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SMR.1508 - 6

SUMMER SCHOOL ON PARTICLE PHYSICS

16 June - 4 July 2003

FLAVOUR PHYSICS

Part 3

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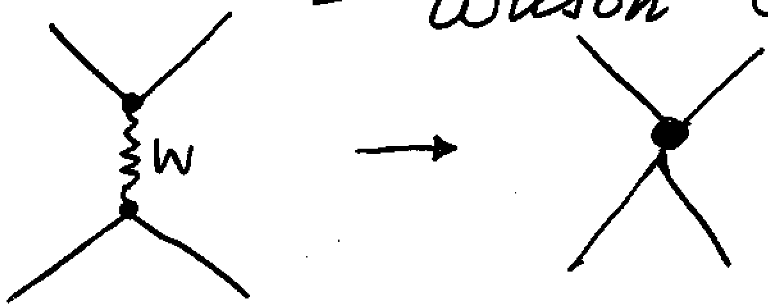
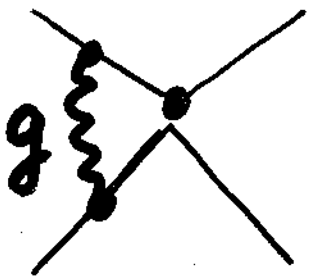
4. Effective Hamiltonian QCD + Renormalization

$$\mathcal{H} = \sum C_i\,(M_W ..)\; O_i\,[q] \quad (p.2)$$

C_i ← Wilson Coefficients

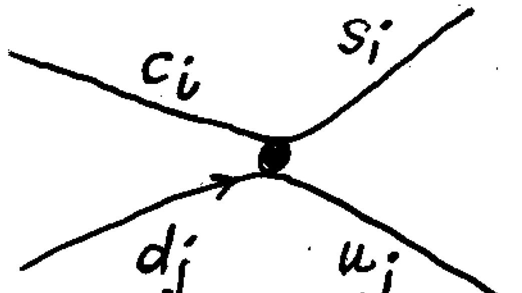


C obtained by calculating "full" diagram ↔ matching



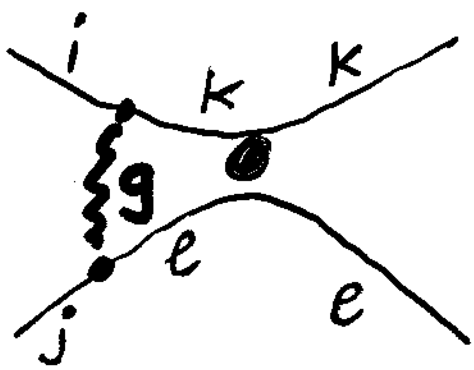
① introduces new operators
② introduces divergencies

①



color structure

$$\underbrace{(\bar{s}_i c_i)(\bar{u}_j d_j)}_{O_2}$$



$$\sum T^a_{ik} T^a_{j\ell} =$$

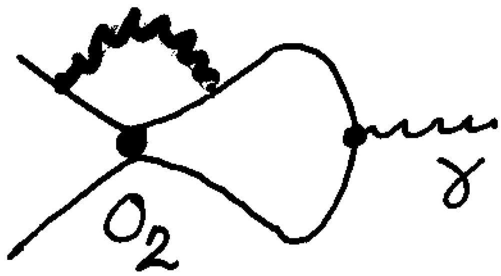
$$\frac{1}{2}\delta_{i\ell}\delta_{kj} - \frac{1}{2N}\delta_{ik}\delta_{j\ell}$$

$$\underbrace{(\bar{u}_i c_i)(\bar{s}_j d_j)}_{O_1} \qquad \underbrace{(\bar{s}_i c_i)(\bar{u}_j d_j)}_{O_2}$$

$$\mathcal{H}_{eff} \simeq C_1 O_1 + C_2 O_2$$

more general possibilities:

example:

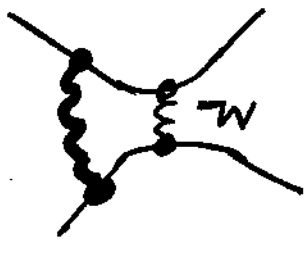


$$O_2 \longrightarrow O_7 = \bar{u}\,\sigma_{\mu\nu}\, c\, F^{\mu\nu}$$

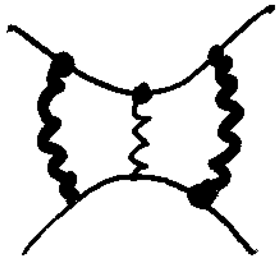
$$O_i \xrightarrow[QCD]{} O_j \quad \text{operator mixing}$$

② Divergencies → Renormalization

higher oder corrections



$\alpha_s \text{Log}\left(P^2/M_W^2\right)$



$\left(\alpha_s \log\left(P^2/M_W^2\right)\right)^2$ etc ↙ sum up

leading log

also $\alpha_s\left(\alpha_s \log\left(P^2/M_W^2\right)\right)$ NLL

Essentials of renormalization

① Field and couplings

$$\mathcal{L} = \bar{\psi} \not{p} \psi + g \bar{\psi} \not{G} \psi \qquad C_F \cdot \frac{\alpha}{4\pi}$$

$$\Rightarrow \quad \bar{\psi} \not{p} \left(\frac{A}{\varepsilon} + \log \frac{\mu^2}{p^2} \, .. \right) \psi \quad \text{D.R}$$

$$\bar{\psi} \not{p} \left(B \log \frac{\Lambda^2}{p^2} + .. \right) \psi \quad \text{C.O.}$$

$(\mu^2 = p^2)$

$$\mathcal{L} \to \bar{\psi} \not{p} \left(1 + \frac{A}{\epsilon}\right) \psi \qquad \epsilon \to 0$$

not a useful $\mathcal{L}$ to start, also scale

$$\to \quad \mathcal{L} = \bar{\psi} \not{p} \left(1 - \frac{A}{\varepsilon}\right) \psi = \bar{\psi}_0 \not{p} \psi_0$$

$$\psi_0 = \sqrt{1 - \frac{A}{\varepsilon}} \, \psi_0 = \sqrt{Z} \psi$$

$$\mathcal{L} = Z \bar{\psi} \not{p} \psi = \bar{\psi} \not{p} \psi + (Z-1) \bar{\psi} \not{p} \psi$$

"Counterterms"

ψ_0 : bare fields

ψ : renormalized fields

μ : renormalization scale

ψ_0 independent of μ

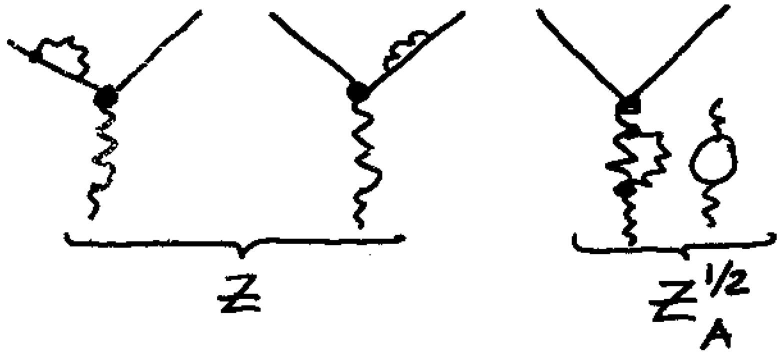
ψ depends on μ

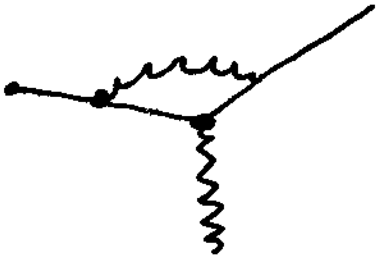
Scale μ essential for renormalizat.

similar for G^μ field Z_A

* Interactions

$$\mathcal{L}_I = g\,\bar{\psi} G \psi$$



not enough:  also divergent

$$\mathcal{L}_I \Rightarrow \mathcal{L}_I + g(Z_1 - 1)\,\bar{\psi} \not{G} \psi$$

new counter term

$$= g Z_1 \bar{\psi} G \psi = \underbrace{g Z_1 Z^{-1} \sqrt{Z_A^{-1}}}_{g_0}\; \bar{\psi}_0 \not{G}_0 \psi_0$$

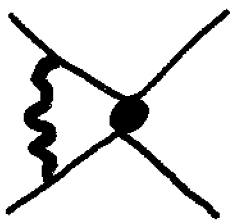
$$g_0 = g\, Z_1 Z^{-1} \sqrt{Z_A^{-1}} = g\, Z_g$$

g_0 scale independent, g dependent: $g(\mu)$

$$g_0 = Z(\mu)\, g_1(\mu)$$

Often $Z_1 = Z \Rightarrow g_0 = \frac{1}{\sqrt{Z_A}} \cdot g$

* Effective Operator O_i



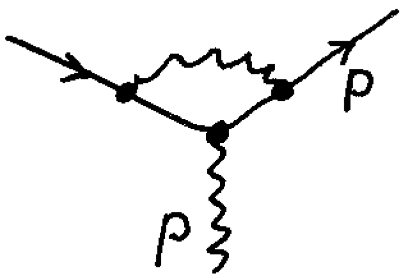
$$O_{i_0} = Z_{ij}\, O_j$$

→ and similar for the coefficients C_i

$$\mathcal{H}_{eff} = \sum C_{i0}\, O_{i0} \equiv C_0\, O_0$$
$$= (C_0\, Z)\,(Z^{-1} O_0) \equiv C\;\, O$$
$$C_0 = Z^{-1}\, C$$
$$\mathcal{H}_{eff} = \sum C_i(\mu)\, O_i(\mu)$$

Renormalization group

① Importance of scale μ :



$$\left(g + g^3 \log \frac{p^2}{\mu^2}\right)$$

$\mu = p$: Result $\sim g$, no large logs

$\mu \gg p$: corrections large

$\langle O_i(\mu) \rangle$: $\mu \sim p$ ($\sim m_B$)

② $g_0 = Z_g(\mu)\, g(\mu)$; $m_0 = -\gamma_m \cdot m$

$C_0 = Z^{-1} \cdot C$ etc.

$$\mu \frac{d}{d\mu} g_0 = \frac{d}{d\log\mu} g_0 = \frac{d}{d\log\mu} Z_g \cdot g + Z_g \frac{d}{d\log\mu} g$$

must vanish (careful

$$\frac{d}{d\log\mu} g = -g \frac{1}{Z_g} \frac{\partial Z_g}{\partial \log\mu} \equiv \beta(g)$$

$$Z_g \hat{=} Z_3 = 1 - \frac{\alpha_s}{4\pi}\left(\frac{11N}{6} - \frac{2f}{6}\right)\frac{1}{\varepsilon} + \dots$$

$$\beta = \underbrace{\frac{-g^3}{16\pi^2}\left(\frac{11N}{6} - \frac{2f}{6}\right)}_{\beta_0 \cdot g^3} - \beta_1 g^5 \dots$$

Similar: $\gamma_m = \gamma_m^0 g^2 + \gamma_m g^4 + \dots$

$$\frac{dm}{d\log\mu} = -\gamma_m \cdot m \qquad dg = \beta \, d\log\mu$$

$$\frac{d\log m}{dg} = -\frac{\gamma}{\beta} \Rightarrow \left(\frac{m}{m_0}\right) = \exp\int dg \frac{\gamma}{\beta}$$

$$\exp \sim \int dg \frac{\gamma_0}{\beta_0} \frac{1}{g}\left(\frac{1 + \gamma_1/\gamma_0 \, g^2}{1 + \beta_1/\beta_0 \, g^2}\right)$$

$$\left(\frac{m}{m_0}\right) = \left(\frac{\alpha}{\alpha_0}\right)^{\gamma_0/2\beta_0}\left(1 + \left(\frac{\gamma_1}{2\beta_0} - \frac{\beta_1\gamma_0}{2\beta_0^2}\right)\alpha\right)$$

LL

NLL

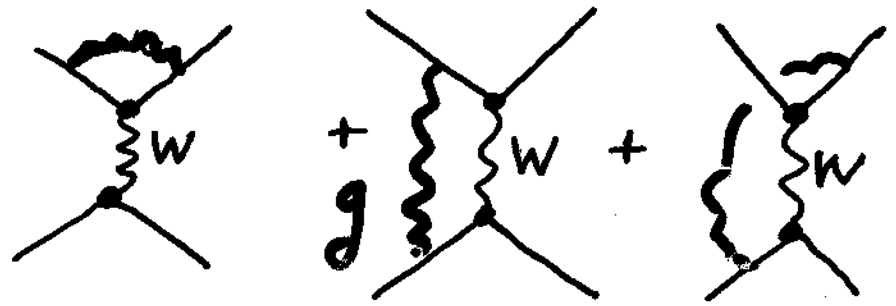
$m_0 = m(\mu_0)$

$\alpha_0 = \alpha(\mu_0)$

sums up all (large) logs → beyond pert. th.

Do the matching (to full theory) and fix matching scale μ_0

$A = \langle \mathcal{H} \rangle$

$A_{full} =$ 

$$A_{ff} = \frac{G_F}{\sqrt{2}} V_{cs}^{*} V_{ud} \Big\{ \Big(1 + 2C_F \frac{\alpha_s}{4\pi}\Big(\frac{1}{\epsilon} + \log\frac{\mu^2}{p^2}\Big)S_2$$
$$+ \frac{3}{N}\frac{\alpha_s}{4\pi} \log \frac{M_W^2}{p^2} S_2 - \frac{3\alpha_s}{4\pi} \log \frac{M_W^2}{p^2} S_1 \Big\}$$

$S_1 = \langle O_1 \rangle_T \qquad S_2 = \langle O_2 \rangle_T$

$\langle O_1 \rangle =$ 

$$= \Big(1 + 2C_F \frac{\alpha_s}{4\pi}\Big(\frac{1}{\varepsilon} + \log\frac{\mu^2}{p^2}\Big)\Big) S_1 + \frac{3}{N}\frac{\alpha_s}{4\pi}\Big(\frac{1}{\varepsilon} + \log\frac{\mu^2}{p^2}\Big) S$$
$$- 3\frac{\alpha_s}{4\pi}\Big(\frac{1}{\varepsilon} + \log\frac{\mu^2}{p^2}\Big) S_2$$

S_2

$$\langle O_2{}^{\circ} \rangle = \Big(1 + 2C_F \frac{\alpha_s}{4\pi}\Big(\frac{1}{\varepsilon} + \log\frac{\mu^2}{p^2}\Big)\Big) S_2 + \frac{3\alpha_s}{N 4\pi}\Big(\frac{1}{\varepsilon} + l\frac{\mu^2}{p^2}$$
$$- \frac{3\alpha_s}{4\pi}\Big(\frac{1}{\varepsilon} + \log\frac{\mu^2}{p^2}\Big) S_1$$

$$A_{eff} = C_1 \langle O_1 \rangle + C_2 \langle O_2 \rangle$$

matching: $A_{eff} = A_{ff}$

compare coefficients!

$$S_1: \; C_1\left(1+2C_F \frac{\alpha_s}{4\pi} \cdot \left(\log\frac{\mu^2}{p^2}\right) + \frac{3\alpha_s}{N4\pi}\log\frac{\mu^2}{p^2}\right) - C_2 \frac{3\alpha_s}{4\pi}\log\frac{\mu^2}{p^2}\Big) = -\frac{3\alpha_s}{4\pi}\log\frac{\mu^2}{p^2}\Big)$$

$$S_2: \; C_2\left(1+2C_F\frac{\alpha_s}{4\pi}\left(\log\frac{\mu^2}{p^2}\right) + \frac{3\alpha_s}{N4\pi}\log\frac{\mu^2}{p^2}\right) - C_1 \frac{3\alpha_s}{4\pi}\log\frac{\mu^2}{p^2} =$$

$$1 + 2C_F\frac{\alpha_s}{4\pi}\log\frac{\mu^2}{p^2} + \frac{3\alpha_s}{N4\pi}\log\frac{M_W^2}{p^2}$$

$$C_1(\mu) = -\frac{3\alpha_s}{4\pi}\log\frac{M_W^2}{\mu^2}$$

$$C_2(\mu) = 1 + \frac{3}{N}\frac{\alpha_s}{4\pi}\log\frac{M_W^2}{\mu^2}$$

$\frac{1}{\epsilon}$ absorbed by renormalizations

$$A \hat{=} C(\mu)\; O(\mu) \hat{=}$$

$$\left(1 + \text{const} \log\frac{M_W^2}{\mu^2}\right)\langle O(\mu)\rangle$$

Important:

Momentum range in loop goes from zero to M_W: The part zero to μ is in $\langle O \rangle$, the rest in C. μ between p and M;

$$\underbrace{\left(1+\alpha_s \log \frac{M_W^2}{p^2}\right)}_{\text{all}} = \underbrace{\left(1+\alpha_s \log \frac{M_W^2}{\mu^2}\right)}_{C} \underbrace{\left(1+\alpha_s \log \frac{\mu^2}{p^2}\right)}_{\langle O \rangle}$$

factorization of L.D. and S.D.

S.D. perturbative $\Longrightarrow$ C's !

L.D. non-perturb. $\Longrightarrow$ $\langle O \rangle$

- C are process independent

choice of μ_0: M_W

avoid large Log M_W/μ_0

Back to effective Hamiltonian

R.G. solution $\swarrow$ from matching

$$C(\mu) = C(\mu_0) \exp \int dg \frac{Z}{\beta}$$

careful about indices (C_i, Z_{ij})

perturbation theory:

$$C(\mu) = \left[C_{(1)}(\mu_0) + \alpha_s(\mu_0) C_{(2)}\right] \left(\frac{\alpha}{\alpha_0}\right)^{Z/2\beta_0} (1 + K\cdot\alpha)$$

L.L: $C_{(1)} \left(\frac{\alpha}{\alpha_0}\right)^{Z/2\beta_0}$

NLL: $\left(C_{(1)} \cdot K + C_{(2)}\right) \alpha \left(\frac{\alpha}{\alpha_0}\right)^{Z/2\beta_0}$

Matrix element:

$$\langle O_i \rangle = \langle O_i \rangle_1 + \alpha \langle O_1 \rangle_2$$

L.L: $C_{(1)} \cdot \langle O_i \rangle_1 \left(\frac{\alpha}{\alpha_0}\right)^{Z/2\beta_0}$

NLL: $C_{(1)} K \langle O_i \rangle_1 + C_{(1)} \langle O_i \rangle_1 + C_{(2)} \langle O_i \rangle_2$

Calculation of physical quantities

① Model → $C_i(\mu_0)$ $\mu_0 \sim M_W$
include tree + loops "Matching"

② Calculate $C_i(\mu)$ by renormalization group (anomalous dimensions)
"running"

③ calculate matrix element $\langle O_i(\mu) \rangle$

② perturbative

③ non-perturbative: Lattice, χPT, quark-models, sum rules, ...

- much work (even 3 loops!) done
  $b \to s\gamma$ best studied example (NLL in very good agreement

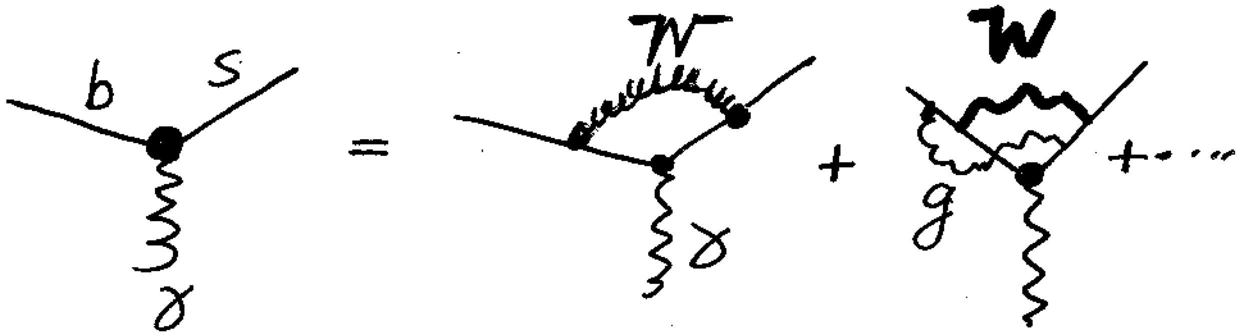
- ③ hard, some processes (inclusive) ok

μ-dependence does not cancel in practice. Even in $b \to s\gamma$ ≈ 15% uncertain

$$\left(\frac{m_b}{2} < \mu < 2 m_b \right)$$

Most successful example:

$b \to s\gamma$



Calculate

3 loops, divergent
2 loops, complete } NLL

$$B \to X_s \gamma \begin{cases} \text{ep}: (3.2 \pm 0.7)\, 10^{-4} \\ \text{th}: (3.6 \pm 0.3)\, 10^{-4} \end{cases}$$

5. CP-violation

- Asymmetry between particles/antiparticles (C-violation not sufficient)
- Relate to (microscopic) time reversal
- SM: not precisely tested => new physics

$\not{C}\not{P}$ seen 1964: $\epsilon \sim 2.3 \cdot 10^{-3}$

1999: ϵ'/ϵ firmly established

$$(\epsilon'/\epsilon) \simeq (17.2 \pm 1.8)\, 10^{-4}$$

2001 $a_{\psi k} \simeq 0.734 \pm 0.054$

other B decays

$\not{C}\not{P}$ possible in: Nucleons, d_n, d_e, K, D, B; universe (Baryon)

all cannot be explained by CKM.

Strong CP- violation

$$\mathcal{L} = \cdots \quad + \quad \theta F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$\tilde{F}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}$

(instantons) $\quad F_{\mu\nu}$: gluonic field strength

roughly : $\vec{E}\,\vec{B}$ (is T odd)

quark mass diagonalization (CKM) :
chiral transformations. give a supplementary term μ = arg det M_Q

$$\bar{\theta} = (\theta + \mu)$$

chiral perturbation, etc...

$$d_E \simeq \bar{\theta} \cdot 10^{-16} \, e\, cm$$

experiment : $d_E \leq 10^{-26} \rightarrow \bar{\theta} < 10^{-10}$

why so small ?

$\rightarrow$ Peccei-Quinn symmetry $\rightarrow$ axion
light pseudoscalar. cosmology...

Susy $\rightarrow$ large $d_E \Rightarrow$ bounds

Mixing formalism (K°, B°)

$$K_1 = \frac{1}{\sqrt{2}}(K^\circ - \bar{K}^\circ) \qquad CP: K_1 = + K_1$$
$$K_2 = \frac{1}{\sqrt{2}}(K^\circ + \bar{K}^\circ) \qquad CP: K_2 \ - K_2$$

} CP - Eigen-states $(|K\rangle)$ *

- CP violation →

$$K_L = \frac{pK^\circ + q\bar{K}^\circ}{\sqrt{p^2+q^2}} = \frac{K_2 + \bar{\varepsilon}K_1}{\sqrt{1+|\bar{\varepsilon}|^2}} \qquad \bar{\epsilon} = \frac{p-q}{p+q}$$

$$K_S = \frac{pK^\circ - q\bar{K}^\circ}{\sqrt{\ldots}} = \frac{K_1 + \bar{\varepsilon}K_2}{\sqrt{\ldots\ldots}}$$

$$i\frac{d}{dt}\begin{pmatrix}K^\circ\\ \bar{K}^\circ\end{pmatrix} = (M - i\Gamma)\begin{pmatrix}K^\circ\\ \bar{K}^\circ\end{pmatrix} \qquad M = \begin{pmatrix}M_{11} & \overset{\Delta S=2}{M_{12}}\\ M_{21} & M_{22}\end{pmatrix}$$

$$\Delta m = m_L - m_S \qquad \Delta m\,\Delta\Gamma = 4\,\mathrm{Re}(M_{12}\Gamma_{12}^*)$$
$$\Delta\Gamma = \Gamma_L - \Gamma_S \qquad (\Delta m)^2 - \frac{1}{2}(\Delta\Gamma)^2 = 4|M_{12}|^2 - |\Gamma_{12}|^2$$

$$\frac{q}{p} = \frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - i\frac{\Delta\Gamma}{2}} \qquad \frac{1-\bar{\varepsilon}}{1+\bar{\varepsilon}} = \frac{\Delta m - \frac{i}{2}\Delta\Gamma}{2M_{12} - i\Gamma_{12}}$$

Kaons: $\bar{\varepsilon} \sim 10^{-3}$: $\Delta m \simeq 2\,\mathrm{Re}\,M_{12}$ $\quad \Delta\Gamma \simeq 2\,\mathrm{Re}\,\Gamma_{12}$ *

B's : $\Delta\Gamma$ small $(B_L, B_S!)$ $\qquad \frac{q}{p} = + \frac{M_{12}^*}{|M_{12}|}$

* general: $K^\circ, \bar{K}^\circ$ arbitrary phases under CP
→ phase of (p/q) not physical → Re $\bar{\epsilon}$!

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* $\Delta\Gamma \simeq -2\,\Delta m$ $\qquad$ experiment

Kaon system:

CP seen in $K_L \to 2\pi$

(2π) system is CP even (Bose)

$K_2 \to 2\pi$ violates CP

$K_1 \to 2\pi$ ok

$\to$ both decays seen: CP-violated

① $K_L \simeq K_2 + \bar{\varepsilon} K_1$ indirect: $\Delta S = 2$

$K_1 \to \pi\pi$

direct: CP $\Delta S = 1$

$K_2 \to \pi\pi$

$$A(K^+ \to \pi^+\pi^0) = \sqrt{\tfrac{3}{2}}\, A_2 e^{i\delta_2}$$

$$A(K^0 \to \pi^+\pi^-) = \sqrt{\tfrac{2}{3}}\, A_0 e^{i\delta_0} + \sqrt{\tfrac{1}{3}}\, A_2 e^{i\delta_2}$$

$$A(K^0 \to \pi^0\pi^0) = \sqrt{\tfrac{2}{3}}\, A_0 e^{i\delta_0} - 2\sqrt{\tfrac{1}{3}}\, A_2 e^{i\delta_2}$$

$$\eta_{00} = \frac{\langle \pi^0\pi^0 | \mathcal{H} | K_L \rangle}{\langle \pi^0\pi^0 | \mathcal{H} | K_S \rangle} \qquad \eta_{+-} = \frac{\langle \pi^+\pi^- | \mathcal{H} | K_L \rangle}{\langle \quad | \quad | \quad \rangle}$$

if only $\bar{\varepsilon}$: $\eta_{00} = \eta_{+-} = \bar{\varepsilon}$

if $A_2 = 0$: $\eta_{00} = \eta_{+-}$

$\bar{K}^0 \to \pi\pi$: $A_I \to -A_I^*$ $\quad \delta_I \to \delta_I$

no CP-viol. in $\Delta S = 1$ if A real

$$\varepsilon = \frac{A(K_L \to 2\pi)_{I=0}}{A(K_S \to 2\pi)_{I=0}} \simeq \left(\bar{\varepsilon} + i\,\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right)$$

$$\varepsilon' = \frac{1}{\sqrt{2}} \, \mathrm{Im}\left(\frac{A_2}{A_0}\right) \exp\Big(i \underbrace{\left[\tfrac{\pi}{2} + \delta_2 - \delta_0\right]}_{\approx \pi/4}\Big)$$

$$\eta_{00} \cong \epsilon - 2\varepsilon' \qquad \eta_{+-} \cong \epsilon + \epsilon'$$

ϵ' : "direct" CP-violation

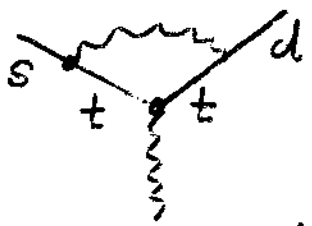
ϵ : indirect CP-violation

with $\quad R(\epsilon'/\epsilon) \sim 2 \cdot 10^{-3} \ll 1$

deep meaning or stand. model effect ?

$\downarrow$ new physics $\qquad \downarrow$ flavors, QCD

Calculation : $\mathcal{H}_{eff}^{\Delta S = 1} = \sum_i c_i(\mu) O_i(\mu)$



$\sim \mathrm{Im}\left(V_{td} V_{ts}^*\right)$

$$\frac{\epsilon'}{\varepsilon} \simeq \mathrm{Im}\left(V_{td} V_{ts}^*\right) \left(\frac{110\,\mathrm{MeV}}{M_S(2)M}\right)^2 \left(\frac{\Lambda_{\overline{MS}}}{340\,\mathrm{MeV}}\right) \cdot \left(B_6^{1/2}\, 0.75 - B_8^{3/2}\right)$$

$$\left\langle (\bar{s}\gamma_\mu d_L)\left(\sum_q \bar{q}\gamma^\mu q\right)\right\rangle = B_6^{1/2} \left\langle \ldots \right\rangle^{VAC}$$

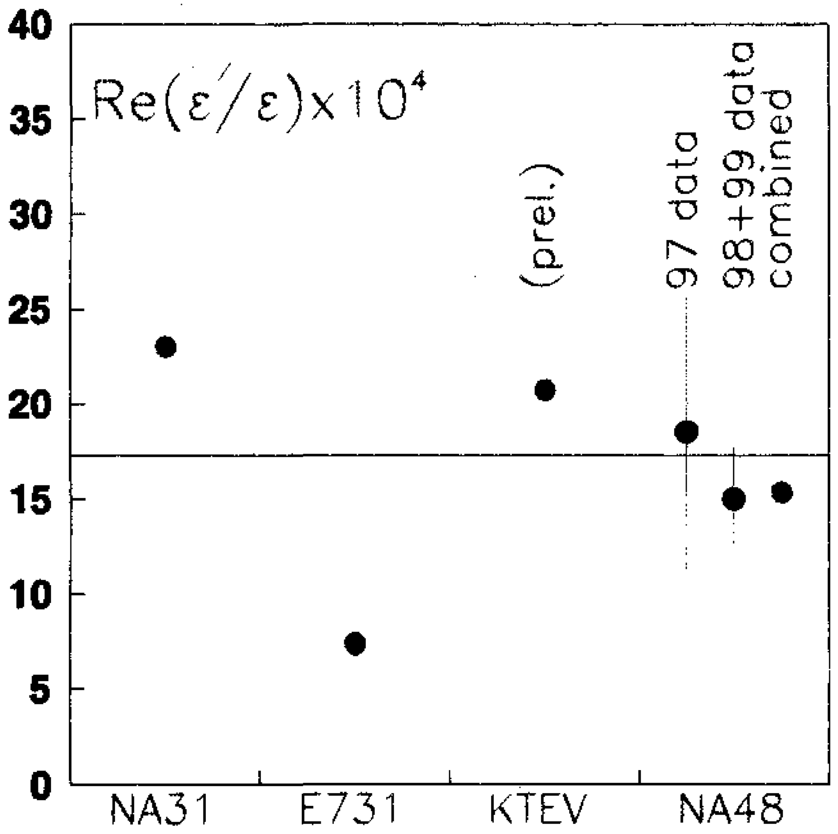
$$\left\langle (\bar{s}\gamma_\mu d_L)\left(\sum_q e_q\, \bar{q}\gamma^\mu q\right)\right\rangle = B_8^{3/2} \left\langle \ldots \right\rangle^{VAC}$$

Many estimates of B's

typical result: $\frac{\epsilon'}{\epsilon} \sim (10 \pm 7)\, 10^{-4}$

new work on rescattering of π ion
indicates higher values
Seems to account for experiment.

CONCLUSIONS



Combined NA48 result :

[redacted]

World average of NA31, E731, KTeV and NA48:

[redacted]

⟹Both Indirect and Direct CP Violation components discovered, measured and confirmed in the kaon system

CP violation in B-physics

$$B_L = \frac{1}{\sqrt{p^2+q^2}}\left(pB^\circ + q\bar{B}^\circ\right) \qquad \frac{q}{p} \simeq \frac{M_{12}^*}{|M_{12}|}$$

$$B_S = \frac{1}{\sqrt{}}\left(pB^\circ - q\bar{B}^\circ\right)$$

$\frac{q}{p} \sim$ phase : ϵ not so interesting

but CP-violation in M_{12}^* ?

Quantum mechanics

$B\bar{B}$ produced: Consider $B^\circ - \bar{B}^\circ$ pair

at $t=0$: $B^\circ(t)\big|_{t=0} = B^\circ$

$\bar{B}^\circ(t)\big|_{t=0} = \bar{B}^\circ$

Pair produced in p state (antisymmetric)

$$\psi \sim \frac{1}{\sqrt{2}}\left[B^\circ(x)\bar{B}^\circ(-x) - \bar{B}^\circ(x)B^\circ(-x)\right]$$

(energy not sufficient for symm. state)

$$B^\circ(t) = g_+ B^\circ + \frac{q}{p} g_- \bar{B}^\circ$$

$$g_+ = \exp\left(-\frac{\Gamma t}{2}\right)\exp\left(-iMt\right)\cos\left(\frac{\Delta m t}{2}\right)$$

$$g_- = i\exp\left(-\frac{\Gamma t}{2}\right)\exp\left(-iMt\right)\sin\left(\frac{\Delta m t}{2}\right)$$

$$A_F = \langle F|H|B\rangle \qquad \bar{A}_F = \langle F|H|\bar{B}\rangle$$

F can be 'reached' from B <u>and</u> $\bar{B}$

$\rightarrow$ only neutrals

$$\langle F | H | B(t) \rangle = A_F (g_+ + \lambda g_-)$$

$$\langle \bar{F} | H | \bar{B}(t) \rangle = \bar{A}_{\bar{F}} (g_- + \bar{\lambda} g_+)$$

$$\lambda = \frac{q}{p} \frac{\bar{A}_F}{A_F} \qquad \bar{\lambda} = \frac{p}{q} \frac{A_F}{\bar{A}_F} \qquad \text{(phase convention indep.)}$$

$$\Gamma^B(t) = |\dots|^2 = |A_F|^2 e^{-\Gamma t} \left\{ \frac{1+|\lambda|^2}{2} + \frac{1-|\lambda|^2}{2} \cos \Delta m t - \mathrm{Im}\,\lambda \sin \Delta m t \right\}$$

$$\Gamma^{\bar{B}}(t) = | \quad |^2 = |\bar{A}_{\bar{F}}|^2 e^{-\Gamma t} \left\{ \frac{1+|\bar{\lambda}|^2}{2} + \frac{1-|\bar{\lambda}|^2}{2} \cos \Delta m t - \mathrm{Im}\,\lambda \sin \Delta m t \right\}$$

For charged B's :

$$\Gamma^B(t) = |A|^2 e^{-\Gamma t}$$

$$\Gamma^{\bar{B}}(t) = |\bar{A}|^2 e^{-\Gamma t}$$

$$\boxed{\mathcal{A}_F(t) = \frac{\Gamma(B^0(t) \to F) - \Gamma(\bar{B}^0(t) \to \bar{F})}{\Gamma(B^0(t) \to F) + \Gamma(\bar{B}^0(t) \to \bar{F})}}$$

charged : $\mathcal{A}_F = \dfrac{|A_F|^2 - |\bar{A}_{\bar{F}}|^2}{|A_F|^2 + |\bar{A}_{\bar{F}}|^2} \quad \forall t$

$\mathcal{A}_F \neq 0$: ① $A_F \neq \bar{A}_{\bar{F}}$ <u>Direct CP-violation</u>

Requires $A_F \sim \sum_i' A_i e^{i\delta_i} e^{i\phi_i}$ $\qquad i \neq 1$ p.51

$$\mathcal{A}_F \sim \sin(\phi_i - \phi_j) \sin(\delta_i - \delta_j)$$

Neutral B's

- Flavor specific: F such that only one of the B's contributes

$\bar{B}^0 \to \ell^- \bar{\nu} X \qquad B^0 \not\to \ell^- \nu X$

$$\bar{A}_F = A^*_{\bar{F}}$$

$$\mathcal{A}_F = \frac{1 - |q/p|^2}{1 + |q/p|^2} \qquad \text{small} \sim 10^{-2}$$

② $\left|\frac{q}{p}\right| \neq 1$ indirect (mixing) CP-violation

- Flavor not specific: $B, \bar{B} \to F$

choose $F = \bar{F} \qquad \bar{\lambda} = 1/\lambda$

$$\Gamma^{\bar{B}}(t) = |A_f|^2 e^{-\Gamma t} \left\{ \frac{1+|\lambda|^2}{2} - \frac{1-|\lambda|^2}{2} \cos \Delta m t + \mathrm{Im}\,\lambda \sin \Delta m t \right\}$$

$$\mathcal{A}_F = \frac{(1-|\lambda|^2)\cos \Delta m t - 2\,\mathrm{Im}\,\lambda \sin \Delta m t}{1 + |\lambda|^2}$$

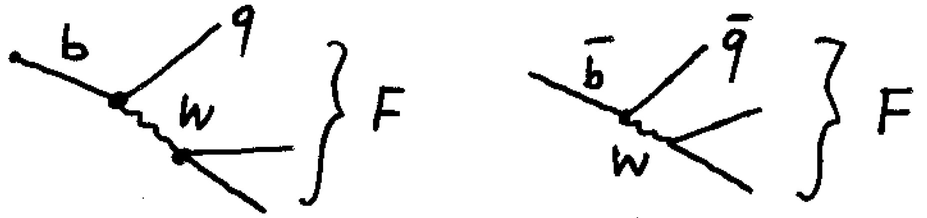
③ $|\lambda| \neq 1$ or $\mathrm{Im}\,\lambda \neq 1$

mixing-decay CP-violation

$$\lambda = \frac{\bar{A}}{A}\frac{q}{p} \qquad \frac{q}{p} = \frac{M_{12}^*}{|M_{12}|} = \frac{(V_{td}V_{tb}^*)^2 + NP..}{|\ |^2 + NP...} = \underset{B_d}{\overset{e^{-2i\phi_M}}{\overset{\downarrow}{e^{-2i(\beta+\chi)}}}}$$

- similar for B_s : $\frac{q}{p} \simeq e^{-2i\chi}$ NP.

- $\frac{\bar{A}}{A}$ in general dificult but
  if 'one process' $\rightarrow$ $\frac{\bar{A}}{A}$ is just a phase!



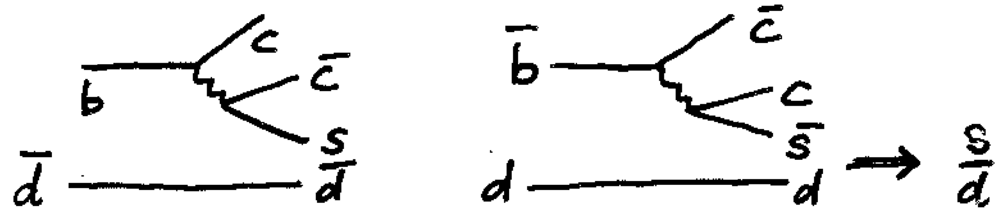
$\sim V_{qb} V_{ij}^*$ $\quad\quad$ $\sim V_{qb}^* V_{ij}$

$$\frac{\bar{A}}{A} \approx \frac{V_{qb}^* V_{ij}}{V_{qb} V_{ij}^*} = e^{-2i\phi_d}$$

$$\rightarrow \lambda = e^{-2i\phi_d} \cdot e^{-2i\phi_M}$$

Need to include final state factor

example : $b \rightarrow c\bar{c}s$ $\quad\quad$ $\bar{b} \rightarrow \bar{c}c\bar{s}$



$= \frac{-\mathcal{A}(K_S)}{\mathcal{A}(K_L)}$

or $B \rightarrow J/\psi + K_L, K_S$

$$\boxed{\lambda = e^{-2i\phi_d}\, e^{-2i\phi_M}\, e^{-2i\phi_F}}$$ *

$K_{L,S} \mathrel{\hat{=}} \frac{1}{\sqrt{2}}(\bar{s}d \pm \bar{d}s)$

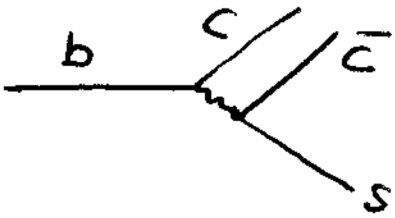
phase convent.

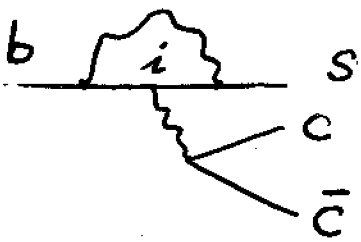
can use various decays: $e^{-2i\phi_d}$

B_d or B_s $\quad\quad$ $e^{-2i\phi_M}$

Final states $\quad\quad$ $e^{-2i\phi_F}$

- best known examples $B \to J/\psi\, K_{(S)}$ $B \to \phi K_{(S)}$





$T: V_{cb} V_{cs}^*$ $\qquad$ $P: \Sigma V_{ib} V_{is}^* f(m_i)$
$\qquad i = u, c, t$

$$A_{tot} = T + P + (NP)$$

$$0 = V_{cb} V_{cs}^* + V_{tb} V_{ts}^* + \underbrace{V_{td} V_{ud}^*}_{10^{-2}}$$

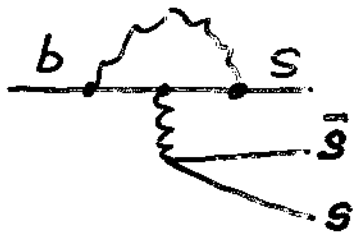
$$A_{tot} = V_{cb} V_{cs}^* (T + P_c - P_T) + (10)^{-2} + NP$$

NP small (Lunghi + W.A.)

$$\frac{\bar{A}}{A} = \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \cong 1 \implies$$

$$\boxed{\begin{aligned} \mathcal{A}_F &= -2\,\mathrm{Im}\,(e^{-2i\phi_M}) \\ &= 2\sin(2\beta + 2\chi) \end{aligned}}$$

Similar: $B \to \phi K_{(S)}$



$$\mathcal{A}_F = 2\sin(2\beta + 2\chi')$$

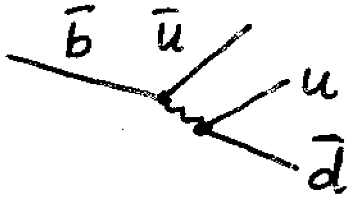
new physics can induce ~ 20 - 30 %

Requires asymmetric B-factory

Similar possibilities in B_s, where $\phi_M \simeq 0$ in S.M.

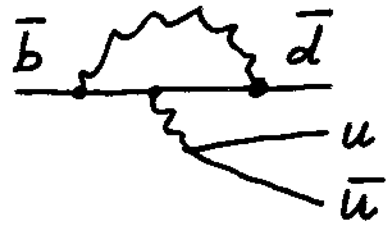
- Angle α (between $V_{td} V_{ub}$)

  needs a decay with V_{ub} $\qquad B \to \pi^0 \pi^0$



$A \sim V_{ub}^* V_{ud} \sim (0.2)^3$

But also



$A \sim V_{tb}^* V_{td} \sim (0.2)^3$

$\left|\frac{\bar{A}}{A}\right| \neq e^{i\phi_d}$ "Penguin pollution"

Various ideas, calculations, ...

Isospin decomposition : $\pi^+\pi^-$, $\pi^0\pi^0$, $\pi^+\pi^0$

isolate $\Delta I = \frac{3}{2}$: Tree graph

- Angle γ

  $B_s \to \omega K_s$, ρK_s, $\pi^0 K_s$ ... but Penguins

  other decays

- λ's are related (see * p. 63)

  $\lambda = 0$ in $B_s \to J/\psi K_s$ ...

- measure γ in amplitude triangles

Useful (+ necessary) references

- A. Buras + M. Lindner
  Heavy Flavours W.S. — Flavour

- M. Battaglia et. al.
  The CKM matrix and the Unitary triangle — CKM
  hep-ph/0304132

- A. Buras; Les Houches lectures 1998 — Eff. Ham.
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- C. Jarlskog;
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- HQET:
  Book by A. Manohar and M. Wise — HQET
  Articles by A. Falk etc
  hep-ph/9610369

- SCET: Articles by
  C. Bauer et. al Hep-ph 0011336

General: BaBar Physics Handbook

New: Ringberg - Phenomenology
workshop 2003

Gauge field theory: T. Muta; QCD W.S.