united nations
educational, scientific
and cultural
organization

international atomic
energy agency

the
abdus salam
international centre for theoretical physics

SMR.1508 - 15

SUMMER SCHOOL ON PARTICLE PHYSICS

16 June - 4 July 2003

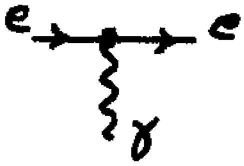
THE STANDARD MODEL AND HIGGS PHYSICS

Lecture II

W. MARCIANO
Dept. of Physics
Brookhaven National Lab.
Upton, NY
U.S.A.

3.) Precision Electroweak Measurements & Radiative Corrections

Applied Renormalization



$-ie_0\gamma_\mu$ coupling   QED coupling

+ "many" other bare couplings & masses $g_{2_0}, g'_0, \sin^2\theta_W^0$
$m_e^0, m_\mu^0, m_\tau^0, m_u^0 \ldots m_W^0, m_Z^0, \sin\theta_{ij}^0 \ldots$

Renormalizable Quantum Field Theory: Short-Distance Divergences $\to g_0, m_0$

generic $\begin{cases} g_0 = g_R - \delta g \\ m_0 = m_R - \delta m \end{cases}$ δg & δm divergent counterterms cancel perturbative infinities

Prescription: Start with $\mathcal{L} + \mathcal{L}_{\text{gauge fix.}} + \mathcal{L}_{\text{ghost}}$

1) Derive Feynman Rules: Couplings & Propagators

2) Compute Quantum Loops

3) Renormalize away infinities & some finite parts

4) Obtain unambiguous predictions in terms of finite measured quantities eg $e_R, m_R \ldots \sin\theta_R$
or other renorm. parameters
eg $\overline{MS}$ Modified Minimal Subtrac

Some arbitrariness in finite parts:

$$g^{0} = g_R - \delta g_R = g_R' - \delta g_R' \qquad g_R = g_R' + \text{finite diff.}$$

Freedom to choose "convenient" (most appropriate) scheme

How to handle short-distance infinities:

Method of choice: Dimensional Regularization

Space-Time $4 \to n$ s.t. for $n<4$ finite loops

U.V. divergence appears at pole as $n \to 4$, $\left(\frac{1}{n-4}\right)^{\ell}$

$\ell = 1$ (one loop), $\ell = 2$ (two loop)...

Preserves Gauge Symmetry

Gives Unambiguous Finite Predictions!

$$\int \frac{d^4k}{(2\pi)^4} F(k,p) \longrightarrow \int \frac{d^nk}{(2\pi)^n} F(k,p,n)$$

$$k \cdot p = k_0 p_0 - k_1 p_1 - k_2 p_2 \ldots k_{n-1} p_{n-1}$$

$$g_{\mu\nu} \Rightarrow g_{00} = 1, \quad g_{ij} = -\delta_{ij} \qquad \text{etc}$$

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2 g_{\mu\nu} I$$

$$\gamma_\mu \gamma^\mu = n I$$

etc.

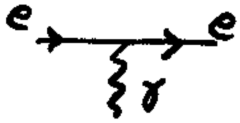
Combine Propagators: $\frac{1}{a^r b^m} = \frac{\Gamma(r+m)}{\Gamma(r)\Gamma(m)} \int_0^1 dx \, \frac{x^{r-1}(1-x)^{m-1}}{[ax+b(1-x)]^{r+m}}$

$$\int \frac{d^nQ}{(2\pi)^n} \frac{(Q^2)^r}{[Q^2-C]^m} = \frac{i(-1)^{r-m}}{(16\pi^2)^{n/4}} \, C^{r-m+n/2} \, \frac{\Gamma(r+n/2)\Gamma(m-r-n/2)}{\Gamma(n/2)\Gamma(m)}$$

$$\Gamma(2-n/2) = \underbrace{-\frac{2}{n-4}}_{\text{U.V. div.}} - \gamma + O(n-4)$$

eg. QED Sector (electrons + photons)

$$\mathcal{L}(x) = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\partial_\mu\gamma^\mu - e_0 A_\mu \gamma^\mu - m_e^0)\psi - \underbrace{\frac{i}{2\xi}(\partial_\mu A^\mu)}_{\text{gauge fixing}}$$



$-ie_0\gamma_\mu$ vertex

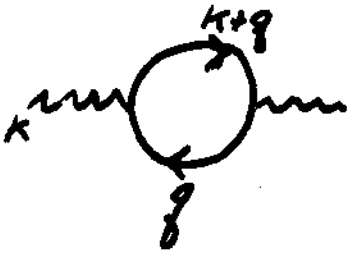
$iS(p) = \frac{i}{\not{p} - m_e^0}$

$-\frac{i}{k^2}\left(g_{\mu\nu} + \frac{(\xi-1)k_\mu k_\nu}{k^2}\right)$

$\xi = 1$ Feynman
$\xi = 0$ Landau

eg Vacuum Polarization

$$\frac{-ig_{\mu\nu}}{k^2} \rightarrow \frac{-ig_{\mu\nu}}{k^2(1+\pi(k^2))} \qquad \pi(k^2) \text{ divergent}$$

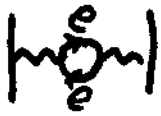


$$= i\left(g_{\mu\nu}k^2 - k_\mu k_\nu\right)\Pi(k^2)$$

$$= (-)(-ie_0)^2 \int \frac{d^n q}{(2\pi)^n} \frac{Tr\, \gamma_\mu (\not{q}+m_e^0)\gamma_\nu(\not{k}+\not{q}+m_e^0)}{(q^2-m_e^{0\,2})((k+q)^2-m_e^{0\,2})}$$

show $$\Pi(k^2) = \frac{8e_0^2}{(16\pi^2)^{n/4}} \underbrace{\Gamma(2-n/2)}_{\text{u.v. pole at } n=4} \int_0^1 dx\, (x-x^2)\left(m_e^{0\,2}+k^2(x^2-x)\right)^{\frac{n}{2}-2}$$

Conventional Charge Renormalization

$+$


$+ \cdots \quad \rightarrow e^2$ (physical)

$$e^2 = \lim_{n\to 4} \frac{e_0^2}{1+\Pi(0)} \quad \text{or} \quad e^2 = Z_3 e_0^2 \quad Z_3 = \frac{1}{1+\Pi(0)}$$

$k^2=0$ corresponds to ∞ distance (Atomic Physics)

Instead $$e^2(k^2) = \lim_{n\to 4}\frac{e_0^2}{1+\Pi(k^2)} \quad \text{or} \quad e^2(\mu)_{\overline{MS}}$$ subtract poles, γ, $\ln 4\pi$

More appropriate for H.E. Physics

$$\alpha \equiv \frac{e^2(0)}{4\pi}$$

$$\frac{1}{\alpha(k^2)} \simeq \frac{1}{\alpha} - \frac{1}{3\pi}\ln\left(\frac{m_e^2}{k^2}\right)$$ + muon loops, quark loops etc

$$\alpha(k^2) > \alpha(0) \simeq 1/137$$

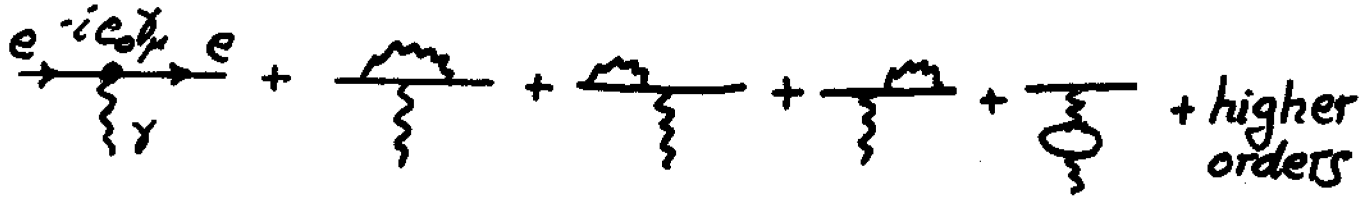
$$\alpha(m_Z^2) \simeq 1/129$$ Running Coupling

i) Fine Structure Constant α & $\frac{g_e-2}{2}$

Dirac Eq. $\rightarrow g_e = 2$ $\quad g_e$ = gyromagnetic ratio

Mag. Moment $\vec{\mu} = \frac{e}{2m_e} g_e \vec{S}$ $\quad \xrightarrow{\mu} \!\!\! \rightarrow_S$ pol. electron

Quantum Field Theory Predicts: $a_e = \frac{g_e-2}{2} \neq 0$



$=$ 

Fully Dress Vertex

$$\langle e(p') | J_\alpha^{em} | e(p) \rangle = \bar{u}_e(p') \Gamma_\alpha u_e(p)$$

$$\Gamma_\alpha = e F_1(q^2)\gamma_\mu + i\frac{e}{2m_e} F_2(q^2)\sigma_{\alpha\beta} q^\beta - F_3(q^2)\sigma_{\alpha\beta} q^\beta \gamma_5$$

$F_1(0) = 1$ Electic Charge $\quad e^2/4\pi \simeq 1/137$

$F_2(0) = a_e = \frac{g_e-2}{2}$ Anomalous Mag. Moment

$F_3(0)$ = Electric Dipole Moment $\quad$ Violates P & T

What is e or $\alpha = e^2/4\pi$ defined at $q^2 = 0$?

Precise Value

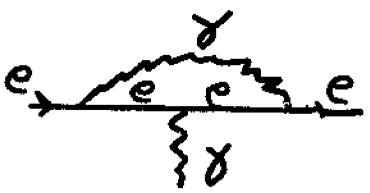
α can be directly measured in Condensed Matter, Atomic...

$\alpha^{-1} = 137.03600300\ (270)$	Quantum Hall Effect
$\alpha^{-1} = 137.03600840\ (330)$	Rydberg ($\hbar/m_n$)
$\alpha^{-1} = 137.03598710\ (430)$	AC Josephson
$\alpha^{-1} = 137.03599520\ (790)$	Muonium HFS

$\alpha^{-1}_{AVE} = 137.03600140\ (183)$ good agreement! QED Well Tested

What is $a_e = \frac{g_e - 2}{2}$ predicted to be?

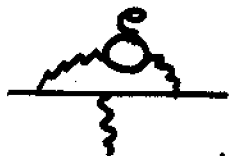
Schwinger



$\rightarrow a_e = \frac{\alpha}{2\pi} = 0.00116$ (Finite)
agreed with exp

Higher Orders

$+ \ldots$



etc.

divergent
$e_0^2 \to e^2$ removes div.

$$a_e = \frac{\alpha}{2\pi} - 0.328478444\left(\frac{\alpha}{\pi}\right)^2 + 1.181234\left(\frac{\alpha}{\pi}\right)^3 - 1.7502\left(\frac{\alpha}{\pi}\right)^4 + \ldots \quad 1.66\times10^{-12}$$

quarks
$W^+, Z \ldots$
tiny

Experiment:

$$\left.\begin{aligned} a_{e^-}^{exp} &= 0.0011596521884(43) \\ a_{e^+}^{exp} &= 0.0011596521879(43) \end{aligned}\right\} \text{H. Dehmelt et al.}$$

Solve for α

$\alpha^{-1}(a_e) = 137.03599877(40)$
vs
$\alpha^{-1}_{Ave} = 137.03600140(183)$
} 1.4 sigma diff

Beautiful test of QED

New Harvard Exp. $\rightarrow$ Factor 15 Improvement in a_e + $\alpha(a_e)$!

What will it tell us? Nothing unless other α_{exp} improve!

"New Physics" $\rightarrow$ $\underbrace{\Delta a_e \sim \frac{m_e^2}{\Lambda^2}}_{\text{suppressed by } m_e^2}$ Λ = scale of NewPhys.

ii) Muon Anomalous Magnetic Moment $a_\mu = \frac{g_\mu - 2}{2}$

$\left(\frac{m_\mu}{m_e}\right)^2 \simeq 40{,}000$ times more sensitive to "New Physics"!

Also 40,000 times more sensitive to strong + weak int.

Experiment is about 1/200 a_e in precision

$\rightarrow$ Real Gain Factor 200

3 CERN Experiments ending in 1977 ("Last g_μ-2 Experiment")

$$\left.\begin{array}{l} a_{\mu^+}^{exp} = 116591100\,(1100) \times 10^{-11} \\ a_{\mu^-}^{exp} = 116593700\,(1200) \times 10^{-11} \end{array}\right\} a_\mu^{exp} = 116592300\,(840) \times 10^{-11}$$

3.) <u>Muon: $a_\mu \equiv \frac{g_\mu - 2}{2}$</u>

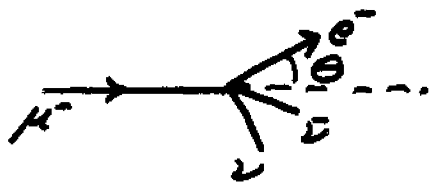
Can one do a precise measurement?

$m_\mu \simeq 106.7\,\text{MeV}$, $\underline{\tau_\mu \simeq 2.2 \times 10^{-6}\,\text{sec}}$ (Eternity!) $\mu^- \to e^- \bar{\nu}_e \nu_\mu$

<u>Muon Miracles</u>: Copious Production
Polarized
Parity Violation
a_μ Precession

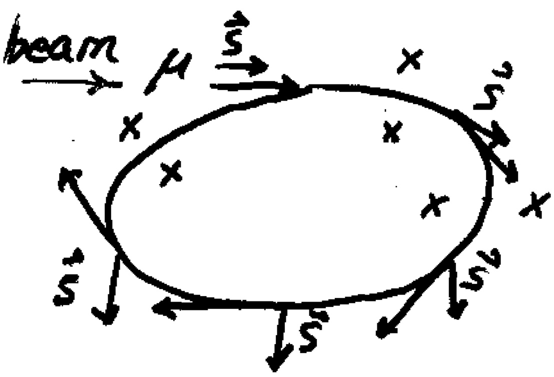
Protons $\longrightarrow$ $\boxed{\text{target}}$ $\xrightarrow[\text{pions}]{}$ $\pi^- \to \mu^- \bar{\nu}_\mu$ Polarized Muons

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$



e^- direction monitors polarization <u>Parity Viol</u>

Put muons in a storage ring (Magnetic Field)



<u>spin precesses</u>
$\sin\left(\frac{e}{2m_\mu} a_\mu B t\right)$

measure a_μ not g_μ!

Precise B + t required

E821 at BNL (Vernon Hughes et al.) Goal factor 20 improvement!

Current $a_{\mu^+}^{exp} = 116\,592\,030(80)\times 10^{-11}$ Factor 10 better than CERN exps.

Results from μ^- data expected soon

Roughly $\rightarrow$ $\pm 60\text{-}65\times 10^{-11}$ Final Error (Assuming CPT)

If funded to run, could go to $\pm 20\text{-}30\times 10^{-11}$ (should)

Elegant + Challenging Exp.
with
Important Implications For Theory (SUSY)

"Last $g_\mu - 2$ Experiment?" At least for a while.

Status of a_μ^{SM} Prediction

$$a_\mu^{SM} = \underbrace{a_\mu^{QED}}_{\substack{4\text{ loops}\\ +5\text{ loop est.}}} + \underbrace{a_\mu^{Hadronic}}_{3\text{ loops}} + \underbrace{a_\mu^{EW}}_{\substack{2\text{ loop}\\ +3\text{ loop leading logs}}}$$ very challenging

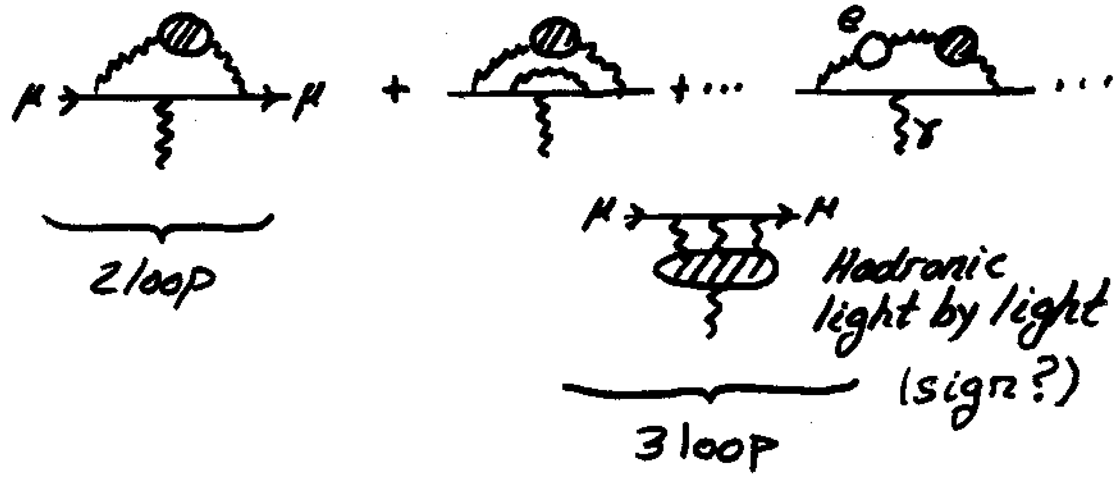
$$a_\mu^{QED} = \frac{\alpha}{2\pi} + 0.765857376\left(\frac{\alpha}{\pi}\right)^2 + 24.05050898\left(\frac{\alpha}{\pi}\right)^3$$


light by light large

$$+\underbrace{126.07\left(\frac{\alpha}{\pi}\right)^4}_{\substack{\text{will change}\\ \text{New Kinoshita}}} + \underbrace{930\left(\frac{\alpha}{\pi}\right)^5}_{5\text{ loop estimate}}$$

$a_\mu^{QED} = 116584716(10)\times 10^{-11}$ Error will shrink

a_μ^{Hadronic}



No Realistic First Principles QCD Calculation Yet

* T. Blum Lattice $\pi(q^2)$ Very Promising (±1% Hard)

Standard Method - Dispersion Relation

Im  ~ $\sigma(e^+e^- \to \text{hadrons})$

$\pi^+\pi^-$ Final State Dominates

Alternative (Check) $\tau \to \nu_\tau + \text{hadrons}$ + isospin corr.

$$a_\mu^{\text{Had.}}(\text{vac. pol.}) \sim \frac{1}{4\pi^2}\int_{4m_\pi^2}^{\infty} ds\, \underbrace{K(s)}_{\sim\frac{1}{s}}\, \sigma^0(e^+e^- \to \text{hadrons})$$

1998 (Davier + Höcker) $e^+e^- \to \pi^+\pi^-, 3\pi, 4\pi, K^+K^- \ldots + \tau \to \nu_\tau 2\pi, 4\pi \ldots$

$$a_\mu^{\text{Had}}(\text{vac. pol.}) = 6924(62) \times 10^{-11}$$

Improvements in $e^+e^- \to \rho \to \pi^+\pi^-$ Novosibirsk

+ tau data

2002 $a_\mu^{Had.}$(vac.pol) $= 6847(70)\times 10^{-11}$ Davier et al
e^+e^- data alone

$7090(59)\times 10^{-11}$ τ data alone (where it exists)

Differ by 243×10^{-11}! Inconsistent at $\sim 3\sigma$

τ data Red Flag or Red Herring

e^+e^- luminosity normalization (change expected)

τ data (additional isospin corrections?)

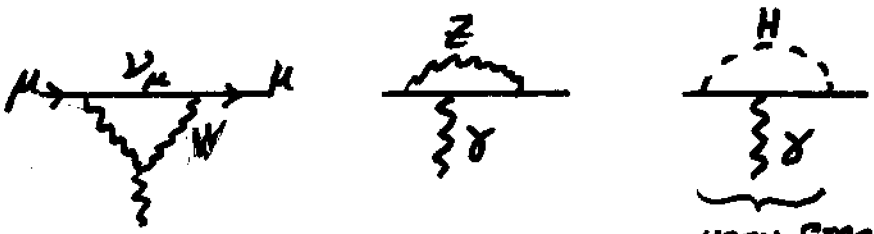
More $e^+e^- \to$ Hadrons $+\gamma$ expected soon KLOE, BaBar

$$a_\mu^{\text{Hadronic}}(3\text{ loop}) = -14(35)\times 10^{-11}$$

$a_\mu^{Had} = 6833(78)\times 10^{-11}$ e^+e^- data

$a_\mu^{Had} = 7076(69)\times 10^{-11}$ τ data

a_μ^{EW} 1 loop (1972)



very small
extra $\frac{m_\mu^2}{m_H^2}$ sup.

$$a_\mu^{EW} = \frac{G_\mu m_\mu^2}{8\sqrt{2}\pi^2}\left\{\frac{10}{3} - \frac{5}{3} + \frac{1}{3}(1-4\sin^2\theta_W)^2\right\}$$

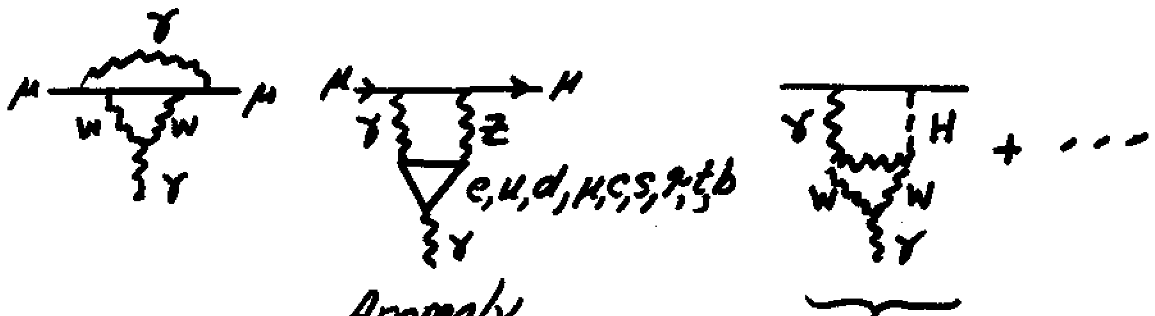
$\simeq \underline{195\times 10^{-11}}$

Original E821 Goal $\pm 40\times 10^{-11}$
5σ effect

$G_\mu = 1.16637\times 10^{-5}\,\text{GeV}^{-2}$

1991 Kukhto et al $\rightarrow$ 2 loop EW relatively large
(Partial Calculation) enhanced by $\ln \frac{m_Z^2}{m_\mu^2} \simeq 13.5$ Factor

<u>Examples:</u>



Anomaly Diagrams Cancellation!

~1000 time one loop Higgs

$\underline{a_\mu^{EW}(2\text{loop}) = -41 \times 10^{-11}}$! -22% of 1 loop!

$a_\mu^{EW}(3\text{loop leading logs}) \sim 0$ (cancellation)

$\underline{a_\mu^{EW} = 154(1)(2) \times 10^{-11}}$

$a_\mu^{SM} = 116\,591\,693\,(79) \times 10^{-11}$ e^+e^- data
$116\,591\,936\,(70) \times 10^{-11}$ τ data

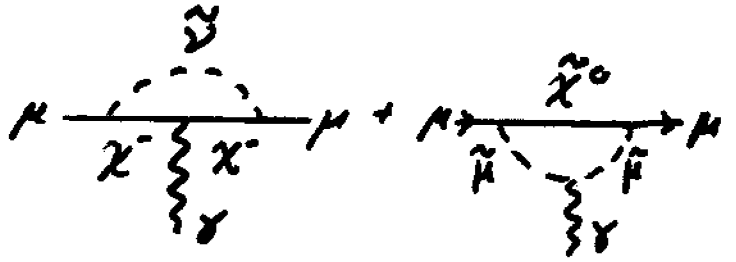
$a_\mu^{exp} - a_\mu^{SM} = a_\mu^{New Physics} \simeq 337 \pm 112 \times 10^{-11}$
$94 \pm 100 \times 10^{-11}$ } Expected to change

3σ or 1σ deviation?

<u>My guess $\lesssim 2\sigma$ deviation when the dust settles</u>

Next Exp. Report?

Supersymmetry and $g_\mu - 2/2$

$a_\mu^{SUSY} =$ 

Potentially Large

Rough Approx $m_{\tilde{\nu}} \simeq m_{\tilde{\mu}} \simeq m_{\chi^-} \simeq m_{\chi^0} = m_{SUSY}$

$$a_\mu^{SUSY} \simeq \overset{+/-}{(\mathrm{sgn}\,\mu)} \times 130 \times 10^{-11} \left(\frac{100\,\mathrm{GeV}}{m_{SUSY}}\right)^2 \underbrace{\tan\beta}_{\text{enhancement factor } \sim 3-40!}$$

+ sign better for Dark Matter Searches

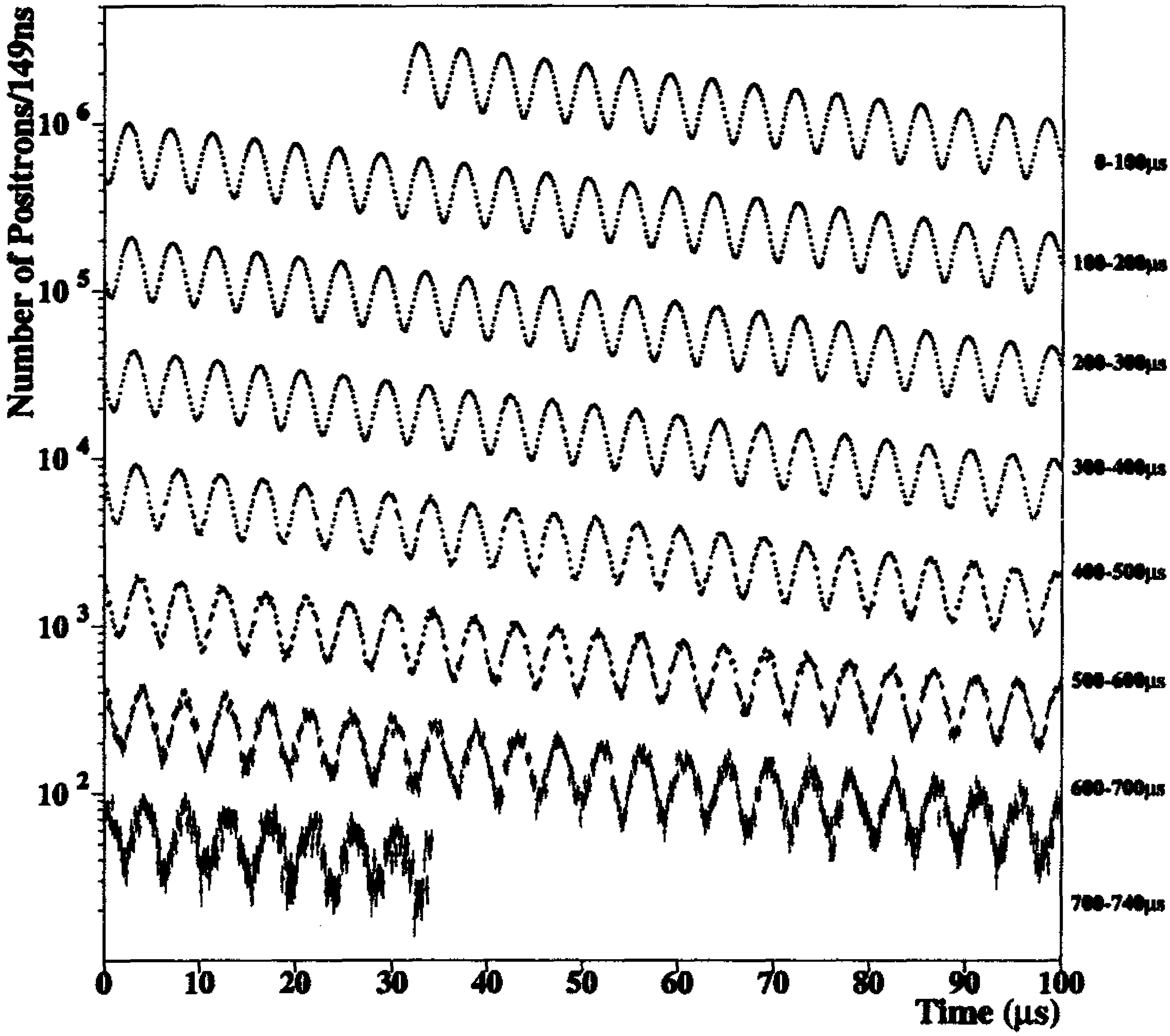
If low energy SUSY (100 ~ 300 GeV) is correct, we should be starting to see it in a_μ.

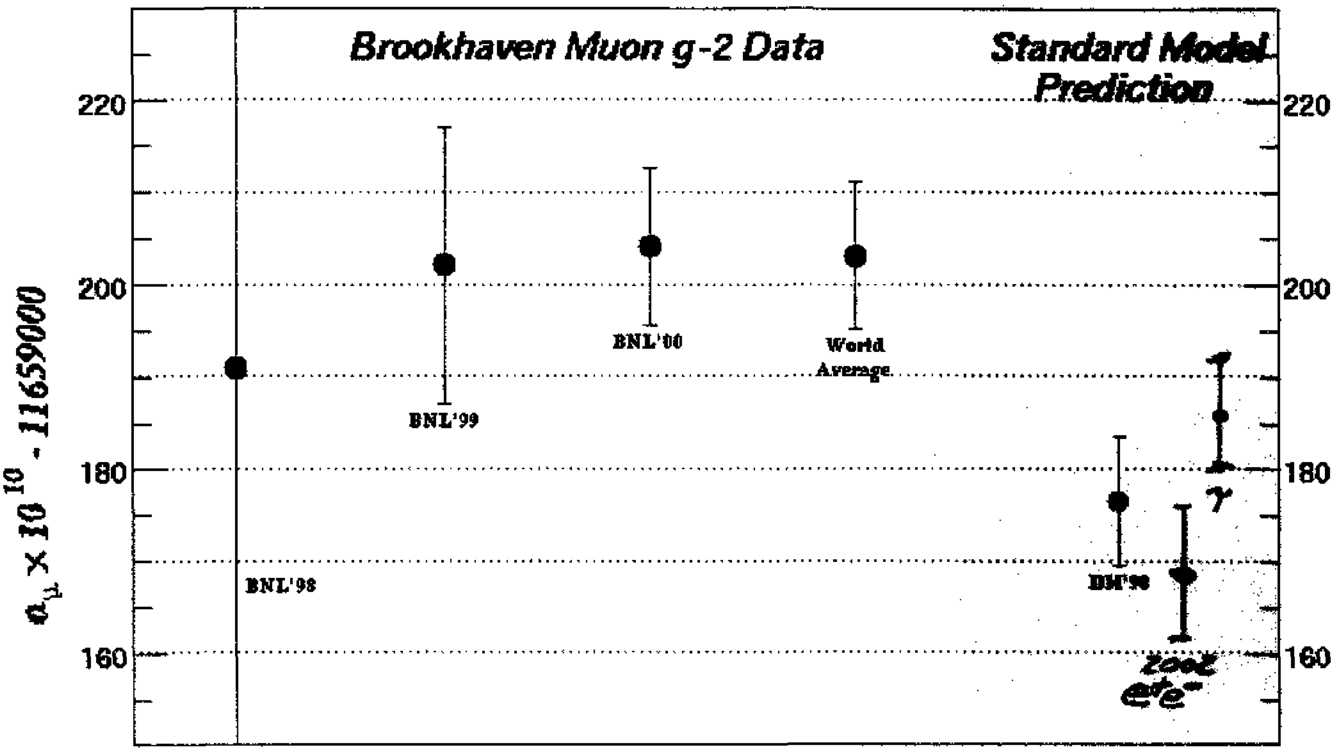
Is there a deviation in $a_\mu^{exp} - a_\mu^{SM}$?

Only Better Exp + a_μ^{Had} will tell.

$g_\mu - 2/2$ Harbinger of Supersymmetry?

Future - Determine $\tan\beta = \frac{v_2}{v_1}$


Number of Positrons/149ns
10^6
10^5
10^4
10^3
10^2
0-100μs
100-200μs
200-300μs
300-400μs
400-500μs
500-600μs
600-700μs
700-740μs
0
10
20
30
40
50
60
70
80
90
100
Time (μs)



References: BNL'98 PRL 86 2227
BNL'99 PR 62D 091101
BNL'00 accepted for publication in PRL

DH'98 a_μ(had;1) from PL 435B 427