

SUMMER SCHOOL ON PARTICLE PHYSICS

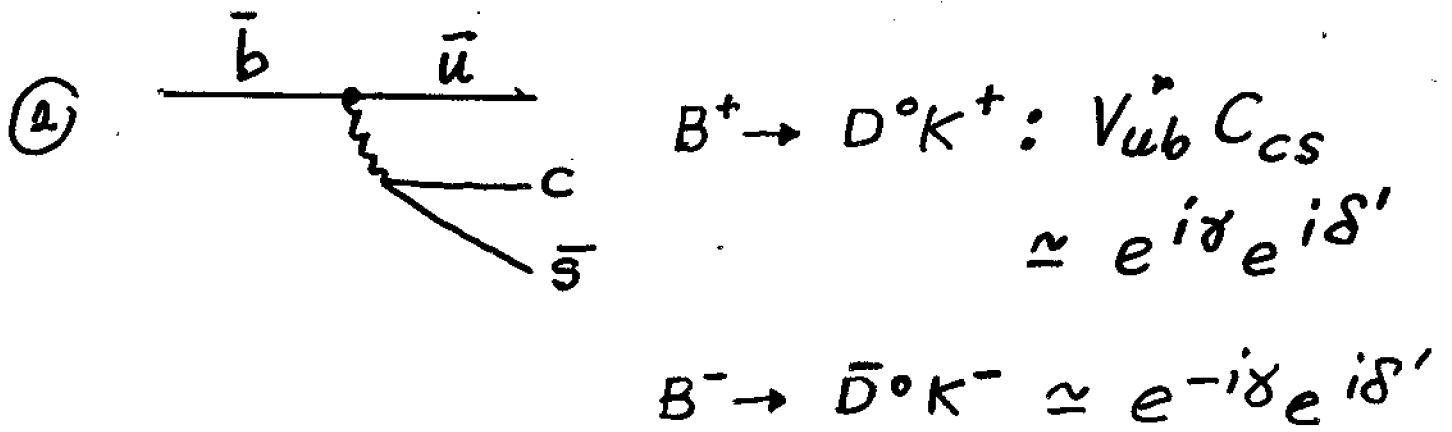
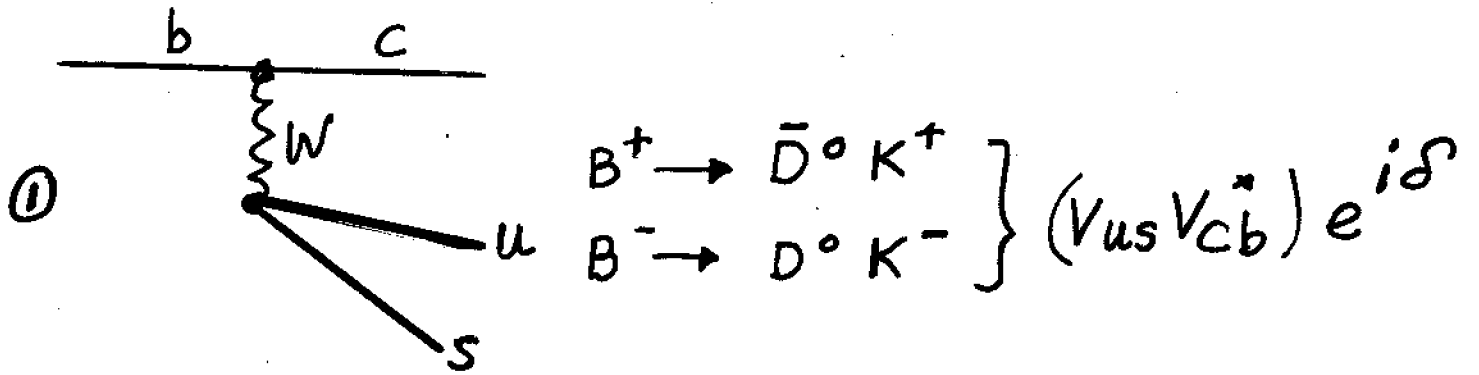
16 June - 4 July 2003

FLAVOUR PHYSICS

Part 4

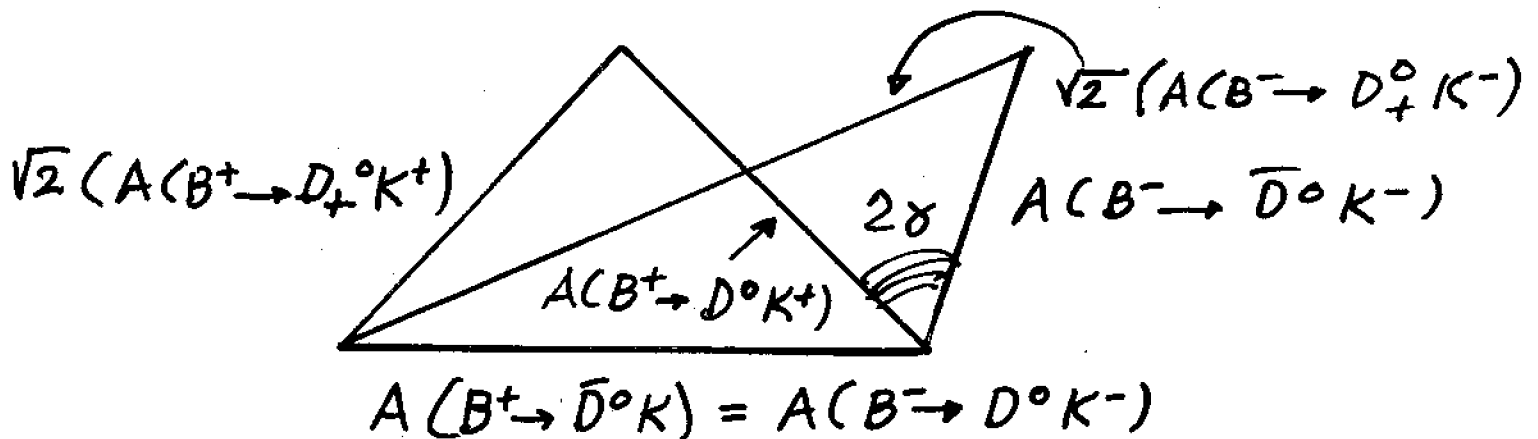
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Idea ($B \rightarrow DK$)



$$D_{\pm}^0 = \frac{1}{\sqrt{2}} (D^0 + \bar{D}^0)$$

$$\Rightarrow A(B^+ \rightarrow \bar{D}^0 K^+) + A(B^+ \rightarrow D^0 K^+) = \sqrt{2} (A \rightarrow D_+^0 K^+)$$

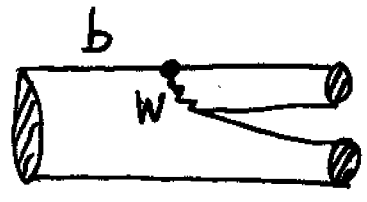


clean, but hard!

6. Heavy quarks; hadrons

a) General comments

- Hadronic decays: $B \rightarrow K\pi$, $B \rightarrow D\pi$, $B \rightarrow \pi\pi$ interesting for CP-violation, QCD, etc.



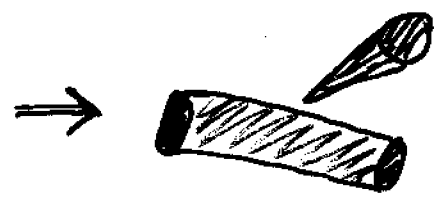
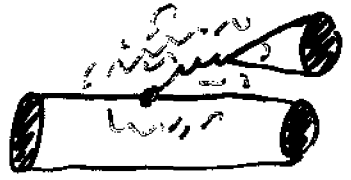
Tree level



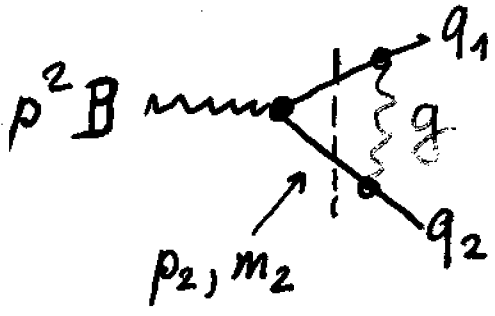
Penguin different decays



- Factorization



- Final state interaction (strong phases)



$$H \rightarrow q_1 q_2$$

diagram gets an imaginary part

$$\text{Im } D \sim \theta(q^2 - 4m^2) \delta(p_1^2 - m_1^2) \delta(p_2^2 - m_2^2) \dots$$

intermed. on mass shell

Amplitude $A \sim A e^{i\delta} \leftarrow$ scattering phase 68

physical picture

$B \xrightarrow{\text{weak}} I \xrightarrow{\text{strong}} F$ phase \leftrightarrow Intermediate states

$$\text{CPT} : \sum_F \Gamma(B \rightarrow F) = \sum_F \Gamma(\bar{B} \rightarrow \bar{F})$$

If only one F : no CP-violation

\Rightarrow need two states
need their phases

p. C61

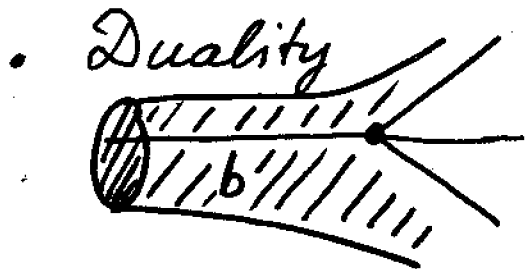
$$A(B \rightarrow F) = (B \rightarrow F_1 \rightarrow F) + (B \rightarrow F_2 \rightarrow F)$$

$$\text{Weak: CKM} \rightarrow A_1 e^{i\delta_1} + A_2 e^{i\delta_2}$$

$$A(\bar{B} \rightarrow F) = (\bar{B} \rightarrow \bar{F}_1 \rightarrow \bar{F}) + (\bar{B} \rightarrow \bar{F}_2 \rightarrow \bar{F})$$
$$A_1^* e^{+i\delta_1} + A_2 e^{+i\delta_2}$$

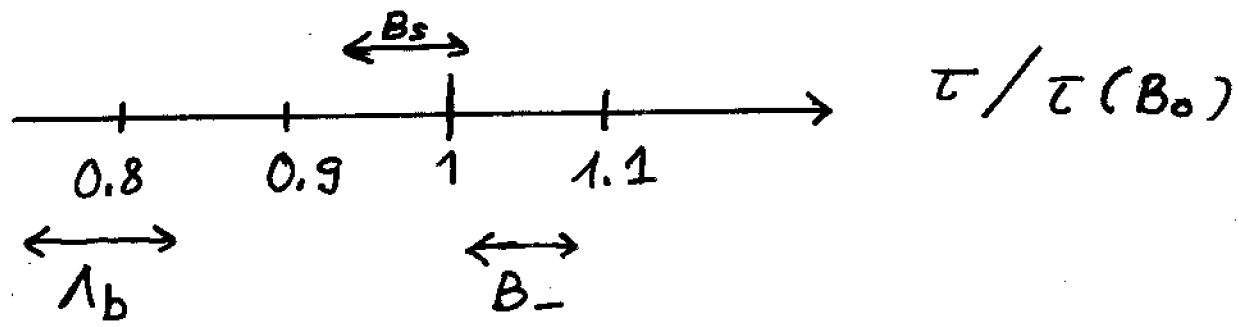
$$A_{CP} \cong \Gamma(B \rightarrow F) - \Gamma(\bar{B} \rightarrow \bar{F})$$

$$\simeq \text{Im}(A_1 A_2^*) \sin(\delta_1 - \delta_2)$$



\bar{b} decays "unaffected" by surrounding (heavy, unless in atom)

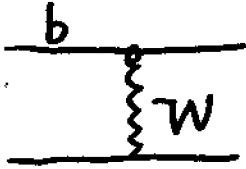
implies equal lifetimes for all Hadrons with a b



"Quark hadron duality"

Physical results: $A_0 \left(1 + \frac{A_1}{m} + \frac{A_2}{m^2} + \dots \right)$

Corrections: exchange + annihilation



small

Problem: $\tau(\Lambda_B)$ too small!

HQET - short exposition

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H_Q : hadron, containing Q
(b, c) and light quark q

$R_{\text{hadron}} \sim \frac{1}{\Lambda_{\text{QCD}}} \leftrightarrow$ Bohr radius e, m

$$\lambda_Q \sim \frac{1}{m_Q} \ll R_{\text{hadron}}$$

* q moves independently of Q
(atom: e "independent of nucleus")

\Rightarrow flavour symmetry
 $b \leftrightarrow c$

compare spectra of B 's and C 's

* Q at rest: only electric colour field
Spin of heavy quark decouples

\Rightarrow Spin symmetry

$$B \leftrightarrow B^* \quad D \leftrightarrow D^*$$

relate spectra of B and B^*

$$m_{B^*} - m_B \sim O(1/m_b)$$

* heavy quark decay constants

$$f_B \sim \frac{1}{\sqrt{m_B}} \quad f_B/f_D = \sqrt{\frac{m_D}{m_B}}$$

Effective Lagrangian

(systematic expansion in $1/m_Q$)

if $E \ll m_Q \Rightarrow$ no antiparticles

4 components \rightarrow 2 components

$$Q \simeq \Psi_{\text{Dirac}} = \begin{cases} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \sim 1 & \text{particle : } \psi_p \\ \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix} \sim \frac{E}{m} & \text{antiparticle : } \psi_{\bar{p}} \end{cases}$$

Dirac equation : $(i\not{\partial} - m_Q) Q = 0$

$\not{\partial}_\mu = \partial_\mu - ig A_\mu$ $|A_\mu| \simeq \Lambda_{QCD}$

$m_Q \gg \Lambda_{QCD}$

Q at rest : $p_\mu = m v_\mu$ $v_\mu = (1, 0, 0, 0)$

$(i\gamma_0 \frac{\partial}{\partial t} - m) Q = ig \not{A} Q$ $(i \frac{\partial}{\partial t} - m) \psi_p \simeq \Lambda Q$

$Q \simeq e^{-imat} \tilde{Q}$ $(i \frac{\partial}{\partial t} + m) \psi_{\bar{p}} \simeq \Lambda Q$

$i \frac{\partial}{\partial t} Q \simeq m \tilde{Q} e^{-imat}$

$m = m_Q \left. \begin{matrix} (m - m) \tilde{\psi}_p = \Lambda Q \\ (m + m) \tilde{\psi}_{\bar{p}} = \Lambda Q \end{matrix} \right\} \psi_{\bar{p}} \sim \frac{\Lambda}{2m_Q} \psi_p$

Idea: use EOM for $\tilde{\psi}_{\bar{p}}$ in terms of ψ_p and get effective Lagrangian in terms of ψ_p (etc).

Recall: $\psi_{\bar{p}} \sim \left(\frac{\vec{p} \vec{\sigma} + \vec{A} \vec{\sigma}}{2m} \right) \psi_p$

systematic: set $p_\mu = mv_\mu + k_\mu$

$v^2 = 1$ 4-veloc.

k small $\sim \Lambda_{\text{QCD}}$

$$(\not{p} - m) Q \approx (m(\not{v} - 1) + \not{k}) Q = 0$$

$$m \rightarrow \infty : m(\not{v} - 1) Q = 0$$

$$Q = \left[\frac{(1 + \not{v})}{2} + \frac{(1 - \not{v})}{2} \right] Q = Q_+ + Q_-$$

Dirac-Eqn: $0 \cdot Q_+ = 0 \rightarrow Q_+$ leading
 $Q_- = 0 \rightarrow Q_- \approx \frac{1}{M}$

$$h_\nu(x) = e^{im_Q v \cdot x} Q_+ \quad \nu: \text{label}$$

$$H_\nu(x) = e^{im_Q v \cdot x} Q_-$$

insert in $\mathcal{L} = \bar{Q} ((\not{p} - eA)^\mu \gamma_\mu - m) Q$

and neglect small terms

$$\mathcal{L}_{\text{HQET}} = \bar{h}_\nu i v \cdot \mathcal{D} h_\nu$$

(use $\frac{(1 + \not{v})}{2} \gamma^\mu \frac{(1 + \not{v})}{2} = v^\mu$)

$$\mathcal{D}_\mu = k_\mu - eA_\mu \quad (\text{no } mv_\mu)$$

Propagator: $\frac{1}{k \cdot v + i\epsilon}$

Including also next term, find

$$(i\nu D + 2m_Q) H_V(x) = i \not{\partial}_\perp h_\nu \quad \not{D}_\perp^\mu = \not{D}^\mu - \not{v}^\mu \not{D}$$

$$\mathcal{L}_{\text{eff}} = h_\nu i\nu \not{D} h_\nu + h_\nu i \not{\partial}_\perp \frac{1}{i\nu D + 2m_Q} i \not{\partial}_\perp h_\nu$$

$$\underbrace{\frac{1}{2m_Q} (h_\nu (i \not{\partial}_\perp)^2 h_\nu + \frac{g}{2} h_\nu \sigma^{\alpha\beta} G_{\alpha\beta} h_\nu + \dots)}$$

use $(\vec{a} \cdot \vec{\sigma})(\vec{b} \cdot \vec{\sigma}) = \vec{a} \cdot \vec{b} + (\vec{a} \times \vec{b}) \cdot \vec{\sigma}$

Effective Lagrangian for
 $m_Q \gg \Lambda_{\text{QCD}}$

currents:

$$\bullet \bar{c} \gamma_\mu b \rightarrow \bar{h}_{\nu^c}^c \gamma_\mu h_\nu^b + \frac{1}{2m_c} \bar{h}_{\nu^c}^c (-i \not{\partial}_\perp) \gamma_\mu h_\nu^b$$

$$+ \frac{1}{2m_b} \bar{h}_{\nu^c}^c \gamma_\mu (i \not{\partial}_\perp) h_\nu^b + \dots$$

$$\bullet \bar{q} \gamma_\mu Q b \rightarrow \bar{q} \gamma_\mu h_\nu^b + \dots$$

Renormalization: replace by

$$C(m_Q, \mu) \bar{q} \gamma_\mu h_\nu^b$$

masses

$$m_H = m_Q + \underbrace{\langle H | \bar{h}_\nu (i\mathcal{D})^2 h_\nu | H \rangle}_{\equiv \lambda_1} \frac{1}{2m_Q} + \underbrace{\langle H | \bar{h}_\nu (\vec{\sigma} \vec{B}) h_\nu | H \rangle}_{\equiv \lambda_2} \frac{1}{4m_Q} + \bar{\Lambda}$$

• \mathcal{D}^2 spin independent

• $\vec{\sigma} \vec{B} \sim \vec{\sigma} (\vec{E} \times \vec{v}) \sim \frac{(\vec{\sigma} \vec{L})}{r^3 m_Q}$

$$\vec{E} \sim \frac{\vec{r}}{r^3}$$

Spin
↓

$$\vec{\sigma} \vec{L} = \frac{1}{2} ((\vec{\sigma} + \vec{L})^2 - \vec{\sigma}^2 - \vec{L}^2) = \frac{1}{2} (J(J+1) + \dots)$$

$$m_B = M_0 - \frac{3\lambda_2}{2m_Q} ; m_{B^*} = M_0 + \frac{\lambda_2}{2m_Q}$$

$$\lambda_2 \approx 0.12 \text{ GeV}^2$$

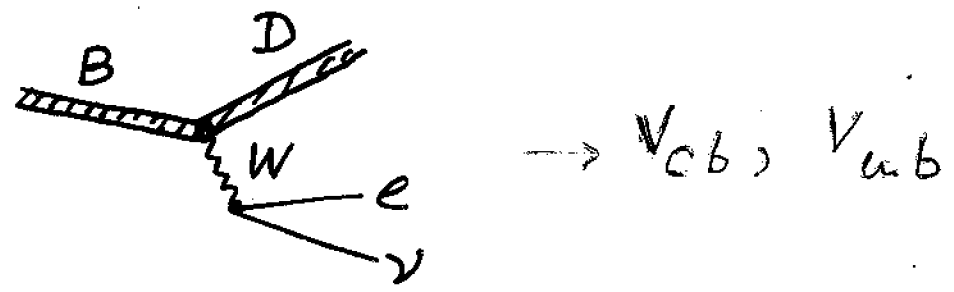
similar for \mathcal{D} .

In this way construct $\mathcal{L}_{\text{eff}}(h_\nu)$.

$$\lambda_1 \approx -0.3 \pm 0.2 \text{ GeV}^2$$

Applications

Semileptonic decays of B



$$\langle D | (\bar{c} \gamma^\mu b) | B \rangle = f_+(q^2) \cdot (p_B + p_D)^\mu + f_-(q^2) (p_B - p_D)^\mu$$

in HQET: $p_B = m_B v$ $p_D = m_D v'$

$$\langle \dots \rangle = \frac{1}{2} \left[(m_B + m_D) f^+ + (m_B - m_D) f^- \right]_1 (v + v')^\mu + \frac{1}{2} \left[(m_B - m_D) f^+ + (m_B + m_D) f^- \right]_2 (v - v')^\mu$$

- $v = v'$: flavor symmetry: $b \rightarrow c$
"no other effect"
- "Dirac-equation" for h_v mass independent: current conserved, only one form factor ($(v - v') \langle \dots \rangle = 0 \rightarrow [\dots]_2 = 0$)

$$\langle D(v') \dots B(v) \rangle = \xi(vv')^* (v + v')^\mu$$

$\xi(1) = 1$ Jsgw/Wise

$$f^\pm = \frac{m_B \pm m_D}{2 m_B m_D} \xi(vv')$$

$q^2 = m_B^2 + m_D^2 - 2 m_B m_D v v'$

* why only vv' ?

$$\frac{d\Gamma}{d\omega} (B \rightarrow D \ell \nu) = (\dots) F^2(\omega)$$

↑ $\xi(\omega) + \text{corrections}$

$$\frac{1}{m_Q^2}, \text{QCD} \longrightarrow F(1) \approx 0.91 \pm 0.03$$

$$\frac{d\Gamma}{d\omega} (B \rightarrow D^* \ell \nu) = (\dots) F^2(\omega)$$

many calculations done

Total (inclusive) rates

$$\Gamma(H_b \rightarrow X) = \frac{G_F^2 m_b^5}{192\pi^2} \left\{ c \left(\frac{1 + \lambda_1 - 3\lambda_2}{2m_b^2} \right) + \dots \right\}$$

X = final state (with some specification)

$$X = (X_H + \ell) \rightarrow c = |V_{cb}|^2 (1 - 8x^2 + \dots)$$

$x = m_c/m_b$

→ find V_{cb}

Latest developments: SCET

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Soft-collinear effective theory

In decays of heavy mesons into light particles: collinear momenta

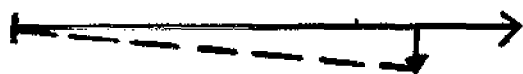
• $p_\mu \sim (1, 0, 0, 1)$: $p^2 \approx \Lambda_{\text{QCD}}^2 \ll m_B^2$

$$n = (1, 0, 0, 1) \quad \bar{n} = (1, 0, 0, -1)$$

$$p_\mu = \frac{(\bar{n}p)}{2} n_\mu + \frac{np}{2} \bar{n}_\mu + p_\mu^\perp$$

$O(m_b) \quad O\left(\frac{\Lambda^2}{m_b}\right) \quad O(\Lambda)$

$n^2 = 0$
 $\bar{n}^2 = 0$
 $n\bar{n} = 2$



"needs QCD to go off n -directions"

quarks with collinear momenta dominate
(similar to particle over antiparticle)
→ collinear particles (q and \bar{q})

Idea: write an effective theory in terms of these fields (+ soft)

soft: $p_\mu \sim (\Lambda, \Lambda, \Lambda)$ $\Lambda = \Lambda_{\text{QCD}}$

recall: $h, H \sim \frac{(1 \pm \not{n})}{2} Q$

here $P_c = \frac{\not{n}\not{\bar{n}}}{4} \not{q}$ $P_a = \frac{\not{\bar{n}}\not{n}}{4} \not{q}$

$P_c^2 = P_a^2 = 1$ $P_c \cdot P_a = 1$ $P_a + P_c = 1$

$\not{p} \cdot (\frac{\not{n}\not{\bar{n}}}{4} \not{q}) \sim \not{n}\not{n}\not{\bar{n}}\not{q} = 0$

$\not{p} (\frac{\not{\bar{n}}\not{n}}{4} \not{q}) \sim \not{n}\not{\bar{n}}\not{n}\not{q} = \not{n}\not{q} \neq 0$

Same ideas as before (p. 72/73)

$\mathcal{L} = \bar{\xi}_c (i\not{n}D + g\not{n}A_c + \not{p}_\perp \frac{1}{\not{n}\not{D}_c} \not{p}_\perp) \xi_c$

$\xi_c = P_c \not{q}$ A_c collin. gluons

Advantage: soft gluons decouple

Feynman rule

• $\bar{\xi}_c i\not{n}D \xi_c \Rightarrow \not{n}_\mu A^\mu \xrightarrow{\text{L.C.G.}} 0$
light cone gauge

• General:

Redefine fields $\xi \rightarrow W(nA) \xi$

$Y = P \exp \int dy \cdot nA_x$

recall: $\bar{\psi}(\partial - iA)\psi \simeq \bar{\psi} \partial (e^{i \int dy A}) \psi$

⇒ Can show several "decouplings"

→ factorization

- quite technical

- not yet best formulation

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