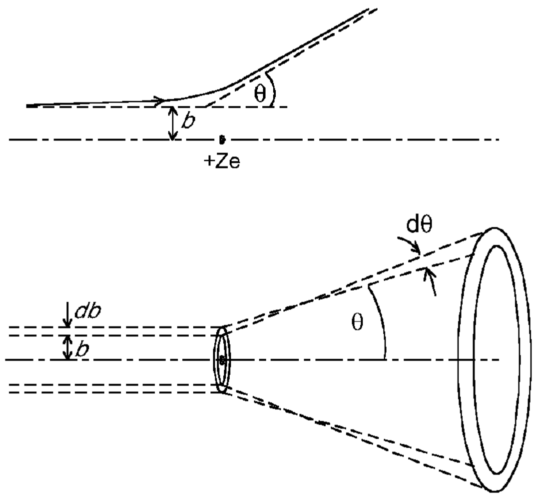

Fig.2

For a single unmoveable nucleus placed on the path of the ion beam of intensity equal to N particles per square cm in 1 sec the number of the ions scattered in the angle interval from θ to $\theta + d\theta$ is $dN = 2\pi b db N$ where b and db are derived from eq.(5). The value

$$d\sigma = \frac{dN}{N} = 2\pi b\, db \qquad (6)$$

is differential cross section which is expressed for the target containing n nuclei per unit area by the well known Rutherford formulae

$$d\sigma = n\left(\frac{Zze^2}{mv^2}\right)\frac{d\Omega}{4\sin^4\frac{\theta}{2}} \quad . \tag{7}$$

Distinct of pure Coulomb scattering the cross section cannot be calculated from an algebraic formulae in case of the nuclear interaction.

Nuclear scattering is considered below for the simplest case of the projectile with no charge.. According to quantum mechanics a particle state is described by the wave function ψ, which is obtained as a solution of the wave equation. For the case of elastic scattering of spinless non-identical particles the wave equation has a form of Schrö dinger equation with a spherically symmetric potential $V(r)$

$$\Delta\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0 \quad , \tag{8}$$

where

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial}{\partial z^2} \quad . \tag{9}$$

Prior to scattering the wave function ψ for the particle with a given momentum p has a form of a plane wave:

$$\psi = \mathrm{e}^{ikz} , \tag{10}$$

where k is a propagation vector

$$k = \frac{p}{\hbar} = \frac{1}{\bar{\lambda}} \quad . \tag{11}$$

Here $\bar{\lambda} = \lambda / 2\pi$, where λ is de Broglie wavelength.
This function is a solution of the eq.(8) in case of $V(r) = 0$, i.e. the equation of the form

$$\Delta\psi + \frac{2m}{\hbar^2} E\psi = 0 \tag{12}$$

and is normalized to correspond to the flux density equal to the projectiles velocity.

In the course of scattering the plane wave interacts with the field of nucleus $V(r)$, that gives rise to a spherical wave divergent from the center of the interaction. This wave has a form of

$$f(\theta)\frac{e^{ikr}}{r} \quad . \tag{13}$$

Thus the last stage of the scattering process (after scattering) is depicted by a superposition of the two waves – plane and spherical ones:

$$e^{ikz} + \frac{e^{ikr} f(\theta)}{r} \quad . \tag{14}$$

Here θ is a scattering angle; *f*(θ) is an amplitude of the divergent wave; the $1/r$ factor stands for decreasing of the flux in reverse proportionality to the square of the distance.

The square of the modulus of the scattered wave amplitude is equal to the differential cross section

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 \quad . \tag{15}$$

This is easy to prove. By definition the differential cross section $d\sigma$ is equal to the fraction dN/N of the initial particles flux N scattered into the given solid angle $d\Omega$. Assuming the density of particles in the primary beam being equal to unity one obtains $N = \upsilon$, where υ is the particles velocity. For dN one obtains

$$dN = \left| f(\theta) \frac{e^{ikr}}{r} \right|^2 \upsilon r^2 \sin\theta \, d\theta \, d\varphi \tag{16}$$

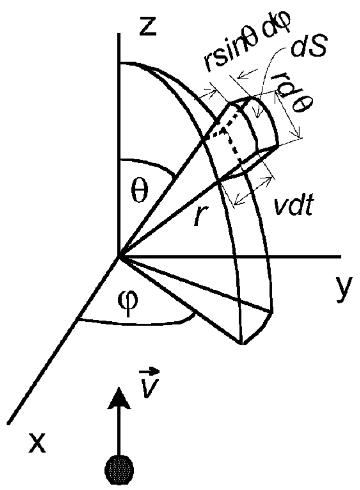

Fig.3

Taking into account that velocity does not change in the elastic scattering and that $\sin\theta d\theta d\varphi = d\Omega$ one finally obtains that

$$d\sigma = \frac{dN}{N} = \frac{|f(\theta)|^2 r^2 \upsilon \, d\Omega}{r^2 \upsilon} = |f(\theta)|^2 d\Omega \quad . \tag{17}$$

or

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 \quad . \tag{18}$$

The angular distribution of the scattered particles is defined by the *f*(θ) function. For the quantitative analysis of the elastic scattering eq. (8) and (12) are considered in spherical coordinates. The general solution of these equations has the form of

$$\psi = \sum_{l=0}^{\infty} A_l P_l(\cos\theta) R_{kl}(r) \quad , \tag{19}$$

where R_l (r) is a radial wave function; $P_l(\cos\theta)$ is Legendre polynomial (P_1=1, P_2=cosθ, P_2=(3cosθ-1)/2, …).

Far from the center of scattering (at large distances r) the radial function for each of l can be represented in form of two partial spherical waves one of which is converging $e^{-i\left(kr-l\frac{\pi}{2}\right)}$ and the other is divergent $e^{i\left(kr-l\frac{\pi}{2}\right)}$.

For the initial stage depicted by a plane wave both the waves have equal amplitudes and

$$R_{kl}(r) \sim e^{i\left(kr-l\frac{\pi}{2}\right)} - e^{-i\left(kr-l\frac{\pi}{2}\right)} \qquad (20)$$

So the plane wave expressed through an expansion over Legendre polynomials has a form of

$$e^{ikz} = \sum_{l=0}^{\infty} \frac{(2l+1)i^l}{2ikr} P_l(\cos\theta)\left[e^{i\left(kr-l\frac{\pi}{2}\right)} - e^{-i\left(kr-l\frac{\pi}{2}\right)}\right]. \qquad (21)$$

Here each of the spherical waves corresponds to the particles moving with given orbital momentum l and is characterized by the angular distribution $P_l(\cos\theta)$ (see Fig.4).

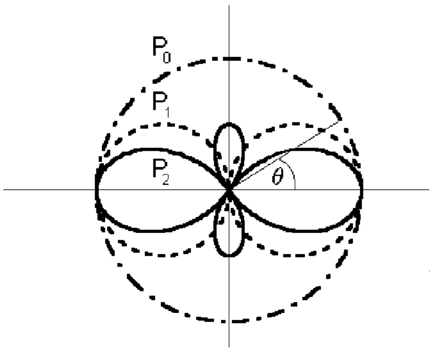

Fig.4

Suppose the projectile posseses kinetic momentum p and angular momentum l. Then from comparison between classical and quantum mechanical relations for the modulus of the angular momentum

$$|l| = \rho p = \hbar\sqrt{l(l+1)} \qquad (22)$$

follows that

$$\rho = \frac{\hbar}{p}\sqrt{l(l+1)} = \lambda\sqrt{l(l+1)} \ , \qquad (23)$$

i.e. the initial beam behaves as if it were subdivided in cylindrical zones with radii defined by eq. (23) as shown in Fig.5.

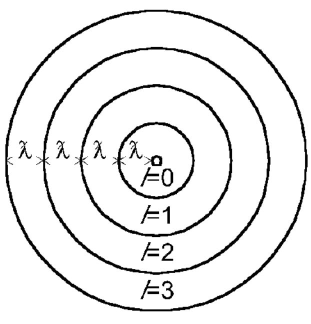

Fig.5

In the process of scattering an additional divergent spherical wave arises. So the ratio between convergent and divergent waves changes. The change of the ratio can be formally taken into account by a coefficient at the divergent wave

$$R_{kl}(r) \sim S_l e^{i\left(kr-l\frac{\pi}{2}\right)} - e^{-i\left(kr-l\frac{\pi}{2}\right)} \qquad (24)$$

In case of the elastic scattering the fluxes for the convergent and divergent waves should be equal to each other for each of l. This means that $|S_l|^2=1$. So the factor S_l can be written as

$$S_l = e^{2i\delta_l} \qquad (25)$$

where δ_l is called a phase shift.

Physically the phase shift is explained by the difference of the wave velocity in the presence of the nuclear forces field and outside the nucleus as is illustrated in Fig.6.

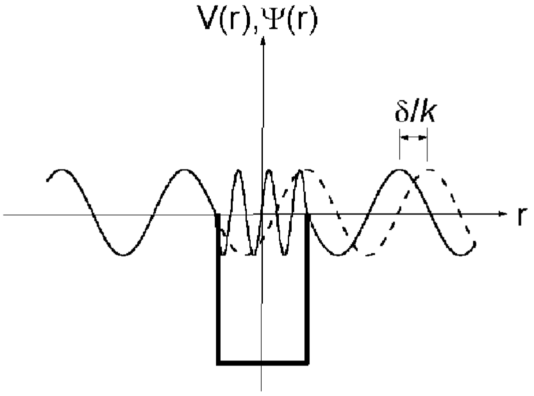

Fig.6

The partial wave after scattering has then a form of

$$R_{kl}(r) \sim e^{i\left(kr - l\frac{\pi}{2} + 2\delta_l\right)} - e^{-i\left(kr - l\frac{\pi}{2}\right)} \qquad (26)$$

The solution of eq. ( ) for the final stage of scattering is

$$e^{ikz} + f(\theta)\frac{e^{ikr}}{r} = \sum_{l=0}^{\infty} \frac{(2l+1)i^l}{2ikr} P_l(\cos\theta)\left[S_l e^{i\left(kr - l\frac{\pi}{2}\right)} - e^{-i\left(kr - l\frac{\pi}{2}\right)}\right]. \qquad (27)$$

The following relation between the scattering amplitude and phases can be derived

$$f(\theta) = \frac{1}{2ik}\sum_l (2l+1)(e^{2i\delta_l} - 1)P_l(\cos\theta). \qquad (28)$$

Summing up, the differential cross section for elastic scattering is calculated from eq. (15), the scattering amplitude being expressed through phase shifts δ_l according to eq.(28). The phase shifts for partial waves are calculated by resolving Schrö dinger equation (8) with assumed potential $V(r)$. This equation is split into angular and radial ones. The asymptotic general solution for radial equation is

$$R_{kl} \approx \sqrt{\frac{2}{\pi}}\frac{1}{r}\sin\left(kr - l\frac{\pi}{2} + \delta_l\right). \qquad (29)$$

The phase shifts δ_l are defined by the edge conditions. The phase shifts are functions of k and l but do not depend on the scattering angle.

If the projectile is charged it interacts with combined Coulomb and nuclear fields of the target nucleus. The relation for the scattering amplitude is then

$$f(\theta) = f_C(\theta) + \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)(S_l - 1)e^{2i\sigma_l}P_l(\cos\theta), \qquad (30)$$

where $f_C(\theta)$ and σ_l are amplitude and phase shift of the Coulomb scattering respectively.

The S_l values defined by eq.(25) can be considered as elements of some diagonal matrix which is called a scattering matrix. In case of pure elastic scattering phase shifts δ_l are real numbers. However they become complex if inelastic scattering is also present in the scattering process. This corresponds to decreasing of the amplitude of the divergent waves i.e. $|S_l|<1$.

In case a projectile possesses non zero spin all the ideology described above retained valid. However, the equations become more complicated since radial wave equation splits into (2s+1) equation. Suppose projectiles are protons which spin is _. Then spin of bombarding particle may be combined with angular momentum l by two ways to produce the total angular momentum $j=l\pm$_.

The proton elastic scattering differential cross section is obtained in this case through resolving of Schrö dinger equations for partial waves as $d\sigma / d\Omega = A(\theta)^2 + B(\theta)^2$, the scattering amplitudes $A(\theta)$ and $B(\theta)$ being defined by the following relations

$$A(\theta) = f_C(\theta) + \frac{1}{2ik}\sum_{l=1}^{\infty}[(l+1)S_l^+ + lS_l^- - (2l+1)]\exp(2i\sigma_l)P_l(\cos\theta);$$

(31)

$$B(\theta) = \frac{1}{2ik}\sum_{l=0}^{\infty}(S_l^+ - S_l^-)\exp(2i\sigma_l)P_l^1(\cos\theta),$$

where $f_C(\theta)$ is an amplitude of Coulomb scattering, σ_l are Coulomb phase shifts, $P_l(\cos\theta)$ are Legendre polynomials, $P_l^1(\cos\theta)$ are associated Legendre polynomials, S_l^+ and S_l^- are scattering matrix elements for different spin orientation, k is a wave number.

The above representation of the elastic scattering process produces the cross section with a smooth dependence on energy. Some rather broad resonances called "shape (or size) resonances" are observed only at energies when conditions for standing waves to form in the nucleus potential well are fulfilled (Fig.7).

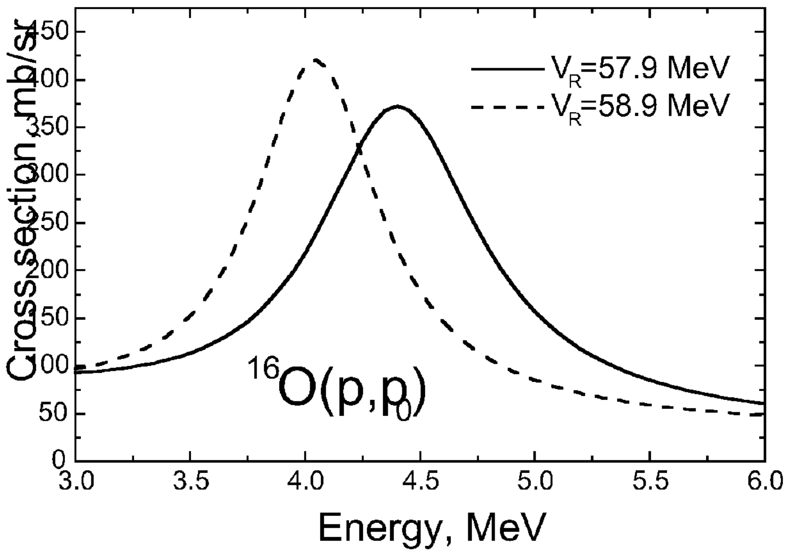

Fig.7

The considered so far mechanism of scattering is called direct or potential scattering since it proceeds through direct interaction of a single bombarding particle with a potential well representing a nucleus. Nuclear interaction at low energies can proceed also in two stages through the mechanism of a compound nucleus (Fig.8). The first stage of the interaction is the absorption of the bombarding particle by the target nucleus and the production of an intermediate, or compound, nucleus. The compound nucleus is always highly excited because the absorbed particle brings both its kinetic energy and the bond energy of the absorbed nucleons into the produced nucleus. The second stage is the decay of the compound nucleus with the emission of this or that particle. The original particle may always be such a particle, and here again the original nucleus is formed. A typical lifetime for a compound nucleus is $\sim 10^{-14}$ sec that is very long as compared with the time of direct interaction defined as a time ($10^{-23} \div 10^{-21}$ sec) needed for the bombarding particle passes through the region occupied by the nucleus potential well.

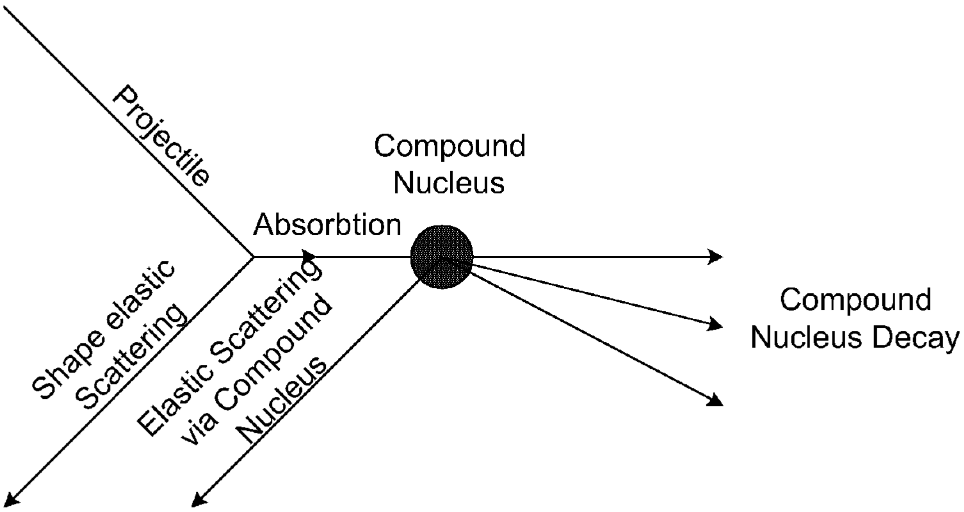

Fig.8

The compound nucleus has discreet energy levels as shown in Fig.9 and so the cross section of the elastic scattering through this mechanism has a resonance structure. Because of the relatively long lifetime and due to the uncertainty relation (written in energy-time coordinates it is $\Delta E \cdot \Delta t \geq \hbar$) the widths of the compound nucleus levels are rather small. So are the widths of the resonances observed in the cross section.

One of the ways to take resonance scattering into account is to add Breit-Wigner resonance terms to the diagonal elements of the scattering matrix:

$$S_l^{\pm} = \exp(2i\lambda_l^{\pm}) \left[\exp(-2\mu_l^{\pm}) + \exp(2i\phi_p) \frac{i\Gamma_p}{E_0 - E - \frac{1}{2} i\Gamma} \right], \qquad (32)$$

where $\lambda_l^{\pm} + i\mu_l^{\pm}$ is the off-resonance nuclear phase shift describing the elastic scattering of protons of energy E from spin zero nuclei. The quantities E_0, Γ, and Γ_p are the energy, total width and partial elastic width, respectively. The subscript l is the relative angular momentum of the proton and the target in units of $\hbar$. The plus sign refers to the case when $J=l+\_$ and the minus sign to the case when $J=l-\_$. The quantity ϕ_p is a resonance phase shift.

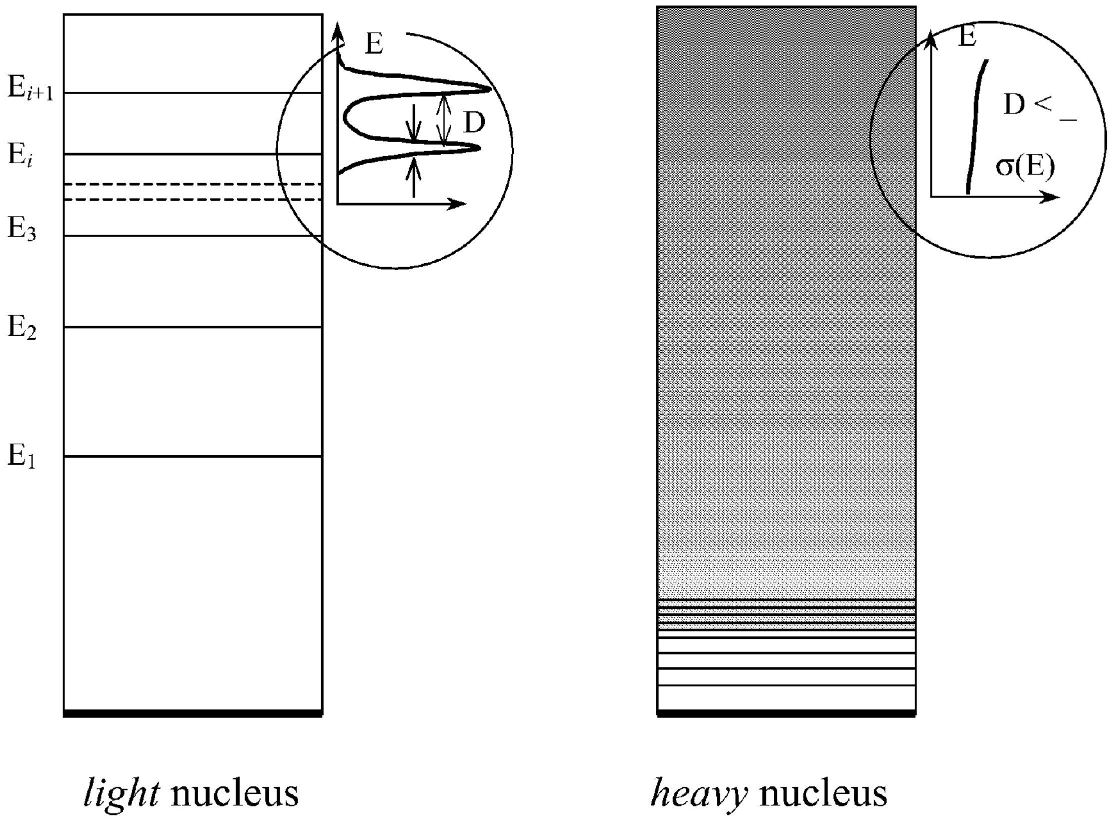

Fig.9

Because of the interference between potential and resonance scattering the excitation function has a typical structure with resonances pictured as dips and bumps rather than as Breit-Wigner functions (Fig.10,11).

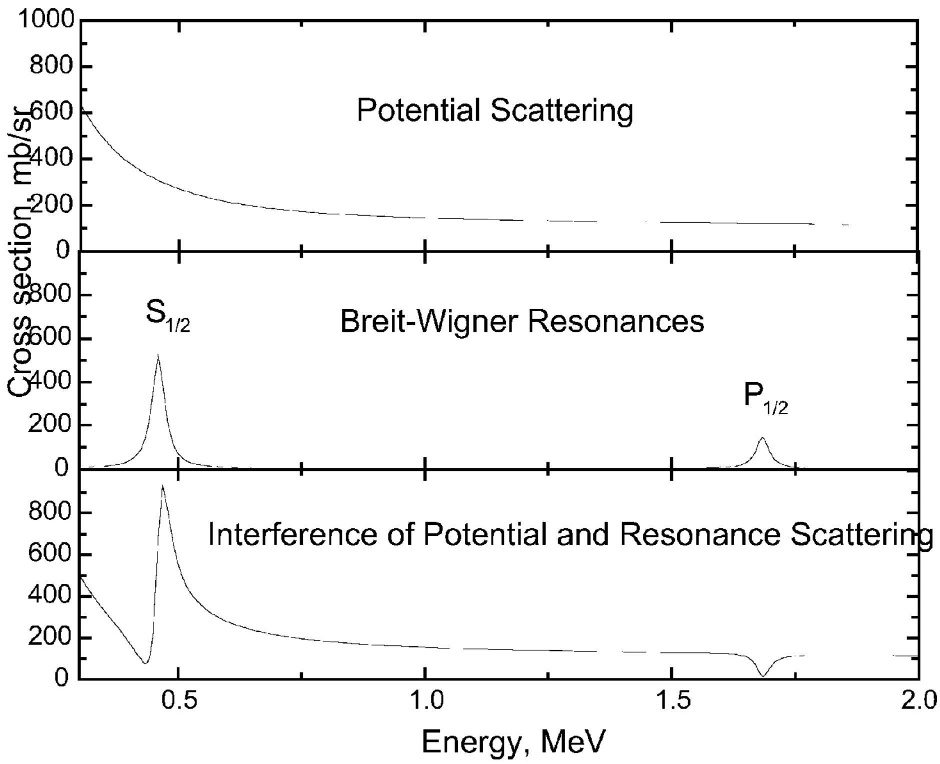

Fig.10

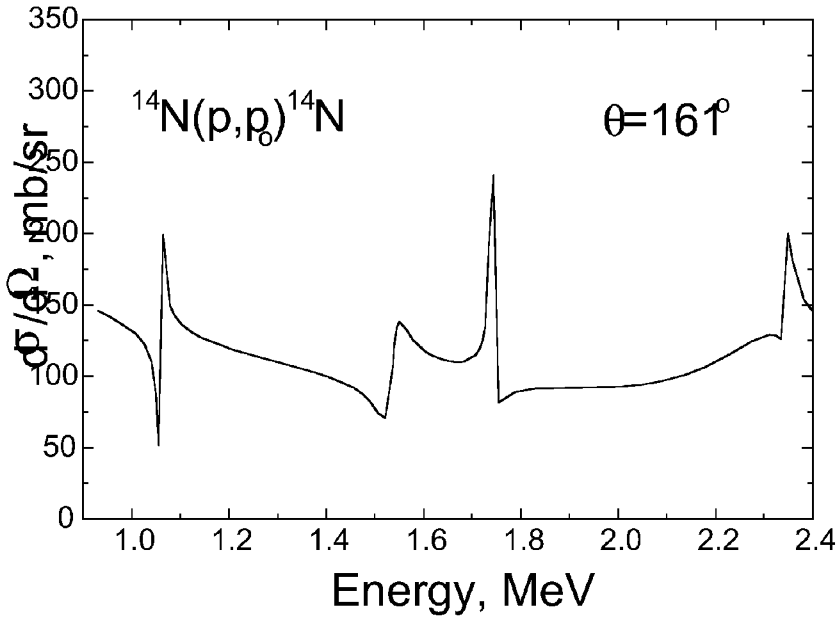

Fig.11

If nuclear reactions contribute to the total cross section along with elastic scattering this should be someway taken into account. Though some progress has been achieved in microscopic theory of nuclear reactions it is practical to apply a phenomenological approach consisting in consideration of the projectile interaction with the nucleus as a whole, the nucleus being represented by an appropriate potential. The potential parameters are found through fitting theoretical calculations to the available experimental data. To make this approach more physical the potential shape is derived from the known features of the nucleon-nucleon interaction and from distributions of matter and charge in the nucleus.

In the so-called optical model nucleus is represented by means of a complex potential. The interaction of the projectile with the nucleus is then reduced to de-Broglie's wave refraction and absorption by a opaque sphere. The name of the model originates from the formal analogy with the light plane wave passing through a semitransparent sphere.

As well as refraction and absorption of the light is described by a complex index

$$n = n_r + i\kappa_a \qquad (33)$$

the complex potential of the form

$$U = V + iW \qquad (34)$$

is used to take into account scattering and absorption of the projectile by the nucleus. The real part of the potential is responsible for scattering whereas the imaginary part stands for absorption.

The standard form of the optical potential is as follows:

$$U(r) = U_C(r) + U_R(r) + iU_1(r) + U_{so}(r)\,, \qquad (35)$$

where U_C is Coulomb potential defined by eq.(2),

$$U_R(r) = -V_R f_R(r) \tag{36}$$

$$U_I(r) = 4a_i W_D \frac{df_i(r)}{dr} \tag{37}$$

$$U_{so} = \left(\frac{\hbar}{m_\pi c}\right)^2 V_{so} \frac{1}{r}\frac{df_{so}}{dr} \boldsymbol{l}\cdot\boldsymbol{s} \tag{38}$$

$$f_R(r) = \left[1+\exp\left(\frac{r-R_x}{a_x}\right)\right]^{-1} \tag{39}$$

$$R_x = r_x A^{1/3} \tag{40}$$

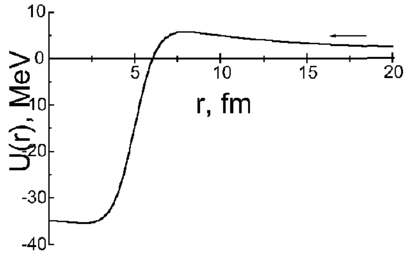

Fig.12

The potential terms represent, in sequence, the real central volume potential of the depth V_R, the imaginary central surface potential of the depth W_D (volume absorption is negligible at low energies), and the surface spin-orbit potential of the depth V_{so}, while $f_x(r)$ is a Saxon-Woods formfactor, R_x is a half value radius, a_x is a diffuseness parameter, A is a target mass number, m_π is a π-meson mass, c is light velocity, $\boldsymbol{l}$ and $\boldsymbol{s}$ denote angular momentum and spin operators respectively.

Due to more than 30 years of application of the optical model the general features of phenomenological optical potential parameters are well established. An intensive study of the low energy anomalies in the optical potential behaviour was made in early 80s. The peculiarities, that were found, are as follows. The strength parameters often have strong energy dependence in the vicinity of the Coulomb barrier. The real potential radial dependence is of more complicated than Saxon-Woods form. The imagine part of potential reveals non-systematic dependence on nucleus mass number. Absorption is peaked at the nucleus surface. The radius of the imaginary potential diminishes with decreasing energy while its diffuseness increases. Calculations in framework of the optical model are very sensitive to the parameters used. So the results obtained with global sets such as the potentials obtained by Perey or Becchetti and Greenlees appear to be unrealistic. Several attempts have been made to develop a global set for low energy region but reliable results may be expected only in case of calculations with parameters fitted to the experimental data. Authors of the recent IAEA co-ordinated research project claime to produce a Reference Input Parameter Library (Fig.13) with optical model parameters for low energy proton scattering included.

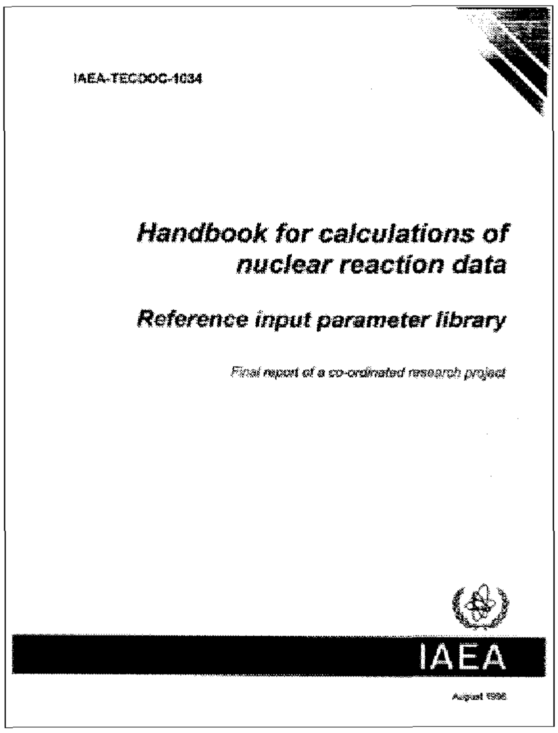

Fig.13

It is evident from Figs.14 and 15 that calculations with RIPL input parameters produce actually cross sections absolutely inconsistent with experimental data.

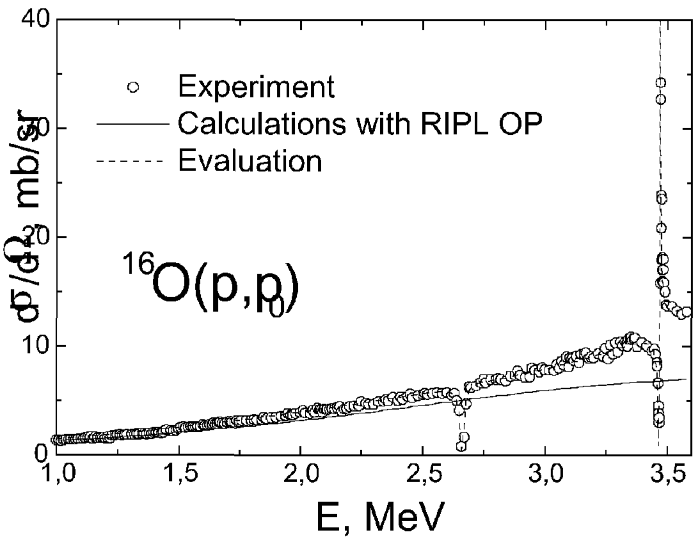

Fig.14

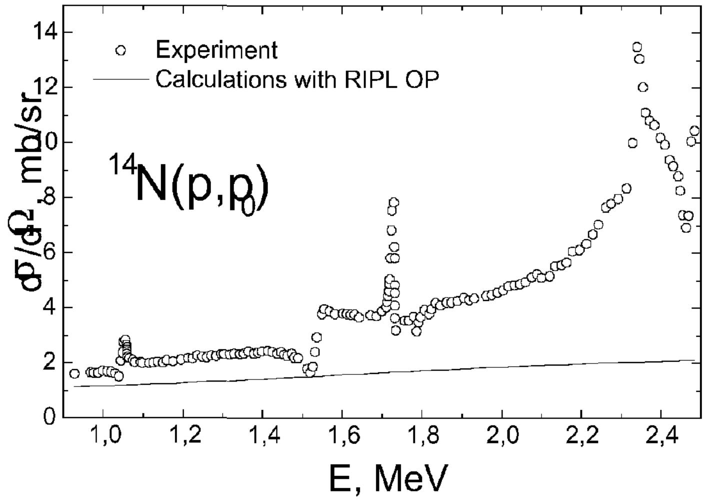

Fig.15

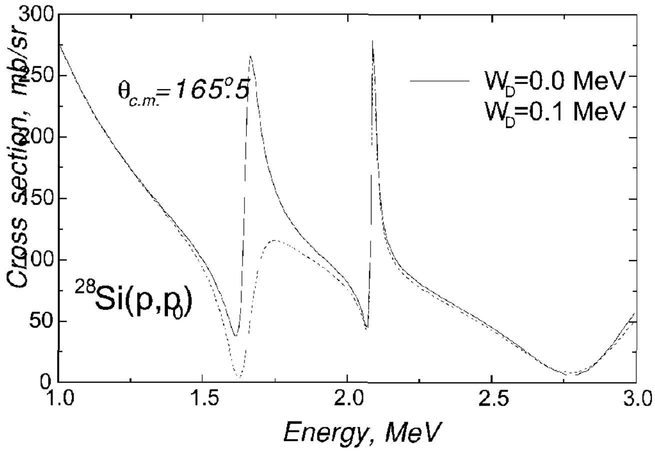

Fig.16

In most cases of charged particles low energy scattering the contribution of the reaction channels is negligible and so the imaginary potential is close to zero. The calculated cross section is extremely sensitive to this parameter as illustrated in Fig.(16). As far as the imaginary part of the potential is equal to zero the cross section is represented by S-matrix formalism rather than by the optical model.

It is interesting to note that the differential cross section at higher energies is insensitive to the spin-orbit potential (38) and it influences only polarization data. In the region of separated resonances such is not the case. Because of the spin-orbit interaction the energy levels split with respect to the total angular momentum as is shown in Fig.17.

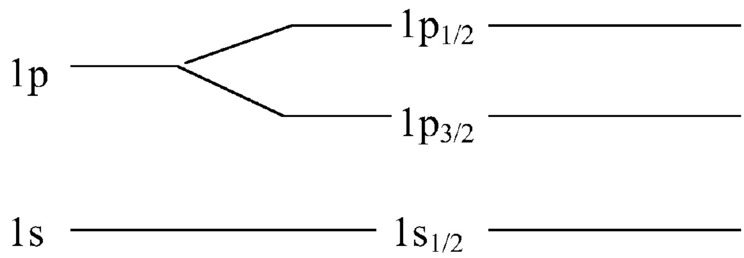

Fig.17

As a result the distance between split resonances strongly depends on the spin-orbit potential (see Fig.18).

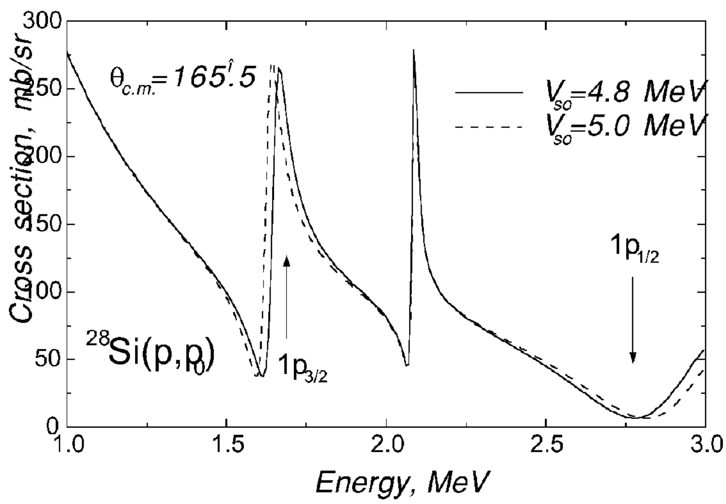

Fig.18

For the reactions induced by low energy deuterons at light nuclei it was assumed that the main contribution to the cross section of the process is given by the following three mechanisms: direct stripping, resonant mechanism and in some cases a compound nucleus mechanism. It was accepted, that the complete amplitude T of process is $T=D+R$, where D is the amplitude of the direct process of stripping, which was calculated within the framework of a method of deformed waves without the account of effects of a recoil nucleus, and R-is the amplitude of resonant process, calculated in frameworks of a single level approximation. The compound mechanism contribution if any is incoherent and it may be simply added. Complete and partial width of formation and disintegration of resonances in the system, which are necessary in order to calculate the amplitude of R, were defined by fitting the model predictions to the available experimental cross sections of elastic deuteron scattering and (d,p)-reaction. The satisfactory description of the experimental data for $^{12}C(d,p_o)^{13}C$ reaction is feasible (see Fig.19). However, for a reliable description of a whole set of (d,p)-reaction data a development of the model in several directions is required.

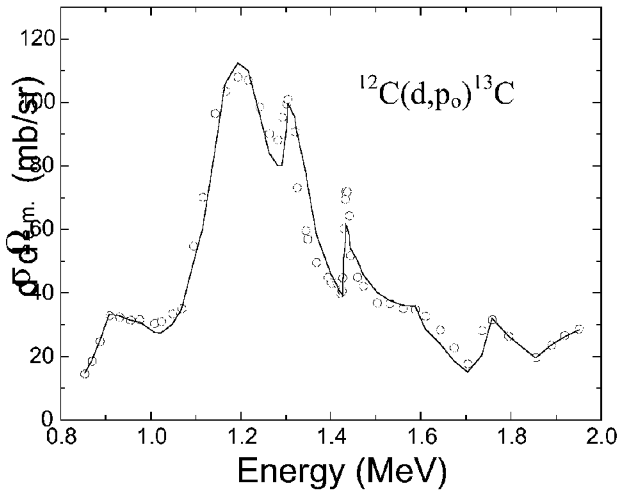

Fig.19

The important source of the nuclear level information for light nuclei is Ajzenberg-Selove's compilation published in several volumes of Nucl. Phys. Now it is available on-line at www.tunl.duke.edu/nucldata/fas/fas_list.shtml. The compilation comprises tables of adopted nucleus level properties, level diagrams, short discussions, and references. An example of the level diagram is shown in Fig.20.

If you want to find out the characteristics of a particular nuclear reaction X(a,b)Y, you may find that reaction discussed in three places in the compilation: under the residual nucleus Y, under the compound nucleus (X+a), or occasionally under the target nucleus X. If you are mainly interested in the excitation curve, resonance, yields, or angular distributions in the resonance region, then look for the reaction under the compound nucleus. If you are mainly interested in emitted-particle groups, then look for the reaction under the residual nucleus.

As an example, consider the energy level diagram for ^{17}F, illustrated by Fig. 20. The energy levels of F are depicted schematically as horizontal lines on what might be thought of as a square potential well. The bottom horizontal line represents the ground state. Broad energy levels are shown crosshatched; uncertain levels are indicated by dashed lines. The small digits at the right end of the energy-level lines indicate the angular momentum comprises parity, and isobaric spin of the level near which they are shown. Vertical lines connecting energy levels represent electromagnetic transitions (usually γ-ray emission).

Scattered about the page are the reactions that lead to formation of ^{17}F-nucleus. For some of these reactions qualitative thin-target excitation curves are depicted above the Q-value lines and alongside the energy-level diagram.

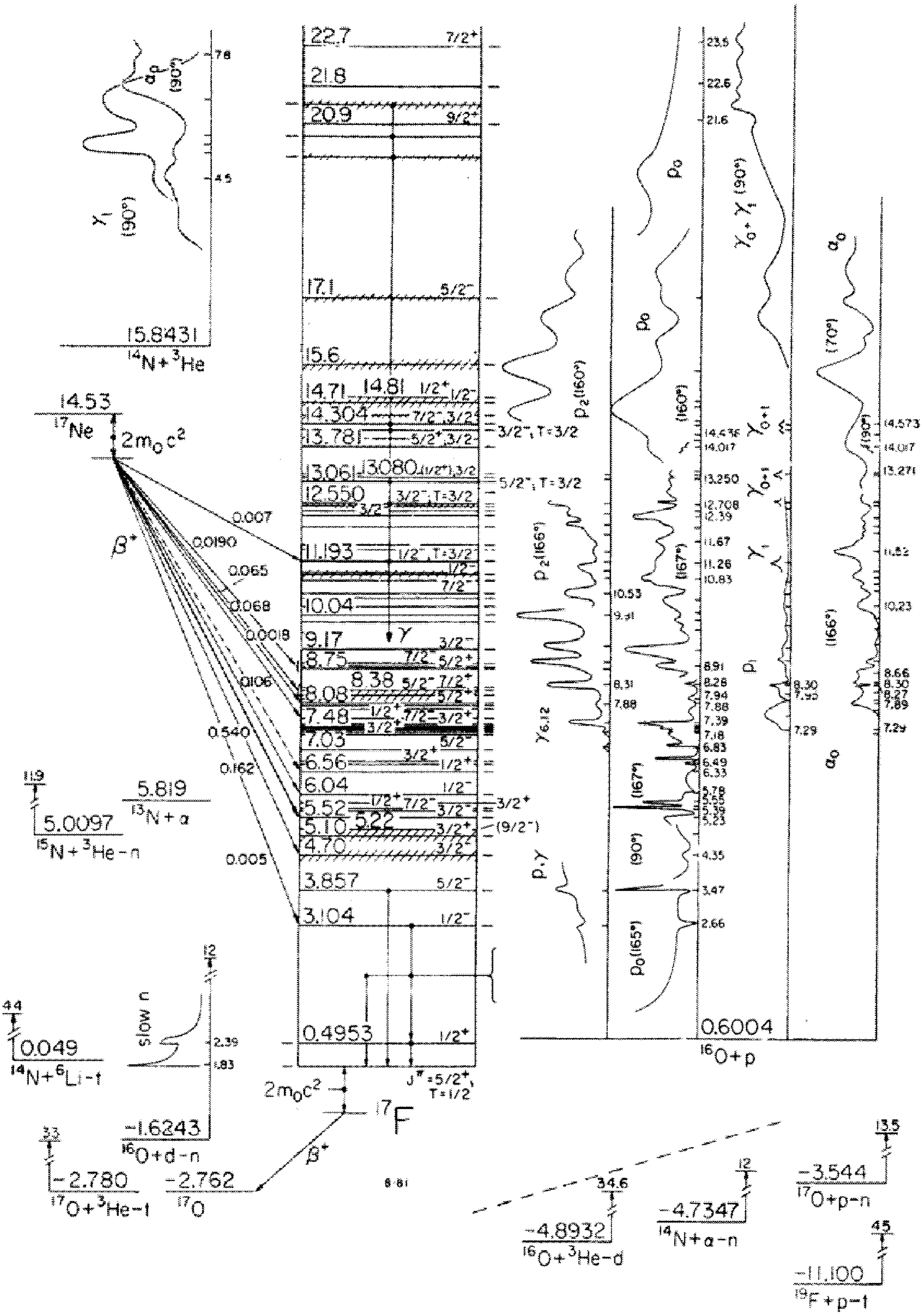

Fig.20